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LOCATION ECONOMICS
APPLIED TO TOMATO PROCESSING
IN NEW ZEALAND

A thesis

submitted in partial fulfilment

of the requirements for the Degree

of

Master of Agricultural Science with Honours

in the

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by

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CHAPTER 1

INTRODUCTION

The processing of raw agricultural products is the largest function of the New Zealand manufacturing industry. It is inevitable that agricultural processing will expand. If New Zealand's economy continues to be based on agricultural production and is to remain viable it is imperative that agricultural processing become more efficient. Technical and economic research should investigate ways to improve this efficiency of processing.

1.1 INCREASED AGRICULTURAL PROCESSING IN NEW ZEALAND

The manufacturing industry is likely to be called upon to increase the volume, the depth and the breadth of its processing of agricultural products.

(1) New Zealand's recent agricultural policy has been aimed at increasing the production of traditional agricultural products. This policy has resulted in rapid production increases and so there are corresponding increases required in the processing of traditional products - butter, cheese, carcass meats, and to a limited extent wool.

(2) In recent years importing countries have demanded increased depth of processing. (An example is the 'boning-out' of carcass beef for the U.S. manufacturing beef market.) In other cases new overseas outlets have been developed based upon further processing of present products. A striking example here has been the rapid development within the dairy industry of specialised products based on milk protein.

Economists have noted for some time that although there is a low increase in demand for agricultural products as incomes rise (income elasticity of demand = 0.4), there is a marked increase in demand for processing and other marketing services associated with the raw product (YED = 0.8).[1] If New Zealand is to take full advantage of rising incomes in her overseas markets, future policy should be aimed at a greater depth of processing of our present agricultural products.

(3) In the face of dropping world prices for New Zealand's present agricultural products, it is likely that some of the future production increases will be in the form of new crops. The processing industry could be instrumental in expanding the breadth of production for processing. By rational development of integrated industries it could be possible to export quantities of tomatoes, peas and other crops at present grown mainly for the domestic market. New products could include a cheap starch product from an intensive potato industry, various industrial extracts derived from the expanding range of chemurgic crops, and many others.

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1. SCHULTZ, T.W. The economic organization of agriculture, Chap. 5 (especially p. 70). McGraw-Hill Inc., New York. 1953.

These three forms of production increase imply an expansion in the size and number of agricultural processing plants in New Zealand.

1.2 EXPANSION ALLOWS EFFICIENT RE-LOCATION OF PROCESSING

The efficiency of a processing industry depends not only on the technical efficiency of the mechanical processing, but on a wide range of technical and economic factors. Many of these factors vary according to the geographical location of the various facets of the overall industry.

During a period of expansion an industry has the opportunity to alter its pattern of production to increase efficiency. At present this is taking place within the dairy industry where large numbers of small companies are amalgamating. The large companies are increasing their efficiency by one-product specialisation of their individual factories, but could increase efficiency even more by taking into account the location effects on this specialisation and on the best sites for establishing new plants.

1.3 SCOPE OF THE PRESENT LOCATION STUDY

The following sections of this thesis will attempt to formalise the analysis of processing plant location problems, so that the future processing expansion may be planned more efficiently. The procedures will be illustrated by referring to the processing plant re-location problem of the tomato processing industry.

Chapter Two will discuss the physical and institutional aspects of the tomato industry location problem. It will then outline the economic considerations and show that within the general decision framework there are specific economic problems which lend themselves to a scientific analysis. Finally the specific economic location problem to be analysed will be defined.

There is an absence of comprehensive texts on the recent developments in the theory of the economics of location. Chapter Three will thus review the literature written on the broad subject of location economics with special emphasis on problem analysis. It will then indicate methods already discussed in the literature which may be relevant to the present study.

The latter sections will be concerned with selecting methods, describing data collection, analysis and the implication of the solutions to the tomato industry's location problem.

CHAPTER 2

THE TOMATO PROCESSING LOCATION PROBLEM

"Land utilization takes place within three frameworks: the physical, the institutional, and the economic."[1]

R.T. Ely and G.S. Wehrwein.

In the economic analysis of production problems the traditional approach has been to first determine the relevant technical (or physical) relationships, then to apply suitable economic choice criteria to derive an optimum level of activity. This 'optimum' is then quoted as the course which policy-makers should follow.

On the other hand, many policy decisions are made at present without any direct reference to the underlying technical or economic relationships. The solution adopted often depends almost entirely on the political strengths of the various interested parties. Such solutions are more likely to reflect past institutional rigidities rather than future technical, economic and social realities.

The author believes that policy decisions in general, and the

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1. ELY, R.T. and WEHRWEIN, G.S. Land economics, p.v. The MacMillan Company, New York. 1940.

decision regarding future location of tomato processing plants in particular, should be made in the light of thorough technical and economic investigation, but taking into account present institutional arrangements which are likely to remain in the future.

The present chapter is concerned with showing briefly the type of institutional pressures present within the tomato processing industry, and the technical relationships which should be studied. More important, the chapter will isolate, within the location problem, the specific aspect which is amenable to economic analysis.

2.1 PRESENT PROCESSING PLANT LOCATION

The present location of tomato processing plants is a result of two main influences:

(1) the historical location of processing until the early 1960's, and

(2) the amalgamation of a number of processing companies in 1963, with subsequent re-organisation of production within these companies.

2.1.1 Historical Location

Historically tomato processing was located along with other fruit and vegetable processing in multi-product plants. These plants have developed principally in Otago, South Canterbury, Christchurch, Nelson, Motueka, Hawkes Bay, Gisborne, Pukekohe and Auckland. The fact that tomato processing has been carried out in only six of these localities

is probably equally a consequence of individual processor's policy, as it is an indication of the comparative advantage of regions as feasible producing areas. The processing plants operating in these six regions in the early 1960's (listed in approximate order of their tomato intake) were: J. Wattie Canneries, Hastings; J. Wattie, Gisborne; Crest Foods, Hastings; Thomson and Hills, Ltd., Auckland and Pukekohe; Kirkpatrick's Ltd., Nelson; New Zealand Foods, Ltd., Motueka; which all handled other crops as well as tomatoes, and Irvine Stevenson, Ltd.; Whittome and Stevenson, Ltd.; and Ellwoods, Ltd.; all of Auckland, which handled mainly tomatoes to produce pickles, relishes, and sauces. The actual tonnage of tomatoes grown in each of these regions is shown by Sanderson.[2]

2.1.2 Company Amalgamation

In 1962 the operations of New Zealand Foods, Ltd. and Crest Foods were amalgamated under the name Unilever (N.Z.) Ltd., and processing of tomatoes at Motueka ceased. Within the next two years both Kirkpatrick's Ltd. (Nelson) and Thomson and Hills (Auckland) were amalgamated with J. Wattie Canneries and an indication was given that tomato processing by the Nelson and Auckland plants was likely to be terminated. For the 1965-66 season this indication proved correct in the case of the Auckland plant, and the Nelson plant was supplied by only 6 growers, compared with 99 (including "glass-house reject") suppliers in 1964-65.

2. SANDERSON, K.T. A field survey and report of costs and returns for tomatoes for processing. Table 1.1, p.8. N.Z. Vegetable and Produce Growers Federation Inc., Wellington, 1965.

Thus in the last two seasons the amalgamation of processing companies into the present state of duopoly (disregarding the smaller processors in Auckland), has caused a further reduction in the number of plants which process tomatoes, and these plants operate only in the two East Coast regions, with limited processing in Nelson. The senior director of one company has further told growers that "his company is considering the whole question of tomato production",[3] which shows that the question of the location of production and processing of tomatoes remains unsolved in the overall problem of company re-organisation.

2.2 GROWER-PROCESSOR RELATIONSHIPS

In recent years the importations of tomato puree have been negligible[4] (due partly to Government intervention through import control) and so the factory requirements have been met from local supplies of raw tomatoes, and inter-seasonal surpluses carried over in bulk as pulp.

2.2.1 Determination of Local Supply and Price

The various companies have ensured an adequate supply of raw tomatoes by making two types of contractual arrangements with growers. The contracts provide for the grower to supply either a given tonnage, or the total production of "reasonable quality" tomatoes from a given acreage planted. Under tonnage contracts as previously operated in

3. WILLS, W.C. Another year of changes and fewer markets for processors. N.Z. Comm. Grower. 22, 1: 9-10, June 1966.
4. SANDERSON, K.T. op. cit. Table 1.2, p.8.

Nelson and Auckland, growers tended to grow a larger acreage than required to fill their contracts, and so they carried the risk associated with disposing of any excess production in a surplus season. Companies on the East Coast, on the other hand, have become closely associated with growers in advisory and other capacities, in an attempt to obtain a reasonably predictable quantity of uniform-quality tomatoes from their contracted acreages.[5] These latter companies are now the "parents" of almost all other processing plants in New Zealand, and so one can expect an intensification of advisory efforts, and a move towards fewer and larger acreage contracts in districts where their plants remain in operation, thus giving a higher degree of certainty to growers.

Prices per ton of 'reasonable quality' tomatoes delivered to the processing factory are negotiated when contracts are signed at the beginning of the season. In practice, this has meant that prices were set by processors such that thay attracted just the required supply of raw tomatoes. In fact, prices had remained constant for ten years, at £13/10/- per ton in the East Coast districts, £16/-/- per ton in Nelson, £19/-/- per ton in Auckland Central and £20/-/- per ton in Pukekohe.[6] These static prices have in the past brought forward an adequate and in fact increasing annual supply of raw tomatoes, despite rising costs.

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5. ibid. pp. 16-18.
6. One would expect companies which buy only a known tonnage to be able to pay a higher price to the raw tomato producer, as the company is saved the costs associated with inter-seasonal storage between surplus and deficit seasons. Sanderson has shown that the Nelson grower would lose little, if the price dropped to £13/10/-, providing that he was allowed the certainty of sale of all his crop through an acreage contract.
   See ibid. p. 51, para. (2).

2.2.2 Price Inelasticity of Supply

The increased quantity supplied, even though money prices have remained constant and real prices have fallen, is probably due to:

(1) a reduction in the crop production possibilities in some districts due to processing plant closures, and the necessity to produce even unprofitable crops in order to cover some of the overhead costs.

(2) an increase in yields and productivity brought about by the farmers and, especially in Gisborne, with advisory aid from the processing companies.

(3) ignorance on the part of growers as to the relative profitability of different crops, and the changes in profitability due to cost changes, and

(4) lack of collective bargaining powers by growers where companies may have controlled such a large amount of local production as to be able to exert some monopoly buying-power.[7]

2.2.3 Improving Individual and Collective Supply Response

Motivated by a recognition of some, or all of the above reasons for the lack of 'the competitive element in settling prices',[8] the delegates

7. This monopoly power would not hold for a single crop unless, either its production required a large capital outlay or there was no alternative use for the land involved.
8. WILLS, W.C. op. cit. P. 9.

to the 1963 Conference of the Process Division of the N.Z. Vegetable and Produce Growers' Federation called for a study of the tomato industry. The presentation of the ensuing report has been partly instrumental in alleviating some of these anomalies in the following ways:

(1) The processing of tomatoes has been continued at least temporarily in the Nelson district. The price has not risen, but the number of growers has been reduced so that, hopefully, these growers will be able to produce on large enough areas to remain economic.

(2) The Process Division executive has called for a Department of Agriculture investigation into production methods so that some of the wide variations in productivity brought out in the report may be eliminated, and thus overall productivity and net returns be raised. On the basis of these yield variations from the survey the Vegetable Research Committee has called for more intensive Department of Agriculture advisory work. Some growers have suggested the establishment of farm improvement clubs for process growers.[9]

(3) A break-down of individual growers' costs, together with Chapter 5 of the report[10] ('Process Tomato Growing as a Profitable Enterprise') have been sent to all growers on the survey panel. Copies were made available to other growers on request, and the findings of the survey were published.[11] (Some uneconomic growers are known to have

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9. 'Process Tomato Prices are too Low', N.Z. Comm. Grower. Vol. 21, 10:p.13. March, 1966,
and:
'District Should Deal With Process Problems - as far as possible.' N.Z. Comm. Grower. Vol. 22, 3: p.13. August, 1966.
10. SANDERSON, K.T. op. cit. pp.38-50.
11. N.Z. Comm. Grower. Vol. 21, 7:p.9. December, 1965,
and:
N.Z. Comm. Grower. Vol. 21, 10: pp.19-25. March, 1966.

ceased tomato production on the basis of this knowledge.)

Thus the national body is pushing ahead with the policy of aiding growers in their individual competitive position. The Federation has also acted as a collective bargaining agent in price negotiations with processing companies. One of the companies accepted and praised the findings of the report and as a result of continued negotiations on this basis, "...both J. Wattie Canneries, Ltd., and Unilever (N.Z.), Ltd., have given assurances that growers in Hawkes Bay and Gisborne will receive £1/-/- per ton more for tomatoes for processing in the ... (1966-67)...season."[12]

A further increase of countervailing power by producers in the face of duopoly will occur if the process growers heed the plea of their chairman and 'insist on a collective price-fixing procedure.'[13]

2.3 RATIONAL LOCATION OF FUTURE PROCESSING

The previous section has highlighted the recent moves towards collective bargaining between two large processing companies and the national Federation of Regional Growers' Associations. Changes in processing plant location are issues which are discussed by these parties, and while these discussions are highly desirable in aiding co-operation, there is no reason to suppose that the solution reached is the optimum for the country, or even for either of the negotiating

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12. 'Processors Lift Tomato Prices.' N.Z. Comm. Grower. Vol. 21, 2: p.3. July, 1966.
13. WILLS, W.C. op. cit. p.9.

parties. The stand taken by Growers' representatives may be affected by parochial, rather than national interests and the processing companies are likely to be very much guided by their respective competitive positions. In fact the processing companies may not know what their optimum locational pattern would be.

Within the present institutional framework, one would recommend that the growers and the companies should, in the interests of their industry, attempt to find the facts relating to the efficiency of the whole industry rather than relying on tactical negotiation.

This would involve studying the technical efficiency of present and all possible future facets of the industry, then finding the best combination of these facets which will maximize economic efficiency, and finally deciding whether this combination is institutionally feasible.

2.3.1 Technical Considerations

One should consider in turn each facet of the whole industry and list and study all possible alternatives. This thesis is concerned in detail with the economic aspects, and so technical considerations will simply be listed.

(1) Raw tomato production : List all possible tomato producing regions and determine the likely productivity (yield per acre) and the maximum area of tomatoes which could be produced per year in each. If possible one should also note the scatter of this production and such factors as labour availability for picking.

(2) Raw tomato assembly : Are facilities available for transport to a regional processing site, and/or for transport to any other possible processing region? List the costs for alternative modes and routes of assembly.

(3) Raw tomato processing : List all possible regions which could process tomatoes and possible sites within regions. What is the availability of labour, fresh water, etc., for the processing plant? Will these limit the size of plant, and if so to what size? What is the technical efficiency of different-sized plants? Could the site be linked to national and international transport systems?

(4) Final product transport : List all possible modes and routes of transport from all possible processing sites to all possible markets.

(5) Final product marketing : Are there any possible markets which are not being exploited by the industry in New Zealand or overseas?

2.3.2. Economic Considerations

Working through all the technical possibilities for all facets of the industry, one should attempt to determine the following:

The present cost per unit of carrying out each facet.

The likely changes in cost if output or quantity handled is increased or decreased (i.e. the likely economies of scale.)

Ideally one would like to know the marginal revenue of a unit of product sold on each market and the marginal cost of producing a unit

from each processing plant.

2.3.3 Economic Analysis

Having defined the technical and economic characteristics of the industry, the following problems remain to be solved:

(1) What would be the optimum locational pattern of the industry?

(2) In what ways does the present deviate from the optimum?

(3) What is the most efficient path of development from the present to the optimal location pattern?

2.3.4 Institutional Feasibility

To achieve a measure of reality one should ensure that the solution as described by the economic analysis is institutionally feasible. This would involve determining whether the present processing companies or some other companies or possibly growers' co-operatives were willing and able to carry out processing and distribution according to the optimum pattern. Also whether the regional Growers' Associations would accept the nationally optimum pattern of production.

2.4 THE SPECIFIC ECONOMIC ANALYSIS

The present study is concerned with formulating a method of solving the tomato processing location problem. It will attempt to derive a method of analysis which will search among all technical production possibilities in order to arrive at an economically optimum solution. There is neither time nor research monies available for a complex investigation of all technical possibilities in this thesis.

However, some technical information is available and this will be used to illustrate execution of the economic analysis.

In order to use this technical data the following assumptions must be made:

(1) The only regions considered as feasible tomato producers are those which grew them during the 1964/65 season, and the yields are as recorded by Sanderson. It is assumed that yields and costs per ton remain constant for each region irrespective of the tonnage grown.

(2) There is no raw tomato transport between regions at present and initially it is assumed to be unfeasible.

(3) All the processing takes place in the present plants although their capacities are unlimited. Again, technical efficiency and cost are assumed independent of volume handled.

(4) Only present forms of final product transport are considered.

(5) Consumption is assumed to be fixed at its 1964/65 level irrespective of retail price. It is assumed that no new export markets are to be opened up.

CHAPTER 3

REVIEW OF LITERATURE ON LOCATION ECONOMICS

"..The economics of resource immobility..(is)..a reasonably accurate description of regional economics.." J. Meyer.[1]

3.1 ECONOMIC THEORY AND LOCATION PROBLEMS

Location problems arise because of immobility of some fixed factors in the long run, for instance minerals, and immobility of most other factors in the short run. An economy is limited in the extent to which it can maximize its given aims in the short run, by the efficiency with which it has solved the long run location problem.

In the past, economic theory has been concerned with the maximizing, for example of gross national product, by studying three main aspects of the economy:

(1) national policy - by the study of macro-economics,

(2) individual consumer and producer action in micro-economics, and

(3) the effect of other economies in international trade.

The first two branches have implicitly ignored the location problem by assuming that an economy is situated at a point. The third branch

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1. MEYER, J. 'Regional Economics: A Survey.' Am. Econ. Rev. 53, 1:25. March, 1963.

has been concerned mainly with location problems between economies, but again has assumed each to be situated at a point.

3.1.1 The Location Aspects of Pure International Trade Theory

International trade theory is commonly separated into the 'monetary' and the 'pure' (or equilibrium') theories.[2] The former is closely related to business-cycle theory and to the modern theory of the determination of income and employment as studied in 'Keynesian' economics. The latter is that part of general price and value theory concerned with equilibrium between economies, whereas micro-economic theory studies this problem within economies. In comparison with micro-economic theory the 'pure' theory of international trade studies in detail the effects of resource immobility on the comparative advantage of different economies as sites for production of different goods and services. For this reason the 'pure' theory has occasionally been identified as a type of 'location theory', for example by Ohlin.[3] However, as Haberler has said: "..The traditional theory of international trade is at a higher level of abstraction..(than true location theory)...; it treats the separate countries or regions as spaceless points (markets) and abstracts (with occasional exceptions) from the spatial characteristics of the domestic markets and from intra-regional transportation costs.."[4]

2. HABERLER, G. 'A Survey of International Trade Theory.' Special papers in International Economics. Princeton Univ. 1961.
3. OHLIN, B. 'Interregional and International Trade.' Cambridge (Mass.) 1935.
4. International transportation costs could readily be incorporated into trade theory by treating the cost as an addition to tariffs imposed by the importing country. However this aspect is not generally included within the compass of international trade theory.

By ignoring transportation costs, then, international trade theory assumes that all countries exist as spaceless points, all at the same location.

3.1.2 Micro-economics and the Spatial Dimension

A complete economic equilibrium theory would involve consideration of the location problem by explicitly including transport costs. Although it is logically possible to proceed towards this complete theory from present international trade theory, much more rapid progress is being made by expansion of micro-economics to include the spatial dimension. The general equilibrium theory of Walras, Pareto, Hicks and Samuelson is now being modified along spatial lines, mainly by workers in the new field of regional economics, but there is no reason why the national and international economy cannot be similarly studied.

In order to explore the so-called 'North-South' problem, regional economics has been partly concerned with macro-economic and welfare parameters, but it has also embarked on micro-economic price equilibrium analysis. The problems of individual firms and industries have drawn attention to the spatial factor as it affects micro-economic decisions within regions, and location decisions between regions. Thus the comparative advantage concept of international trade theory has been incorporated into a general theory of spatial price equilibrium.[5]

There have been an increasing number of empirical analyses based on these theoretical foundations, but including the spatial factor, and

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5. ISARD, W. and REINER, T. 'Comparative Costs and Industrial Complex Analysis,' Chapter 1, pp.26-28. In Isard, W. and Cumberland, J. (Eds) 'Regional Economic Planning'. OEEC, Paris, 1961.

these analyses form the basis of what has now been isolated as 'regional analysis'.

3.1.3. Regional Micro-economics Describes Location

Modern regional analysis is based on developments in general economics which occurred in the 1940's. These developments have been described by Meyer[6] as coming under the four headings:

(1) a revitalization of location theory particularly as contained in Losch's[7] work;

(2) international and interregional multiplier theory as illustrated by the work of Metzler[8], Goodwin[9], Chipman[10], and others;

(3) Leontief[11] interindustry input - output analysis; and

(4) mathematical programming.[12]

Regional analysis could be thought of as two sections contained under the four headings - firstly the multiplier theory of (2) which is really

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6. MEYER, J. op. cit. pp.29-30.
7. LOSCH, A. The economics of location, Translated by Woglam, W.H. and STOPLER, W.F. Yale University Press, NewHaven. 1954.
8. METZLER, L.A. A multiple region theory of income and trade. Econometrica, 18:329-354. Oct., 1950.
9. GOODWIN, R. The multiplier as a matrix. Econ. J. 59:537-555. Dec., 1949.
10. CHIPMAN, J.S. The theory of intersectoral money flows and income formation. Baltimore, 1950.
11. LEONTIEF, W.W. The structure of the American economy, 1919-1939. New York, 1951.
12. For bibliography and discussion of literature on mathematical programming see e.g.
DORFMAN, R. Mathematical or linearprogramming; a non-mathematical exposition. Am. Econ. Rev. 43:797-825 Dec., 1953.
For literature on spatial programming see
STEVENS, B.H. A review of the literature on linear methods and models for spatial analysis. J. Am. Inst. Planners. 26, 3:253-259 Aug., 1960.

an extension of macro-economics to include the spatial element; secondly, location theory which is a Loschian 'theory of a firm' expanded towards a theory of general spatial equilibrium, using the methods alluded to in (3) and (4). It is the latter section of regional analysis which is relevant to this study. Losch and Isard[13] have gone a long way towards developing a general equilibrium system of location, and when this location theory does succeed in developing such a system, then international trade will become merely a special case within such a general location theory framework.[14]

3.2 LOCATION THEORY

"Location theory is largely a function of transportation."

R. Renne.[15]

Modern location theory, as outlined in the previous section, is based upon the early location theory developed by von Thunen, Weber, and Losch, but is being rapidly extended from a Marshallian series of partial analyses towards more complete consistent spatial equilibrium models of the Walrasian, 'general equilibrium' type. The main reason for the rapid extension of the theory has been attributed by Meyer to the development of Leontief's input-output analysis and of mathematical programming, both of which enable much more complicated empirical analyses than was previously possible.

13. ISARD, W. Location and space economy. John Wiley and Sons, Inc., New York, 1954.
14. HABERLER, G. op. cit. pp. 4-5
15. RENNE, R. Land economics, Rev. Edn., p. 167. Harper Bros., New York, 1958.

Modern location theory will now be discussed in the following sections:

(1) classical theory of location, and the additions made to this theory during its 're-vitalization' in the 1940's, and subsequently:

(2) mathematical partial analyses of the production of raw materials, their assembly, and the processing and distribution of the final product, based directly on this location theory,

(3) partial analyses and finally spatial equilibrium analyses based on the transportation model, linear programming and other forms of mathematical programming, and

(4) the application of Leontief's input - output interindustry matrix to regional industry location problems with the inclusion of the spatial element, i.e. transportation cost vectors.

3.3 CLASSICAL LOCATION THEORY RE-VITALIZED

"The traditional location theory of the Launhardt, Weber, Palander, and Hoover type has posed the problem of finding the point of minimum cost for assembling raw materials, processing them, and distributing the finished product to the market point or area. For the most part, demand has been taken as given, or its variation as of minor consequence for determining the optimum plant location. Even agricultural location theory of the Thunen type takes prices and hence demand at the city market as set."[16]

Isard, W. and J.M. Peck.

16. ISARD, W. and PECK, J.M. Location theory and international and interregional trade theory. Q. J. Econ. 68, 1:105. Feb., 1954.

3.3.1 von Thunen's Theory of Agricultural Location

The first notable work on economic location theory was developed to solve an empirical agricultural problem. A German farmer and economist, Johann Heinrich von Thunen, when faced with the problem of determining the best farming pattern for his estate 'Tellow', recognized that distance from the market would affect the optimum type of activity to be carried out on his different blocks of land. He formalized these ideas by drawing up a general normative model for determining what crops should be grown on the land surrounding a central market or city in an isolated state, and first wrote his expanded theory of agricultural location in 1826.[17]

In his 'isolated state' von Thunen assumed initially that the city was the sole consuming centre and that prices for all products on the city market were known. He assumed that a uniform plain surrounding the city produced all the goods for these markets, and that the volume of outputs of each crop were known and were the same at any point on the plain. There was no developed transport network, but all products were hauled to the city by waggons, transport costs being solely labour costs. The cost per mile was the same for all products, and costs per mile were constant irrespective of distance from the market. Thus effective price per ton for each crop could be calculated at any given distance from the market by subtracting

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17. von THUNEN, J.H. Der isolierte staat. Gustav Fischer, Jona. 1910. from the original: Der isolierte staat in beziehung auf landwirtschaft and nationalokonomie, (OR: The isolated state in relation to agriculture and the national economy.) Hamburg, 1926.

transport costs (cost/ton mile x distance) from actual market price. All livestock were driven to market on-the-hoof and so transport costs were zero for meat and wool. In accordance with cultural practice, von Thunen postulated six possible land utilization patterns and, assuming rational behaviour by farmers, said that the crop which is either more perishable and bulky, or which earns the highest return per acre, will be situated closest to the market. Further, that when the return per acre (or economic rent) from the next best use was just equal to that of the best, there would be a margin of transference, and beyond this, the second best use would be undertaken, and so on. Thus the optimum production pattern consisted of land uses arranged in concentric rings around the city, with the most valuable, perishable crops situated at the market, and purely livestock enterprises situated at the furthest distance or extensive margin.[18]

In order to introduce more reality, von Thunen postulated the existence of:

(1) a river flowing past the city, and

(2) a small outlying town.

The river provided cheaper transport for producers along its banks, and so the circular zones were distorted as the more bulky crops were

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18. ibid. p. 387. The actual order of land utilization from the central market was: (1) No regular order of crops. Perishable crops planted to suit climate and market.
(2) Man-made forest zone for construction and fuel. High transport costs.
(3) Grain and leafy crop rotation culture, e.g. Norfolk rotation (i) winter grain (ii) roots (iii) summer grain (iv) clover.
(4) Grain, pasture, fallow, rotation culture.
(5) Three field system. Permanent pasture, rye, barley, fallow.
(6) Grazing.

now the 'best use' further from the market. The small town distorted the circular pattern again by having its own small concentric pattern of production.

Recent texts on agricultural location have generally been based on von Thunen's concentric ring theory. Ely and Wehrwein explicitly showed that it was the economic rent which each land-use was capable of paying at each location, which determined the optimum crop to be grown.[19] They graphed rent per acre against distance from the market for different crops and showed that the distance indicated at the intersection of these lines was the margin of transference between two crops.[20] The location economist Hoover added few modifications to von Thunen's theory in his treatment of land use competition[21], while Renne indicated some of the effects of modern transport systems on the simplified concentric patterns.[22] More recent workers have used much the same rationale as von Thunen but have dropped the 'isolated state' assumption.[23] Heady dropped the uniform productivity assumption and considered the competition between regions with different production functions, for different products, and allowed for transport costs by using von Thunen's effective prices to producers in each region.[24]

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19. ELY, R.T. and G.S. WEHRWEIN. Land economics, p. 113. MacMillan, New York. 1958.
20. ibid. pp. 135-136
21. HOOVER, E.M. jr. The location of economic activity, Chapt. 6. McGraw-Hill, New York, 1948.
22. RENNE, R. op. cit. pp. 170.
23. BARLOWE, R. Land resource economics, pp. 255-260. Prentice-Hall Inc., New Jersey. 1958.
24. HEADY, E.O. The economics of agricultural production and resource use, pp. 650-659. Prentice-Hall, Inc., New Jersey, 1952.

His theoretical analysis thus directly considered both comparative advantage and transport costs as locators of agricultural production. It is difficult to understand, then, why transport costs have apparently been omitted in a recent empirical analysis of interregional agricultural production in India by Heady and Randhawa.[25]

3.3.2 The Theory of Processing Plant Location

The simplest form of processing imaginable is the extraction of an 'earth-bound' raw material and distribution of it to many markets. This involves basically the inverse of the agricultural firm's location problem and has been solved by a von Thunen-type analysis, where the effective price to the consumer at any point equals the cost of extraction plus transportation cost from the source.[26] In fact Hoover drew up a map with lines joining all points of equal delivered price.[27] These lines were called isotims and the map showed the same concentric configuration as von Thunen's analysis, with the extractive industry located at the centre of the circle. Where the quantity demanded at all points is known, this analysis will show the quantity which each source of material will supply to the total market, i.e. it was really a size of plant analysis.

Another main body of theory has been developed to answer the problem of location of plants for processing mobile raw materials.

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25. RANDHAWA, N.S. and HEADY, E.O. An interregional model for agricultural planning in India. J.Fm.Econ.46, 1:137-149. Feb., 1964.
26. ELY, R.T. and WEHRWEIN, G.S. op. cit. p. 140 and p. 394.
27. HOOVER, E.M. jr. Location theory and the shoe and leather industries, Chapt. 2. Harvard University Press, Cambridge. 1937.

Weber[28] was the first economist to postulate that a processing plant will be located at the point at which combined raw material, processing and final product distribution costs are a minimum.

Weber assumed that:

(1) there were many consuming centres, with given demand at each;

(2) there was an uneven distribution of raw materials; but that

(3) raw material costs were the same at all deposits; and finally,

(4) that there were equal transportation costs per ton for all materials and products.[29]

In the absence of production cost differentials for various sites, i.e. assuming mainly labour costs constant, the minimum cost location would be that at which transport costs were minimized. This was either at the source of raw materials or at the market depending on the ratio between the weight of the localized raw material, and the weight of the product. If this ratio, the 'material index', was greater than one, the industry would be 'material oriented' and if less than one, 'market oriented'. Where labour costs varied between raw material site and market site there would be production cost differentials, and where these were greater than the transport cost differential of processing at the two sites, labour costs would affect the location.[30]

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28. WEBER, A. Uben den standort der industrien, erster teil, Rein theorie des standorts, (OR: On the location of industries, part 1, Pure theory of location.) Tubingen. 1909.
English translation by: FRIEDRICH, C.J. Alfred Weber's theory of the location of industries. University of Chicago Press, Chicago. 1920.
29. FRIEDRICH, C.J. ibid. p.xxiii.
30. ibid. pp. 60-61.

If transportation costs were constant, i.e. there was neither weight loss nor weight gain in manufacturing, all production would go to the point of lowest labour costs.

Isard[31] developed Weber's theory along more eclectic lines, and both are fully covered in Hoover's book.[32] Hoover considered in detail locational preferences of producers and consumers, the structure of the transfer costs (assembly and distribution), the processing costs and labour market, and the process of locational change. He showed graphical methods of finding the minimum cost point of location, by adding assembly and distribution costs, but he did not include demand effects.

Anticipating Isard and Peck's quotation at the beginning of this section, Losch postulated that:

"In a free economy the correct location of the individual enterprise lies where the net profit is the greatest."[33]

Thus Losch was the first to include demand and market area effects. However, to quote Meyer:

"Losch's theory not only is highly idealized and stylized but has few immediate or obvious empirical possibilities."[34]

Thus the more general revisions of Weberian location theory by Isard and Hoover are being used in preference, in many of the mathematical applications of processing plant location problems.[35]

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31. ISARD, W. and PECK, M.J. op. cit.
32. HOOVER, E.M. Jr. (1948) op. cit.
33. LOSCH, A. op. cit.
34. MEYER, J. op. cit. p. 30.
35. The more advanced models outlined in the following two sections do include the effects of regional demand, however.

3.4 RECENT ANALYSES BASED ON TRADITIONAL LOCATION THEORY

Many of the theoretical concepts outlined in section 3.3 were illustrated empirically within the respective texts, but more recently workers have built much more meaningful models based on two or more stages of the generalized problem. This is the problem of assembling raw materials, processing them and distributing the final product to the consumer so as to gain maximum profit.

3.4.1 Development of the Mathematical Form of Analysis

The simplest form of model is that which assumes the processing plant to be situated, like von Thunen's city, at the centre of a plain producing the product in question at a uniform density over it. An assembly function can be drawn up to show the total cost of collecting all the raw material within any radius of the plant, i.e. for any size of plant. The pioneering work in this field was a study of milk assembly by Bressler and Hammerberg.[36] Progressing one step from pure assembly functions, Olson[37] included processing costs in his analysis of location of milk processing plants. Following von Thunen he assumed that:

(1) milk was available for processing evenly at a given rate over an area;

(2) transport costs per mile were uniform throughout the area

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36. BRESSLER, R.G. and HAMMERBERG, D.O. Efficiency of milk marketing in Connecticut. No. 3, Economics of the assembly of milk. Storrs. Agric. Expt. Stn. Bull. 239. 1942.
37. OLSON, F.L. Location theory as applied to milk processing plants. J. Fm. Econ. 41, 5:1546-1555. Dec., 1959.

(3) there was perfect competition on the selling side of the market for the manufactured product; and

(4) the same price existed for butter, milk powder and cheese at all locations. (Zero transportation cost of finished product.)

Thus, since total production was given, the relative location of plants to each other would determine their respective capacities. As demand was taken as constant, cost minimization would be the same as profit maximization, and by postulating a co-operative industry, Olson defined his objective as finding the location and number of processing plants which minimized the total of assembly costs plus processing costs.

He then derived a formula for calculating the total assembly cost of a given volume of milk, by integrating a marginal assembly cost function. The minimum-cost assembly area would be circular because of the constant density of production and even transport costs. The radius of the circle would increase as total volume handled by the plant was expanded. The total variable transport cost could be calculated for a range of plant volumes.[38]

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38. Where c = marginal cost/mile/cwt. of milk
r = modal distance of farms from the plant, given capacity V
$\therefore$ marginal cost for plant of capacity (V) = r x c ....(i)
Now volume (V) = density (D) x πr^2
$\therefore$ $r = \left(\frac{V}{\pi D}\right)^{\frac{1}{2}}$ ....(ii)
From (i) to (ii), Marginal cost of assembly for plant capacity
$V = \left(\frac{V}{\pi D}\right)^{\frac{1}{2}} \times c$
$\therefore$ total variable cost $= \int c V^{\frac{1}{2}} (\pi D)^{-\frac{1}{2}} \, DV$
$= \frac{2}{3} c (\pi D)^{-\frac{1}{2}} V^{\frac{3}{2}}$
Given D and c solve for total variable cost for a range of values for V.

Processing costs: He determined a long run planning curve (or Chamberlin's 'envelope' curve) by drawing up cost curves for several hypothetical plant sizes, using the 'synthetic' method outlined by Knudtson.[39] He assumed total processing cost could be of the form

Processing cost = aV^b

Where a = ratio of costs to volume

b = rate of change of this ratio

Combining the assembly and processing analyses Olson solved for the optimum volume or size of a processing plant at any given production density, and back-substituted to find the radius which each plant should serve.

Olson modified the collection areas by recognizing that there would be overlapping of adjacent circles and so re-calculated for hexagonal assembly areas.

He showed that optimum size differed:

(a) at different milk production densities and handling costs,

(b) for different types of processing plants because of different cost structures, and

(c) where there was marked seasonality.

He mentioned, but did not pursue analysis of the Weberian type, i.e. the extent of market or raw material orientation or the effect of transportation costs of final product on the location, bearing in mind the weight-gaining or weight-losing properties of different possible

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39. KNUDTSON, A.C. Estimating economies of scale. J. Fm. Econ. 40,3: 750-756. Aug., 1958.

processes. He did not explore the assumptions of;

(1) even density of production,

(2) even transportation costs, or

(3) perfect competition on the market for manufactured products.

In discussing Olson's paper, C.E. French[40] stated that the basic plant location problem lay in the concept of a market itself and thus he took strong issue with Olson's virtual dismissal of the demand side as a locator. Even though the region was largely a producing one there would be some localized areas of demand both for raw milk and for the processed products. Also plants located at different points within this region would have different locational advantages in respect to the main markets outside the area. This interregional competition is hardly manageable in Olson's model. Finally, because of the basic assumption (1) of even production density this method is unable to handle the already-present, rather odd shape of procurement areas. French suggested that many of these problems could be overcome by integrating this analysis into a mathematical programming model and possibly supplementing it with simulation techniques to handle stochastic characteristics. He made a final plea for a model capable of giving "..reasonable criteria for defining market areas in the full economic context."[41]

Despite French's plea there have been more recent papers which use Olson's basic technique but introduce a few refinements. In studying

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40. FRENCH, C.E. Discussion: Milk processing plant location. J. Fm. Econ. 41, 5:1556-1558. Dec., 1959.
41. ibid. p. 1558.

the location of broiler production, Henry and Seagraves[42] derived a transport cost function which included not only the cost/100lb. of birds procured, but also the costs of visits by fieldmen, feed-delivery, etc., all expressed as cost/100lbs. of live birds procured. They added this to a processing cost function as Olson had, for a range of sizes of plants and thus assembly areas. However Henry and Seagraves repeated this exercise for many different uniform densities of broiler production. They were thus able to draw some conclusions about the costs of increasing production by 100lbs. at the extensive margin (and thus increasing transportation costs); or at the intensive margin, (and thus increasing individual producers' labour costs.) As the industry was not completely vertically integrated, and processors buy the broilers at the farm gate, it was in their interest to encourage expansion at the intensive margin and thus reduce transportation costs.

Apart from the glaring omission of demand effects on these partial analyses, the assumption of uniform production density throughout the area is an over-simplification, and Ben. C. French[43] has gone some way towards overcoming this and has also looked at the structure of assembly costs. He postulated a supply plain dotted with 'islands' rising to different heights corresponding to the different production densities. Thus to find the total volume available within a given radius of a proposed plant, French found the double integral $\int_0^{2\pi}\int_0^{r}$ of all production

42. HENRY, W.R. and SEAGRAVES, J.A. Economic aspects of broiler production density. J. Fm. Econ. 42,1:1-17. Feb., 1960.
43. FRENCH, B.C. Some considerations in estimating assembly cost functions for agricultural processing operations. J. Fm. Econ. 42,4:767-778. Nov., 1960.

islands located at any point (r,θ). This is very valuable especially where there are localized pockets of production (as one would imagine to be the case with broiler production). In analysing assembly costs, French considered those which were concerned with volume and were constant per unit irrespective of production density, e.g. loading, and those concerned with distance, e.g. fuel. These latter 'variable' per unit costs were the only ones included in the transport function. He indicated the use of hypothetical processing curves of the Henry and Seagraves type.

3.4.2 Applications of this Analysis

The basic model postulated by Olson and elaborated by other workers is capable of including a number of institutional realities in application to specific, long run, normative analyses of the most efficient organization of an agricultural processing industry within a region.

The work of Ben. French enables the model to handle discrete pockets of raw material production and to make some allowance for existing transportation routes.

In terms of mode of assembly it is capable of handling 'point-pick-up' of raw materials which involves a separate trip from the plant to each supply point, as used by Henry and Seagraves, and Ben. French. Alternatively where applicable it can handle the 'route assembly' of raw materials as typified by the milk processing industry of Olson's model. The practical implications of these two modes of assembly were discussed

by Boutwell and Simmons.[44] Consideration can also be given to transport of other services, e.g. advisory coverage, and to the dichotomy of fixed and variable costs of assembling one unit of production from different distances.

A range of processing costs per unit processed can be determined by postulating a series of possible plant sizes and deriving 'synthetic' cost curves, or by plotting the cost per unit processed in a range of existing plants, i.e. a 'statistical' cost curve. A full discussion of these methods, a comparison of their usefulness, and relative literature on them is available in Knudtson's paper.[45]

3.4.3 The Stollsteimer Technique

An American, Stollsteimer, has developed a technique which is very similar to those outlined above in its general scope, but which adopts the more realistic approach of comparing actual transport costs rather than deriving a transportation function.[46] Like the models above, Stollsteimer's model set about solving the location and size of plant problem by minimizing the assembly and processing costs or the distribution and processing costs, but not, (in its present non-programming form) of assembly, processing costs, and distributing costs, simultaneously in the one model.

On the processing side, Stollsteimer assumed a linear cost function (constant marginal processing cost in any given plant), with

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44. BOUTWELL, W.K. and SIMMONS, R.L. Estimating route assembly costs. J. Fm. Econ. 46, 4:841-848. Nov., 1964.
45. KNUDTSON, A.C. op. cit.
46. STOLLSTEIMER, J.F. A working model for plant numbers and locations. J. Fm. Econ. 45, 3:631-654. Aug., 1963.

a positive intercept. He assumed that in its simplest form,[47] the function was horizontal (no economies of scale in plant operation) and was the same for all plant locations.[48] Thus as the number of plants was increased, the horizontal function became stepped by the addition of the positive intercept (the fixed costs) of each successive plant introduced. The method is similar to those above in that it develops cost functions for processing plants, and, as will be shown later, is even more flexible in that it can accommodate linear cost functions with a different slope at each location.

Transport costs were dealt with in a much more direct way than even that suggested by B. French.[49] In fact, Stollsteimer drew up a transportation tableau[50] identical to that used in the transportation model discussed next, but he solved it by the simple method of inspection of row vectors to find the minimum actual transport cost from any of the sources to each of the possible plant locations in turn.[51] He then calculated the minimum total transport cost for assembling goods at each plant in turn and selected the minimum. Then he found the two

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47. ibid. pp. 638-639, Case III.
48. An economies of scale function used later in his analysis was:
    TPC = 13408 + 3056.39 V
    where V = processing rate for a 250-hour operating season.
    This was derived in:
    STOLLSTEIMER, J.F. Bulk containers for deciduous fruits: cost and efficiency in local assembly operations. Giannini Found. of Agric. Econ. Research. Report 237. 1960.
49. FRENCH, B.C. op. cit.
50. Where available, actual transport costs should be used in this tableau, but Stollsteimer had developed an hypothetical set from economic-engineering principles prior to the analysis here discussed. See: STOLLSTEIMER, J.F. (1960) op. cit.
51. He does not require an iterative programming technique as he has no quantitative limit on the capacities of each plant, corresponding to the demand limitations in the transportation model.

plants which together gave the minimum total transport cost, by considering all possible combinations of plants taken two at a time. For each combination he decided which plant each of the sources in turn would supply, by inspection of the transport costs to either. He repeated this procedure for 3, 4, 5 plants and so on, and thus was able to draw up a function of total transfer cost and number of plants, this curve being an envelope to a set of minimum total transfer cost points for each number of processing plants.

On the same axes he drew his linear function corresponding to the addition of the successive processing plants and finally totalled the processing plus transfer cost for each successive number of plants, and the plant number which minimized the total transfer and processing costs was indicated by the lowest point on that curve. The plant locations which were utilized were found by referring back to the combination of plants which gave the minimum transportation cost for that number of plants.

The method could allow for economies of scale in plant operations, both where these were the same for all locations and where they varied for each location, i.e. the slope of the linear cost curve was different at each location. The latter, more complicated alternative was handled in Stollsteimer's Case II,[52] by adding to each column of the transfer cost matrix, the slope co-efficient of the processing cost function applicable for each particular plant site. Thus the minimum combined transfer and variable processing costs were determined as above from the

52. STOLLSTEIMER, J.F. (1963) op. cit. p. 638.

modified transportation matrix. The fixed processing cost function was derived by adding the positive intercepts of the processing cost curves at the individual locations indicated by the plants included at each successive number in the minimum transport costs/plant number solution.

Taking a supply of raw materials as given, the transport costs from each source to each plant location, and the slope and intercept of the linear processing cost curve at each plant location, Stollsteimer could for each possible number of plants operating, say which actual plants operated and the rate of operation (or level of intake of each). He could determine the haulage pattern corresponding to the minimized transport cost solution, and the actual minimum transport bill for that number of plants. He could separately determine the total processing cost for each number of plants and by adding these to the successive minimum transport cost solutions could show the minimum total transport and processing costs for each number of plants and thus the overall minimum cost number of plants, their location, capacity, and raw material assembly pattern.

A similar analysis could be carried out for processing and distribution, where demand patterns were known, but the location solution would not necessarily correspond to that from the assembly side, and there is no readily apparent method of reconciling these solutions without the use of programming techniques.

A useful extension to this basic model has been made by Polopolus,[53] who moved from the single raw material/single product firm, to the firm

53. POLOPOLUS, L. Optimum plant numbers and locations for multiple product processing. J. Fm. Econ. 47, 2:287-295. May, 1965.

which buys many raw materials, each being processed to form its own final product. The important step by Polopolus was the recognition of joint processing costs which reduce the effective cost of processing each individual product in a multiproduct firm.[54] This was shown to be especially relevant to plants which may be able to process other crops in the off-season of their main crop. In terms of Stollsteimer's model, if two different crops were processed in two different plants they would incur fixed costs equal to twice the intercept of the cost curve of one plant. If they were processed sequentially in only one plant, they would incur the fixed costs of only one plant (plus some small additional fixed cost to allow for overhead on those processing machines which were not common to both processes). This consideration is particularly relevant to the generalization of any single product analysis to take in a multi-product problem.

3.4.4 Limitations of These Models

These models are useful in that they focus attention directly on the alternative to the processing firm of encouraging extended production at the intensive margin by offering higher prices close to the factory, or at the extensive margin by paying increased assembly costs to cover a wider area.

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54. Workers at the Regional Science Research Institute associated with the University of Pennsylvania, are currently doing research on 'programming with external economies', which would presumably include product interactions like those mentioned by Polopolus, only treated at the industry level.
STEVENS, B.H. (President of Regional Science Research Inst.) pers. comm. June, 1966.

However the main objection still stands. The models discussed handle only the effects of the source of raw materials and the processing costs on location, and ignore the effect of the location of demand centres. The fact that these are important is ironically illustrated in a paper by Cobia and Babb[55] who used the same basic model of Henry and Seagraves but ignored assembly costs and determined the location of milk processing plants according to distribution costs. To this extent, then, these models have not complied with the theoretical requirement of finding points of minimum total transfer costs as laid down by Weber and Hoover.[56] This basic limitation makes these models capable only of handling location within a single region, and incapable of studying the simultaneous location of plants in many different regions which compete for sale of their products on the same markets.

In order to obtain more meaningful analyses, a series of these models concerned with the production and processing of a crop in different regions could be linked to a processing-distribution model by using one of the mathematical programming techniques outlined in the following section, thus simultaneously taking into account the marketing factors of distribution of, and demand for, the final product. This two-stage analysis would be less efficient than single analyses which included all three facets simultaneously. Such comprehensive analyses using mathematical programming will now be developed.

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55. COBIA, D.W. and BABB, E.M. An application of equilibrium size of plant analysis to fluid milk processing and distribution. J. Fm. Econ. 46, 1:109-116. Feb., 1964.
In assuming a uniform demand pattern, this model is a direct application of the theory of location of extractive industries, discussed in Section 3.3.2 above.

56. HOOVER, E.M. Jr. (1948) op. cit. Chapters 3 and 4.

3.5 MATHEMATICAL PROGRAMMING AND LOCATION ACTIVITY ANALYSIS[57]

"All's fair in love and war."
Anonymous.

The rapid development of mathematical programming was precipitated by the need for evolving least-cost methods of logistics during the Second World War. This initial preoccupation of programme researchers with economic problems in general and transportation or location economic problems in particular has largely been continued, thus the development of tools for location analysis has not been allowed to lag far behind the development of new programming methods. Because of this close relationship, the historical development of location and spatial analyses in general shows the succeeding levels of abstraction which may be attained using these programming methods.

3.5.1. The Transportation Model

The simplest form of mathematical programming, and the first postulated, was the transportation model. This was first formulated and solved by the American, Hitchcock[58] and independently formulated, but not solved by the Russian, Kantorovitch.[59] It was aimed at outlining the problem of distribution of a homogeneous product from spatially separated sources (e.g. warehouses) to spatially separated

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57. See especially STEVENS, B.H. op. cit. for full review of literature.
58. HITCHCOCK, F.L. The distribution of a product from several sources to numerous localities. J. Math. Phys. 20: 224-230. 1941.
59. KANTOROVITCH, L. (1942)
English translation: On the translocation of masses.
Management Sci. 5,1:1-4. 1958.

localities (e.g. consumers) in such a way as to minimize the transport costs incurred.

Mathematically the problem was expressed thus:

Given: C_i = capacity of warehouse, at location i, (i=1...m)

D_j = demand at location j, (j=1...n)

t_{ij} = transport cost from warehouse at i to consumer at j, for all i, j.

Assume: total warehouse capacity = total consumer demand,

i.e. $\sum_{i=1}^{m} C_i = \sum_{j=1}^{n} D_j$

There is no net storage in the period considered.

To find: X_{ij} = the amount of product shipped from warehouse at i to consumer at j (for all i, j), which will

Minimize: T = total transport cost.

Hitchcock and Kantorovitch formulated the mathematical system thus:

Minimize: $T = X_{11}t_{11} + X_{12}t_{12} + \ldots + X_{21}t_{21} + \ldots + X_{mn}t_{mn}$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} \cdot t_{ij}$$

Total cost equals volume hauled times cost per unit.

Subject to: $\sum_{j=1}^{n} X_{ij} = C_i$ for all i.

$\sum_{i=1}^{m} X_{ij} = D_j$ for all j.

Amount transported is just equal to the capacity of the warehouse on the one hand, and the amount consumed on the other.

And to: all $X_{ij} \geqslant 0$ There are no negative flows of goods.

This system was not solved as a set of simultaneous linear equations but by an iterative, systematic approach to the minimization of the objective function, T, subject to the restraining equations. The 'transportation' technique of solution was advanced by Professor..... Koopmans[60] in 1947 independently of Hitchcock's work, and this general model has become known as the 'Koopmans-Hitchcock Transportation Model'. At approximately the same time Dantzig, working in a U.S. Air Force study group solved the Hitchcock formulation using the Simplex method, which was capable of handling also the more complex linear programming formulation of inequalities. Dantzig's findings were not made public until later and were finally published in 1951.[61] For the very reason that this model is simpler than the linear programming model, the simplified computing routines of the Koopman's type are more efficient than the Simplex for transportation solutions.[62]

By assuming that both the quantity supplied and the quantity demanded are given, and fixed, this model would appear too restrictive for anything but the simplest analysis of a transportation system. However, by including various modifications, Koch and Snodgrass, Snodgrass and C.E. French, have applied this model to analysis of the

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60. KOOPMANS, T.C. Optimum utilization of the transportation system. Proc. International Statist. Conf., Washington D.C. Vol. 5. 1947. (Reprinted as supplement to Econometrics, 17. 1949).
61. DANTZIG, G.B. Application of the Simplex method to a transportation problem, Chapt. 23. In Koopmans, T.D. (Ed.) Activity analysis of production and allocation. Wiley and Sons, New York, 1951.
62. For discussion of 'stepping-stone' and 'MODI' methods of solution; HEADY, E.O. and CANDLER, W.V. Linear programming methods. Iowa State University Press, Iowa. 1958.

tomato processing and dairying industries respectively.[63] Koch and Snodgrass added processing costs to the transport costs from each region, and used marginal values thrown up in the solution to show the extra cost or saving of shifting one unit of production or consumption between regions. They also used the model to study the 'efficiency' of the actual marketing system. By selecting one area as the base, and the actual price in that area as the base price, they added the relevant transport costs according to the least cost pattern, and thus determined the 'equilibrium' price at each other consumption point. Using a regression of actual prices on their 'equilibrium' prices, they obtained an R^2 of 0.5 and thus assumed that only one half of the regional price variation was explained by the model. They then adjusted the transport costs using subjective estimates of the effect of consumer preferences, product differentiation and pricing inefficiencies due to isolation of some regions, all in an attempt to fully explain the actual prices.

The juggling of the t_{ij}'s according to subjective considerations are of little use in a model which purports to show the relative inefficiency in the actual marketing system in finding the true equilibrium prices. These subjective conditions should have been reduced to objective measures and included (ideally, as regional demand

63. KOCH, A.R. and SNODGRASS, M.M. Linear programming applied to location and product flow determination in the tomato processing industry. Pap. Proc. reg. Sci. Ass. 5:151-162. 1959.
SNODGRASS, M.M. and FRENCH, C.E. Linear programming approach to interregional competition in dairying. Agric. Expt. Stn. Purdue University. 1958.

curves) in the formation of the original model which would then throw up more accurate, true 'spatial equilibrium' prices, for comparison with the actual prices.

In his book on the use of programming methods in industry, Vazsonyi[64] again added production costs to the t_{ij}'s, assumed some overall excess of production capacity, but limited the total capacity of each plant. He studied the sensitivity of the solution by increasing the output of fully-utilized plants using overtime running with consequent increased costs per unit. This is really an iterative method of including at least some of the increased costs associated with an inelastic production function in the short run.

Even less ambitious but probably more ideally suited to this specific method is a study which restricts itself purely to a close study of the transportation system.[65] Some specific problems can be tackled by this method, namely activity analyses of transportation systems for firms with (to them) infinitely elastic supply functions or infinitely elastic markets. However, it would be dangerous to attach much weight to 'spatial equilibrium prices' obtained from the rigid assumptions imposed by this method.

3.5.2. The Transhipment Problem

The transportation model above is limited to pair-wise connections

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64. VAZSONYI, A. Scientific programming in business and industry. pp. 41-44. Wiley and Sons, New York. 1961.

65. see for example:
BECKMAN, M. et al. Studies in the economics of transportation. Yale University Press, New Haven. 1956.

between one point which is purely a "source", and another point which acts as a "sump". This limitation is overcome by a model which can treat each point as both a sump and/or a source, and provides each point with a stockpile so that it can effectively act as a transhipping point. The model should then solve for optimum route transportation and for the minimum stockpiles required at each point to enable this transhipment pattern to function. Such a modification to the transportation model was developed by Orden.[66] His model used the same basic data as the transportation problem but allowed a predominantly source point to import a quantity and he thus constrained his system so that the amount shifted from each point equalled the amount produced at that point plus inshipments. The non-negativity constraint was satisfied by adding an arbitrary stockpile to the availability at each point initially and then subtracting it from the final solution to give actual amounts shipped and received from each point, and thus the amount shipped along each route.

King and Logan used this rationale to include the processing function into a transportation model.[67] They assumed known supplies of slaughter animals and known demands for beef at different points, and included the required processing plants as transhipment points by adding slaughter costs to the cost of live animal shipment to that point. They had a further submatrix to handle costs of shipping beef from the

66. ORDEN, A. The transhipment problem. Management Sci. 2,3:276-285 1956.
67. KING, G.A. and LOGAN, S.H. Optimum location, number and size of processing plants with raw product and final product shipments. J. Fm. Econ. 46,1:94-108. Mar., 1964.

slaughterhouse to the demand centres. By including only the present location of slaughterhouses, but allowing each to have a very high capacity, King and Logan derived an optimum size of plant for each location and the optimum shipping pattern of slaughter cattle and of beef implied by this.

Using a separately derived economies of scale curve for slaughter plants,[68] they then iteratively introduced new processing cost matrices where the cost per unit processed for each plant was that cost on their economies of scale curve which corresponded to the volume handled by that plant in the solution to the previous iteration. The final solution may include some of the present plants at a zero production level, so this model in fact indicates the optimum size and number of plants given the present range (or a hypothetical range) of locations of plants.

The basic King and Logan model has been further simplified by Hurt and Tramel to obviate the need for the introduction of artificial 'stockpile' variables.[69] They also expanded the multi-region, multi-plant model above, to include multi-processing of the single raw product, to give a multi-product shipment pattern.

These models are the first to have been capable of mathematically showing the Hooverian interaction of assembly costs, processing costs and distribution costs on location of the firm. They are capable of

68. LOGAN, S.H. and KING, G.A. Economies of scale in slaughter plants. Giannini Foundation Research Report 260, University of California. 1962.

69. HURT, V.G. and TRAMEL, T.E. Alternative formulations of the transhipment problem. J. Fm. Econ. 47,3:763-773. Aug., 1965.

handling quite complex problems, and as their authors pointed out, these models have a great computational advantage over the linear programming spatial equilibrium model for problems of comparable size. They have the disadvantage of not showing so readily the sensitivity of the final solution to changes in supply, demand or processing costs, but as outlined, King and Logan have used a fairly efficient iterative method for exploring at least the supply side of the problem.[70]

3.5.3 Generalized Linear Programming

Probably the most versatile mathematical programming technique developed is linear programming. In its general form - as distinct from the transportation programme - linear programming can incorporate many facets of location problems simultaneously, in arriving at the minimum-cost point for processing. A formulation for the spatial analysis of the processing activity taking into account assembly, processing and distribution costs in true Hooverian fashion, was published in King and Logan's paper.[71]

The formulation included activities representing live animal shipment, livestock slaughter and meat shipment with equations to bring about reconciliation at each stage. Supply of live animals, and demand for meat were single-valued and given, as were capacities of processing plants. However King and Logan used a transhipment model to solve their

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70. Dr Hurt is also carrying on further work using linear demand functions for the markets to solve for equilibrium prices and distributions, and has "...enjoyed some limited success in this area." pers. comm. Mar., 1966.

71. KING, G.A. and LOGAN, S.H. op. cit. pp.95-96.

problem because of the high computer capacity requirement of the linear programming form.[72]

An empirical activity analysis of a 26-region model for beef has been carried out by Judge, Havlicek, and Rizek,[73] who incidentally claimed computation to be no problem because even their 72 equation x 1326 activity model required only two minutes on an I.B.M. 7094 electronic computer, and "... modern computers are capable of handling 1000 equations and a very large number of activities."[74]

Judge et. al. assumed as given:

(1) the regional availabilities of raw materials designated S_i^h for the hth type of raw material in region i;

(2) the regional demands for final products d_i^k for the kth final product in region i;

(3) the unit transport costs of shipment from i to j of raw materials and final products t_{ij}^h and t_{ij}^k respectively;

(4) the unit cost of processing the kth type of final product in region i - C_i^k;

(5) the rate at which the qth type of product (raw material or intermediate product) is converted per unit of process into the mth type of intermediate or final product in region i: i^{qm} (where q = k, h, r; m = k, h.)

(6) the capacity of the rth processing plant in region i: S_i^r.

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72. ibid. p. 96.
73. JUDGE, G.G., HAVLICEK, J. and RIZEK, R.L. An interregional model: its formulation and application to the livestock industry. Agric. Econ. Res. 17, 1:1-9. Jan., 1965.
74. ibid. p.9.

By minimizing total costs, their model solved for the quantity of the hth raw or intermediate product shipped from i to j to produce the kth final product (X_{ij}^{h}) and the quantity of the kth type of final product shipped between i and j (X_{ij}^{k}) and thus the optimum assembly, processing and distribution pattern, and the total cost of this total activity. The solution to the dual ascribed values or rents to the restrictions.

They formulated the model and minimized costs using the Kuhn - Tucker saddle point theorem,[75] then expressed the primal-dual problem in a conventional Simplex tableau. (See Figure 1.)

Fig. 1. Interregional Simplex Tableau

Values and Rents	Flow and Processing Activities		Restrictions
	Final Products	Primary and Intermediate	
u	X_{ij}^{k}	X_{ij}^{h}	P_o
u_i^k	i^{kk}		d_i^k
u_i^h	i^{hk}	i^{hh}	$-S_i^l$
u_{ir}	i^{rk}		$-S_i^{hr}$
C_j's	$(t_{ij}^k + C_i^k)$	(t_{ij}^h)	

75. KUHN, H. and TUCKER, A.W. Non-linear programming, pp. 481-491. In Neyman, J. (Ed.) Proc. Second Berkeley Symp. math. Statist. Probab. Univ. of California Press. Berkeley. 1951.

Note that:

(1) In the Heady and Candler notation, the u's are equivalent to C_i's, the P_o column to the b_i (availability) column, and the bottom row to the C_j row.

(2) In this formulation, cost of transport of final product is added to the processing cost.

The flexibility of the model was stressed and possible modifications which were mentioned were:

(1) extension to include the production and flow or intermediate products;

(2) the inclusion of processing costs where they differ between regions, rather than plant capacities, and thus solve for optimum long run location;

(3) the inclusion of regional prices in the objective function (now: $P_i^k - C_i^k - t_{ij}^k$) and thus solve a profit maximizing model. An important feature of this model in comparison with preceding types, is that internal prices of raw material and differential regional values for final products, and the rent of over-utilized plants are all indicated directly in the minimum cost solution.[76] Thus this method contains the rudiments of a spatial equilibrium model in its concern about equilibrium prices, however it is not truly a spatial equilibrium as demand is assumed inelastic at the present price (given or not).

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76. The relationship between the dual prices in multi-location linear programming and the classical theory of location rent is discussed fully in:
STEVENS, B.H. Linear programming and location rent. J. reg. Sci. 3,2:15-26. 1961.

At a slightly higher level of abstraction is an activity analysis model which Takayama and Judge developed for the agricultural sector.[77] It involved a similar formulation to that of Judge, Havlicek, and Rizek except that it included given linear demand relations for final products. The aim was to maximize net consumer surplus, i.e. "...the summation of areas under the individual regional demand relations, minus the total cost incurred in shipping the mobile commodities between regions, and processing the secondary intermediate commodities."[78]

Supply was not dealt with in functional form but both models really derived a normative supply pattern, given the cost specifications outlined. The Takayama and Judge formulation solved for market prices given average production and transport costs and given linear demand relationships. Thus it took into account most of the conditions for spatial price equilibrium.

3.6 INPUT-OUTPUT THEORY AND SPATIAL EQUILIBRIUM

In Section 3.2 it was stated that much of the rapid expansion of location theory has been due to the development of mathematical programming and of Leontief's input-output analysis. Having extensively discussed the programming and its many possible applications to the spatial analysis of separate industries, or even many industries simultaneously, there remains to be discussed the application of Leontief's

77. TAKAYAMA, T. and JUDGE, G.G. An interregional activity analysis model for the agricultural sector. J. Fm. Econ. 46,2:349-365. May, 1964.
78. ibid. p. 354.

much simplified inter-industry analysis so that it can include the spatial functions. However, as the problem to be handled in this thesis is, initially at least, that of a single industry, and as some of the programming models discussed are capable of handling in more detail the location problem of two or more industries, it will suffice to mention here that the method of including interregional transportation cost vectors into a series of regional input-output matrices has been developed by Moses.[79] The model is operational and is applied to a 9-region, 20-industry model of the U.S. However, like the basic Leontief input-output model which it incorporates, Moses' model adopts single valued input-output coefficients and other rigidities which are avoided by more detailed programming analyses.

3.7 THE SPATIAL PRICE EQUILIBRIUM PROBLEM

The methods outlined in the previous sections (3.4 and 3.5) discussed the problems of flow determination when demand and supply relations are restricted to a greater or lesser extent, i.e. they tackle spatial activity analysis problems. The more general problem is one of determining equilibrium prices at different locations when demand and supply information is in a functional form. If positive historical supply and demand functions were available it would be theoretically feasible to determine a unique spatial equilibrium solution for all

79. MOSES, L.N. A general equilibrium model of production, interregional trade, and location of industry. Rev. Econ. Statist. 42, 4:373-397. Nov., 1960.

prices and flows, and further, to explain the interaction of all economic forces in a wholly normative general equilibrium model, including the space, time, and form dimensions.

The foundation worker in this field of spatial comparative statics, was Stephen Enke. He defined the basic equilibrium problem thus: "There are three(or more) regions trading in a homogeneous good. Each region constitutes a single or distinct market. Each possible pair of regions are separated - but not isolated - by a transportation cost per physical unit which is independent of volume. There are no legal restrictions to the limit the profit-seeking traders in each region. For each region, the functions which relate local production and local use to local price are known, and consequently the magnitude of the difference which will be exported or imported at each local price is known. Given these trade functions and transportation costs, we wish to ascertain:

(1) the net price at each region;

(2) the quantity of exports or imports for each region;

(3) which regions export, import, or do neither;

(4) the aggregate trade in each commodity;

(5) the volume and direction of trade between each possible pair of regions."[80]

Probably the most important aspect of this initial conceptual formulation of the problem of interconnected markets, is the explicit incorporation of separate regional demand and supply functions. Basing his analysis on positive, historical, regional demand and supply functions,

80. ENKE, S. Equilibrium among spatially separated markets: solution by electric analogue. Econometrica. 19:41. 1951.

and interregional transport costs, Enke solved the spatial price equilibrium problem using electric analogue.

3.7.1 Solution by Linear Programming

Samuelson, in discussing the Enke formulation, showed that it contained within it a Koopmans - Hitchcock minimum transportation cost problem.[81] He re-wrote the Enke formulation as a linear programme which iteratively moved towards maximization of 'social payoff'. The solution gave the five sets of information specified by Enke which were consistent with the supply and demand functions and the transportation costs, and which maximized social payoff.

Baumol presented a similar solution to that of Samuelson, whereas Beckman extended the formulation to accommodate the case of continuous geographic distribution of production and consumption. This latter refinement would seldom be used because of lack of data - there is usually insufficient data for deriving regional demand curves without breaking it down still further, unless uniform distribution was assumed, as in the models in section 3.4.1 above.

The Enke-Samuelson formulation and solution is an efficient approach to spatial pricing and geographical flow systems and has been used in some very useful empirical studies, notably by Fox and Judge.[82]

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81. SAMUELSON, P.A. Spatial price equilibrium and linear programming. Am. Econ. Rev. 42:283-303. 1953.

82. FOX, K.A. A spatial equilibrium model of the livestock-feed economy. Econometrica. 21:547-566 1953.
_____ and TAEUBER. C. Spatial equilibrium models of the livestock-feed economy. Am. Econ. Rev. 45. 1955.
JUDGE, G.G. Competitive position of the Connecticut poultry industry (7). A spatial equilibrium model for eggs. Storrs. Agric. Expt. Stn. Bull. 318. University of Connecticut. Jan., 1956.
_____ and WALLCE, T.D. Estimation of spatial price equilibrium models. J. Fm. Econ. 40,4:801-820. Nov., 1958.

Different workers have explored the effects of regional demand, the transportation system and regional supply using linear programming to solve for the spatial equilibrium.

3.7.1.1 With Emphasis on Regional Demand

Samuelson illustrated the basis of his analysis in the simple form of the two-regional case by means of graphs.[83] He drew a common price axis vertically with quantity axes going to left and right for the two regions, and thus drew 'back-to-back' demand and supply curves. Assuming a perfect market, he said that the demand curve of the deficit region will be shifted downwards by exactly the cost of transportation per unit, read off the price scale. The spatial equilibrium prices and flows could then be found by iteratively altering the price until the excess supply from one region was just equal to the deficit in the other. The prices differed by the cost of transportation from surplus to deficit region.

Judge and Wallace[84] developed algebraically an N-regional model for beef, in which regional supply was pre-determined; total beef production equalled total beef consumption in any time period; there was no cross-hauling of beef and the usual assumptions of homogeneity of product, etc., were made. They showed the problem in three parts:

(1) Determination of regional prices, consumption, excess demands and supplies, by drawing up demand functions. The simplest form of these functions would be:

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83. SAMUELSON, P.A. op. cit. p. 286.
84. JUDGE, G.G. and WALLACE, T.D. op. cit. pp. 805-808.

$$Y_{1i} = \beta_i (Y_{20} + d_i)$$

where Y_{1i} = quantity demanded in region i,

β_i = price elasticity of demand in region i,

Y_{20} = price in base region,

d_i = price differential of region i from the base region.

They extended the basic function for each region to include other demand determinants of income, population, competing products and the stochastic residual, and summed these regional demand functions. As $\sum_{i=1}^{N} Y_{1i} = \Sigma S$ (total supply), it was then possible to express the determination of the base price in the form:

$$Y_{20} = \frac{1}{\sum_{i=1}^{N} \beta_i} \left[\Sigma S - \sum_{i=1}^{N} \beta_i d_i - \sum_{i=1}^{N} \sum_{j=1}^{K} \alpha_{ij} Z_{ij} - \sum_{i=1}^{N} \alpha_{io} \right]$$

where new terms are:

Z_{ij} = quantitative measure of jth consumption-affecting factor (j = 1..k) i.e. income, etc.

α_{ij} = elasticity of demand associated with jth factor, i.e. YED, etc.

α_{io} = stochastic residual in the ith region.

Where price differentials were known, one could solve for the base price (Y_{20}), then add the individual differentials and substitute back into the original respective regional demand equations to solve for regional consumption. Supply was assumed pre-determined, so regional surpluses or deficits could be calculated.

(2) The minimum cost flows of beef among regions could then be

found by entering the surpluses and deficits in an N x N transportation model and solving by the Simplex method.

In actual fact the price differentials (d_i's) mentioned in (1) above would not be known, because the price differential of the ith region from the base region would not always be equal to the direct transportation cost between the two. Thus the third section (usually as the first step) will be to:

(3) Determine regional price differentials (d_i's). Find which regions are likely to be surplus or deficit regions by roughly calculating consumption from per capita consumption, and population figures. Now find approximate price differentials by:

(i) If in reality one region ships to another, prices differ by per-unit transport cost:

(ii) If two surplus regions ship to the same deficit region, the difference in prices in the surplus regions equals the difference between per-unit transport costs to the deficit regions (starting with the base region and calculating other differentials).

Using these approximate differentials prices and flows could be calculated from steps (i) and (ii) above. Now by the dual linear programme Judge and Wallace derived a unique set of price differentials consistent with the equilibrium solution. If these new differentials differed from approximate ones they substituted the new differentials into steps (i) and (ii) and continued this process until the differentials generated in the dual of the last optimum transportation model agreed with the differentials used to determine the equilibrium prices

and flows which had been fed into that model. The solution thus found was the competitive equilibrium solution resulting from supply points disposing of their fixed production (in the short run) at a maximum price, and was arrived at simultaneously with the solution of the value and flow problem.

Judge and Wallace indicated that this model was particularly valuable because it was operational, in that it utilized many readily available data (elasticities of demand) from commodity or sector analyses, and it was computationally manageable. In discussing this model, Stout[85] has pointed out the fact that as a locator of processing (slaughter-houses in this case) transport costs are relatively unimportant and probably account for not more than 15% of the total costs of processing. More generally one could say that the model would have more appeal if supply, rather than being pre-determined, was expressed as a function of processing costs, and raw material costs especially. However as a very short-run model for perishable goods, this is very useful and particularly its handling of determination of price differentials on the demand side could well be integrated with a more complete raw material assembly, processing, and distribution model to get a short-run or long-run method of analysis for multi-regional industries.

3.7.1.2 With Emphasis on the Transportation System. The work by Goldman[86] contains an erudite discussion on the possible levels of

85. STOUT, R.G. A note on analytical models: the need for caution in assumptions and data use. J. Fm. Econ. 42,4:890. Nov., 1960.
86. GOLDMAN, T.A. Efficient transportation and industrial location. Pap. Proc. reg. Sci. Ass. 4:91-105. 1958.

abstraction of locational enquiry, concluding that some workers have ignored the transportation system as an entity in itself. His paper thus deals largely with finding a location solution consistent with the present transportation system; in particular he solves for usage of different transport types the transportation pattern and equilibrium transportation costs.

3.7.1.3 With Emphasis on Regional Supply. The spatial equilibrium analyses as outlined are capable of handling historical linear supply relations derived from annual supply data from the region. Some of the earlier works developed hypothetical supply curves by investigating economies of scale in hypothetical firms of different sizes. However, Cunningham says that these are of little use to the individual farmer as they do not tell what kind of farm should make the change within the region if profits are to be maximized.[87] He calls for more use of data from systematic farm studies (presumably finding supply response to price by linear programming), and for more study of intra-regional competition by farmers. Specific aspects would be the effect of technological innovators on surrounding farmers, and assessment of the changes of profitability from some of the institutional changes which are at present taking place, e.g. ".. in New York State there is a concentration of cows into fewer and larger units of 40 - 60 cows..."!

A method of studying supply response which fulfils Cunningham's

87. CUNNINGHAM, L.C. Interregional competition as viewed by operating firms and institutions. J. Fm. Econ. 46, 2:460-465. May, 1964.

requirements and which holds much intuitive appeal is that of Recursive Programming, developed by Day.[88] This method derives present optimum patterns from a series of linear programming models. It incorporates institutional rigidities, rates of technological adoption and uncertainty in a sequential chain of linked linear programmes, which explain the optimum adjustment path towards a long run optimum production pattern given the present state of knowledge. Thus the present supply is derived only insofar as it is consistent with a feasible and ultimately optimum supply pattern.

Conceivably, a recursive supply programme could be coupled with an activity analysis of the Judge, Havlicek, and Rizek-type, or Takayama and Judge-type[89] to give a very useful linear programming spatial equilibrium model.

3.7.2 Solution by Iterative Inspection

The method used by West and Brandow[90] in solving a dairy industry problem concentrated on satisfying price restrictions, rather than on maximizing or minimizing an objective function. Working with supply and demand elasticities derived from regression analyses, and with transport costs, West and Brandow used a map and desk calculator to iteratively move towards the equilibrium. The steps of each iteration were to

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88. DAY, R.H. An approach to production response. Agric. Econ. Res. 14, 4:134-148. Oct., 1962, and:
_____. Recursive programming and production response. North Holland, Amsterdam. 1963.
89. See Section 3.5.3, p. 48 - 52.
90. WEST, D.A. and BRANDOW, G.E. Space product equilibrium in the dairy industry in the Northeastern and North-Central regions. J. Fm. Econ. 46, 4:719-731. Nov., 1964.

establish product prices, solve the demand and supply functions for quantities consumed and produced, allocate these quantities among areas and finally to readjust product prices. Iterations were repeated until all equilibrium conditions were satisfied. They found this technique more efficient than the use of quadratic programming. This method corresponds roughly to the handling of demand by Judge and Wallace, but appear to be a retrograde step back into the non-mathematical-programming 'woods' and the manual work involved would achieve enormous proportions for complex analyses.

3.7.3 Solution by Quadratic Programming

The quadratic programming format alluded to in 3.7.2 above is that developed by Takayama and Judge.[91] They show how spatial equilibrium problems of the Enke - Samuelson variety can be handled in a single-stage analysis (cf two- or three-stage methods in 3.7.1 above) using quadratic programming.

"Assuming the existence of linear regional demand and supply relations, models are formulated and algorithms specified which may be used to obtain directly and efficiently the interregional price and flow solutions for the single and multi-product, n-regional cases."[92] This method is thus of use mainly in conjunction with demand and supply relations determined by regression from historical data.

3.7.4 Solution by Reactive Programming

Reactive programming solves the equilibrium flow problem, and by

91. TAKAYAMA, T. and JUDGE, G.G. Spatial equilibrium and quadratic programming. J. Fm. Econ. 46, 1:67-93. Feb., 1964.
92. ibid. p. 67.

back-substitution, the equilibrium prices for the Enke - Samuelson formulation. As explained by Tramel and Seale,[93] the model achieves the equilibrium by equating the marginal revenues per unit of product shipped from one region to all other regions. (Apparently the same operation could be carried out equating these marginal revenues for each region to the marginal costs indicated by regional supply curves to find a complete equilibrium.) The formulation presented assumes fixed regional supplies and expresses the demand curve in consuming region j for the product from each producing region i as:

Price in region j is some function of the total quantity supplied to region j by all producing regions i=1, 2..m.

$$P_j = F_j \left(\sum_{i=1}^{m} Q_{ij}\right). \qquad (j=1, 2....n) \qquad ....(1)$$

From the data, (m X n) equations are now formed to denote the 'net' average revenue per unit of product produced in i and sold in j.[94] These equations are of the form: Net revenue per unit produced in region i and sold in region j (R_{ij}), equals the price in region j (P_j) less the transport costs from region i to region j (T_{ij}).

$$R_{ij} = F_j \left(\sum_{i=1}^{m} Q_{ij}\right) - T_{ij}. \quad (i=1,2,...m, \; j=1,2,...n) \qquad ....(2)$$

This equation system is then solved according to the restrictions of:

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93. TRAMEL, T.E. and SEALE, A.D., Jr. Reactive programming of supply and demand relations - applications to fresh vegetables. J. Fm. Econ. 41, 5:1012-1022. Dec., 1959.
TRAMEL, T.E. Reactive programming - an algorithm for solving spatial equilibrium problems. Mississippi Agric. Expt. Stn. Tech. Pub. No. 9. State College. Nov., 1965.

94. Note that this is 'net' of transport costs, but not of raw material, or processing costs in this formulation.

(1) non-negative quantities;

(2) in any region i, all R_{ij}'s are equal for all regions to which shipments are made in the final solution, and all are greater than R_{ij}'s for all regions to which shipments are not made;

(3) all net revenues (R_{ij}'s) are positive for i to j combinations which appear in the optimum flow patterns; and

(4) the availability of product at i is not exceeded by the solution flow pattern from i. $\left(S_i \geqslant \sum_{j=1}^{n} Q_{ij} \right)$ ....(3)

The authors note that: (i) transportation costs can be included in equation (2) in a functional form;

(ii) production or cost functions can be substituted for $\sum_{i=1}^{m} Q_{ij}$ (i.e. S_i) in equation (2).

Perhaps the most important property of this method is that it explains the derivation of equilibrium prices and flows by directly taking account of the Marshallian principle of profit maximization by marketing firms equating marginal revenues and marginal costs in assembling and distributing a product to spatially separated markets, i.e. "marginal revenue pricing".

It is probably for this very reason that the reactive programming method has ".. more intuitive appeal than does linear programming."[95]

3.7.5 Solution by Dynamic Coupling[96]

The regional application of this technique has been recommended by

95. FOOTE, R.J. A comment on reactive programming. J. Fm. Econ. 41, 5:1023-1024. Dec., 1959.

96. GOODWIN, R.M. Dynamic coupling with especial reference to markets having production lags. Econometrica, July, 1947.

R.H. Day[97] who suggests that previous works cannot be extended very much without tackling two major aspects of economic structure:

(1) Inter-temporal structure or dynamics, and

(2) Inter-connectedness of production (or more generally, supply), transportation and demand.

The second point is very pertinent to price equilibrium in spatial markets.

Day formulated a structure for analysis which consists of two sub-models:

(i) An Enke - Samuelson model for determining the temporary interregional market equilibrium (prices and flows, including storage). This model uses exogenous data for final demand functions and transportation costs, and uses the actual supply available in that time period as determined in the second sub-model;

(ii) A firm decision model which uses prices received as solved for by the marketing model, to incorporate into linear programming models of regional farm types, which are then solved to find the expected output, and by including stochastic weather effects, the actual supply available for distribution by the marketing model.

From this brief outline it will be seen that Day's formulation would adequately handle Enke's demand functions and transport costs, and extends the supply side to include a production model which incorporates the complete 'production decision-making milieu' using

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97. DAY, R.H. Dynamic coupling, optimizing and regional interdependence. J. Fm. Econ. 46, 2:442-452. May, 1964.

linear programming or the even more realistic recursive programming.

3.8 SPACE-TIME AND SPACE-TIME-FORM EQUILIBRIUM

The final step in deriving complete price equilibrium models has been the incorporating of the time factor and the product-form factor, along with the spatial factor as three interacting forces in market determination of equilibrium prices.

There has been some general work of a 'comparative dynamic' nature which compared the long-run optima at two points in time. Day has called for extension of this type of thinking to include the inter-temporal structure, or the way in which the system moves towards an ever-changing equilibrium over time.[98] (Again, doubtless referring to the uses of recursive programming, but in an inter-temporal sense in this instance.) An actual model which includes the space and time dimensions has been developed by Berman.[99] His model is basically a linear programming production and shipment model which includes activities for regional expansion of production capacity, and transportation capacity (rolling stock, tractive units, and terminal capacity being treated separately) over time. This model answers Day's criticism in studying the process of economic growth of the production and the transportation sectors over time. It is designed to maximize a vector of final demand deliveries in the end period through an optimal pattern of growth - specifically, an optimal allocation of capital resources

98. ibid. pp. 442-444.
99. BERMAN, E.B. A spatial and dynamic growth model. Pap. Proc. reg. Sci. Ass. 5:143-150. 1959.

between the expansion of productive capacity, and expansion of transportation capacity. This model is very similar to a dynamic version of Goldman's[100] study of the 'impact' effect of shifts in final demand, and/or capacities.

Some of the models discussed in previous sections have included the form dimension in spatial analyses, notably the multi-product models of Polopolus, Hurt and Tramel, and Judge, Havlicek and Rizek.[101] However, the most general and all-embracing of model formulations was that developed by Hassler.[102] He showed mathematical frameworks for models of increasing complexity from the single product, spatial model; to the single product, space-time model; and then to the multiproduct space-time model, or the space-time-form model. He said that such model progression could be extended to finally formulate a multi-raw material, multiproduct space-time-form model with a sector to explain the supply function of raw materials, and the inclusion of the risk and uncertainty factors at the farm and marketing levels. If such a model was developed; if the data (including demand functions) were available; and if the whole was computationally feasible, then location theory would indeed have achieved what Harberler doubted possible:[103] analysis of a general equilibrium system of location, flows and prices.

3.9 SUMMARY OF THE LITERATURE ON LOCATION ECONOMICS

(1) Location economics could be described as regional microeconomics.

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100. GOLDMAN, T.A. op. cit.
101. See Sections 3.4.3, p.35, 3.5.2, p.45, and 3.5.3, pp. 48 - 52.
102. HASSLER, J.B. Interregional competition in agriculture: principle forces, normative models and reality. J. Fm. Econ. 41, 5:959-968.
103. See Section 3.1, pp. 17 - 18.

It allows that the transport costs of raw materials and final products as well as physical aspects may affect the comparative advantages of different places (or regions) as sites for the optimum location of the firm.

(2) The foundation workers in the field, von Thunen, Weber, Losch and Hoover have shown logically that the production, and processing costs and the transport cost to the final markets all exert a specific influence on the optimum location of economic activity.

(3) They have further shown that some industries are market-orientated and others raw material-orientated, depending upon the weight-gain or weight-loss in the raw material processing. The actual optimum location will depend upon labour costs and transport costs at different processing points.

(4) Analyses based upon this theory have proceeded to increasingly complex levels. The early formulation of the problem has been maintained by some workers doing simple activity analyses using algebraic formulae for solution; more complex activity analyses have been done using mathematical programming algorithms to solve them; and finally attempts have been made to fully describe the attainment of spatial price equilibrium using positive and normative analyses.

(5) The simpler activity analyses studied the effects of two of the marketing functions only - assembly and processing, or processing and distribution. They assumed the pattern of raw material production and of consumption was given. The mathematical functions describing processing and transport varied. Olson used the integral calculus to

find total transport cost, and used a synthetic Chamberlin's curve to describe processing scale effects. Stollsteimer in his model used actual transport costs and a set of linear production functions for the processing costs.

(6) Mathematical programming techniques have enabled workers to solve problems which contain a number of inter-related, constrained activities. The simple Koopmans-Hitchcock Transportation Model solved purely for the minimum cost transport pattern, but the transhipment extension to this model enabled King and Logan to include processing costs at each point - given quantity supplied and quantity demanded by each region though. Using more general Linear Programming, Judge et. al. specified a model in which supply of raw material is not necessarily given, but can be solved for within the model, according to costs. Thus programming allowed activity analyses to solve for most of the flows of goods and costs - given the final demand pattern.

(7) Recently, a number of economists have attempted to describe the attainment of spatial price equilibrium. A wide range of programming tools have been used in a search for ever-better descriptions. These tools include:

(a) A linear programme included in an iterative 'Cobweb'-like procedure by Judge and Wallace,

(b) Quadratic programming working from demand and supply functions (Takayama and Judge).

(c) Reactive programming which uses a method of equating marginal revenues between regions (Tramel and Seale).

(d) Dynamic Coupling and Recursive Programming which R.H. Day suggested would allow for the dynamic and inter-temporal nature of attaining price equilibria.

(8) The suggestion by Day has been further investigated by Hassler and it is likely that the location models will be extended to analyse space-time and space-time-form problems in an attempt to describe general price equilibrium of a 'Walrasian' type.

(9) In general, there have been developed a large number of mathematical models which specify a wide range of location problems. However, these models have not always been tested under actual conditions, and it remains to be seen which of them will be operational for the many people making important location decisions. From a practical point of view - from the individual policy-maker's viewpoint - some of the activity analysis models appear very good and could be very useful in making future location decisions.

CHAPTER 4

LOCATION ANALYSIS APPLIED TO THE TOMATO PROCESSING INDUSTRY

This section of the thesis will be concerned with selecting forms of analysis from existing literature and developing special models to analyse the economics of tomato processing plant location in New Zealand. The approach to the economic analysis follows on from the technical and institutional description in Chapter 2. To reiterate, the economic problems this section will explore are:

(1) What would be the optimum locational pattern of industry?

(2) In what ways does the present pattern deviate from this optimum? and

(3) What is the most efficient path of development from the present to the optimal location pattern?

A form of analysis will now be selected and a model specified which initially will logically analyse the first two questions.

4.1 THE FORM OF ANALYSIS

The first and main question above is concerned with finding what the organization of the industry <u>should</u> be, and this implies the use of

normative analysis. Normative models determine what should have resulted from a set of specified initial conditions, i.e. institutional organization and available physical techniques, if the objective of a system is known. They determine the efficiency of the actual system insofar as it succeeds in attaining this objective, whereas the positive - or 'descriptive-predictive' - class of models are used to predict future values for structural variables, if the structure remains unchanged. In comparing these classes of models, Hassler[1] has said that: "Efficiency model analyses are on the higher level of significance - they are diagnostic and active, instead of passive a appraisers of economic systems." In essence, these normative analyses do not take the present, possibly inefficient structure of the system as given, but in fact attempt to describe the optimum structure, and optimal structural changes over space, time and form dimensions, if the system is to attain its objective.

Ideally, when considering the industry as an entity, the rigidities of institutional organization and physical techniques of tomato production, assembly, processing, distribution and consumption should be included in a spatial equilibrium model using normative supply functions (involving linear programmes at the farm level), and demand functions. Thus the equilibrium prices and rents, locations and capacities could be derived under conditions of perfect competition. Such a model would be beyond the scope of a research project of this nature, and in selecting the simpler activity analysis approach, one is guided by

1. .HASSLER, J.B. op. cit. p. 960.

similar thoughts to those expressed by Judge, that: "There are too many research grants of $50,000 to solve $100 problems."[2]

The objective of the industry (as distinct from the national objective of maximizing utility) should be to maximize overall profit. In this analysis regional quantities demanded and wholesale consumer prices are assumed fixed. Thus the total revenue of the industry will be fixed and so the industry will maximize profits by minimizing costs. The economic objective of the industry would be to minimize the total cost of raw production, assembly, processing and final product distribution. The problem of efficiently attaining this minimum cost objective could be studied by extending the two-stage activity analysis models of Stollsteimer, and Hurt and Tramel[3] toward the industry spatial equilibrium model mentioned above. Within the framework of the Hurt and Tramel modified transhipment formulation however, regional supplies of raw products must be specified, and not developed within the model in accordance with attaining overall cost minimization. The Stollsteimer technique could study such a problem only if production, assembly and processing costs in each region were combined into a mixed regional function, describing economies of scale of production, assembly and processing. The model would then be unable to handle shipment of raw tomatoes between producing regions. An even more important basic objection is that, as presented by Stollsteimer, the technique is incapable of handling limits on regional capacities of production or

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2. JUDGE, G.G. Discussion: impact of electronic computing on farm economic research. J. Fm. Econ. 40, 5:1236. Dec., 1958.
3. See Sections 3.4.3, pp. 35 - 39, and 3.5.2, pp. 45-48 above.

processing. Such capacity limitations can only be handled by mathematical programming methods.

Therefore, in order to incorporate separate production costs, assembly costs, processing costs and distribution costs as locators, allowing that maximum capacities may be imposed, but not restricting each plant to a fixed capacity or output, a generalized linear programming model was developed. This model is basically very similar to that developed independently by Judge, Havlicek and Rizek, and published subsequently.[4] It includes more facets than their model and in fact verifies the flexibility claimed for this type of model by these authors.

4.2 SPECIFICATION OF THE MODEL

Discussing a different analytical problem, the econometrician, Klein has suggested that results can be improved more by paying more attention to *a priori* assumptions and data than by designing more complex analytical tools.[5] He said that the calculating time saved by the use of computers could well be spent in specifying the models better than is done at present.

There is a corollary. It is often said that many methodologists 'use a sledgehammer to crack a nut'. Where there is an automated sledgehammer readily available it may be preferable to use it rather than a manual nutcracker, because it saves time. To extend the metaphor even further, the time saved in the cracking process may be used to find a

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4. JUDGE, G.G., HAVLICEK, J. and RIZEK, R.L. *op. cit.*
5. KLEIN, L.R. Single equation vs. equation system methods of estimation in econometrics. *Econometrica*. 28, 4:866-871. Oct., 1960.

much larger nut which cannot be cracked manually anyway.

In terms of the present problem, the author recognises that it could be solved by a simple method of iterative inspection of cost matrices. However the same solution can be obtained much more rapidly using the available linear programming computer programme. The time saved in computing rather than inspection and manual calculation, may be used to specify further constraints on the problem, and the problem will then need the linear programme to solve it.

4.2.1 The Basic Model

The linear programming model developed has individual activities to describe production of the raw material from each producing to each processing region. It also has activities corresponding to each processing plant, and for transport of the final product from each processing to each consuming region.

The model assumes that the industry, faced with fixed gross revenue will aim to maximize profits by minimizing costs. In fact the objective of the model is to find an optimum solution in which the total of all the individual costs is a minimum.

The regions, activities and costs are now defined and the mathematical formulation outlined.

Given:

i is a possible raw tomato producing region $(i = 1,2...m)$

j is a possible raw tomato processing region $(j = 1,2...n)$

k is a region demanding processed tomatoes $(k = 1,2...q)$

c_i = cost/ton of producing raw tomatoes on region i.

x_{ij} = cost/ton of transporting raw tomatoes from region i to region j.

p_j = cost/ton of processing raw tomatoes on region j.

t_{jk} = cost/ton of transporting processed tomato products from region j to region k, (expressed in cost/ton of raw tomato content).

D_k = consumption of processed tomato products in region k, (expressed in tons of raw tomato content).

Required to find:

C_i = tons of raw tomatoes produced in region i.

X_{ij} = tons of raw tomatoes transported from region i to region j.

P_j = tons of raw tomatoes processed in region j.

T_{jk} = tons of processed tomato products transported from region j to region k (expressed in tons of raw tomato content).

The solution values of C_i, X_{ij}, P_j and T_{jk} must describe that location pattern which will minimize:

Z = the total cost of producing, transporting and processing raw tomatoes in, and distributing processed tomato products between all regions, such that the consumption pattern is satisfied.

Mathematical Formulation:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} C_i c_i + \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} x_{ij} + \sum_{j=1}^{n} P_j p_j + \sum_{j=1}^{n} \sum_{k=1}^{q} T_{jk} t_{jk}$$

$$\text{Subject to} \quad C_i \geq \sum_{j=1}^{n} X_{ij} \quad \text{for all } i \qquad \ldots.(1)$$

$$\sum_{i=1}^{m} X_{ij} \geq P_j \quad \text{for all } j \qquad \ldots.(2)$$

$$P_j \geqslant \sum_{k=1}^{q} T_{jk} \quad \text{for all } j \qquad \ldots.(3)$$

$$\sum_{j=1}^{n} T_{jk} \geqslant D_k \quad \text{for all } k \qquad \ldots.(4)$$

And to C_i, X_{ij}, P_j and $T_{jk} \geqslant 0$. $\ldots.(5)$

These constraints ensure that:

(1) The raw production in region i is equal to (or greater than) the tomatoes transported from that region.

(2) The tomatoes transported from all producing regions (including producing region j) to region j is equal to (or greater than) the amount processed in region j.

(3) The amount of processed tomato products processed in region j is equal to (or greater than) the amount transported to all consuming regions.

(4) The processed tomato products transported from all processing regions to region k is equal to (or greater than) the amount consumed in region k, and

(5) There are no negative flows or negative amounts produced or processed.

Solution is by the Simplex method, using normal profit maximizing routines, but expressing costs as negative net returns, and maximizing a positive objective function, Z. (i.e. minimizing a negative Z)

4.2.2 The Simplex Formulation

The system of inequalities described above is readily handled by the cost-minimizing simplex routines for solution of the primal linear

programming problem. The pivotal part of the programme is the fulfilling of regional demand from a chain of production and transport processes.

The quantity consumed is required to be just equal to the amount of final product transported to the consuming region, and so demand should not go into 'disposal'. Thus artificial activities corresponding to each consumption equation are included. The artificial activities can be forced out of the solution and the consumption ones come in, using the 'm technique' of giving each artificial activity a very high cost (m) and each consuming activity a very low cost (-m).[6]

The level at which each activity comes in is indicated by the subscripted capital letters and the cost per unit of the activity (c_j), the corresponding subscripted small letter.

In Figure 2, the disposal activities have been omitted for clarity. Thus the constraints are still in the form of inequalities rather than equations. The first set of constraints ensures that the quantity of raw tomatoes produced in each region is just equal to the total quantities transported to all processing regions. From the table we can write the first constraint as:

$$0 \geqslant -1C_1 + 1X_{11} + X_{12}$$

or

$$C_1 \geqslant X_{11} + X_{12}.$$

This does ensure that raw production is at least equal to total outshipments from region 1.

The second set of constraints ensures that the quantity of raw

6. HEADY, E.O. and CANDLER, W. op. cit. pp. 131-136.

Fig. 2. Hypothetical Simplex Tableau for Basic Model
(2 producing, 2 processing and 3 consuming regions)

	Activity	Raw Prod'n.		Raw Product Transport				Processing		Final Product Transport						Artificial		
	Level	C_1	C_2	X_{11}	X_{12}	X_{21}	X_{22}	P_1	P_2	T_{11}	T_{12}	T_{13}	T_{21}	T_{22}	T_{23}	Q_1	Q_2	Q_3
	c_j	c_1	c_2	x_{11}	x_{12}	x_{21}	x_{22}	p_1	p_2	t_{11}	t_{12}	t_{13}	t_{21}	t_{22}	t_{23}	m	m	m
Constraint	c_i b_i																	
(1)	(0 0	-1		+1	+1													
	(0 0		-1			+1	+1											
(2)	(0 0			-1		-1		+1										
	(0 0				-1		-1		+1									
(3)	(0 0							-1		+1	+1	+1						
	(0 0								-1				+1	+1	+1			
(4)	(-m D_1									+1			+1			-1		
	(-m D_2										+1			+1			-1	
	(-m D_3											+1			+1			-1

Note: All quantities and prices relate to one unit of raw material. Thus final product transport costs and consumption quantities must be converted to 'raw material equivalents'.

tomatoes shipped into each processing region is at least equal to the amount processed in that region. (In the final solution the raw tomatoes produced will never be greater than the amount required for processing, because the linear programming algorithm is being used here to minimize costs. If surplus tomatoes were to be produced, costs would not be a minimum, and so in the final solution these constraints should all be equalities with no raw production in disposal.) The constraint for processing region 1 (the third row down) can be written:

$$0 \geqslant -1X_{11} - 1X_{21} + 1P_1$$

or

$$X_{11} + X_{21} \geqslant P_1.$$

This does ensure that the total quantity of tomatoes transported to region 1 is at least as great as the quantity processed in region 1.

The third set of constraints ensures that the amount processed in each region is just equal to the amount of final product transported from that region. Again for processing region 1,

$$0 \geqslant -1P_1 + 1T_{11} + 1T_{12} + 1T_{13}$$

or

$$P_1 \geqslant T_{11} + T_{12} + T_{13}.$$

Finally the fourth constraint ensures that the quantity of final product transported to each region is at least equal to the quantity consumed in that region. Leaving out the artificial activities and returning to the inequality form, the constraint for consuming region 1 reads: $D_1 \leqslant T_{11} + T_{21}$.

In fact one can picture the model as a causal chain moving from consumer back to producer. In this chain each link requires at least enough from the last link to finally fulfil total demand:

Consumption $\leqslant$ Final transport $\leqslant$ Processing $\leqslant$ Raw transport $\leqslant$ Raw production.

This formulation, with each function (production, processing and transportation) entered as a separate activity, enables extensive and rapid exploration of the stability of the solution to cost changes or capacity constraints imposed on any of these activities. (No capacity constraints are included in this 'long-run normative' examples of the formulation.)

4.2.3 Economies of Scale

If an economies of scale curve was available for any of the activities (regional production, processing or transportation) then economies could be handled by the iterative method used by King and Logan[7] and mentioned in Chapter 3. Economies of scale curves are extremely difficult to construct, especially for multiproduct plants.[8]

The matrix of this model is symmetrical and consequently computationally very efficient. Once the first solution is achieved, the solution levels of e.g. each processing activity could be read off, and the actual cost per unit of operating at these levels obtained from the economies-of-scale curves for the respective plants. These new costs per unit (p_j's) could be typed into the computer, which is capable

7. KING, G.A. and LOGAN, S.H. op. cit.
8. REED, R.H. and BOLES, J.N. Non-linear programming of field and plant vegetable processing activities. Agric. Econ. Res. 15, 3:89-93. July, 1963.

of very rapid re-solution where such changes are made. The total time for each iteration would be less than ten minutes even on the relatively slow IBM 1620. The model outlined is well suited to such iterative treatment of economies of scale because it includes each phase in the marketing chain as a separate activity.

4.3 APPLICATION OF THE MODEL

Having developed the basic formulation of the model, some modifications to the classification of activities are necessary in order to conform to the institutional framework of the tomato processing industry. Also, a method must be developed to reduce all measurements to one dimension - tons of raw tomatoes - bearing in mind that it is a multi-product industry. Finally the regions must be defined in order to arrive at a workable formulation.

4.3.1 Raw Material Activities

The basic model is capable of handling many raw materials simultaneously, but this study was concerned specifically with the effect of regional advantages in raw tomato production, and so other raw materials were not treated separately. Where other raw materials were not equally available at the same cost in all processing regions, then these cost differentials would be included as differentials in the overall processing cost per unit in each processing region, in this model.

4.3.2 Transport of Raw Tomatoes

The cost of transporting raw tomatoes within a region was included

in the raw tomato production costs, because the factory contract price to growers is 'per ton of satisfactory fruit delivered to the factory.' At present there is no transporting of raw tomatoes for processing from one producing region to another processing region and so the corresponding activities have been omitted from this model. This facet could be re-introduced if computationally feasible within the limited computer capacity, and if inspection of the final solution matrix showed that such re-introduction was justified.

4.3.3 Incorporation of Multi-product Processing

The formulation above implicitly assumes that the cost of processing the raw material in any one processing region is constant, irrespective of the region to which the final product is shipped. Thus for each processing region there is a single processing activity with a cost per unit (P_j) associated with it. However raw tomatoes are processed to form many final products:- spaghetti, baked beans, tomato soup, tomato sauce and canned whole tomatoes mainly. The final product-mix demanded by each consuming region differs, and the cost of processing one ton of raw tomatoes into each final product differs. Thus the cost of processing one ton of raw tomatoes in any given processing region, for shipment to a consuming region, will vary according to the product-mix demanded by that consuming region.

This could be accommodated in the model by including separate activities for each product in the processing, final product transport stages and in the demand constraints. The matrix size would then be increased beyond the available computer capacity, and so an alternative

was developed. For each processing region, activities were inserted to represent processing of raw tomatoes into the product-mix demanded by each consuming region in turn. The unit cost associated with each processing activity was specific for each processing and each consuming region, and was designated p_{jk} for products processed in region j, for sale in region k.

4.3.4 Definition of Regions

4.3.4.1 Raw Tomato Producing Regions ($i = 1...m$). The regions included were those regions actually producing tomatoes in 1964/65, except that the Central Auckland region was excluded because urban expansion of Auckland city has already eliminated most producers in this region. Possible producing regions included are thus:

South Auckland (Pukekohe)	i = 1
Gisborne	i = 2
Hawkes Bay	i = 3
and Nelson - Motueka	i = 4

4.3.4.2 Processing Regions ($j = 1...n$). These correspond to the producing regions and are:

Auckland	j = 1
Gisborne	j = 2
Hastings	j = 3
and Nelson	j = 4

4.3.4.3 Consuming Regions ($k = 1...q$). The regional classification

of the country which was adopted uses the 'Statistical Areas' as defined by the New Zealand Department of Statistics. Consumption was assumed to take place at the point of wholesale, and in calculating transportation costs it was assumed that consumption in each of these areas took place at the wholesaling store in the largest centre in each area. In most statistical areas there is one dominant consuming centre but even where there are other fairly large centres with wholesaling facilities, the intra-regional distribution costs would be small when compared with the inter-regional costs of transporting from processing to consuming regions.

Because of the small quantity consumed in Westland, Christchurch was assumed to serve the Canterbury and Westland statistical areas.

Classification of the consuming regions and their respective distributing centres were thus:

Statistical Area	Distribution Centre	Classification
Northland	Whangarei	k = 1
Central Auckland	Auckland	k = 2
South Auckland - Bay of Plenty	Hamilton	k = 3
East Coast	Gisborne	k = 4
Hawkes Bay	Hastings	k = 5
Taranaki	New Plymouth	k = 6
Wellington	Wellington	k = 7
Marlborough	Blenheim	k = 8
Nelson	Nelson	k = 9
Canterbury and Westland	Christchurch	k = 10
Otago	Dunedin	k = 11
Southland	Invercargill	k = 12

4.3.5 The Modified Formulation

The activities in the model are now:

(a) raw tomato production in four regions,

(b) four regions processing tomatoes into final product-mixes consumed in twelve regions,

(c) final product transport from four processing regions to twelve consuming regions.

The objective is to minimize the total cost of carrying out these activities.

$$\text{Minimize: } Z = \sum_{i=1}^{4} C_i \cdot c_i + \sum_{j=1}^{4} \sum_{k=1}^{12} P_{jk} \cdot p_{jk} + \sum_{j=1}^{4} \sum_{k=1}^{12} T_{jk} \cdot t_{jk}$$

However the total consumption must be fulfilled, and this is achieved by ensuring that the total final product shipped from each processing region to the kth consuming region is at least equal to consumption in that region,

$$D_k \leqslant \sum_{j=1}^{4} T_{jk} \quad (k = 1,2 \ldots 12) \ ;$$

that the amount processed in region j for the consumption in region k is greater than the final product transported from j to k,

$$T_{jk} \leqslant P_{jk} \quad (j = 1,2 \ldots 4) \quad (k = 1,2 \ldots 12) \ ;$$

and finally that the amount produced in each producing region is at least as great as the amount processed in the corresponding processing region for consignment to all consuming regions,

$$\sum_{k=1}^{12} P_{jk} \leqslant C_i \quad (i = j = 1 \ldots 4) \ .$$

Also there can be no negative production levels, or flows.

C_i, P_{jk} and $T_{jk} \geqslant 0$ for all i, j, k.

4.4 DERIVATION OF THE DATA

The generalized linear programming model outlined above is capable of producing a solution from relatively simple data, or may be used in conjunction with detailed data for any one or all of the stages along the marketing chain. In an exploratory study of this nature, where resources for data collection are limited, the quality and complexity of the data has been determined by the type of data already collected and which has been made available from different sources.

The viewpoint adopted above is that of maximizing efficiency, i.e. minimizing costs for the whole industry. Thus the costs required at each stage were the average costs of carrying out the operation with no monopoly profits or institutional price rigidities included. These costs were assumed to be single-valued with respect to the scale of operation in each region.

Some of the data collected was expressed in tons of raw tomatoes, and some in tons of final product. Further, it was sometimes necessary to break down measurements into tons of each individual final product. A system of subscripting and superscripting was adopted and is now explained using the consumption symbol D_k as an example.

The subscripts i, j, and k are retained to describe the producing, processing and consuming regions. As will be seen later the

consuming regions may be grouped into main areas I, II and III and these are used as subscripts where applicable.

The superscripts F and R are used to distinguish between data which applies to a ton of packed final product, or to a ton of raw tomatoes respectively.

Thus D_k^F is consumption in region k expressed as packed final product,

D_k^R is consumption in region k expressed as raw tomatoes.

Finally, the superscripts x and X are used to express data in terms of individual products (x = 1, 2, 3, 4, 5 for spaghetti, baked beans, tomato soup, tomato sauce and canned tomatoes respectively), or as a total of all products (X),

i.e. $D_k^{XF} = \sum_{x=1}^{5} D_k^{xF}$, or $D_k^{XF} = D_k^F$.

4.4.1 Regional Costs of Raw Tomato Production (c_i)

The regional average costs of raw tomato production were collected by means of a field survey, and have been fully described in a previous publication.[9] Briefly, the method of data collection used was similar to other such surveys.[10] For costing purposes the direct costs were determined in detail for tomatoes, while the overhead costs were taken as a fraction of the total farm overhead costs. Overheads were allocated

9. SANDERSON, K.T. op. cit.

10. WESNEY, D.A. Study of the financial returns to process pea growers in Hawkes Bay. Unpublished M. Agr. Sci. thesis. Massey College, 1964.
SUTER, R.C. and D.E. WILSON. Tomatoes - costs and returns. Agric. Expt. Stn., Lafayette, Indiana. Res. Bull. 750. Purdue Univ. Oct., 1962.
NELSON, J.B. Tomatoes for processing: production costs. Ontario Farm Economic Studies. Ontario Dpt. Agric., Toronto. 1954.

to the tomato growing enterprise according to the fraction of total gross revenue earned by the tomatoes. A detailed explanation of costing procedures is included in the survey report.[11]

Apart from the obvious assumption implied in any sampling survey - that the sample is representative - a further assumption was included to remove an institutional rigidity from the survey data.

Yields recorded for Nelson in the survey report were lower than the potential supply in that season because of the operation of tonnage contracts with consequent waste of surplus production. Acreage contracts are now in operation in Nelson, and under these conditions a conservative estimate of what could have been harvested in 1964/65 is 18.4 tons per acre, compared with the 15.2 tons per acre which was actually accepted by the factory.[12] The figure of 18.4 tons per acre was used in this model. A re-calculation of the total production cost/acre for panel growers in Nelson was made by adding the total growing cost/acre (£139.1) to the cost of harvesting 18.4 tons i.e. 18.4 x £4.39 = £80.8. The total production cost/acre now equals £219.9 for 18.4 tons. Figures for other regions were taken directly from the survey report,[13] and the calculation of the average production cost/ton is now shown in Table 4.1.

Table 4.1 Regional Raw Tomato Production Costs 1964/65

Region	South Auckland	Gisborne	Hastings	Nelson
i =	1	2	3	4
Average production cost/ acre (£)	239.10	174.40	201.70	219.90
Average yield (tons/acre)	13.00	14.03	17.29	18.40
Average cost/ton (£)	18.42	12.43	11.67	11.95
(c_i)	(c_1)	(c_2)	(c_3)	(c_4)

11, 12, 13. See overleaf.

4.4.2 Average Costs of Processing (p_j^{xF})

The costs required here are the processing costs per ton of each final product, (x), for each processing region, (j). These individual product costs were not available, nor were the regional cost differentials, or costs/ton at different output levels.[14] The processing costs used were derived in cost/ton of processed product from the figures for the 'Fruit and Vegetable Processing Industry' published by the New Zealand Department of Statistics.[15] In fact, the figure derived was the cost/ton of processing all canned and bottled products (expressed /ton of finished product).

Assumptions:

(1) The labour costs and overheads are equal per ton of all processed fruit and vegetable products.

(2) Material costs (apart from raw fruit) include 'sugar', 'meat', and 'other materials'. It was assumed that 'meat' and 'sugar' were used in the production of other main products, e.g. canned sausages and jam. The costs of 'other materials' were thus spread over the total output of the industry.

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11. SANDERSON, K.T. op. cit. pp. 13 - 15.
12. ibid. p. 51. Para. (2).
13. ibid. Table 4.7 Summary of direct costs by operations. p. 30.
14. This non-availability is partly due to the multiproduct nature of the two concerns involved (Unilever (N.Z.) Ltd, and J. Wattie Canneries Ltd.) and attendant problems of allocation of overheads among products. Those figures which the companies do have are unavailable for publication because of commercial considerations, i.e. grower-processor relations and processor competition. Individual companies would be capable of extracting more exact figures for their own specific costs.
15. New Zealand Industrial Production Statistics. 1963/64. New Zealand Department of Statistics, Wellington. 1965.

(3) Tinplate, can, and bottle costs were spread over all canned and bottled products.

(4) Other packing materials would include cartons etc., and these would be used in greater quantity for non-canned and bottled products, e.g. frozen foods. The assumption was made that the total packing of one ton of non-canned and bottled products would be twice as costly as the provision of labels, cartons, etc., for canned products, thus the total costs of 'other packing materials' was divided by the total tonnage of canned and bottled products, plus twice the tonnage of other products.

Average processing cost/ton of finished canned and bottled products for all New Zealand Fruit and Vegetable Processors 1963/64

1.	Total wage and salary payments = £1,916,000	
	Other expenses of manufacture[16] = £1,717,000	
	Labour and other expenses, total cost	= £3,633,000
	Tonnage of all fruit and vegetable products[17]	= 61,893.2 tons
	Labour and other expenses, cost/ton	= £58.698/ton
2.	Other materials, total cost	= £577,000
	Tonnage of all fruit and vegetable products	= 61,893.2 tons
	Other materials cost	= £9.323/ton
3.	Total cost of tinplate, cans bottles, etc.	= £1,788,000
	Tons of canned and bottled products.[17]	= 48,262.3
	Tinplate, cans, bottles, etc. cost	= £37.048/ton
4.	Total cost of other packing materials	= £846,000
	Tons of canned and bottled products	= 48,262.3 ...(a)
	Tons of other products	= 13,630.9 ...(b)
	(a) + (2x (b))	= 75,524.1
	Other packing materials, cost/ton	= £11.202/ton
	Processing cost/ton of finished canned and bottled products	= £116.271/ton

16, 17, See overleaf.

In the absence of more specific data for individual regions and individual products, this figure was used as the cost/ton of packed final product, for processing each of the final tomato products in each of the regions.

Thus, $p_j^{xF} = 116.271$ for all products (x) and for all regions (j). As this figure holds for all x = 1...5, it holds also for X. It can thus be written as p_j^{XF}, of more simply $p_j^F = 116.271$.

4.4.3 Cost of Final Product Transport (t_{jk}^{XF} or t_{jk}^F)

The transport costs were determined from f.o.r. or f.o.b. at the point of processing to f.o.r. or f.o.b. at the distributing centre. The costs were all expressed per ton of packed final product. Sources of data are stated, and where applicable the special local rate for the railways was used. Some estimation was required to convert shipping figures (which apply to 40 cubic feet) to costs per ton. To do this, the shipping company figures were compared with processing company figures for the Gisborne - Christchurch haul. In terms of costs, it was found that one ton of product could be assumed to occupy 1.41 x 40 cubic feet, and so other shipping costs per 40 cubic feet were multiplied by 1.41 in order to arrive at the cost/ton. Shipping freights included wharfage dues.

Where there were alternative modes of transport, the lowest cost mode was selected. In many cases data were obtained from more than one

16. Includes: coal, electricity, coke, gas, oils, etc., insurance, interest on loans, etc., depreciation, rent, repairs and maintenance, other.
17. See Appendix I, Table 1 for breakdown of output.

source and so cross-checking was possible. The final transport cost matrix is shown in Table 4.2. Note that these figures are per ton of packed final product, i.e. are t^F_{jk} for all j and all k.

Table 4.2 Transport Costs of Processed Products (£/ton of packed, final product)

Distribution Centre	j = k	Processing Region: Auckland 1 £/ton	Gisborne 2 £/ton	Hastings 3 £/ton	Nelson 4 £/ton
Whangarei	1	5.18[r]	13.69[r]	11.28[r]	17.50[r]
Auckland	2	0.00	7.31[r]	8.70[w]	7.80[w]
Hamilton	3	4.15[r]	7.00[r]	7.36[w]	11.95*
Gisborne	4	8.80[r]	0.00	6.10[r]	14.45*
Hastings	5	9.00[r]	6.10[r]	0.00	12.80*
New Plymouth	6	7.90[r]	9.27[r]	7.55[r]	13.80[r]
Wellington	7	8.50[r]	8.40[w]	5.35[w]	6.40[w]
Blenheim	8	13.07[r]	12.97[r]	9.92[r]	3.90[r]
Nelson	9	14.55*	14.45[w]	12.80[w]	0.00
Christchurch	10	11.68[u]	11.31[w]	11.31[w]	7.00[r]
Dunedin	11	11.68[u]	11.35[w]	11.25[w]	7.60[r]
Invercargill	12	12.72[u]	12.47[u]	13.37[u]	12.20[r]

Sources of Data:

r: New Zealand Railways Department, Christchurch.

w: J. Wattie Canneries, Ltd., Hastings.

u: Union Steam Ship Company (N.Z.)Ltd.Christchurch.

Adjusted as outlined above.

*: Informed estimates.

4.4.4 Consumption of Tomato Products (D_k^{xF})
(Expressed in tons of final products.)

Dispositions of processed tomato products were not available from the processing companies, large chain-stores or the Statistics Department. A private marketing research organization (A.C. Nielsen Pty. Ltd.) was able to supply figures for the consumption of individual products for New Zealand, broken into three main areas, over a period of four years from 1963 to 1966.

The three areas were:

Area 1 containing consuming regions 1, 2, 3, 4.
Area 11 " " " 5, 6, 7.
Area 111 " " " 8, 9, 10, 11, 12.

The data supplied represented sales to large retail outlets and excluded institutional sales i.e. restaurants, hospitals, hotels, etc., and excluded those retail groceries which do less than £5,000 per annum in pure grocery. They also omitted the consumption of canned whole tomatoes. See Appendix I Table 2 for actual data obtained, expressed in tons.

4.4.4.1 Correction of data to include all outlets. As the model analysed the total distribution and consumption of processed tomato products, the Nielsen figures were corrected to allow for consumption through all outlets. This was done by comparison of Nielsen figures with total production figures. To make this correction it was assumed that:

(1) the tomato products distributed through institutions and small

grocers had the same regional disposition as those sold through large retail outlets and recorded in the Nielsen figures.

(2) over a four-year period, the total amount of each product produced in New Zealand just equalled the total amount consumed in New Zealand over that period. This implies that exports are negligible (which is quite accurate[18]) and that stocks of goods held before and after are equal.

The first assumption is not very damaging, because the non-retail disposition is quite a small fraction of the total, and also one would expect the incidence of restaurants, etc. to be approximately proportional to the population in each main area.

The level of stocks could vary quite widely and this variation could be quite a high proportion of the total production in a 'good' or 'bad' growing season; however by spreading this change over four years' production, it would become quite a small fraction of the whole.

Production figures for the four years in question were obtained from New Zealand Department of Statistics' publications and are shown in Appendix I, Table 3.

The fraction of the total consumption of each product in 1964/65 which was recorded in Nielsen's 1964/65 retail uptake figure is thus assumed to be equal to the fraction of the total of four years production of each commodity which was included in Nielsen's retail uptake figures over the same four years. Using the data from Appendix I, Tables 2 and 3, these fractions are derived in Table 4.3, and their

18. See SANDERSON, K.T. op. cit. p. 9.

reciprocals can be used as correction factors to adjust the Nielsen figures to include all outlets.

Table 4.3 Consumption Correction Factors

Product	4 Year Total		Uptake/Production	Correction Factor
	Retail uptake (1)	Production (2)	(3) = (1)/(2)	(4) = (1)/(3)
Canned Spaghetti	15,024	16,851	0.892	1.120
Baked Beans	12,685	14,364	0.883	1.132
Tomato Soup	7,941	9,858	0.806	1.241
Tomato Sauce	8,899	12,808	0.695	1.439

This correction is made to data for retail uptake in 1964/65 from Appendix I, Table 2, and the corrected data appear on Table 4.4. The table shows the calculations separately for each of the three main consuming areas.

4.4.4.2 Calculation of canned tomatoes consumption data. As noted above there is no consumption data available for canned, whole tomatoes, and so the total four years' production (6,842 tons) was spread over the years in the same ratios as the retail uptake of tomato soup.[19] The fraction of the four years' soup consumed in 1964/65 was 2044.5/7941.4,

19. Canned tomato production has been very erratic and it would be unrealistic to assume that consumption has followed a similar erratic pattern.
Tomato soup has similar consumption characteristics to canned tomatoes - a low total consumption, luxury product with rapidly increasing consumption.

and so the consumption of canned tomatoes in 1964/65 was

(2044.5/7941.4 x 6842) = 1762 tons.

This total consumption was regionally distributed, again assuming this to be in the same ratios as tomato soup in 1964/65. Referring again to Appendix I, Table 2, the fraction consumed in Area I was 867.8/2004.5.

This consumption of canned tomatoes in Area I

= (867.8/2044.5 x 1762) = 747.0 tons.

Similarly consumption in Area II = (596.4/2044.5) x 1762) = 515.0 tons

And in Area III = (580.3/2044.5 x 1762) = 500.0 tons

These data are used to complete Table 4.4 below.

Table 4.4 Consumption of Processed Tomato Products for 1964/65 (Expressed in tons of packed final product)

Product	Consuming Area I	II	III
Canned Spaghetti:			
Retail uptake 1964/65 (tons)	1964.6	995.0	910.7
Correction factor	1.122	1.122	1.122
Total consumption $D^{1F}_{I,II,III}$ (tons)	2204.3	1116.4	1021.7
Baked Beans:			
Retail uptake 1964/65 (tons)	1629.4	791.0	804.0
Correction factor	1.132	1.132	1.132
Total consumption $D^{2F}_{I,II,III}$ (tons)	1844.4	895.4	910.1
Tomato Soup:			
Retail uptake 1964/65 (tons)	867.8	596.4	580.3
Correction factor	1.241	1.241	1.241
Total consumption $D^{3F}_{I,II,III}$ (tons)	1076.9	740.7	720.2
Tomato Sauce:			
Retail uptake 1964/65 (tons)	1136.6	626.3	646.2
Correction factor	1.439	1.439	1.439
Total consumption $D^{4F}_{I,II,III}$ (tons)	1635.4	901.2	929.7
Canned Tomatoes:			
Retail uptake 1964/65 (tons)	n.a.	n.a.	n.a.
* Total consumption $D^{5F}_{I,II,III}$ (tons)	747.0	515.0	500.0
Total Final Product Consumption $D^{XF}_{I,II,III}$	7508.0	4168.7	4081.7

* See Section 4.4.4.2 for derivation of these figures

4.4.5 Conversion to Raw Tomato Equivalents

The data expressed in tons of final product must be converted to tons of raw tomatoes. Processing companies record 'recovery factors' for the tons of packed final product 'recovered' from one ton of raw tomatoes, and one of the companies supplied such figures. The figure needed here, however, is the quantity of raw tomatoes required to produce one ton packed of each of the final products, so the reciprocal of the recovery factor shall be called the requirement factor for each product. Table 4.5 lists the requirement factors.

Table 4.5 Raw Tomato Requirement for all Products

Product	x	Recovery Factor	Requirement Factor (rx)
Spaghetti	1	1.544	0.648
Baked beans	2	1.689	0.592
Tomato Soup	3	0.722	1.385
Tomato Sauce	4	0.924	1.082
Canned Tomatoes	5	1.196	0.836

As outlined in Section 4.3.3 above, the raw tomato content per ton of final product will depend upon the product mix which in turn will differ for each consuming region. The important figures required then are the raw tomato content of one ton of final product in the product-mix demanded by each consuming region, or in this case by each main consuming Area. These ratios can be found in two stages:

1. Calculate the breakdown of one ton of final product sent to

each area.

2. Calculate the raw tomato requirement of each fraction of a ton using the requirement factor, thus arrive at total raw requirement of one ton of final product consumed in each Area.

Table 4.6 Raw Tomato Content of Area Product-Mixes

Product		Fraction of Final Product	Raw Tomato Req. Factor	Raw Tomato Req. Per Ton Consumed
AREA I				
Spaghetti	$\frac{1}{7508}$ x	2204.2 = 0.2936	0.648	0.190
B. Beans	" x	1844.6 = 0.2457	0.592	0.145
T. Soup	" x	1076.9 = 0.1434	1.385	0.199
T. Sauce	" x	1635.4 = 0.2178	1.082	0.236
Canned Tom.	" x	747.0 = 0.0995	0.836	0.083
		7508.0 1.0000		0.853 tons
AREA II				
Spaghetti	$\frac{1}{4168.7}$ x	1116.4 = 0.2678	0.648	0.174
B. Beans	" x	895.4 = 0.2148	0.592	0.127
T. Soup	" x	740.7 = 0.1777	1.385	0.246
T. Sauce	" x	901.2 = 0.2162	1.082	0.234
Canned Tom.	" x	515.0 = 0.1235	0.836	0.103
		4168.7 1.0000		0.884 tons
AREA III				
Spaghetti	$\frac{1}{4081.7}$ x	1021.7 = 0.2503	0.648	0.162
B. Beans	" x	910.1 = 0.2230	0.592	0.133
T. Soup	" x	720.2 = 0.1764	1.385	0.244
T. Sauce	" x	929.7 = 0.2278	1.082	0.246
Canned Tom.	" x	500.0 = 0.1225	0.836	0.102
		4081.7 1.0000		0.887 tons

These figures (0.853, 0.884, and 0.887) can now be used directly to convert physical quantities of final product to raw tomatoes for Areas I, II, III. Cost figures for final products will be converted to cost per ton of raw tomatoes by using the reciprocals of these figures (1.172, 1,132 and 1.127).

4.4.6 Consumption by Area and Region - Raw Tomato Equivalents

Referring back to Table 4.4 the total final product consumption in each main area is:

D_I^{XF} = 7508.0 tons

D_{II}^{XF} = 4168.7 "

D_{III}^{XF} = 4081.7 "

Using the conversion factors from Section 4.4.5 these can be converted to total area consumption in terms of raw tomatoes:

$D_I^{XR} = D_I^{XF} \times 0.853 = 6405.8$ tons of raw tomatoes

$D_{II}^{XR} = D_{II}^{XF} \times 0.884 = 3684.1$ " " " "

$D_{III}^{XR} = D_{III}^{XF} \times 0.887 = 3622.8$ " " " "

The total tonnage of raw tomatoes consumed in each main Area D_I^{XR}, D_{II}^{XR} and D_{III}^{XR} was then broken down into tonnage consumed in each of the twelve consuming regions, using population figures. This assumes that consumption per capita is constant throughout each of the three main consuming Areas, as is the product-mix demanded.

The regional consumption of raw tomatoes D_k^{XR} was calculated from the population figures as shown in Table 4.7.

Table 4.7 Raw Tomato Consumption 1964/65, By Regions

Region	Population 1/4/65	Fraction of Area Popn.	Area Consumption $D_{I, II, III}^{XR}$	Regional Consumption D_{k}^{XR}
	(1)	(2)	(3)	(2) x (3)
Northland	91,400	0.0824	6405.8	527.8
Central Auckland	586,097	0.5284	"	3384.8
South Auckland - Bay of Plenty	384,000	0.3462	"	2217.7
East Coast	47,700	0.0430	"	275.5
	1,109,097	1.0000	6405.8	6405.8
Hawkes Bay	124,600	0.1672	3684.1	616.0
Taranaki	104,100	0.1397	"	514.7
Wellington	516,700	0.6931	"	2553.4
	745,400	1.0000	3684.1	3684.1
Marlborough	29,700	0.0378	3622.8	136.9
Nelson	67,700	0.0862	"	312.3
Canterbury & Westland	398,820	0.5077	"	1839.3
Otago	186,400	0.2373	"	859.7
Southland	102,900	0.1310	"	474.6
	785,520	1.0000	3622.8	3622.8

Sources of Data: 1. Supplement to Monthly Abstract of Statistics, June 1965. N.Z. Department of Statistics, Wellington.
2. Section 4.4.6.

4.4.7 Processing Costs Per Ton of Raw Tomatoes (P^R_{jk})

Section 4.4.2 above was concerned with finding the processing cost per ton of final product P^F_j. The cost per ton of raw tomatoes processed will depend upon the product-mix demanded and so in the general case would differ for each of the twelve consuming regions.[20] In this model, however, there are only three possible product-mixes - those mixes consumed in main Areas I, II and III respectively.

The cost expressed per ton of raw tomatoes P^R_j will be the cost per ton of final product P^F_j times the tons of final product obtained from one ton of raw tomatoes (the reciprocals of the conversion factors in Section 4.4.5). Thus the processing cost for a ton of raw tomatoes consigned to each main consuming Area will be:

AREA I, $p^R_{jI} = p^F_j \times 1.172 = 116.271 \times 1.172 =$ £136.27/ton.

AREA II, $p^R_{jII} = p^F_j \times 1.132 = 116.271 \times 1.132 =$ £131.62/ton.

AREA III, $p^R_{jIII} = p^F_j \times 1.127 = 116.271 \times 1.127 =$ £131.04/ton.

In the absence of known regional processing cost differentials, these costs will be used for all processing regions, i.e. for j = 1,2,..4.

20. In the general case, (where processing costs were known for each product and for each processing region) one could calculate the total cost of supplying the total demand of any consuming region k. This total cost would be the packed tons of each final product demanded by region k, times the cost per ton of processing in region j. i.e. Total cost of processing for region k from region j $= \sum_{x=1}^{5} \left[D^{xF}_k \cdot P^{xF}_j \right]$

This total processing cost can be expressed per ton of raw tomatoes by dividing by the total raw tomato requirement of region k, (D^R_k) i.e. processing cost/ton raw $P^R_{jk} = \dfrac{\text{Total cost}}{D^R_k} = \dfrac{\sum_{x=1}^{5} D^{xF}_k \cdot P^{xF}_j}{D^R_k}$.

The data required to find the processing cost per raw ton would be the consumption of each final product in each processing region...

4.4.8 Final Product Transport Costs Per Ton of Raw Tomatoes (t^R_{jk})

Using the same logic as that put forward for processing costs above, the final product transport cost matrix developed in Section 4.4.3 can be converted to a cost/ton of raw tomato content by multiplying each t^F_{jk} by the ratio $\frac{D^{XF}_k}{D^{XR}_k}$. These ratios have been calculated for the three main consuming Areas in Section 4.4.5 and so these ratios can be applied to regions lying within the respective main Areas.

i.e. $$t^R_{jk} = t^F_{jk} \times \frac{D^{XF}_I}{D^{XR}_I}$$

$$= t^F_{jk} \times 1.172, \qquad \text{for } k = 1,2,3,4. \quad \text{(Area I)}$$

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20. continued ... and the raw tomato recovery factor for each product. The last figures are required to convert the consumption from final product to total raw tomato equivalents. (D^R_k).

In this study, p^{xF}_j is constant for x's and j's, and may be called simply $p^F_{\cdot\cdot}$.

The formula is now reduced to: $$P^R_k = \frac{p^F \sum_{x=1}^{5} D^{xF}_k}{D^R_k}$$

But $\sum_{x=1}^{5} D^{xF}_k = D^{XF}_k = D^F_k$ $\quad \therefore p^R_k = p^F \cdot \frac{D^F_k}{D^R_k}$,

calculated for k = I, II, III.

The factor $\frac{D^F_k}{D^R_k}$ is in fact the reciprocal of the conversion factor.

e.g. $\frac{D^F_I}{D^R_I} = \frac{7508.0}{6405.8} = 1.172$, which is the same as the ratio derived in Section 4.4.5.

Similarly $t^R_{jk} = t^F_{jk} \times 1.132$, for k = 5,6,7. (Area II)

And $t^R_{jk} = t^F_{jk} \times 1.127$, for k = 8,9,10,11,12. (Area III)

The final product transport cost matrix expressed per ton of raw tomato content (the t^R_{jk} matrix) is derived from Table 4.2 using these equations, and is shown in Table 4.8.

Table 4.8 Transport Costs of Processed Products (£/ton of raw tomato content)

		Processing Region			
	j=	Auckland 1	Gisborne 2	Hastings 3	Nelson 4
Distribution Centre	k	(£/ton)	(£/ton)	(£/ton)	(£/ton)
Whangarei	1	6.07	16.04	13.22	20.51
Auckland	2	0.00	8.57	10.20	9.14
Hamilton	3	4.86	8.20	8.63	14.00
Gisborne	4	10.31	0.00	7.15	16.93
Hastings	5	10.19	6.91	0.00	14.49
New Plymouth	6	8.94	10.49	8.55	15.62
Wellington	7	9.62	9.51	6.06	7.24
Blenheim	8	14.73	14.62	11.18	4.39
Nelson	9	16.40	16.29	14.43	0.00
Christchurch	10	13.16	12.75	12.75	7.89
Dunedin	11	13.16	12.79	12.68	8.57
Invercargill	12	14.34	14.05	15.07	13.75

The required data have now all been collected and converted into a common dimension. These data will be brought together to show the actual formulation of the problem.

4.4.9 The Adjusted Data in a Simplex Tableau

The modified mathematical formulation from Section 4.3.5 can be drawn up as a Simplex Tableau similar to the generalized one shown in Figure 2, page 79. The initial tableau is shown in Appendix II, page 163 with relevant data included.

The model has four raw production activities (i = 1 ...4), twelve processing activities (one from each processing region j = 1 ...4 to each consuming area, k = I, II, III) and 48 final product transport activities (one from each processing region j = 1 ...4 to each consuming region k = 1,2 ... 12). There are four constraints to equate production and processing volumes, twelve constraints to equate processing and transport volumes, and twelve constraints to ensure that quantity demanded is supplied by one or other of the processing regions. There are twelve artificial activities corresponding to these consumption constraints.

The information contained in this tableau was punched onto cards and the problem solved using a linear programme to give an initial set of results.

4.5 RESULTS OF LOCATION ANALYSIS FROM INDUSTRY VIEWPOINT

The linear programming model outlined in the previous section was

solved on an IBM 1620 computer using IBM Library Program 10.1.006. This program[21] employed normal profit maximizing routines and so minimized a negative total cost. The time taken for solution was approximately seven minutes.

4.5.1 The Computer Solution

The program used was designed so that the solution matrix was analysed and all relevant information printed out in a logical form. All figures are expressed in tons of raw tomatoes, and all results are shown in Appendix III. The activities in the solution matrix fall into three main classes.

(1) real activities included in the basis (or activities which do take place in the minimum-cost pattern);

(2) non-basis real activities (or real production, processing or transport activities which do not occur in the solution pattern); and

(3) non-basis disposal activities. If costs could be minimized with production greater than consumption, then some consumption would go into disposal. In this case the consumption disposal activity would enter the basis. This will not happen when production can be increased continuously.[22] The significance of each class of activity in the solution to this formulation will now be discussed separately.

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21. In accordance with general usage, the spelling 'program' is used to describe a specific computer program. (A list of statements in FORTRAN, COBOL or some other programming language). The traditional spelling 'programme' is maintained for description of development programmes, linear programmes, etc.

22. For full description of linear programming jargon used in this section, see HEADY, E.O. and CANDLER, W., op. cit. Chapter 3.

4.5.1.1 Real Activities Included in the Basis. For these activities, the level at which the activity is included (X_j in conventional notation) is shown, and the actual cost per ton is also shown. Most important, the stability of this solution level (X_j) to changes in costs is shown, and the 'competing' non-basis real activities which define these stability limits are indicated. For example, in Appendix III, Table 1, page 164, activity 001 is included at a level of 3912 tons. The cost/ton is £18.42, and the activity will remain at a level of 3912.6 tons for all costs over the range of £15.44 to £18.82 per ton only. If the cost falls below £15.44 then non-basis real activity 019 will be included in the basis, and the level of 001 will rise above 3912.6 tons. If the cost rises above £18.82/ton then non-basis real activity 041 will be included in the basis, displacing activity 017 and the level of 001 will fall below 3912.6 tons.

A full list of the basis, real activities, their levels, cost stability ranges and competing activities is shown in Appendix III, Table 1.

4.5.1.2 Non-Basis Real Activities. Appendix III, Table 2 shows these activities in numerical order, along with their cost/ton, and stability information. The most important information is the 'shadow price' ($Z_j - C_j$). This figure is the 'marginal cost' of adding one unit of the activity to the basis - it is the increase to total cost if one unit of this non-basis activity was forced into the basis. Alternatively one could describe it in this context as the reduction

in cost per unit necessary before this activity would occur in the basis.

The remaining two columns describe the upper limit of the activity if the cost per unit were reduced by the shadow price, and finally the real activity which would be displaced from the basis by the non-basis activity in question. Specifically the first activity in Table 2, 006 would enter the basis if its costs dropped by £7.14 to £124.48 and the level it would enter at is 514.7 tons. It would displace activity 046 from the basis.

Any non-basis real activities which are 'next best' to an activity in the basis are shown as the competing or limiting activity in the basis real activity figures, in Table I.

4.5.1.3 Non-Basis Disposal Activities. In profit-maximizing routines, where the constraints apply to the availabilities of limiting resources, the shadow prices ($Z_j - C_j$'s) indicate the value of the marginal product (VMP) of these resources. The shadow price is the decrease in profit caused by forcing one unit of the disposal activity into the basis, i.e. forcing one unit of the resource into disposal.

In the present formulation, the constraints 1 to 16 are used to reconcile production, processing and transport, and so the shadow prices are purely the cost of carrying out these activities. They yield no new information, and for that reason are not shown in the Appendix.

The last twelve constraint equations (i.e. 17 to 28) ensure that the amount consumed in each region is provided by the transportation

activity to that region. An arbitrarily high cost (£999.999) was ascribed to each of these activities to ensure that they would not be included in the basis. The decreased cost due to decreasing consumption in each region by one unit will thus be £999.999 minus the actual saving from not supplying that one unit. This is the $Z_j - C_j$ figure shown in the computer solution, and in order to find the actual saving the $Z_j - C_j$ is subtracted from £999.999. This actual saving is the marginal cost or true shadow price of supplying the last ton to the specific region and, assuming that the activity level does not coincide with a vertex on the convex hull, this will also be the marginal cost of supplying an additional ton to that consuming region.

These marginal costs and the level by which the activity can be decreased without the marginal cost changing, are shown in Appendix III, Table 3 below. The basis activity which limits the level of possible decrease for the given marginal cost is also shown.

4.5.2 The Normative Location Pattern

The minimum cost pattern of production, processing and distribution of tomato products is described fully by the tables in Appendix III. The more important aspects of these tables can be shown more simply by listing the quantities of raw tomatoes handled by each stage in the marketing chain, as in Table 4.9.

This is a relatively simple pattern because it is a long-run normative model and there are no capacity constraints on any of the activities.

Table 4.9 Normative Solution Levels of Activities
(In tons of raw tomatoes)

Producing and Processing Region	Total Tonnage	Consuming Region Supplied	Tons Supplied
1. Auckland	3912.6	Northland	527.8
		Auckland	3384.8
2. Gisborne	275.5	East Coast	275.5
3. Hastings	5901.8	South Ak. - Bay of Plenty	2217.7
		Hawkes Bay	616.0
		Taranaki	514.7
		Wellington	2553.4
4. Nelson	3622.8	Marlborough	136.9
		Nelson	312.3
		Canterbury-Westland	1839.3
		Otago	859.7
		Southland	474.6
	13712.7		13712.7

The functional or total cost of all of the industry's operations if the industry was to follow this long-run normative pattern is:

$$Z = £2,091.436$$

In fact this long-run solution could have been found by a simpler technique than linear programming. The linear programming formulation is necessary for more complex explorations of the problem, however. One very convenient aspect of using this algorithm is that it indicates the

sensitivity of the solution to individual cost changes.

4.5.3 Stability of the Normative Pattern

The pattern of tomato production, processing and distribution described in Table 4.9 will hold good only at the specific costs per unit used in this analysis. It is important to know how stable this solution is to cost changes and the effect of different changes on the volumes handled by each facet of the industry. This information can also be obtained by studying all the limits on costs as shown in Appendix III, Tables 1, 2, and 3.

4.5.3.1 Cost Stability Ranges Apply to Each Link in the Supply Chain

The mass of information in Appendix III can be simplified by bearing in mind that the production, processing and distribution activities bringing about supply from one producing region to one consuming region, are all part of the one causal chain. Thus the tolerance to cost changes in final product transport activities will equally apply to the production and processing activities which provide that final product.

For example, referring to Appendix III, Table 1, activity 017 (distribution from Auckland to Northland) will remain at the solution level unless the cost rises from £6.07 to £6.47 per ton. If this rise of £0.4 per ton transported does take place, then activity 041 will come into the solution, i.e. Hastings will transport final product to Northland. Similarly, looking at activity 005 (Auckland processing for

supply to consumption Area I) and activity 001 (raw tomato production in Auckland) we find that these activities will remain at their solution levels unless processing costs rise from £136.27 to £136.67 per ton, or production costs rise from £18.42 to £18.82 per ton respectively. If either processing or production costs do rise by more than this £0.4 per ton, then activity 041 will come in and again Hastings will supply Northland. Thus if there is a rise of more than £0.4 per ton in the cost of producing, processing or distributing tomatoes from Auckland to Northland, or if there are two or more cost changes in the supply chain which in combination give a net rise of £0.4 per ton, then the least-cost solution pattern will change.

In general then, there is a minimum change in costs which will bring about a given change in the supply pattern, and this cost change can take place for any one or a combination of all facets in the supply chain.

4.5.3.2 Graphical Expression of Stability to Cost Changes. Detailed study of the cost ranges in Appendix III enables one to determine the cost changes in each producing-processing region, which are necessary to bring about the addition or subtraction of each consuming region to its list of 'customers'. These cost limits and the corresponding changes in raw tomatoes produced can be used to draw up 'normative supply curves' for each region. These stepped functions in fact describe the amount of raw tomatoes which should be produced in a region if the costs in all other regions remain constant, but the costs of production,

processing or distribution to individual demand regions is allowed to vary. These quantities 'should be produced' if the industry is to minimize costs.

An example of such a graph for a hypothetical producing region and five consuming regions is shown in Figure 3, page 114.

4.5.3.3 Interpretation of the Hypothetical Volume/Cost Graph.

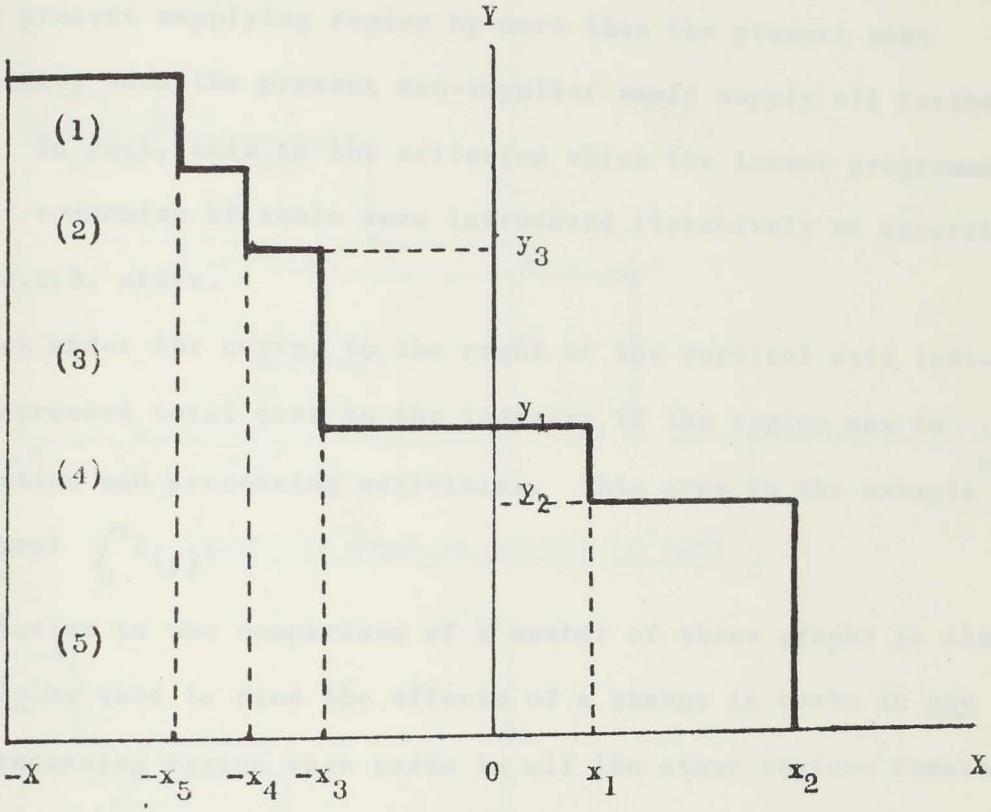
(a) This graph shows in general the stability of the solution to changes in costs of production, processing and distribution. The steeper the graph, the less stable the solution. This hypothetical example is reasonably stable around the actual cost used, but if that cost was reduced by - £x_3 per ton, then the tonnage handled would increase rapidly - the solution would be very unstable.

(b) It shows specifically for this producing region, the amount by which the costs would have to rise before this region would lose each market to another supplying region. In the example, the region is producing y_1 tons of product to supply region 5 (y_2 tons) and region 4 ($y_1 - y_2$ tons). If costs in the supply chain to region 4 rise by £x_1 per ton then the region 4 market will be lost and production will be reduced to y_2 tons.

The graph conversely shows by how much the region must reduce costs to gain other markets, e.g. if costs are reduced by - £x_3 per ton the region will gain the large market of region 3 and total production will rise to y_3 tons.

(c) The graphs of a number of regions could be used along with economies

Fig. 3. Hypothetical Volume/Cost Stability Graph.



Change in Cost of Supply.

of scale or marginal cost curves to decide which producing region the increased supply to a given consuming region should come from. If the marginal cost in the non-supplying region was less than the marginal cost in the present supplying region by more than the present cost stability limit, then the present non-supplier would supply all further quantities. In fact, this is the criterion which the linear programme would use if economies of scale were introduced iteratively as described in Section 4.2.3. above.

(d) The area under the curve, to the right of the vertical axis indicates the increased total cost to the industry if the region was to cease production and processing activities. This area in the example is the integral $\int_0^{x_2}(y)$.

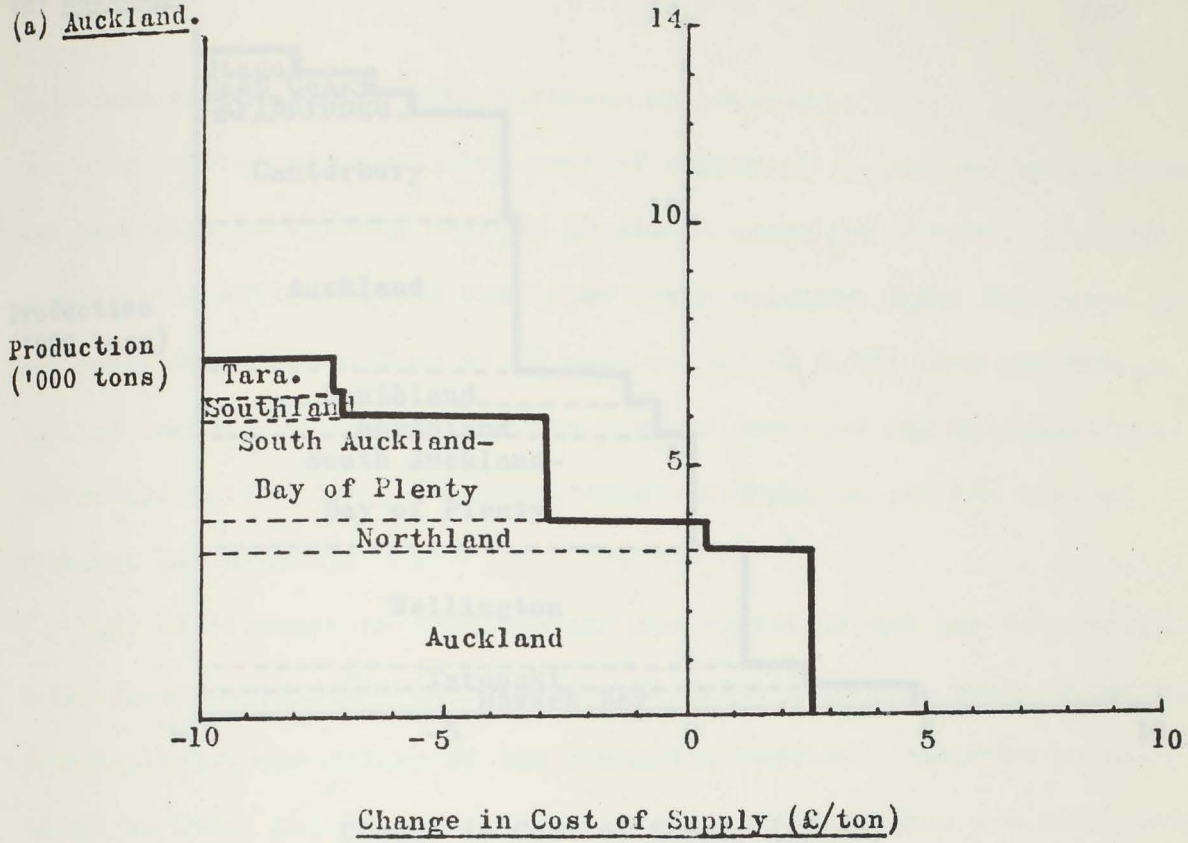
(e) A limitation to the comparison of a number of these graphs is that they can only be used to find the effects of a change in costs in one producing-processing region when costs in all the other regions remain constant.

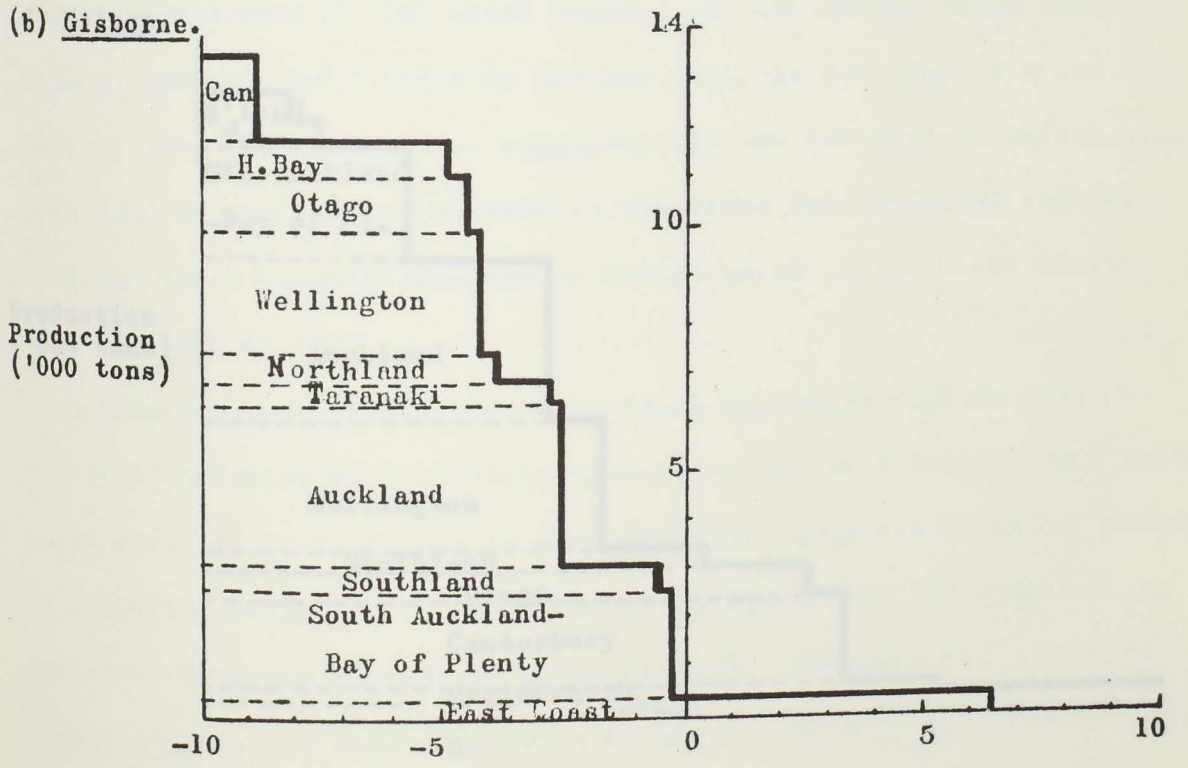
4.5.3.4 Interpretation of Stability Graphs for Regional Cost Changes

The volume/cost stability graphs derived from the long-run solution for each of the producing-processing regions are shown in Figure 4 (a), (b), (c) and (d).

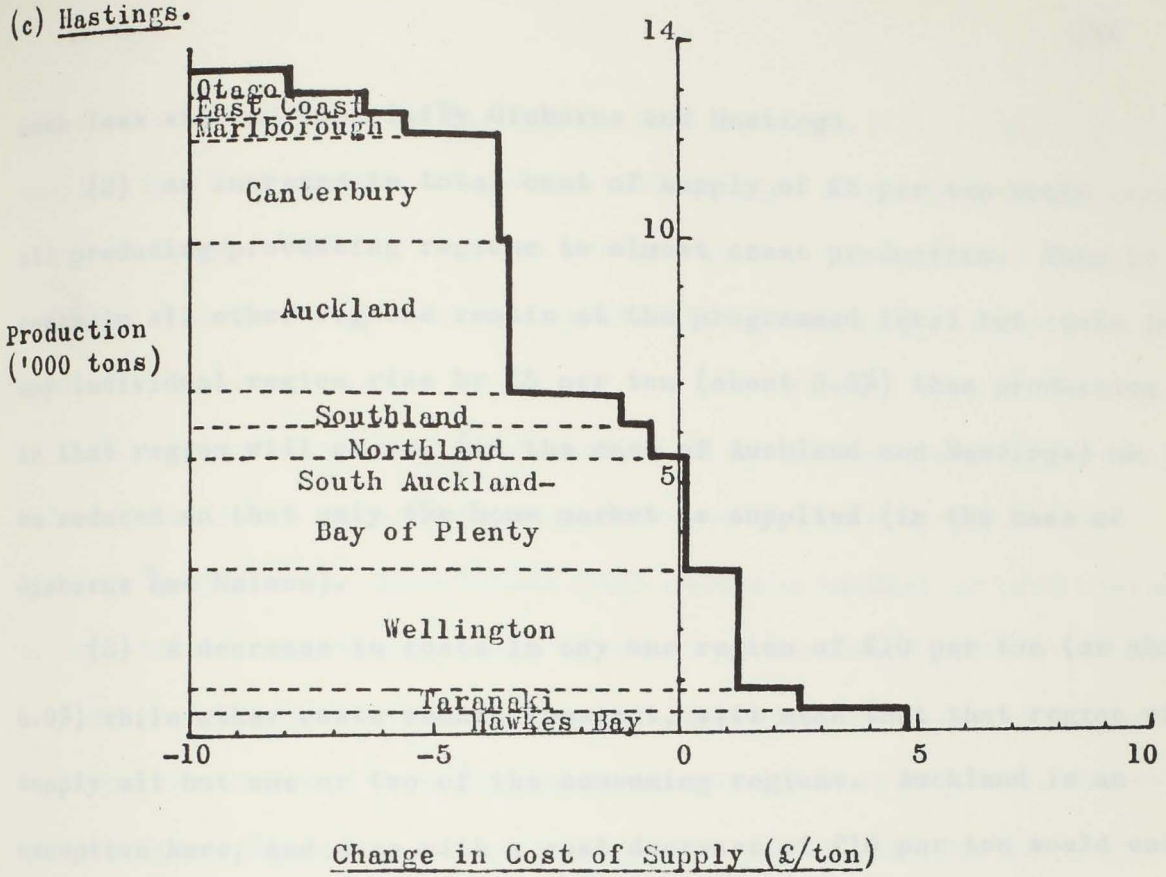
In general, the aspects of the present solution shown by the graphs are: (1) The Auckland graph is not very steep and so Auckland's tonnage is reasonably stable to cost changes, but the other three regions are

Fig. 4. Volume/Cost stability Graphs: Industry Viewpoint.

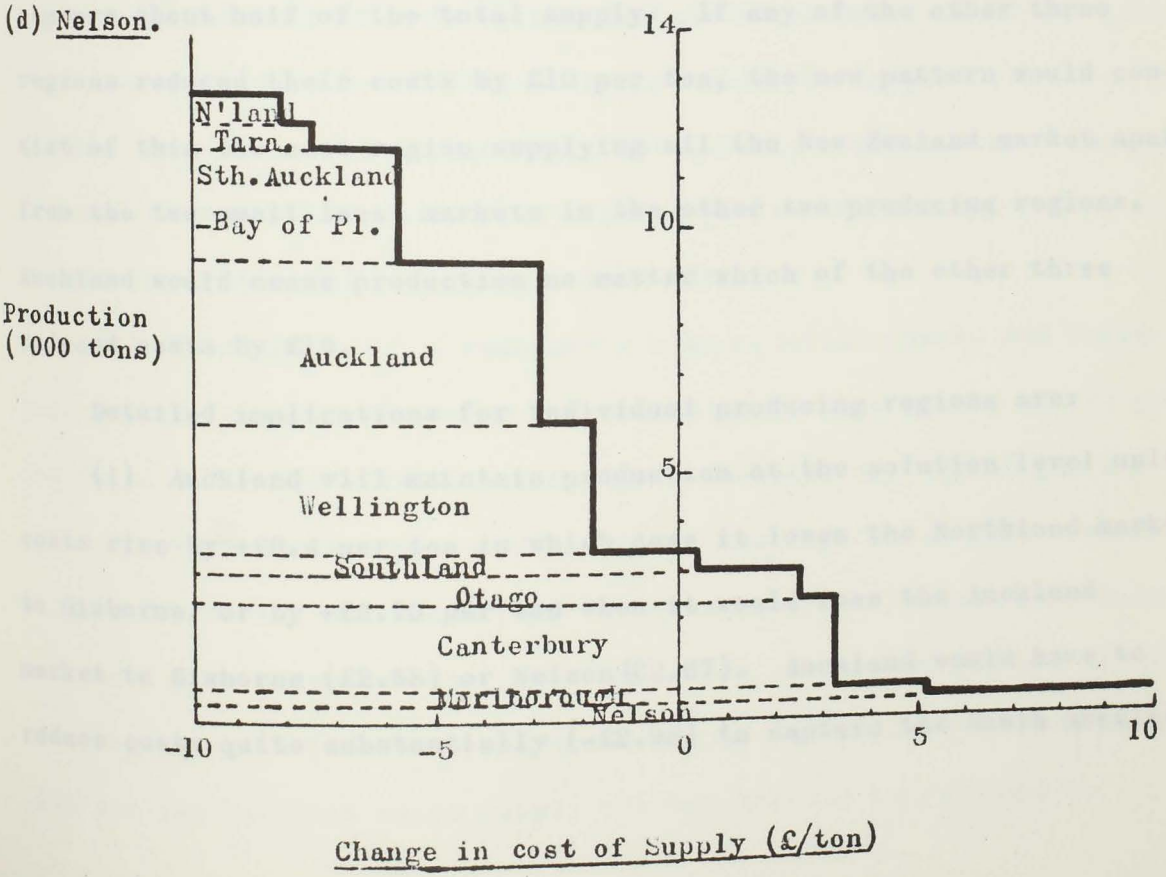




(c) Hastings.



(d) Nelson.



much less stable, especially Gisborne and Hastings.

(2) an increase in total cost of supply of £5 per ton would cause all producing-processing regions to almost cease production. Thus if costs in all other regions remain at the programmed level but costs in any individual region rise by £5 per ton (about 3.3%) then production in that region will cease, (in the case of Auckland and Hastings) or be reduced so that only the home market is supplied (in the case of Gisborne and Nelson).

(3) A decrease in costs in any one region of £10 per ton (or about 6.6%) while other costs remain constant, will mean that that region will supply all but one or two of the consuming regions. Auckland is an exception here, and even with a cost decrease of £10 per ton would only capture about half of the total supply. If any of the other three regions reduced their costs by £10 per ton, the new pattern would consist of this low cost region supplying all the New Zealand market apart from the two small local markets in the other two producing regions. Auckland would cease production no matter which of the other three reduced costs by £10.

Detailed implications for individual producing regions are:

(1) Auckland will maintain production at the solution level unless costs rise by +£0.4 per ton in which case it loses the Northland market to Gisborne, or by +£2.70 per ton when it would lose the Auckland market to Gisborne (£2.58) or Nelson (£2.67). Auckland would have to reduce costs quite substantially (-£2.98) to capture the South Auckland -

Bay of Plenty market from Hastings. Even if costs were reduced by £10 per ton Auckland would only produce 7119.6 tons compared with its solution level of 3912.6 tons at present costs. In summary, Auckland's solution tonnage is relatively unstable to increase but stable to decrease in costs.

(2) Gisborne contrasts in that it is at a very low solution level and would maintain this production unless costs rose by £6.39 and Hastings supplied the East Coast. The solution tonnage is very unstable to cost reductions, however, and if costs were reduced by only -£0.33 per ton production would rise from 275 tons to 2,493 tons. If costs in Gisborne were reduced further, production would rise rapidly. At -£4.97 per ton Gisborne would produce 11,424 tons and at -£10 per ton would produce 13,263 tons of a total New Zealand requirement of 13,713 tons. This instability is due to the fact that Gisborne is just a little further from markets than Hastings, but quickly overcomes this hurdle; and also that sea transport from Gisborne to southern South Island is little dearer than rail transport from Nelson.

(3) Hastings is at a reasonably high solution level, but would lose its two large markets for reasonably small cost rises, +£0.33 per ton would allow Gisborne to supply South Auckland and +£1.45 per ton would allow Nelson to supply Wellington. Hastings could gain two small markets (Northland and Southland) for relatively small cost reductions, but would have to reduce its costs by -£3.54 per ton to capture the other two large markets (Auckland and Canterbury). If costs were reduced by -£10 per ton Hastings would supply all New Zealand except Nelson.

Thus the Hastings solution production level of 5902 tons is relatively stable to cost reductions, but unstable to any Hastings cost increases.

(4) Nelson's solution level of production is relatively stable to either increase or decrease in costs. It enjoys a relative advantage in supplying all South Island regions because of the costs of shipping across Cook Strait from the North Island. This advantage is least for the most southern South Island regions as the cost of transport from even Nelson is quite high, and shipping costs from the North Island increase only slightly as the distance hauled increases. As Nelson costs increase then it will quickly lose the Southland market (+£0.78 per ton), then the Otago market (+£2.78 per ton) and finally its main market, Canterbury, is lost if costs increase by +£3.54 per ton. Nelson will maintain supply of its small home market unless costs rise by +£13.11 per ton. Nelson could capture the Wellington market quite readily with a decrease of -£1.46 per ton but would require marked decreases to supply the other main North Island markets, (-£2.67 per ton for Auckland and -£5.65 for South Auckland - Bay of Plenty). Finally a decrease of £10 per ton would enable Nelson to supply all New Zealand apart from the home markets of Gisborne and Hastings - a total production of 12,821 tons.

4.5.3.5 Increase in Cost Due to Plant Closure. The increase in total costs to the industry due to ceasing production in each region can be found by calculating the total area under the curve to the right of the vertical axis. In this solution the total cost of supplying the New Zealand market was £2,091,436. The change in this cost if each region ceased production, can be found.

(1) If Auckland region ceased production the increased cost would be (+£0.4 x 527.8) + (£2.58 x 3384.8)

= 211.12 + 8732.78

= £8943.9.

(2) If Gisborne ceased production industry costs increase by:

275.5 x 6.39

= £1760.4.

(3) If Hastings ceased production, industry total cost would rise by: (0.33 x 2217.7) + (1.46 x 2553.4) + (2.70 x 514.7) + (4.97 x 616.0)

= 731.84 + 3728.00 + 1389.69 + 3061.52

= £8911.05.

Finally,

(4) If Nelson ceased production industry cost would rise by:

(0.78 x 474.6) + (2.79 x 859.7) + (3.54 x 1839.3) + (5.47 x 136.9) + (13.11 x 312.3)

= 370.19 + 2398.56 + 6511.12 + 748.84 + 4094.25

= £14,122.96.

These calculations also show the increased costs of transferring the supply of each consuming region from its present supplier to the

next lowest cost supplier. The cost of any shift of supplier is readily calculated from the individual graphs as change in total cost equals volume supplied times the shadow cost per ton.

It is interesting to note that none of these total cost increases due to plant closure are very large fractions of the total cost, and so, if there are economies of scale to be reaped in any one region, presumably these increases above could be readily overcome.

4.5.4 The Normative Pattern Expressed in Tons of Final Product

The production of each region as shown in Table 4.9 is expressed in tons of raw tomatoes but these can readily be converted to tons of final product. The actual amount of each final product gained from a ton of raw tomatoes depends upon the consuming Area it is produced for. The tonnage of final product recovered from one ton of raw tomatoes is shown in Table 4.6, page 99.

This factor can be used to find the total tonnage of final product supplied to each Area by each processing region, and the tonnage of final product can be divided among individual products using the 'Fraction of Final Product' information, also from Table 4.6. An example of such a calculation would be the raw tomatoes processed in Auckland for consumption in regions contained in main Area I.

Tonnage of raw tomatoes processed = 3913 tons

Final product recovery $= 3913 \times \frac{1}{0.853}$

$= 3913 \times 1.172$

$= \underline{4585}$ tons final product

Spaghetti as fraction of final product = 0.2936.

= 4585 x 0.2936.

= 1346 tons

Similarly, tons of baked beans = 4585 x 0.2457

= 1126 tons

These calculations were carried out for all processing regions for all Areas which each region supplied and for all final products and the tons of final products processed are shown in Table 4.10.

Table 4.10 Normative Pattern in Tons of Final Product

	Raw Tomatoes (tons)	Final Prod. (tons)	Spaghetti. (tons)	Baked Beans (tons)	Tomato Soup (tons)	Tomato Sauce (tons)	Canned Tomatoes (tons)
Auckland							
to Area I.P_{II}	3913	4585	1346	1126	658	999	456
Gisborne							
to Area O.P_{21}	275	323	95	80	76	70	32
Hastings							
to Area I.P_{31}	2218	2599	763	639	372	566	259
to Area II.P_{311}	3684	4170	1117	896	741	901	515
Total:	5902	6769	1880	1535	1113	1467	774
Nelson							
to Area III.P_{4111}	3623	4083	1022	910	721	930	500

4.6 COMPARISON OF NORMATIVE WITH ACTUAL LOCATION PATTERN

One of the aims of this economic analysis is to see in what ways

the normative, least cost location differs from the present actual pattern. It would also be very interesting to determine the savings in total cost which could be achieved by adopting the normative pattern.

4.6.1 The Tonnages Produced and Processed in Each Region

The actual tonnage of raw tomatoes produced and processed in each region are shown by Sanderson.[23] The overall total tonnage produced was 14,536 tons, whereas the total tonnage of raw tomatoes demanded for consumption during 1964/65 was 13,712.7 tons. The excess of production over consumption would presumably be the net amount stored as tomato pulp and puree - it would also include any errors of measurement in the consumption data.[24] In the absence of other evidence it will be assumed that the same fraction of total production was stored in each region, and thus the total disposals and storage for each region can be calculated. These are shown in Table 4.11.

Table 4.11 Actual Production, Disposals, and Storage 1964/65

Producing Region	Total Production	Disposals	Net Storage
Auckland	2715	2561	154
Gisborne	2541	2397	144
Hastings	8380	7906	474
Nelson	900	849	51
	14,536 tons	13,713 tons	823 tons

23. SANDERSON, K.T. op. cit. Table 1.1, p 8.
24. This net storage figure of 823 tons is approximately 5.7% of total production. One of the processing companies stored 6.4% of total production in 1964/65 as pulp and puree, and so the total consumption figure must be reasonably accurate.

A regional comparison can now be made directly between these disposal figures and the tonnages produced in the normative pattern, as both deal with the same total tonnage consumed.

Table 4.12 Actual and Normative Tonnages Produced

	Actual Regional Production 1964/65 (tons)	Normative Regional Disposal 1964/65 (tons)
Auckland	2,561	3,913
Gisborne	2,397	275
Hastings	7,906	5,902
Nelson	849	3,623
	13,713	13,713

The normative analysis thus indicates that tomato production and processing should be greatly increased in Nelson and to a lesser extent in Auckland. Hastings and Gisborne should each reduce production by approximately 2,000 tons per year - this is a very drastic cut for Gisborne.

As Auckland actually only produced 2,561 tons, there would still be (3,385 - 2,561) = 824 tons required for the Auckland market and 528 tons for the Northland market. Similarly as Nelson actually produced only 849 tons and these would go mostly to Nelson and Marlborough, Canterbury would receive (849 - (137 312)) = 400 tons only from Nelson. Thus Canterbury would require 1,439 tons, Otago require 860 tons and Southland require 475 tons.

Gisborne and Hastings actually produced greater than the least-cost

quantity - 2,122 and 2,004 tons respectively. This surplus will be used to fulfil requirements which Auckland and Nelson were unable to meet. The cost increases above the normative minimum level will determine whether Gisborne or Hastings supply each. The relevant increases are shown in Table 4.13.

Table 4.13 Costs of Gisborne and Hastings Supplying Deficit Regions

Consuming Region	Quantity Required (tons)	Extra cost £ per ton of:- Gisborne Supplying	Hastings Supplying
Northland	528	3.98	0.40
Auckland	824	2.58	3.45
Canterbury	1,439	8.88	3.54
Otago	860	4.69	7.97
Southland	475	0.78	1.04

It is logical to fulfil those deficits in which each region has a marked advantage first. Thus:

Hastings supplies Canterbury with 1,439 tons at cost of 1,439 x 3.54 = £5,094

Hastings still has 2,004 - 1,439 = 565 tons surplus.

Hastings then supplies Northland with 528 tons at cost of 528 x 0.4 = £211

Hastings still has 565 - 528 = 37 tons surplus.

Hastings supplies Southland with 37 tons at cost of 37 x 1.04 = £38

This leaves Gisborne to supply Southland 438 tons at cost of 438 x 0.78 = £342,

Auckland 824	....824 x 2.58	= £2,125,
and Otago 860	....860 x 4.69	= £4,032.
The total increase in costs over normative solution		£11,842

This is not a very large cost inefficiency - $\frac{11,842}{2,091,436}$ = 0,6% - but this is only the minimum inefficiency necessary because of the actual production pattern. This calculation assumes that distribution follows a cost minimizing pattern whereas in fact there is probably a lot of cross-haulage with products from all districts being sold throughout the country.

4.7 TONNAGE CONSTRAINTS FOR AUCKLAND AND NELSON

The normative pattern indicated a very marked increase in tomato production and processing in Nelson and Auckland. In a full location study it would be possible to do a farm management and soil-capability survey to determine the maximum tonnage which could reasonably be grown in each region. This figure could then be used as a maximum constraint on the Auckland and Nelson growing activities.

It was not possible to completely survey the region in this study. The use of institutional constraints such as this tonnage constraint can nevertheless be demonstrated adopting the following assumption.

It was assumed that the maximum possible tonnage which Auckland or Nelson could provide is the maximum area grown over the last four years, times the panel average yield for 1964/65.

4.7.1 Maximum Feasible Tonnage For Auckland

Largest area grown over last four years = 278[25] acres in 1962/63

Average panel yield 1964/65 = 13.0[26] tons/acre

Maximum feasible tonnage = 3,614 tons

4.7.2 Maximum Feasible Tonnage For Nelson

Largest area grown over last four years = 140[25] acres in 1962/63

Average panel yield 1964/65 = 18.4[27] tons/acre

Maximum feasible tonnage 18.4 x 140 = 2,576 tons

4.7.3 Formulation of the Constraints

These maximum feasible tonnage constraints can be imposed upon the Auckland and Nelson production activities by introducing two resources A_1 and A_2, and giving them availabilities of 3,614 and 2,576 respectively. Tomato growing in Auckland is constrained by giving one ton of raw production a requirement of 1 unit of A_1. This can be written quite simply as: $A_1 \geqslant C_1$, or: $(A_1 - A_1 \text{ in disposal}) = C_1$

In this case: 3,614 - Disposal activity 9,929 $= +1.C_1$

Similarly, 2,576 - Disposal activity 9,930 $= +1.C_4$

These can now be written directly in the Simplex notation as constraints 29 and 30 with availabilities (b_i's) of 3,614 and 2,576, and with +1 in the C_1 and C_4 columns respectively. The resources A_1 and A_2 were given zero costs.

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25. SANDERSON, K.T. op. cit. Table 1.1, p. 8.
26. ibid. Table 4.7, p.30.
27. ibid. p.51.

4.7.4 Normative Pattern With Tonnage Constraint

The modified formulation was punched on cards and solved on the IBM 1620 computer using the same program as above. The solution pattern of production is shown in Table 4.14.

Table 4.14 Normative Location Pattern With Tonnage Constraints

Producing and Processing Region	Total Tonnage	Consuming Region Supplied	
Auckland	3614.0	Northland	229.2
		Auckland	3384.8
Gisborne	750.1	East Coast	275.5
		Southland	476.6
Hastings	6772.6	Northland	298.6
		South Auckland - Bay of Plenty	2217.7
		Hawkes Bay	616.0
		Taranaki	514.7
		Wellington	2553.4
		Otago	572.2
Nelson	2576.0	Marlborough	136.9
		Nelson	312.3
		Canterbury	1839.3
		Otago	287.5

The constraints on Auckland and Nelson have brought about only three main changes. The production in Auckland and Nelson is as expected, right up to their tonnage constraints, but (a) Auckland is able

to supply only 229.2 tons to Northland. The remaining 298.6 tons are supplied by Hastings.

(b) Nelson is unable to supply any of Southland's requirement and so Gisborne takes over the Southland supply.

(c) Also Nelson can supply only 287.5 tons of Otago's requirement and so Hastings supplies the remaining 572.2 tons.

Hastings and Gisborne solution volumes are very unstable to cost changes because their costs of supplying South Auckland - Bay of Plenty and Southland are so similar. (See Appendix IV, activities 052 and 031).

4.8 FROM PROCESSING COMPANY'S VIEWPOINT

The normative location pattern found in Section 4.6 assumed that the industry as a whole aimed to minimize total costs. In fact much of the location decision rests with the processing companies. There are only two main companies operating at present and there is likely to be a certain inefficiency of distribution due to each company supplying each region. However it is important to see whether the location pattern which a monopolistic processing company would aim at, differs markedly from that optimum industry pattern. If so, then it may be in the country's interest for the Government to reimburse the processors for adopting the - to them - more costly pattern.

The monopolistic processor does not pay the average cost of producing raw tomatoes, but the present ruling supply prices. It is assumed that the processor does not use his monopsonistic powers to reduce the

price of raw tomatoes, nor for transport services. It is also assumed that demand is infinitely inelastic, and so profit maximization is equivalent to cost minimization.

4.8.1 The Data Changes

The only changes to the original data are the replacement of average costs per ton of growing raw tomatoes in Auckland, Gisborne, Hastings and Nelson (£18.42, £12.43, £11.67 and £11.95 respectively) with the actual cost per ton to the processor - £20.0, £14.5, £14.67 and £16.0 respectively.[28] In actual fact, these prices changes were typed into the computer immediately it had finished the solution from the Industry's viewpoint. The time for re-solution was about four minutes.

4.8.2 Total Cost

The functional in this case was: Z = £2,128,665.
This is an increase of £37,229 over the total cost using average regional costs, and occurs because, using the prices above, the growers received a margin of profit over and above their total growing costs. The margins per ton received by the 'average' grower in Auckland, Gisborne, Hastings and Nelson were, £1.58, £2.07, £3.0 and £4.05 respectively.

4.8.3 Minimum Cost Location - Processor's Viewpoint

The solution pattern achieved by the linear programme is shown in simplified form in Table 4.15.

28. The prices used in Gisborne and Hastings are £1.0 higher than the prices operating in 1964/65. These prices were raised subsequent to the publishing of the author's report (see page 12).

Table 4.15 Location Pattern - Processor's Viewpoint

Producing and Processing Region	Total Tonnage	Consuming Region Supplied	Tons
Auckland	3912.6	Northland	527.8
		Auckland	3384.8
Gisborne	2967.8	South Auckland - Bay of Plenty	2217.7
		East Coast	275.5
		Southland	474.6
Hastings	3684.1	Hawkes Bay	616.0
		Taranaki	514.7
		Wellington	2553.4
Nelson	3148.2	Marlborough	136.9
		Nelson	312.3
		Canterbury	1839.3
		Otago	859.7

The important differences here are that, by accepting a lower profit margin than Hastings and Nelson growers, the Gisborne growers enable their factory to capture the South Auckland - Bay of Plenty and the Southland markets from the other two regions.

The pattern thus described however, still differs widely from the actual pattern in that production by Nelson and Auckland is still very much higher in the normative than the actual pattern. If this minimum-cost solution from the processors viewpoint was adopted, the actual tonnage grown in Hastings would be more than halved.

4.8.4 Stability of Processor's Solution

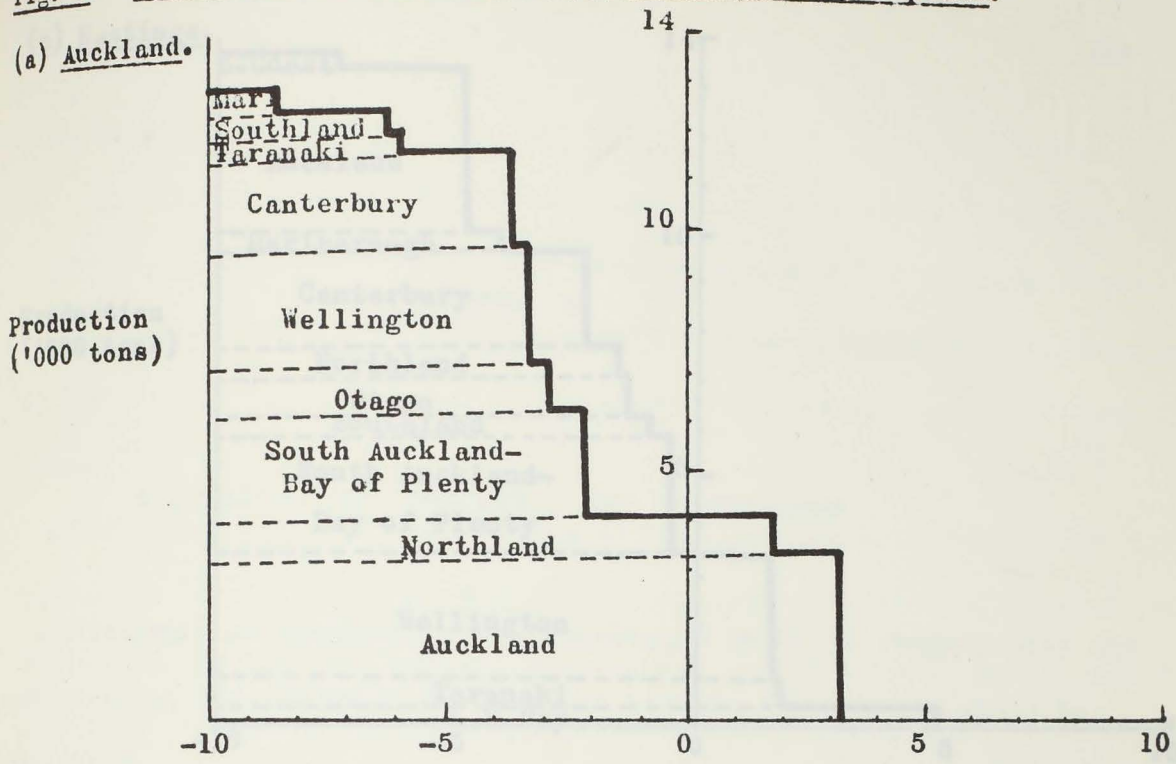
The Volume/Cost stability graphs in Figure 5[29] (a), (b), (c), and (d) show that when the actual prices facing the processor are used the solution levels for each region are very unstable to cost changes. All regions have very steep graphs to the left of the vertical axis, showing that very little cost reduction would be required for that region to take over most of the processing.

This instability is greater than that shown in the earlier analysis from the industry viewpoint. Especially in the case of Auckland and Gisborne. The reason for the increased instability is that the raw tomato price differential between regions is less than the average cost of production differential. Thus Auckland and Gisborne are better able to compete for markets.

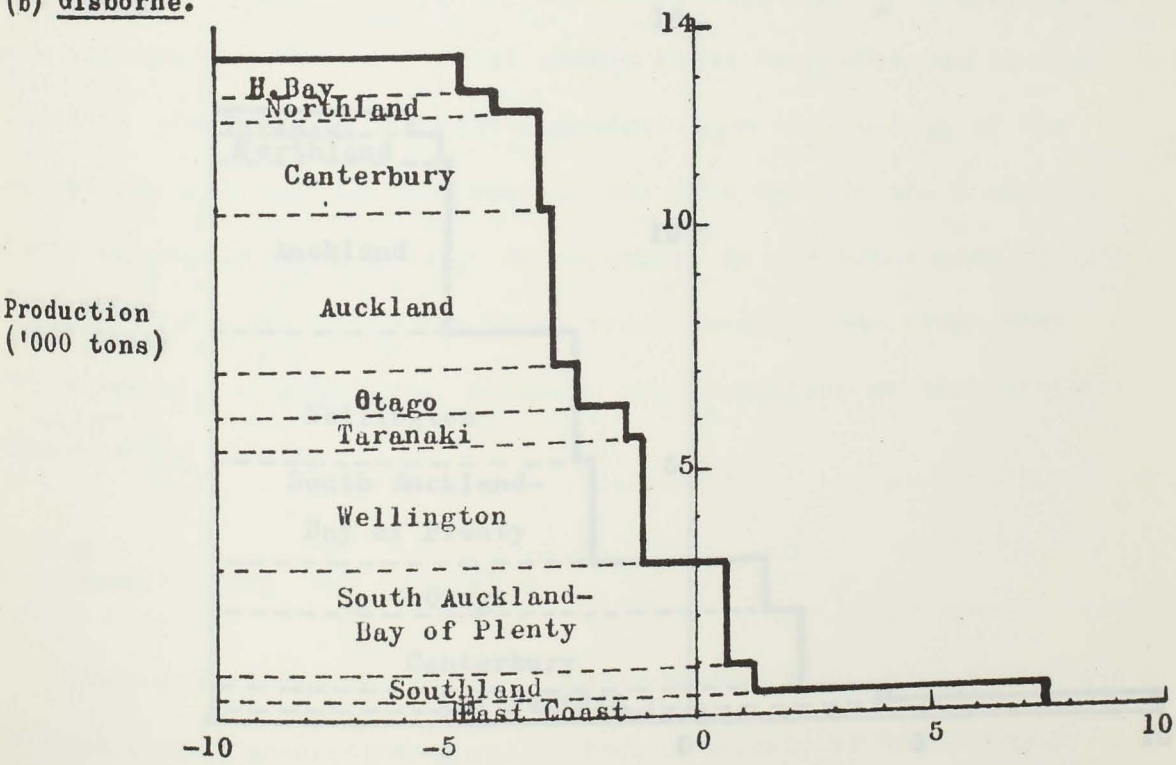
29. The full stability information is available in Appendix V.

Fig. 5. Volume/Cost Stability Graphs: Processor's Viewpoint.

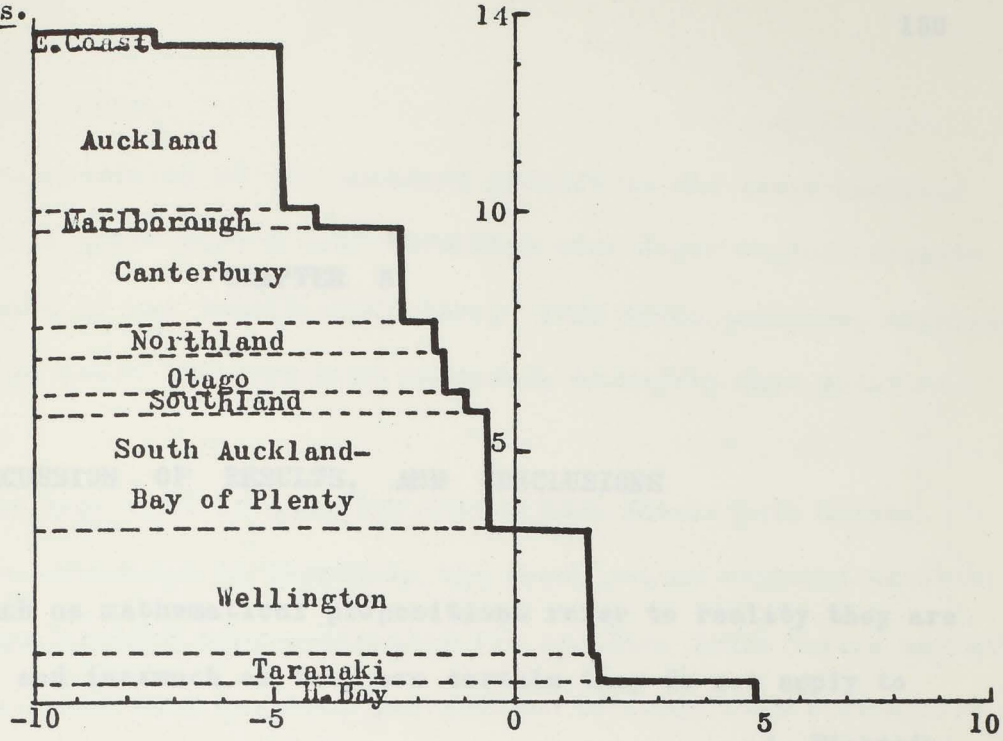
(a) Auckland.



(b) Gisborne.

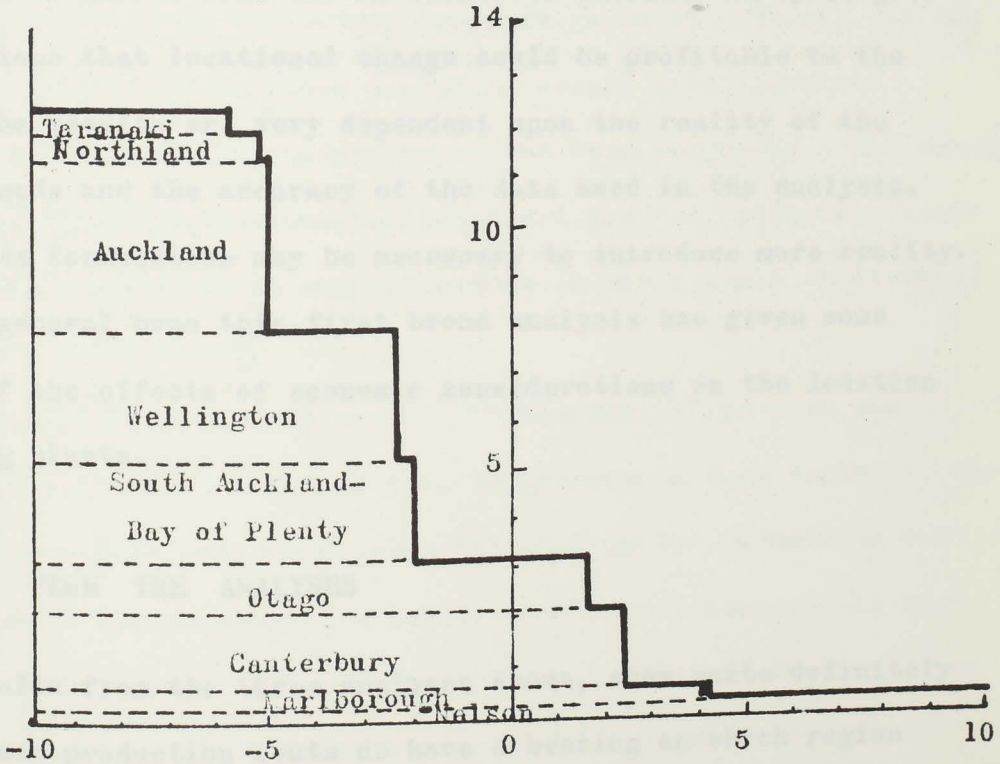


(c) Hastings.



Change in Cost of Supply (£/ton)

(d) Nelson.



Change in Cost of Supply (£/ton)

CHAPTER 5

DISCUSSION OF RESULTS, AND CONCLUSIONS

"Inasmuch as mathematical propositions refer to reality they are not certain, and inasmuch as they are certain they do not apply to reality."

A. Einstein.

The results gained from the mathematical location analysis give some indications that locational change could be profitable to the industry. The results are very dependent upon the reality of the assumptions made and the accuracy of the data used in the analysis. A more complex formulation may be necessary to introduce more reality. However, in general even this first broad analysis has given some indication of the effects of economic considerations on the location of processing plants.

5.1 RESULTS FROM THE ANALYSES

The results from the three analyses above, show quite definitely that raw tomato production costs do have a bearing on which region should supply each regional market if the industry or the processor

is to minimize costs.

(a) The proximity of the Auckland growers to the large Auckland regional market gives them a cost advantage over other regions despite their high cost of raw tomato production. From these analyses, Auckland should grow at least the 3384 tons necessary to supply this Auckland market.

(b) The high cost of shipping across Cook Strait puts Nelson growers at an advantage in supplying the South Island regional markets. This advantage becomes less pronounced for southern South Island markets as transport costs from Gisborne are similar to those from Nelson over this distance.

If industry costs were to be minimized, the tonnages grown in each region would differ from the actual tonnages grown in 1964/65. Nelson and Auckland, according to these analyses, should grow increased tonnages rather than cease production as has been suggested. One should note, however, that the increase in transport and raw production costs due to ceasing production in any one region is but a small fraction of the industry's total costs.

From the point of view of a single processor, faced with the present price per ton of tomatoes paid to growers in each district, the minimum-cost location pattern differs little from the industry's normative pattern. The only changes in the pattern are because the cost of tomatoes from Gisborne is reduced relative to that of tomatoes from Hastings and Nelson. Thus the processor would have Gisborne supply South Auckland - Bay of Plenty and Southland, whereas Hastings and Nelson

supplied them before. Nelson and Auckland would still produce more than they did in 1964/65 if the processor was to minimize his costs.

The important aspect of all the results is that the tonnages produced in each region are very unstable to cost changes. Processing costs make up a large fraction of the total costs of supplying from any region. For these reasons, a relatively small percentage change in processing cost in a region would overcome all the cost advantages or disadvantages which it enjoyed due to its production efficiency or its distance from regional markets.

5.2 EFFECTS OF ASSUMPTIONS ON THE RESULTS

In this exploratory analysis, comprehensive data was not available and so some assumptions were made. The likely effects of relaxing these assumptions are now considered.

5.2.1 Raw Production Assumptions

(1) Raw tomato production takes place only in South Auckland, Gisborne, Hastings and Nelson.

This assumption is probably not very damaging to the solution, although economic production may be possible around Christchurch or in the former potato-growing districts of South Canterbury.[1] These regions would experience quite large transport cost advantages in supplying Canterbury and the southern South Island.

1. Tomatoes have very similar soil requirements to potatoes, but the former require warmer, dry summers for ripening of fruit. South Canterbury may fulfil these requirements.

(2) Raw tomato production costs per ton are single-valued with respect to total tonnage grown in each region.

This assumption is unlikely to hold. South Auckland tomatoes are grown in an area which is quite intensively cropped and so any increased production would probably be more costly. If the tonnage grown in Gisborne or Hastings was reduced, then costs per ton would probably decline as less efficient growers were forced out. If production increased much above the 1964/65 level, costs could well rise, because further production would be grown further from the factories - possibly as far away as Waipukurau from the Hastings factories. On the other hand, if tonnage increased in Nelson and this was accompanied by advice and contract services from the processors as happened in Gisborne, then tomatoes may be grown by fewer, large growers and thus be produced more efficiently than at present.

The economies of scale of raw production could thus affect the solution, probably by increasing normative supply from Hastings and/or Gisborne to the northern North Island.

5.2.2 Raw Product Transport

It is assumed that no inter-regional transport of raw tomatoes takes place. Because of the perishable nature of raw tomatoes it is unlikely that it would pay to transport them from one producing to another processing region unless the former had a large raw production cost advantage, but had a capacity constraint on its local processing plant. A wide difference in processing cost could make this worthwhile.

It may be possible to transport tomatoes as an intermediate product in bulk - pulp or puree - from a remote producing region. The further ingredients of the final product could be added at the market. Again, this would depend on relative processing costs in the producing and consuming regions.

5.2.3. Tomato Processing Cost Assumptions

(1) Tomato processing costs per ton are single-valued with respect to volume processed (for individual final products or for all products).

(2) There are no processing cost differentials between processing regions.

(3) Processing is feasible only in the present processing regions.

These assumptions are very sweeping and the relaxation of any of them could be expected to have a profound effect on the optimum location pattern. This study developed organically from an investigation of tomato production, and it was thus orientated towards the location effects of raw tomato production costs and the distance of the production from the main markets. The instability of the solutions shows that production and transport costs do not have an over-riding effect on location, and that processing characteristics of different regions could be extremely important. The next logical step in the development of this study would be a detailed study of regional processing, including all the institutional and technical characteristics listed in Chapter 2. Of particular importance would be the technical efficiency, capacity and

utilization of present plants and the efficiency, capacity and cost of new plants. From this information, and from labour costs in each region, economies of scale curves could be drawn up.

If a single production function was assumed to apply to all regions and if labour costs were the same in all regions, then from the processor's point of view, it is interesting to note that the normative pattern allowing for economies of scale is likely to be similar to the normative pattern with single-valued processing costs. This is because the level of processing is similar in all regions. Once regional cost differentials were introduced however, it would be most important to also take account of scale economies.

5.2.4 Final Product Transport

(1) It was assumed that only present forms of final product transport were possible.

In fact, in the near future it is likely that New Zealand transport industry will undergo change. One specific development which has been mooted is the use of tugs and barges for very low-cost coastal shipping. The introduction of such a service could reduce the cost especially of long hauls, and it is likely that Nelson would have less advantage in supplying southern South Island markets. In general, however, the ranking of regions would change little, and the transport cost differentials would probably be reduced, so that the final solution would be even less stable and location would exert even less influence on the size of processing plants.

5.2.5 Regional Demand

(1) It was assumed that the quantity demanded in each region was constant irrespective of price.

This is obviously incorrect, and it would also be incorrect to assume that the same demand curve applies to all individual regions, because consuming Areas demanded different quantities per capita despite the similar retail prices throughout New Zealand. The existence of this price levelling throughout the country would enable one to draw up a single, aggregated demand curve with total New Zealand demand expressed as a function of the levelled price. The profit-maximizing volume would be obtained by equating the marginal revenue from this curve with a composite marginal cost figure made up of fractions of a ton consigned along each of the solution supply routes. Because of price-levelling in the past, however, very little information would be available for more meaningful historical individual regional demand curves. The true spatial equilibrium pattern of production, transport and consumption could well be quite different from the activity analysis solutions described above.

5.3 POSSIBLE EXPANSIONS OF THE ACTIVITY ANALYSIS MODEL

The present model, with improved data especially for the processing function adequately describes the effects of the spatial dimension on the long-run normative location and size of tomato processing plants. The solution obtained is likely to differ widely from the actual

production pattern in respect of the total capacities of different processing plants, and of the final product-mixes processed by these plants. It would thus be desirable to expand the specification of the model to include the form and the time dimensions to study the mechanisms of change from the present actual, to the normative pattern.

5.3.1 Inclusion of the Form Dimension

Factors which one would want to include are:

(1) The requirement of different final tomato products for critical processing inputs.

Most tomato products require the tomatoes to be dehydrated and reduced to some form of pulp or puree. They can then be stored as this intermediate product and processed further to their final product during the slack season. Canned whole tomatoes, and tomato juice are both made directly from the raw product and thus require a relatively high degree of processing during the harvesting season. Technical constraints on the fraction of tomatoes which can be converted to the latter products may be necessary.

(2) The economies of processing long runs of individual products in the different regional plants.

Although the detailed information is confidential, it is known that one of the processing companies carries out a degree of regional specialization in the processing of different final products. One would thus assume that economies of scale do occur, but it would be useful to measure these and allow them to interact with other locating factors in deriving the normative pattern.

(3) The inclusion of intermediate products.

Processing could be separated into different stages to allow the transport of intermediate products between regions depending upon the relevant weight-losses or -gains at each stage, and the relative processing costs.

All of these factors could be taken account of by creating separate activities for the processing of each final product in each processing region. Constraints could be formulated to allow for individual product requirements and economies of scale could be handled iteratively. These formulations would require a larger computer capacity than was available to the author, however Lincoln College now has an IBM 1130 which is capable of handling probably 1000 activities in a linear programming problem of this type, and processing companies are acquiring even larger computers. There appears to be no barrier to including the form dimension in the tomato processing location problem.

In fact, however, tomato processing takes place in multi-crop processing plants, and it may be necessary to expand the formulation to explicitly allow for mutually exclusive or independent requirements of different crops for the available processing capacity and labour availability.

5.3.2. Inclusion of the Time Dimension

In any normative analysis, the main problem of interpretation is in deriving a feasible, yet optimum path of development from the present, actual pattern of production to the desired normative pattern. This problem occurs in any expanding industry and is further complicated by

re-location of the various sections of the industry.

A tool which promises to be very useful in analysing problems of this nature is Day's Recursive Programming. This method solves a sequential chain of recurring linear programming problems in which structural components of each year's problem depend upon the solution for the preceding year. The important aspect of the method is that it specifically allows for the flexibility of the present industry to alter supply between seasons and also allows for new investment to take place at less than some maximum rate. It is the difficulty of measurement of the relevant flexibility coefficients and investment coefficients which is likely to limit the application of this method.

A more complete study of the capital structure and utilization of present plants, the salvage value of redundant plants, and technical aspects and economics of plant expansion would be necessary, and this could possibly be achieved using methods similar to Berman's analysis of the transport industry. In fact much of the information required would already be available to the processing companies who are constantly renewing and expanding their plants. Regarding the flexibility of growers to expand regional production, in existing regions some idea of acreage flexibilities could be gained from studying the extent to which companies have been able to vary acreages over recent years. In the case of new, or greatly expanded growing regions, some study of the innovation and rate of adoption of tomato growing may be necessary.

In any case, both Day's and Berman's analyses are based upon linear programming and so it would be possible to include the formulation

contained in this thesis as a sub-matrix within a more complex space-time or space-time-form linear programming model.

5.4 CONCLUSIONS

The work in this thesis allows one to make some general conclusions as to the use of location economics in decisions about the expansion and re-location of agricultural processing industries.

(1) Physical, institutional and economic factors can all affect the final location decision reached.

(2) There are already some methods available, and further methods can be developed to measure the actual economic effects on processing plant expansion and re-location.

(3) The linear programming algorithm is very flexible and can be used to solve a simple, basic specification of the problem.

(4) This initial analysis serves to pinpoint critical factors which will affect the optimum processing plant location.

(5) Those factors which are critical to the solution can be studied more fully and the relevant sections of the basic model expanded to allow for their effect on the normative location solution.

(6) The basic formulation as specified in this thesis could be used as a cost-minimizing sub-routine within a much more complex model of the industry. This model could include detailed analysis of the time and form dimensions and could even include demand functions in a general price equilibrium model of the industry.

SUMMARY

(1) The author introduced the study by outlining the importance of agricultural processing to New Zealand and by suggesting that in the future, processing will take place in greater volume, in greater depth and in greater breadth.

(2) Relevant details of the New Zealand tomato processing industry were outlined. The various technical and institutional location factors were listed, and the actual economic location analysis to be attempted was specified.

(3) The historical and logical development of a theory to describe the location of agriculture, processing and extractive industries was traced.

(4) Recently-derived methods of location analysis were described and discussed in order of the increasing complexity of problems they were capable of analysing.

(5) The New Zealand tomato location problem was re-stated and a general linear programming model was developed to analyse the problem. The general formulation was modified to describe the specific problem.

(6) Data already collected by the author was referred to, and data for other sections of the analysis was collected and converted into a form capable of use in the model.

(7) The model was solved by the computer, using the above data, and a normative location pattern was arrived at. Stability of this normative solution was explored.

(8) A technical constraint was used to limit the total acreages grown in Nelson and Auckland, and the institutional rigidity of raw tomato prices was introduced to solve the problem from the processor's viewpoint.

(9) The implications of results from all analyses were discussed and some suggestions made for the improvement of data and the expansion of the basic model to include more reality.

(10) Finally, some general conclusions were drawn about the use of location economics in general and linear programming models in particular, as an aid in making decisions about the future location of agricultural processing industries.

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APPENDIX I. BASIC DATA

Table 1. New Zealand Fruit and Vegetable Preserving Industry. Output by Products 1963/64.

Item No.		Products	Units	Quantity
1	FRUIT:	Canned pears	(cwt.)	34,679
2		" peaches	"	88,154
3		Other canned fruit	"	151,595
4		Pulped - for sale as such	"	2,978
5		Quick frozen	"	1,155
6		Natural juices	(gals.)	456,054
7	VEGETABLES:	Canned beans in sauce	(cwt.)	76,808
8		" peas	"	89,604
9		" green beans	"	28,717
10		Other canned vegetables	"	99,029
11		Quick frozen peas	"	199,216
12		" " beans	"	43,722
13		" " other	"	25,548
14	SOUPS		(gal.)	328,770
15	TOMATOES:	Canned whole or halves	(cwt.)	14,974
16		Soup	(gal.)	501,423
17		Pickles	"	147,467
18		Chutney and Relish	"	18,780
19	SAUCES:	Tomato	"	642,683
20		Worcester	"	110,749
21	JAMS and JELLIES		(cwt.)	74,611
22	CANNED SPAGHETTI		"	90,223
23	CANNED MEAT		"	197

Tonnage of all products = $\frac{\text{Items (1) (5) (7) (13) (15) (21) (23)}}{20}$

$+ \frac{\text{Items (6) (14) (16) (20) x 11 lb./gal.}}{2240}$

$= \frac{1,021,210}{20} + \frac{2,205,926 \times 11}{2240}$

$= 51,060.5 + 10,832.7$

$= 61,893.2$ tons

Non-canned or bottled products (tons) = $\frac{\text{Items (4) (5) (11) (13)}}{20}$

$= \frac{272,619}{20}$

$= 13,630.9$ tons

Canned or bottled products $= (61,893.2 - 13,630.9)$

$= 48,262.3$ tons

Table 2. Retail Uptake of Processed Tomato Products by Three Main Areas 1962/63 - 1965/66

Product	Year	Area I (tons)	II (tons)	III (tons)	N.Z. (tons)
Canned Spaghetti	1962/63	1737.0	858.0	808.3	3403.3
	63/64	1924.4	887.3	878.0	3683.7
	64/65	1964.6	995.0	910.7	3870.3
	65/66	1934.3	1148.3	984.8	4067.2
					15024.5
Baked Beans	1962/63	1483.4	669.6	722.7	2875.7
	63/64	1587.8	658.9	758.9	3005.6
	64/65	1629.4	791.0	804.0	3224.3
	65/66	1753.9	935.2	890.1	3579.2
					12684.8
Tomato Soup	1962/63	825.8	465.1	466.5	1757.5
	63/64	867.8	486.6	516.0	1870.4
	64/65	867.8	596.4	580.3	2044.5
	65/66	949.0	678.1	641.9	2269.0
					7941.4
Tomato Sauce	1962/63	888.5	535.3	534.3	1958.2
	63/64	973.0	531.5	564.5	2069.2
	64/65	1136.6	626.3	646.2	2409.0
	65/66	1103.7	692.7	666.3	2462.6
					8899.0
Canned Tomatoes		n.a.	n.a.	n.a.	

Source: A.C. Nielsen, Pty, Ltd.

Table 3. New Zealand Production of Processed Tomato Products 1962 - 1965

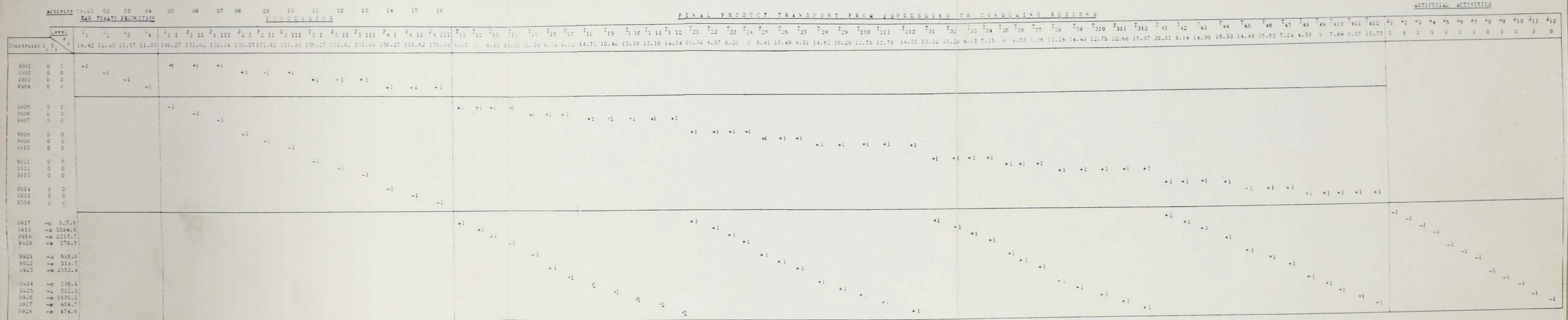
Product	1962	1963	1964	1965	Total
Canned Spaghetti (tons)	4311	4422	4314	3804	16,851
Baked Beans (tons)	3324	3534	4038	3468	14,364
Tomato Soup ('000 gals.)	566	457	498	582	
" " (tons)	2653	2142	2335	2728	9,858
Tomato Sauce ('000 gals.)	540	572	664	832	
" " (tons)	2652	2809	3261	4086	12,808
Canned Tomatoes (tons)	1710	888	2801	1443	6,842

Note: 1 gallon of tomato soup weighs approximately 10.5 lbs.,
1000 " " " " " " 4.688 tons.

1 gallon of tomato sauce weighs approximately 11.0 lbs.,
1000 " " " " " " 4.911 tons.

Source: Tables 29 and 30, Monthly Abstract of Statistics, 31 Jan., 1966. N.Z. Dept. of Statistics, Wellington. 1966.

APPENDIX II. THE INITIAL SIMPLEX TABLEAU



APPENDIX III. NORMATIVE SOLUTION: INDUSTRY VIEWPOINT

Table 1. Basis Real Activities - Levels and Cost Limits.

Activity	Cost (£/ton)	Activity Level (tons)	Upper Cost Limit (£/ton)	Limiting Activity	Lower Cost Limit (£/ton)	Limiting Activity
001	18.42	3912.6	18.82	041	15.44	019
002	12.43	275.5	18.82	044	12.10	031
003	11.67	5901.8	12.00	031	11.27	041
004	11.95	3622.8	12.73	010	10.49	015
005	136.27	3912.6	136.67	041	133.29	019
008	136.27	275.5	142.66	044	135.94	031
011	136.27	2217.7	136.60	031	135.87	041
012	131.62	3684.1	133.08	015	-9.73*	9909
016	131.04	3622.8	131.82	010	-10.63*	9913
017	6.07	527.8	6.47	041	-154.69**	065
018	0.00	3384.8	2.58	030	-2.98**	055
022	8.94	0.0	12.11	029	1.80	006
028	14.34	0.0	18.34	027	7.28	007
032	0.00	275.5	6.39	044	-148.70**	068
034	10.49	0.0	12.00	035	7.79	009
040	14.05	0.0	17.97	039	13.27	010
043	8.63	2217.7	8.96	031	-147.94**	067
045	0.00	616.0	4.97	033	-143.29**	069
046	8.55	514.7	11.25	009	7.04	035
047	6.06	2553.4	7.52	015	0.17	058
052	15.07	0.0	17.86	051	14.03	013
054	9.14	0.0	12.12	055	6.47	014
059	7.24	0.0	13.13	058	5.78	015
060	4.39	136.9	9.86	048	-142.99**	072
061	0.00	312.3	13.11	049	-142.99**	073
062	7.89	1839.3	11.43	050	-142.99**	074
063	8.57	859.7	11.36	051	-142.99**	075
064	13.75	474.6	14.53	010	10.96**	051

* These limits have little meaning because the processing activities are supplying all the consumption in the respective demand Areas, therefore a reduction in the cost of these activities will not increase their levels unless the total cost becomes a negative.

** Similar reasoning to * above, except that in the present cases, these indicate the total costs of production and processing.

Table 2. Non-Basis Real Activities.

Activity	Cost (£/ton)	Shadow Price $(Z_j - c_j)$ (£/ton)	Upper Limit on Activity (tons)	Limiting Basis Activity
006	131.62	7.14	514.7	046
007	131.04	7.06	474.6	064
009	131.62	2.70	514.7	046
010	131.04	0.78	474.6	064
013	131.04	1.04	474.6	064
014	136.27	2.67	3384.8	018
015	131.62	1.46	2553.4	047
019	4.86	2.98	2217.7	043
020	10.31	16.30	275.5	008
021	10.19	17.06	616.0	045
023	9.62	10.31	2553.4	047
024	14.73	16.81	136.9	060
025	16.40	22.85	312.3	061
026	13.16	11.74	1839.3	062
027	13.16	11.06	859.7	063
029	16.04	3.98	527.8	017
030	8.57	2.58	3384.8	018
031	8.20	0.33	2217.7	043
033	6.91	4.97	616.0	045
035	9.51	4.21	2553.4	047
036	14.62	10.99	136.9	060
037	16.29	16.77	312.3	061
038	12.75	8.88	1839.3	062
039	12.79	4.69	859.7	063
041	13.22	0.40	527.8	017
042	10.22	3.45	3384.8	018
044	7.15	6.39	275.5	008
048	11.18	5.47	136.9	060
049	14.43	13.11	312.3	061
050	12.75	3.54	1839.3	062
051	12.68	2.79	859.7	063
053	20.51	7.97	527.8	017
055	14.00	5.65	2217.7	043
056	16.93	16.45	275.5	032
057	14.49	14.77	616.0	045
058	15.62	7.35	514.7	046

Table 3. Non-Basis Disposal Activities

Constraint	$Z_j - c_j$	Shadow Price $999.9-(Z_j-c_j)$	Basis Activity	Level of Basis Activity
9917	839.239	160.76	017	527.8
9918	845.309	154.69	018	3384.8
9919	843.429	156.57	043	2217.7
9920	851.299	148.70	008	275.5
9921	856.709	143.29	045	616.0
9922	848.159	151.84	046	514.7
9923	850.649	149.35	047	2553.4
9924	852.619	147.38	060	136.9
9925	857.009	142.99	061	312.3
9926	849.119	150.88	062	1839.3
9927	848.439	151.56	063	859.7
9928	843.259	156.74	064	474.6

APPENDIX IV. NORMATIVE SOLUTION WITH ACREAGE CONSTRAINTS (NELSON AND AUCKLAND)

Activity	Cost (£/ton)	Activity Level (tons)	Upper Cost Limit (£/ton)	Limiting Activity	Lower Cost Limit (£/ton)	Limiting Activity
001	18.42	3614.0	18.82	9929	9999.99	000
002	12.43	750.1	12.69	052	12.10	031
003	11.67	6772.6	12.00	031	11.41	052
004	11.95	2576.0	15.78	9930	9999.99	000
005	136.27	3614.0	136.67	9929	9999.99	000
008	136.27	275.5	142.66	044	135.94	031
010	131.04	476.6	131.30	052	130.17	039
011	136.27	2516.3	136.60	031	135.87	9929
012	131.62	3684.1	134.32	009	9.73	9909
013	131.04	572.2	131.91	039	130.78	052
016	131.04	2576.0	134.87	9930	9999.99	000
017	6.07	229.2	6.47	9929	3.89	030
018	0.00	3384.8	2.18	030	3.38	055
022	8.94	0.0	12.11	029	1.40	006
028	14.34	0.0	15.29	027	7.66	007
032	0.00	275.5	6.39	044	148.70	068
034	10.49	0.0	12.00	035	7.79	009
040	14.05	474.6	14.31	052	13.10	027
041	13.22	298.6	15.40	030	12.82	9929
043	8.63	2217.7	8.96	031	147.94	067
045	0.00	616.0	4.97	033	143.29	069
046	8.55	514.7	11.25	009	7.04	035
047	6.06	2553.4	7.57	035	0.17	058
051	12.68	572.2	13.43	050	9.63	064
054	9.14	0.0	12.52	055	3.04	014
059	7.24	0.0	13.13	058	1.95	015
060	4.39	136.9	7.07	048	146.82	072
061	0.00	312.3	10.32	049	146.82	073
062	7.89	1839.3	8.64	050	147.82	074
063	8.57	287.5	11.62	064	7.82	050

APPENDIX V. NORMATIVE SOLUTION: PROCESSOR'S VIEWPOINT

Activity	Cost (£/ton)	Activity Level (tons)	Upper Cost Limit (£/ton)	Limiting Activity	Upper Cost Limit (£/ton)	Limiting Activity
001	20.00	3912.6	21.82	041	17.84	019
002	14.50	2967.8	15.11	043	12.91	051
003	14.67	3684.1	16.44	009	14.07	043
004	16.00	3148.2	17.59	051	14.80	064
005	136.27	3912.6	138.09	041	134.11	019
008	136.27	2493.2	136.87	043	133.20	030
010	131.04	474.6	132.23	013	129.45	051
011	136.27	0.0	9999.99	000	135.67	043
012	131.62	3684.1	1333.90	009	-12.73	9909
016	131.04	3148.2	132.63	051	129.84	064
017	6.07	527.8	7.89	041	-156.27	065
018	0.00	3384.8	3.07	030	2.16	055
022	8.94	0.0	12.11	029	3.22	006
028	14.34	0.0	17.14	027	8.55	007
031	8.20	2217.7	8.80	043	-150.77	067
032	0.00	275.5	7.32	044	-150.77	068
034	10.49	0.0	12.00	035	8.72	009
040	14.05	474.6	15.24	013	12.46	051
045	0.00	616.0	4.97	033	-146.29	069
046	8.55	514.7	10.32	009	7.04	035
047	6.06	2553.4	7.57	035	0.17	058
052	15.07	0.0	16.66	051	13.88	013
054	9.14	0.0	11.30	055	4.00	014
059	7.24	0.0	13.13	058	4.73	015
060	4.39	136.9	8.66	048	-147.04	072
061	0.00	312.3	11.91	049	-147.04	073
062	7.89	1839.3	10.23	050	-147.04	074
063	8.57	859.7	10.16	051	-147.04	075