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Abstract

During drying, heat and mass transport in timber results in deformation. Heat transport in timber has long been an important topic in drying technology. In this paper, Lykov's formulas, which were originally developed for homogenous capillary porous bodies, are expanded by incorporating the orthotropic properties of wood. Finite element analysis is used for the numerical solutions. The empirical models are used to generate specific heat, conductivity and convection heat transfer coefficients on this basis of experimental drying conditions and wood properties. Typical heat transport phenomena, temperature profile and thermal gradient, are illustrated using three-dimensional graphs.

Introduction

Lykov's theory (1966) has often been used in the studies of heat transport in timber (Gui *et al.*, 1990). However, Lykov's formulas are only applicable to homogenous and isotropic capillary porous bodies and timber is known as an orthotropic material with three mutually perpendicular conductive directions. In this article, a more realistic heat transport model for orthotropic capillary porous materials is derived as an extension of Lykov's theory. The solutions of this model are obtained using the finite element method to apply the model to wide variety of situations. The scenario of this simulation is low temperature drying and *Pinus Radiata* is chosen as the sample material.

In simulation, the quality of data inputs is equally important as governing equations to achieve satisfactory accuracy. In this study, drying conditions and basic wood properties are obtained from real experiments; boundary conditions, specific heat and heat conductivity are calculated using widely accepted empirical models. Finally, simulation outcomes are illustrated using three-dimensional figures and the distributions of temperature and thermal gradient are visualised.

Development of the governing equations

After neglecting chemical reactions and dimensional changes, the energy and mass conservation in capillary porous bodies can be expressed by

$$c\rho_o \frac{\partial T}{\partial t} = -\nabla \mathbf{J_h} - H\,I\,, \qquad [1a]$$

and

$$\rho_o \frac{\partial M}{\partial t} = -\nabla \mathbf{J_m}\ . \qquad [1b]$$

Equation [1a] denotes the conservation of thermal energy, where T is the temperature , K; t is the time, s; c is the specific heat of moist solid, J kg^{-1} K^{-1}; ρ_o is the density of bone-dry solid, kg m^{-3}; $\mathbf{J_h}$ is the heat flux vector, W m^{-2}; H is the specific enthalpies of water, J kg^{-1}; I denotes the volumetric mass disappearance (or formation) rate of liquid and vapour during phase change, kg m^{-3} s^{-1}; H I can be considered as a heat source having dimensions, J m^{-3} s^{-1}. Equation [1b] refers to the mass balance, where M is the dry basis moisture content, kg kg^{-1}; $\rho_o M$ approximately equals the concentration of water, kg m^{-3}; $\mathbf{J_m}$ is the mass flux vector, kg m^{-2} s^{-1} (Lykov, 1966).

Lykov (1966) further developed his equations for heat and mass transport in isotropic materials and those equations have been adopted in the study of timber drying (Gui *et al.*, 1994; Thomas *et al.*, 1980; Lykov, 1966). However, wood is naturally an orthotropic material with three major diffusion directions. Hence, more realistic and accurate equations should be developed for the study of timber drying.

Heat flux $\mathbf{J_h}$ is strongly related to the temperature gradient and weakly related to moisture gradient (Dufour effect) (Thomas *et al.*, 1980). After neglecting the effect of moisture gradient, heat flux in orthotropic material may be written as

$$\mathbf{J_h} = -\begin{bmatrix} K_{hx} & 0 & 0 \\ 0 & K_{hy} & 0 \\ 0 & 0 & K_{hz} \end{bmatrix} \nabla T = -[K_h]\ \nabla T\ . \qquad [2]$$

where $\mathbf{J_h}$ is the heat flux vector, W m^{-2}; (K_{hx}, K_{hy}, K_{hz}) are the heat conductivities, W $m^{-1} K^{-1}$; $\nabla T = \left\{ \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial y} \right\}^T$ is the temperature gradient vector, K m^{-1}; (x, y, z) are mutually perpendicular and along the tangential, radial and longitudinal direction of wood (Carslaw and Jaeger, 1990).

Mass flux is known to be related to both temperature gradient and moisture gradient (Scoret effect) (Thomas *et al.*, 1980). Therefore, the mass flux in orthotropic material may be expressed by

$$\mathbf{J_m} = -\rho_o \begin{bmatrix} D_{mx} & 0 & 0 \\ 0 & D_{my} & 0 \\ 0 & 0 & D_{mz} \end{bmatrix} \nabla M - \rho_o \begin{bmatrix} D_{mhx} & 0 & 0 \\ 0 & D_{mhy} & 0 \\ 0 & 0 & D_{mhz} \end{bmatrix} \nabla T$$

$$= -\rho_o \{ [D_m] \nabla M + [D_{mh}] \nabla T \} \quad . \qquad [3]$$

where $\mathbf{J_m}$ is the mass flux, kg m^{-2} s^{-1}; (D_{mx}, D_{my}, D_{mz}) are the mass diffusion coefficients, m^2 s^{-1}; (D_{mhx}, D_{mhy}, D_{mhz}) are the mass diffusivities in terms of thermal gradient, m^2 s^{-1} K^{-1}.

Because the heat sorce is defined as the product of mass change rate and specific enthalpy, the following relation exists:

$$H I = -R \varepsilon \frac{\partial M}{\partial t} \qquad [4]$$

where R is the specific enthalpy of phase change, J kg^{-1}; and ε is the phase change coefficient, kg m^{-3}. Here the relation $M = m/m_o$ is used, i.e. moisture content (M) is defined as the mass of moisture (m) divided by the oven-dried capillary porous sample (m_o).

Substituting Equations [2], [3] and [4] into [1a] and [1b], then we have the heat transport equation,

$$\frac{\partial T}{\partial t} = \nabla [D_h] \nabla T + (R\varepsilon / c\rho_o) \nabla [D_m] \nabla M \, , \qquad [5a]$$

$$\frac{\partial M}{\partial t} = \nabla \{[D_m] \nabla M + [D_{mh}] \nabla T\} \qquad [5b]$$

where $[D_h] = ([K_h] + R\varepsilon\, [D_{mh}])/ (c\rho_o) = \begin{bmatrix} D_{hx} & 0 & 0 \\ 0 & D_{hy} & 0 \\ 0 & 0 & D_{hz} \end{bmatrix}$; ($D_{hx}$, D_{hy}, D_{hz}) are heat (or thermal) diffusivities, m^2s^{-1}. Lykov's theory (Lykov, 1966) is therefore extended to orthotropic materials.

Heat convection is a frequently used boundary condition for the solution of heat transport equations, because assuming convective heat transfer between the boundary and environment is more realistic than assuming a constant boundary temperature. In fact, the temperature on a sample surface arises from web-bulb temperature and then approaches dry-bulb temperature during kiln drying. The heat convection transfer is described by the convection coefficient (also called film coefficient or transfer coefficient), that is the mean of heat flux over the temperature difference between the boundary and environment, as

$$h_h = - J_h / [(T_a - T_s)\, \Phi_E] \qquad [6]$$

where h_h is the heat convection coefficient, W m^{-2} K^{-1}; J_h is the heat flux through the boundary, W m^{-2}; T_a is the bulk temperature of airflow, K; T_s is the temperature of the moist wood surface, K; Φ_E is the Ackermann correction factor for the influence of moisture-vapour flux and approximates to unity in normal cases (Keey, 1978).

When drying begins, the surface of a board instantly drops to equilibrium moisture content and remains constant under a steady drying condition. The constant boundary condition is an accurate description of mass transport in timber drying. In addition, the constant boundary condition is far simpler than the convection boundary condition. As a result, the constant boundary condition is normally applied and can be expressed as

$$M = M_e \qquad (t>0) \qquad [7]$$

where M is moisture content, kg kg^{-1}; M_e is the equilibrium moisture content under a certain drying condition, kg kg^{-1}.

Numerical solution

The finite element method is employed for the numerical solution to extend the applicability of the developed model to a wider realty. Following the general procedures of Galerkin's method (Reddy *et al.,* 1988; Zienkiewicz, 1977), the solution domain Ω of a capillary porous body is divided into N elements of r nodes each. The approximate values of temperature and mass over each element are defined as

$$T^{(e)}=\sum_{i=1}^{r} N_i(x,y,z)T_i(t)=\{T_1,T_2,\ldots T_r\}\bullet\{N_1,N_2,\ldots N_r\}^T=\{T\}\bullet\{N\}^T \quad , \qquad [8a]$$

and

$$M^{(e)}=\sum_{j=1}^{r} N_i(x,y,z)M_i(t)=\{M_1,M_2,\ldots M_r\}\bullet\{N_1,N_2,\ldots N_r\}^T=\{M\}\bullet\{N\}^T \qquad [8b]$$

where N_i are the shape functions, T_i and M_i are the approximate functions for temperature and moisture content respectively at node i. In each element, the gradient of temperature and the moisture content can be written in matrix notation

$$\nabla T^{(e)}=\{T\}\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial z} \\ \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial z} \\ \frac{\partial N_r}{\partial x} & \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial z} \end{bmatrix}=\{T\}\,[B] \quad . \qquad [9a]$$

Similarly,

$$\nabla M^{(e)}=\{M\}[B] \, . \qquad [9b]$$

After some manipulation, the expansions of all r algebraic equations can be written in matrix and vector forms as

$$\{\frac{dT}{dt}\}\int_{\Omega^e}\{N\}^T\{N\}dV+\{T\}\int_{\Omega^e}[B][D_h][B]^T dV+\{M\}\int_{\Omega^e}[B][D_{hm}][B]^T dV$$

$$-\{T\}\int_S h_h\{N\}^T\{N\}dS=\int_S M_e\{N\}^T dS-\int_S h_h T_a\{N\}^T dS \qquad [10]$$

where $[D_{hm}] = (R\varepsilon/ c\rho_o)\,[D_m]$, Ω^e is the integral domain, V is the volume, S is the surface of the sample.

Modelling boundary conditions and heat- diffusivities of radiata pine

Even though a developed mathematical model permits a wide applicability for drying problems, the simulation outcomes may be unreliable if the inputs are inappropriate. Previously, many researchers have simply assigned some values to their models without a solid experimental or theoretical basis, which definitely reduced the accuracy of the results (Kamke and Vanek, 1994; Kouali *et al.*, 1992). In this article, the empirical models of convection boundary condition and heat- diffusivities of radiata pine are presented on the basis of previous experimental studies. The effect of moisture gradient on heat transport is normally negligible in timber drying (Kulasiri and Samarainghe, 1996; Siau, 1995; Lykov, 1966), the corresponding coefficients are hence not presented.

Heat convection is mainly strongly related to the air-velocity and drying temperature, and weekly related to water-vapour (Pang, 1966; Salin; 1988; Keey, 1978). Stevens *et al.* (1956) studied the heat convection under different air velocities in a laboratory kiln. They recorded the heat convection coefficients at air speeds ranging from 0.45 m s^{-1} to 2.7 m s^{-1} under constant 60 °C/ 50 °C dry bulb and wet bulb temperatures. A linearly regressed relationship of the obtained coefficients with respect to air velocities was suggested. Stevens and Johnston (1955) also investigated the influence of temperature on heat convection coefficient using a fixed air speed of 1.2 m s^{-1}.

Incorporating the effect of temperature and air velocity, heat convection coefficients for timber drying may be expressed as

$$h_h = 8.9417\ V + 0.1242\ T + 1.3687 \qquad [11]$$

where T is the dry bulb temperature of the conditioned air stream, C. This model is applicable for a temperature below 100 °C and humidity around 60% ~ 70%, above which there may be deviation.

Heat diffusivity is the capacity of a material to absorb heat from environment. It is determined by specific heat, heat conductivity and density. A model of specific heat was given by Siau (1995). In his formula, the specific heat is determined by both temperature and moisture content. When the moisture content is lower than 0.05 kg kg^{-1}, the specific heat can be calculated by

$$c = \frac{1260[1 + 0.004(T_c - 30)] + 4185M}{1 + M} \qquad (M < 0.05) \qquad [12a]$$

where c is the specific heat, J kg^{-1} K^{-1}; T_c is the Celsius temperature, °C; M is the moisture content, kg kg^{-1}.

When the moisture content is between 0.05 to 0.30, the specific heat of wood increases due to the influence of bound water. After adding the increment, specific heat is now written as

$$c = \frac{1260[1 + 0.004(T_c - 30)] + 4185M + 1674(M - 0.05)}{1 + M} \qquad (0.05 \le M \le 0.3)\ [12b]$$

Once the moisture content exceeds the fibre saturation point, the maximum increment of specific heat is added and the expression of specific heat may be simplified as

$$c = \frac{1260[1 + 0.004(T_c - 30)] + 4185M + 418.5}{1 + M} \qquad (0.3 \le M)\ [12c]$$

The heat conductivity measures the heat flux through a sample. Maclean (1941) tested large amount of samples and proposed an empirical model of heat conductivity. This model was quoted by Walker (1993) and Siau (1995) as

$$K_{TR} = \frac{\rho_w}{(1+M)1000}(0.200 + 0.38M) + 0.024 \qquad (M \leq 0.4) \qquad [13a]$$

$$K_{TR} = \frac{\rho_w}{(1+M)1000}(0.200 + 0.52M) + 0.024 \qquad (M > 0.4) \qquad [13b]$$

$$K_L = 2.5\,K_{TR} \qquad [13c]$$

where K_{TR} and K_L is heat conductivity in a transverse section and longitudinal direction, respectively, W m^{-1} K^{-1} ; ρ_w is the wet-basis density of moist sample, kg m^{-3}; M is moisture content of samples, kg kg^{-1}.

Simulation examples

The setting of drying conditions and wood properties are recorded in Table 1. A group of corresponding simulation inputs is shown in Table 2. Simulated temperature profile and thermal gradient are illustrated in Figures 1 and 2.

Experimental values	**Coefficient**	**Unit**
Dry-bulb temperature	60	°C
Humidity	0.7	Dimensionless
Air speed (V)	2	m s^{-1}
Initial temperature	25	°C
Initial moisture content	0.871	kg kg^{-1}
ρ_w	695	kg m^{-3}
ρ_o	413	kg m^{-3}

Table 1. A set of experimental values of drying conditions and wood properties.

Simulated parameters	Coefficient	Unit
Specific heat	2461	J kg^{-1} K^{-1}
Heat convection coefficient	30	W m^{-2} K^{-1}
Transverse heat conductivity	0.2665	W m^{-1} K^{-1}
Longitudinal heat conductivity	0.6662	W m^{-1} K^{-1}
D_{hT}: D_{hR}: D_{hL}	2.2: 2.2: 5.5 (10^{-7})	m^2 s^{-1}

Table 2. A set of simulated inputs for the heat transport equations based on empirical models.

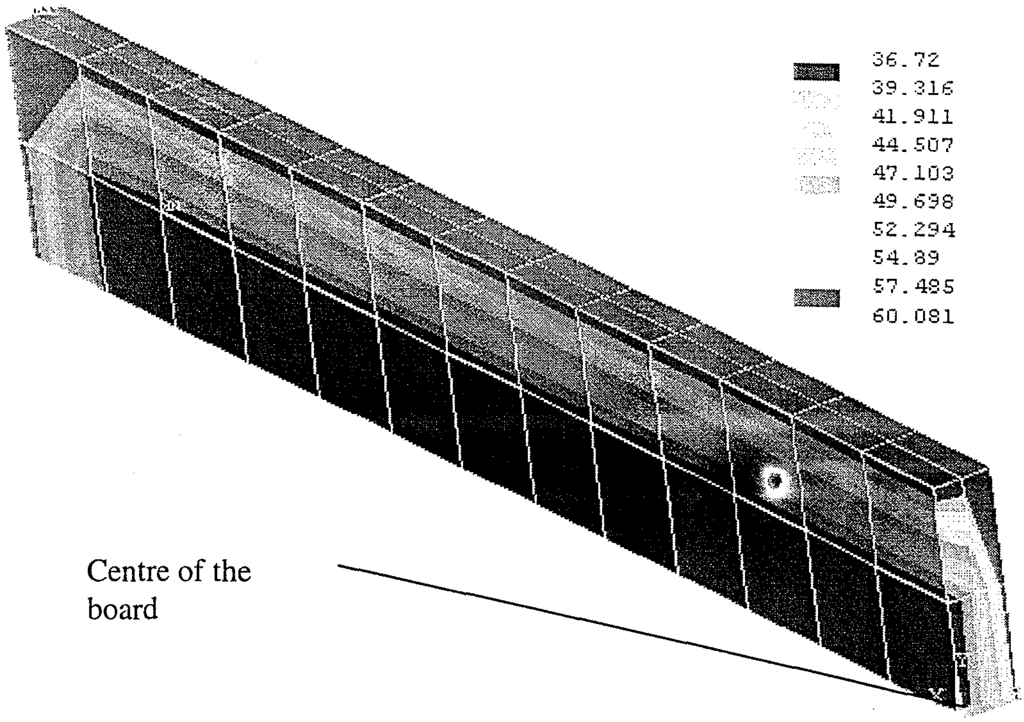

Figure 3.6 (a) Temperature profile in a 1/8 section of a clearwood board after 8 minutes of drying. (Longitudinal × radial × tangential = 1 m × 0.04 m × 0.15 m; inputs are shown in Tables 1 and 2; the unit of the legend is $^{\circ}$C.)

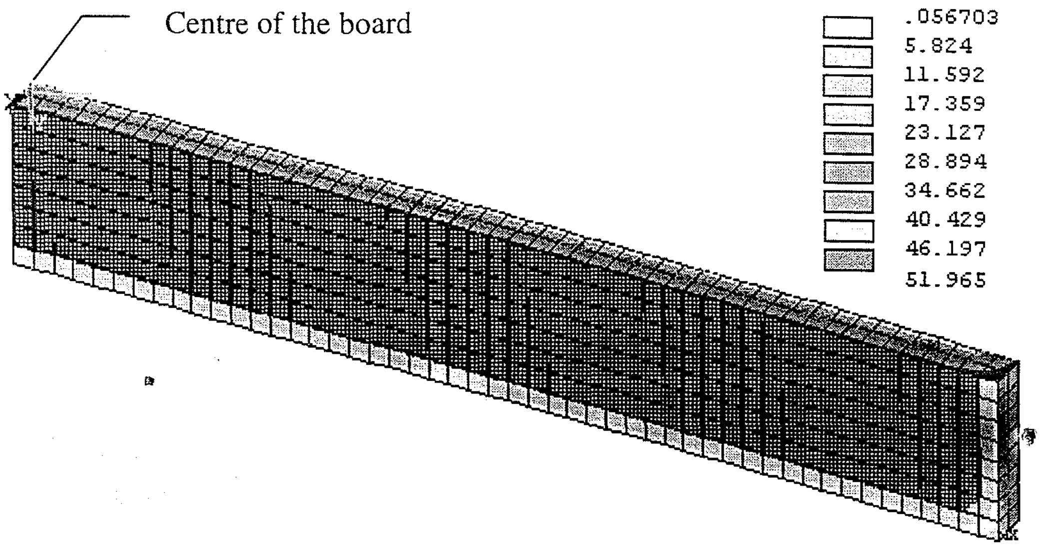

Figure 3.6 (b) Thermal gradient in a 1/8 section of a clearwood board after 3 minutes of drying. (Longitudinal × radial × tangential = 1 m × 0.04 m × 0.15 m; inputs are listed in Tables 1 and 2; the unit of the legend is K m^{-1}.)

Discussion

The heat transfer model was developed on the basis of energy conservation and experimental results. The solutions of formulae were obtained using the finite element method. Hence, this developed model should be applicable to all drying conditions and orthotropic capillary porous materials. The scenario of this simulation is low temperature drying. In this case, the influences of moisture profile are negligible both on boundary connective heat transfer and interior heat transport (Keey, 1978; Kulasiri and Samarasinghe, 1996). However, the effect of moisture profile could be significant in extreme conditions; for instance, in the study of fire resistance. In these cases, heat diffusivity models should be redeveloped, since the models were originally obtained from normal drying conditions.

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