CENTRE FOR
COMPUTING AND BIOMETRICS

Mathematical Models for Computer Generation of Knotty Boards and Panels

Huawu Liu and Don Kulasiri

Research Report No: 97/01
January 1997

RESEARCH REPORT

ISSN 1173-8405

LINCOLN
UNIVERSITY
Te Whare Wānaka O Aoraki

Centre for Computing and Biometrics

The Centre for Computing and Biometrics (CCB) has both an academic (teaching and research) role and a computer services role. The academic section teaches subjects leading to a Bachelor of Applied Computing degree and a computing major in the BCM degree. In addition it contributes computing, statistics and mathematics subjects to a wide range of other Lincoln University degrees. The CCB is also strongly involved in postgraduate teaching leading to honours, masters and PhD degrees. The department has active research interests in modelling and simulation, applied statistics and statistical consulting, end user computing, computer assisted learning, networking, geometric modelling, visualisation, databases and information sharing.

The Computer Services section provides and supports the computer facilities used throughout Lincoln University for teaching, research and administration. It is also responsible for the telecommunications services of the University.

Research Report Editors

Copyright

Correspondence

This paper represents work to date and may not necessarily form the basis for the authors' final conclusions relating to this topic. It is likely, however, that the paper will appear in some form in a journal or in conference proceedings in the near future. The authors would be pleased to receive correspondence in connection with any of the issues raised in this paper. Please contact the authors either by email or by writing to the address below.

Any correspondence concerning the series should be sent to:

The Editor
Centre for Computing and Biometrics
PO Box 84
Lincoln University
Canterbury, NEW ZEALAND

Email: computing@lincoln.ac.nz

MATHEMATICAL MODELS FOR COMPUTER GENERATION OF KNOTTY BOARDS AND PANELS

Huawu Liu
Don Kulasiri

Centre for Computing and Biometrics
Lincoln University, New Zealand
Email: Liuh1@whio.lincoln.ac.nz

ABSTRACT

Most wood boards contain knots. Knots are known as the most weakening factor affecting mechanical properties and the major cause of drying deformation. Some useful mathematical models were developed in this article for later computer simulation on the behaviours of knotty boards in service and in processes. Geometrical features of knot wood including growth angle, position and three dimensional shape were modelled. All sawn patterns and cutting positions were generated. Formulae and modelling steps were introduced and geometrical features of knots were discussed in detail using projective geometry.

Keywords: knotty boards, timber drying, sawn pattern, mathematical modeling, computer simulation.

NOMENCLATURE

a	semi-minor axis coefficient of elliptic cone
a_0	length of the semi-minor axis of the base ellipse of knot wood
A, B, B_1, C, C_1, E, F, F_1	coefficients of cutting-plane equations
b	semi-major axis coefficient of elliptic cone
b_0	length of the semi-major axis of the base ellipse of knot wood
D	distance from the lower end of board to the lower end of log
D_1	distance of a thickness surfaces of board to the axis of log
D_2	distance of a width surfaces of board to the axis of log
H	hight of the vertex of knot wood
H_0	hight of the centre of the base of knot wood
k	enlargement scale
L	axis of global Cartesian coordinate system, along the longitudinal direction of log
L_0	length of board
o, $\tilde{o}$	origin of local coordinate systems
O	origin of the global Cartesian coordinate system
R	axis of the global Cartesian coordinate system, ROL crosses the major symmetric plan of knot wood, which is determined by the axes of knot wood and log
r_0	distance from the centre of base ellipse to the axis of log
r_1	distance from the knife to axis of log during veneer peeled
T	axis of the global Cartesian coordinate the system
T_0	thickness of board
x, y, $\tilde{x}$, $\tilde{y}$, z	axes of the local Cartesian coordinate systems
W_0	width of board

α	angle of the thickness side of board to the major symmetric plane of knot wood
Φ	angle between the diameter of knot on the log and the axis of log
θ	growth angle

1. INTRODUCTION

Most sawn boards contain knots and they were reported to be the most weakening factor of wood mechanical properties in service and the major cause of drying deformation (Grant et al., 1984; Walker, 1993). Knot size and positions directly affect the wood strength. Failure generally occurs in the surrounding area of knots during testing the mechanical properties of knotty boards (Zandbergs and Smith, 1987; Cramer and Goodman, 1985). During seasoning, the maximum warping was found below the oval part of margin or arris knots as shown in Figure 1, when typical dimensions of knots were greater than the half of the board where the oval presented. Moreover, the most severe check in a knotty board was found to be knot check, which ran normally along the symmetric axes of the broken knots (Liu et al., 1996).

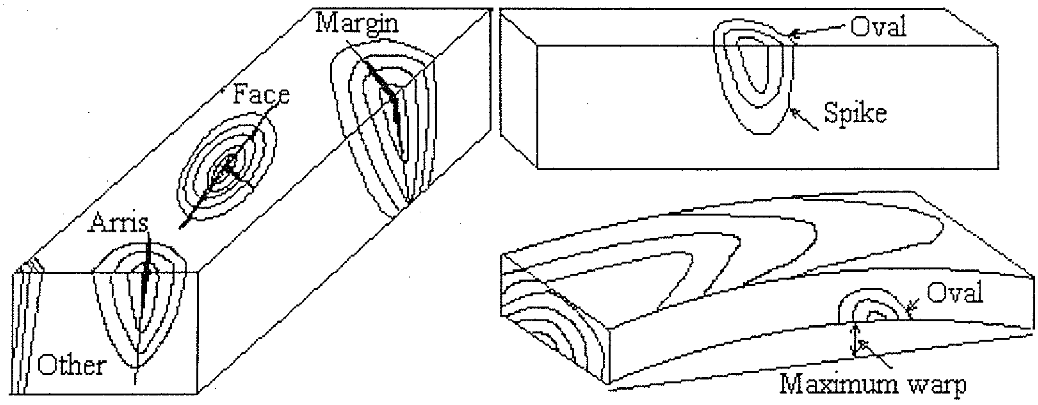

FIG. 1. Knot shapes, types and associated drying defects

Due to the dominant influences of knots on mechanical properties and drying deformation, finite element analysis has been employed to study the stresses associated with knots. In several previous studies, knots were modelled as circles (Zandbergs and Smith, 1987; Cramer and Goodman, 1983). Their study can be further extended if all shapes and positions of knots are included.

Considering that the sawn patterns and the knot geometry can lead to wide variations in physical properties of boards, practical mathematical models for generating knotty boards and panels should have the following properties:

(1) All geometrical properties of knot wood should be taken into account, including growth angle, location and 3-dimensional shape;

(2) The boards dimensions, thickness, width and length, should be included;

(3) All sawn patterns such as flat sawn, quarter sawn, intermediate sawn and veneer peeled need to be included;

(4) Formulae of knot shapes should be given and geometrical features of knots in boards thus can be studied.

(5) Parameters of equations can easily be measured in practice.

Samson (1993), after a thorough literature review, found models satisfying the above criteria were not available. One hundred and twenty radiata pine boards were observed during this study. The information of previous studies was obtained from the Forest Research Institute of New Zealand (Fenton, 1967; Kinimonth and Whitehouse, 1991).

The objective of this study is to develop relevant mathematical models to aid in later computer simulation of the behaviour of knotty boards in drying process and in service.

2. MODELLING GEOMETRY OF KNOT WOOD

Nature of knot

The base of a branch embedded in a stem is called knot wood, which is a horn or cone-like solid with the vertex at the pith (Pugel, 1980). Knot wood may be approximately described as an elliptic cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z^2 \qquad [1]$$

where {x, y, z; o} are the local coordinate system, z is along the symmetric axis of knot wood, and the origin is at the vertex of knot wood (Figure 2). There are two natural symmetrical axes in the base ellipse of knot wood. The minor axis is along the horizontal, whereas the major axis is along the vertical directions of the tree (Figure 2). The elliptic cross-section of a branch produces lease wind resistance and allows it to bear more weight, which can be seen from the formula of moment of intertia of solid beam with elliptic cross section given by Pilkey (1994):

$$\mathbf{I}=\frac{\pi}{4}a_0b_0^3 \qquad [2]$$

where a_0 is the semi-minor axis of ellipse in the horizontal direction and b_0 is the semi-major axis in the vertical direction of ellipse. The strength of the branch is proportional to **I** and thus proportional to a_0 and b_0^3. The two symmetric axes of base ellipse of knot wood are an important geometric feature and will significantly reduce the complexity of formulae for later modelling.

To prevent stress concentration, the intersection of branch and stem is filleted smoothly. The diameter of the branch varies there abnormally. Therefore, when measuring the axes of base ellipse of knot wood for calculating a and b in Equation [1], the position of measurement should be taken immediately under the bark.

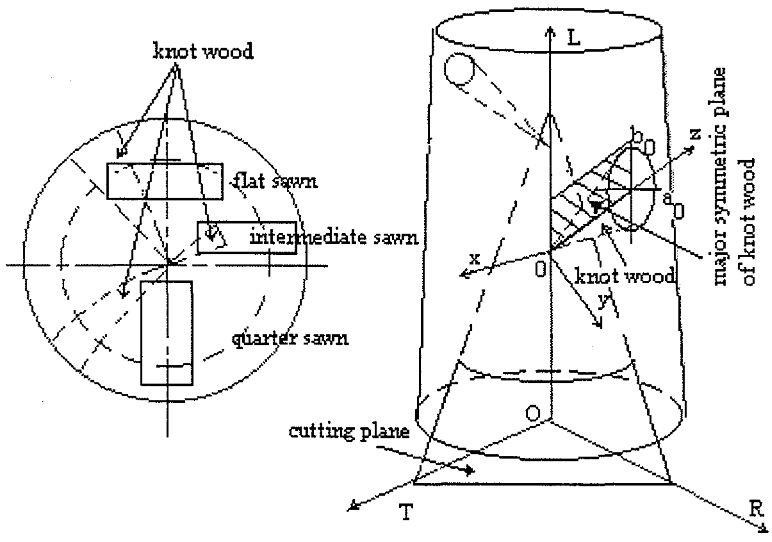

FIG. 2. Knot wood and sawn patterns

The remains of knot wood in sawn board are called knots, whose two-dimensional shapes are decided by the cutting angle of the cutting plane to the axis of knot wood. If the cutting plane is in the region between parallel to the lateral surface of knot wood and perpendicular to the axis of knot wood, then the knot is called an oval, otherwise it is called a spike (Figure 1). An oval can have closed elliptic or circular rings and all rings on spike are unclosed. However, knot shapes have not been rigorously defined. The long narrow elliptic knots are called spikes in practice as well.

Knot types refer to the three dimensional shapes of knots and position in boards (Figure 1). Knot types are classified as face, margin, arris and other. The boundary of a face knot is entirely inside the surface. An arris intersects an edge of a board and consists of a partial oval and a spike, or sometimes two ovals. A margin intersects the edges of a board and appears on three adjacent surfaces of this. A margin knot contains two ovals and one spike, sometimes one oval and two spikes. The other type is defined as the knots excluded from the descriptions of face, margin and arris.

For the investigation of knot shapes in sawn board, a cutting plane (Figure 2) is given in the local Cartesian coordinate system {x, y, z; o} of knot wood as

$$z = Ax + By + C \qquad [3]$$

Knot shapes as the intersection of a knot wood and the cutting plane can be obtained by substituting Equation [3] into [1]. The intersection now can be expressed by

$$(\frac{1}{a^2} - A^2)x^2 + (\frac{1}{b^2} - B^2)y^2 - 2ABxy - 2ACx - 2BCy - C^2 = 0 . \qquad [4]$$

Equation [4] is a typical two dimensional quadratic equation and represents all conical curves. Therefore, the two dimensional shapes including oval and spike can look like a circle, an ellipse, a parabola or a hyperbola depending upon the position of the cutting plane given by Equation [3].

3. MODELLING GEOMETRY OF KNOTS ON SAWN BOARDS AND PEELED PANELS

Modelling the geometrical properties of knot wood

In order to discuss the shapes of knots presented in boards, the Equation [1] must be transformed to a global coordinate system {T, R, L; O}, where O is at the centre on the bottom of a log, L is the axis of the log, ROL crosses the major symmetric plane of knot wood, which is determined by the axes of knot wood and log (Figure 2), T is perpendicular to L and R. In global coordinate system, Equation [1] can be transformed into

$$\frac{T^2}{a^2}+\frac{(R\cos\theta-L\sin\theta+H\sin\theta)^2}{b^2}=(R\sin\theta+T\cos\theta-H\cos\theta)^2 \qquad [5]$$

where H is the height of knot wood vertex in the log and θ ($0<\theta<\pi/2$) is the growth angle of knot to the axis of stem (Figure 3 (a)). The transformation from Equations [1] to [5] can be obtained by rotating local coordinate system{x, y, z; o} to {x', y', z'; o'} and then moving {x', y', z'; o'} to coincide {T, R, L; O}. The rotational transformation from {x, y, z; o} to {x', y', z'; o'} can be expressed by (Figure 3 (b)) the following equation:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \qquad [6]$$

And the translation from {x', y', z'; o'} to {T, R, L; O} can be described by the equation (Figure 3 (c)):

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & H\sin\theta \\ 0 & \sin\theta & \cos\theta & -H\cos\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} T \\ R \\ L \\ 1 \end{bmatrix} \qquad [7]$$

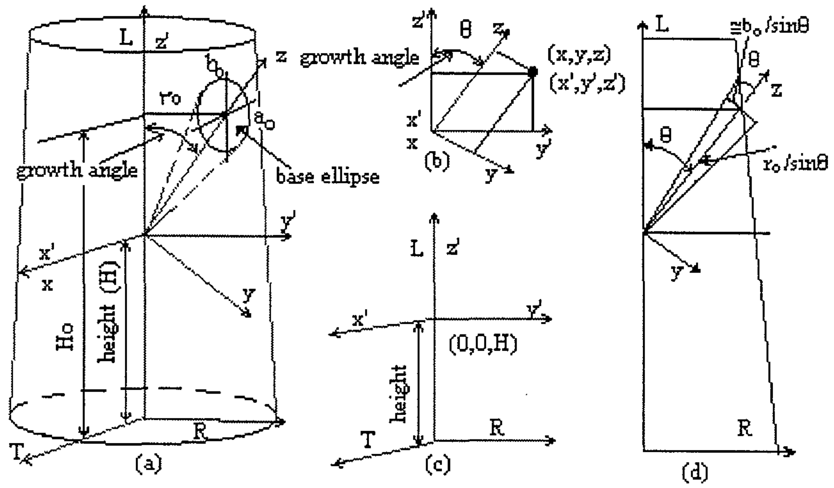

FIG. 3. Coordinate systems and knot wood properties

In practice, the base of a knot wood can more easily be measured than the semi-vertex angle of elliptic cone and thus the symmetrical axes of base ellipse are adopted as the parameters of knot wood. Assuming that a_0 and b_0 are the semi-minor and semi-major axes of the base ellipse immediately under the bark respectively, and that H_0 and r_0 denote the vertical and radial position of the centre of the base ellipse respectively (Figure 3 (a)), the parameters of Equation [5] can be replaced by the measurements as (Figure 3 (d))

$$H = H_0 - r_0/\tan(\theta) \qquad [8]$$

$$a = a_0 \sin(\theta)/r_0 \qquad [9]$$

$$b \cong b_0 \sin^2(\theta)/r_0 \qquad [10]$$

Equation [8] is immediately obtained using trigonometry. In order to obtain Equations [9] and [10], we know that :

1. The intersection of plane $z = r_0/\sin(\theta)$ and the elliptic cone [1] is an ellipse,
2. The major and minor axes of this intersection are on the two symmetric planes of the elliptic cone, and
3. The error of [10] is less than $b_0\sin(\theta)\sin2(\theta)/(2 r_0^2)$ and is negligible.

The above results are illustrated in Figure 3 clearly and will not be discussed in detail.

Equation [4] can now be rewritten as

$$\frac{T^2}{(a_0 \sin\theta / r_0)^2} + \frac{[R\cos\theta - L\sin\theta + \sin\theta(H_0 - r_0/\tan\theta)]^2}{(b_0 \sin^2\theta / r_0)^2}$$
$$= [R\sin\theta + L\cos\theta - \cos\theta(H_0 - r_0/\tan\theta)]^2 \qquad [11]$$

All geometrical properties of knot wood are now included in Equation [11].

Generating swan pattern and cutting positions

Note that the longitudinal direction of boards is parallel to the L axis, opposite surfaces of boards are parallel to each other, and adjacent surfaces are perpendicular. Therefore, the corresponding cutting planes (Figure 4) of the two ends of board can be given as

$$L= D \qquad [12]$$

$$L= D+ L_0 \qquad [13]$$

where D is the distance from the lower end of a board to the bottom of the log, and L_0 is the length of the studied board.

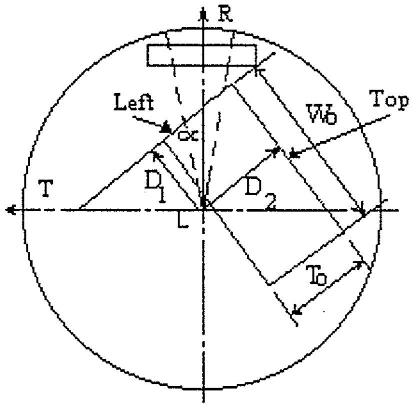

FIG. 4. Geometry of cutting planes

The cutting plane of side surfaces can be given as

$$T= AR+ B \qquad [14]$$

$$T= AR+ B_1 \qquad [15]$$

$$T=-\frac{1}{A}R+C \qquad [16]$$

$$T=-\frac{1}{A}R+C_1 \qquad [17]$$

where A, B, B_1, C and C_1 are coefficients of cutting planes and will be determined by measurements later. Equations [14], [15], [16] and [17] denote the left, right, top and bottom surfaces of board respectively in Figure 4, where the L axis is towards the plane of the paper.

Assuming that the cutting angle of the left side surface to the knot wood is α, which is also the planar angle of the major symmetric plane of knot wood to the cutting plane (Figure 4), we can have

$$A= -\tan \alpha \qquad [18]$$

Introducing T_0 and W_0 to represent the thickness and width of board during flat sawn and intermediate sawn, or the width and thickness during quarter sawn, D_1 and D_2 to denote the distances of the left surface and the top surface of board to the axis of log (Figure 4), the rest of the parameters of Equations [12] to [16] can be expressed by measurements as

$$B = D_1 / \cos\alpha \quad [19]$$

$$B_1 = (D_1 - W_0)/\cos\alpha \quad [20]$$

$$C = -D_2 / \sin\alpha \quad [21]$$

$$C_1 = -(D_2 - T_0)/\sin\alpha \quad [22]$$

Three dimensional geometrical features of knots in sawn boards

The fraction of knot area within a rectangle over the area transverse section of board (rectangle) is called the Knot Area Ratio (KAR), which is used in the grading rules to describe the knotty ratio of structural boards.

The remains of knot wood presented on a pair of adjacent surfaces of a board contain at least one oval. The adjacent surfaces are perpendicular to each other and the vertex angle of knot wood never exceeds 90^0. Hence it is impossible to cut two spikes on a pair of adjacent surfaces of a board. In other words, there is at least one oval in a pair of adjacent surfaces. Large oval knots are responsible for maximum warping during drying, which implies that large margin and arris knots are more likely to produce maximum warping.

The remains of knot wood presented on a pair of opposite surfaces of board are similar figures. One is the enlargement of another. The scale of enlargement can be obtained by mathematical calculation. Assume that A pair of cutting planes parallel to the axis of knot wood in the local coordinate system {x, y, z; o} as

$$y = Ex + F \quad [23]$$

$$y = Ex + F_1 \quad [24]$$

where E, F_1 and F are the coefficients of cutting planes. Substituting [23] and [24] into [1], the shapes of knots on cutting plane [24] and [25] can respectively be expressed by

$$\frac{(x+N^2EF)^2}{N^2E^2(1-F^2N^2)} - \frac{z^2}{E^2(1-F^2N^2)} = 1 \quad [25]$$

$$\frac{(x+N^2EF_1)^2}{N^2E^2(1-F_1^2N^2)} - \frac{z^2}{E^2(1-F_1^2N^2)} = 1. \quad [26]$$

where $N^2=a^2b^2/(b^2+a^2E^2)$. The lengths of the principle axes of Equation [25] over their counterparts in Equation [26] make the enlargment scale, k, is given by

$$k = \frac{1-F^2N^2}{1-F_1^2N^2}. \quad [27]$$

Therefore, Equations [25] and [26] present similar figures and the scale of enlargement equals to k. Now knot shapes on a pair of opposite surfaces of board are proved to be similar. This result greatly simplified the work of constructing knotty boards during modelling stress concentration in the surrounding a knot. This conclusion also can be proved with projective geometry. Regarding the opposite surfaces as a pair of parallel planes and the knot on one plane as a perspective projection of another. The centre of this perspective projection is the vertex of the knot wood. According to projective geometry, a planar curve and its perspective projection on a parallel plane are similar figures (Xu, 1980).

Knot shapes in veneer peeled panels

During veneer peeling, the log is rotated as if a cantilevered cylinder and the knife cuts at the gradient direction of the circular boundary of logs (Walker, 1993). The cutting locus is a cylindrical surface and can be expressed by

$$T^2 + R^2 = r_1^2 \qquad [28]$$

where r_1 is the radius of locus cylinder or the distance from the edge of knife to the centre of rotation. Combining Equations [28] and [5], the shape of the knot as a three-dimensional curve can be expressed as

$$T^2 + R^2 = r_1^2 \qquad [28]$$

$$\frac{r_1^2 - R^2}{a^2} + \frac{(R\cos\theta - L\sin\theta + H\sin\theta)^2}{b^2} = (R\sin\theta + T\cos\theta - H\cos\theta)^2 \qquad [29]$$

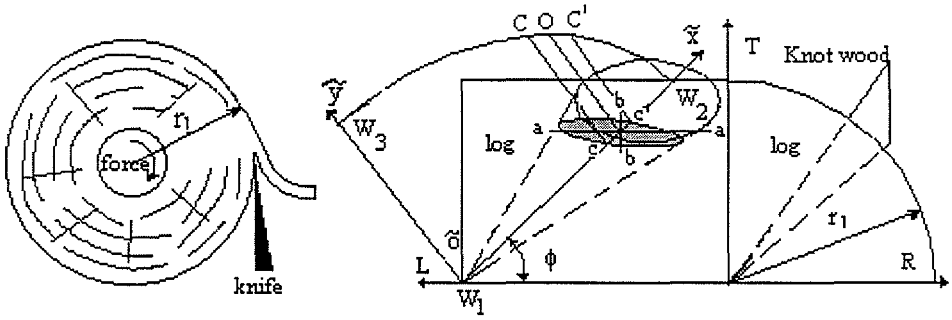

FIG. 5. Formation of knot shapes on veneer peeled panel

The projection of the knot on TOL plane as shown in Equation [29] is a quadratic curve with a closed boundary (see Figure 5), thus representing an ellipse. When the knot on the surface of the cylinder is spread out on a panel, it is an egg-like oval (Figure 6).

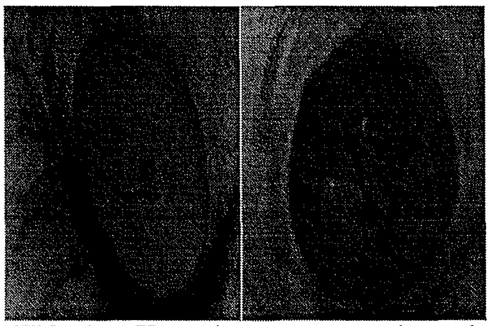

FIG. 6. Knot shapes on spread panel

For the convenience of discussion, knot shapes on the surface of the cylinder and the spread panel are termed as the object and the image respectively. Looking at Figure 5, diameters aa and bb of knot on cylinder are parallel and perpendicular to the axis of log respectively, so their images still bisect each other on the panel. $W_1W_2W_3$ is a given plane perpendicular to TOL and crosses through diameter cc'. The intersection of the cylinder and the plane W is an ellipse shown in Figure 5 as $W_1W_2W_3$. The true figure of cc' on the cylinder is shown as arc COC' on the circumference of $W_1W_2W_3$. Note that OC and OC' have different length, which are the image of oc and oc'. Therefore, the semi-diameters of the knot on the spread panel have different lengths and the knot shape is no longer an ellipse, though it still has an enclosed boundary. The pith of knot wood is now off the centre of the knot (Figure 6).

Assuming the angle of W to log axis is ϕ, then the ellipse $W_1W_2W_3$ can be expressed as

$$\frac{\tilde{x}^2}{(r_1/\sin\phi)^2}+\frac{\tilde{y}^2}{r_1^2}=1 \qquad [30]$$

where $\{\tilde{x}, \tilde{y}; o\}$ is a local coordinate system shown in Figure 6. The image of diameter cc' can be calculated using

$$S=\int_{COC'}\sqrt{1+\tilde{y}'^2}\,d\tilde{x} \qquad [31]$$

where S is the true length of coc', COC' is the locus of integral, $\tilde{y}$ is the subject of $\tilde{x}$ and determined by Equation [30], $\tilde{y}'$ is the differential of $\tilde{y}$. The knot shape on the spread panel can be calculated using Equations [28] to [31].

4. GENERATE KNOTS ON SAWN BOARDS

Generating two-dimensional shapes

If $H< D < H_0$ or $H< D+L_0 < H_0$, the knot on the butt or upper end can be generated in mathematical software Maple V (Char et al., 1994) by combining Equations [11] and [12] or [11] and [13].

All side surfaces have the form

$T=ER+F$ $\quad$ ($E\neq0$ and $F\neq0$) $\quad$ [32]

or $T=F$ $\quad$ [33]

or $R=F$ $\quad$ [34]

The conditions for Equations [32], [33] and [34] to form a knot are $0< -F/E < r_0+b_0/\tan\alpha$, $-b_0< T < b_0$, and $0< R <r_0$ respectively.

Samples of knot shapes and types

Parameters of a knot wood sample are given as

H_0=3 m	hight of the centre of base ellipse
r_0=0.2 m	distance from the centre of base ellipse to the axis of log
a_0=0.04 m	length of semi-minor axis of base ellipse
b_0=0.05 m	length of semi-major axis of base ellipse
$\theta=45^0$	growth angle

When one end of the board is at
L=2.9 (m)

then an oval knot on the end can be drawn using Maple V (Char et al, 1994) as

$$\frac{T^2}{(0.02)^2}+\frac{(R-0.103)^2}{(0.025)^2}=1 \qquad [35]$$

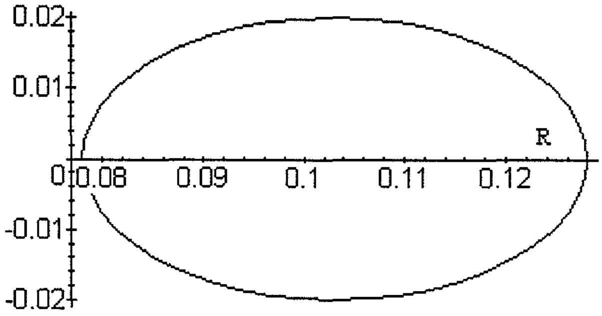

FIG. 8. An oval knot on the end of board

When the cutting plane is

7R+9L-25=0,

We can simplify it into $y=31.26x^2-0.00625$ and describe the true figure of this spike knot with equation

$$Y=252.03\,X^2 \quad X\in(-0.04,0.04) \qquad [36]$$

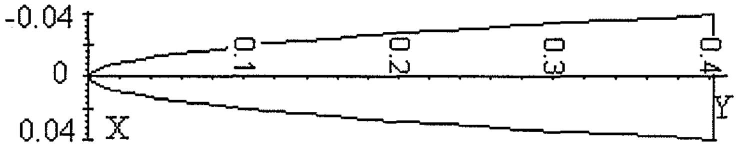

FIG. 9. An spike on a side surface of board

Images and figures in Figure 10 represent different types of knots. These figures are created by the graphic interface software AUTOCAD Release 13 (Hood, 1996). First step of generating a knotty board in AUTOCAD is to produce a cylinder as a log and an elliptic cone as a knot wood according to their measurements. The cutting process may be modelled by the slice function of AUTOCAD. The created knotty board can be used for further structural and thermal analysis. If the geometrical features of knotty boards are well understood, the work of creating a knotty board can be greatly simplified.

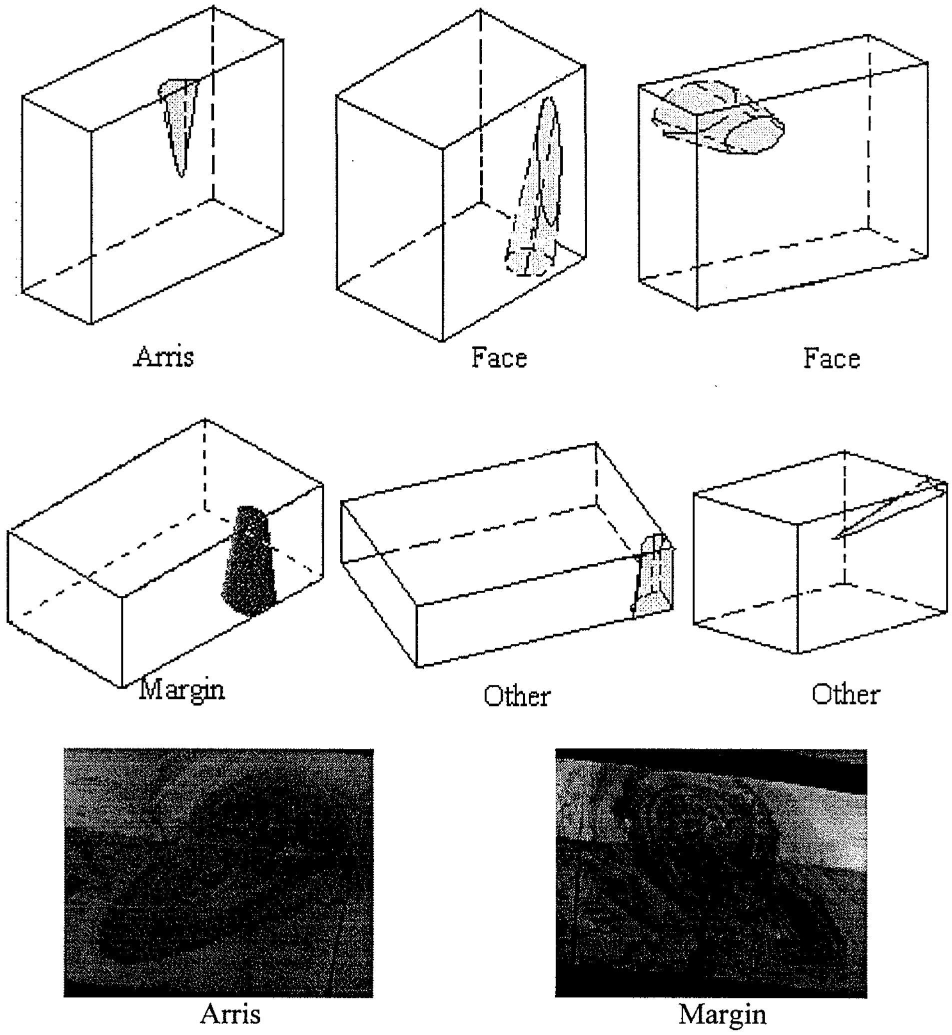

FIG. 10. Samples of different knot types

5. SUMMARY AND DISCUSSION

As the majority of the products of sawmills, knotty boards will attract more attention in wood science in the future. Knots as the dominant factor weakening mechanical properties and causing drying deformation have not been well studied and modelled. A study of generating knotty boards lead to the following results:

1. The base of knot wood on cambia is an ellipse or a circle with two natural axes along the horizontal and vertical direction of tree. This biological feature can help a branch to bear more weight and significantly reduces the complexity of mathematical models of knot wood.

2. The remains of knot wood in sawn board are called knots and the two-dimensional shapes of knots are classified into oval knot and spike. A complete oval knot has closed elliptic or circular rings, whereas a spike does not have any closed ring. In practice, long narrow elliptic knots may be called spikes as well.

3. Referring to the three-dimensional shapes and positions on boards, knots are divided into different types, ie. face, margin, arris and other. A face knot has a complete

boundary which lies inside board surface and shapes as an oval. A arris knot consists of a partial oval and a parabolic or hyperbolic spike, or sometime just two ovals. A margin has two similar ovals on opposite surfaces and one spike on the adjacent surface, or sometimes two spikes and one oval.

4. A knot can not have two spikes on a pair of adjacent surfaces of board. The remains of a knot wood on a pair of opposite surfaces of board are similar figures, which implies that margin and arris knots contain at least one oval. Therefore, large margin and arris knots are liable to cause large warps, because the maximum warp is normally below a large oval in knotty boards.

5. The major knot crack is along the symmetric axes of the studied knot, where the zones of stress concentration exist. The broken direction of knots can be given more precisely now as the direction determined by the symmetric axes of spike and oval.

6. The knots on the peeled panel are egg-shaped ovals like two half ellipses pieced together. Their shapes are determined by Equations [30] to [33].

Some sawmills use automated saw machines and program their sawn patterns before cutting. After the knot wood properties are scanned or measured, the factors affecting boards quality, such as the slope of grain orientation, knot position and the symmetric directions of knots can be estimated by the above equations. It is therefore possible to diagnose the quality of sawn boards and propose an optimised sawn pattern using a small artificial intelligence program.

REFERENCES

Char, B.W.; Geddes, K. O.; Gonnet, G. H.; Leong, B. L.; Monagan, M. B.; Watt, S.M. 1994. Maple V (release 3.0) Language Reference Manual, Waterloo Maple Software and University of Waterloo, Canada.

Cramer, S. M.; Goodman, J. R. 1986. Failure modeling: a basis for strength prediction of lumber, Wood and Fibre Sci., 18(3): 446-459.

Cramer, S. M.; Goodman, J. R. 1983. Model for stress analysis and strength prediction of lumber, Wood and Fibre Sci., 15(4): 338-349.

Fenton, R. R. 1967. A timber grade study of first rotation pinus radiata (D. Don) from Kaiangarroa State Forest, NZFS FRI Technical Paper No. 54, Wellington.

Grant, D. J.; Anton, A.; Lind, P. 1984. Bending Strength, Stiffness, and Stress-grade of Structural Pinus Radiata: Effect of Knots and Timber Density, New Zealand J. of Forestry Science, 14(3):331-48.

Hood, J. D. 1996. Easy AutoCAD: Release 13 for Windows, McGraw-Hill. Auckland.

Kininmonth, J. A. 1961. Checking of Intergrown knots During Seasoning of Radiata Pine Sawn Timber, New Zealand Forest Service Technical Paper (unpublished), NO.30.

Kininmonth, J, A.; Whitehouse, L. J. 1991. Properties and Uses of New Zealand Radiata Pine, Ministry of Forestry and Forest Research Institute, New Zealand.

Liu, H. 1996. Nature of knot checks developed during seasoning of radiata pine boards, Research report No. 96/13, Dept. of Computing and Biometrics, Lincoln University, New Zealand.

Pilkey, W. D. 1994. Formulas for Stress, Strain and Structural Materials, John Wiley & Sons, INC, New York.

Pugel, A.D. 1980. Evaluation of selected mechanical properties of coniferous knotwood, M.S. thesis, Dept. of Forest and Wood Sci., Colorado State University.

Samson, M. 1993. Modelling of Knots in Logs, Wood Sci. And Technol., 27:429-437.

Walker, J. C. F. 1993. Primary Wood Processing: principles and practice, Chapman & Hall, London.

Xu, Hongwen. 1980. Constructive geometry, Tianjing University, China.

Zandbergs, J. G.; F. W. Smith. 1987. Finite element fracture prediction for wood with knots and cross grain, Wood and Fibre Sci., 20(1): 97-106.