The Keynesian Multiplier
Liquidity Preference
And Endogenous Money

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Abstract

An extension of Meade’s (1993) process analysis diagram is used to analyse the consequences of investment expenditure financed by credit-money, and to comment on the Keynesian multiplier theory recently challenged by Moore (1988), on Keynes’s theory of the revolving fund of investment finance and endogenous money as analysed by Davidson (1968), and on the debate initiated by Asimakopulos (1983) about whether liquidity preference and inadequate saving can restrict investment. This leads to an analysis of the issues recently debated by Cottrell (1994) and Moore (1994) about the compatibility of Post Keynesian theories of the multiplier, liquidity preference and endogenous money.

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1. Introduction

The Journal of Post Keynesian Economics recently carried a very important exchange between Alvin Cottrell (1994) and Basil Moore (1994) concerning the impact of endogenous money theory on the soundness of the Keynesian multiplier. Their exchange followed an earlier statement by Moore (1988, p. 312) that “the equality of planned investment and saving does not occur through the adjustment of income, as the Keynesian income-multiplier approach asserts” and that “the Keynesian multiplier analysis is thus fundamentally flawed”. Cottrell defends the multiplier analysis, and indeed points out that endogenous money theory should strengthen that analysis by implying constant interest rates after an increase in autonomous expenditure (because of the horizontal LM schedule). Moore accepts this latter point, but argues that in a nonergodic world models of macroeconomic equilibrium such as the IS-LM model must be discarded, and with them the concept of the Keynesian multiplier.

It cannot be over-emphasised how important the outcome of the Cottrell-Moore debate is for Post Keynesian macroeconomics. Many leading commentators on Keynes have argued previously that the multiplier analysis is the fundamental innovation in The General Theory (see, for example, Hicks, 1936, p. 239, Robinson, 1937, Chapter 2, Meade, 1975, p. 82, Patinkin, 1976, p. 65, and Trevithick, 1994, p. 77), while the theory of endogenous money is now widely accepted as a major distinguishing characteristic of the Post Keynesian paradigm (see, for example, Sawyer, 1988, p. 2, Arestis, 1992, Chapter 8, Lavoie, 1992, Chapter 4, and Davidson, 1994, pp. 135-6). If these two theories are indeed not compatible, as Moore argues, this represents a serious blow to the internal coherence of the Post Keynesian project.

The Cottrell-Moore debate takes place in the context of efforts to analyse the consequences of an increase in investment expenditure financed endogenously by an increase in credit-money. As was first formalised by Davidson (1968, p. 314; see also his 1978, Chapter 11, and 1986 developments of this insight), this event implies that:

\[
\Delta I_t = \Delta M_t
\]

(1)

where \(I_t\) and \(M_t\) are investment expenditure and the stock of money respectively. If a constant propensity to consume, \(c\), is assumed, and if for heuristic purposes the model is restricted to two sectors with no supply-side constraints, then the increase in investment expenditure produces a multiplied increase in real income, \(Y\), according to Keynes’s (1936, p. 115) standard formula:
The difficulty arises because, as Keynes (1936, p. 166) first recognised, “the psychological time-preferences of an individual require two distinct sets of decisions”; that is, as well as the propensity to consume in equation (2), consideration must also be given to the aggregate “liquidity preference” of agents in the economy. If it is further assumed that the price level is constant (and for simplicity normalised to equal one), and that ceteris paribus liquidity preference results in agents wishing to hold money balances in some constant proportion, \( h \), of their income (where \( h \) can also be modelled as the inverse of the long-run income velocity of money), then this consideration produces the following equation:

\[
\Delta Y_t = \Delta M/h
\]

Kregel (1988) describes the multiplier theory (equation 2) and the liquidity preference theory (equation 3) as “two sides of the same coin”, but the addition of endogenous money theory in equation (1) means that the two equations lead to different predictions for the impact on real income, unless by chance \( h = (1-c) \). Moore’s answer to this inconsistency is to argue that the multiplier theory in (2) is “fundamentally flawed”, while Cottrell implicitly argues that equation (1) does not tell the complete story since either the interest rate will change (affecting \( \Delta L \) or the money supply will change (affecting \( \Delta M \)) until (2) and (3) are brought into equality. Resolving this conflict, therefore, requires either new insights into Kregel’s integration of the multiplier and liquidity preference or into Davidson’s model of endogenously financed investment expenditure. This is the purpose of this present paper.

Even this brief introduction, however, reveals how difficult is the analysis of this problem. The analyst is required to carefully distinguish between the real flows that produce the multiplier effect and the accompanying money flows (see, for example, Chick, 1985). Also, care must be taken to distinguish between the demand for money to finance new investment projects (implicit in equation 1) and the liquidity preference of wealth holders (implicit in equation 3), as Wray (1990, p. 20 and pp. 162-70) has emphasised in his important study. In this respect, it is unfortunate that the same symbol, \( \Delta M \), is used for the two types of money demand, and this paper will introduce new notation reflecting this in the following section. Finally, “time” is an important consideration, both because the multiplier process takes time to have its effect (Moore, 1994) and because it cannot be assumed that the money created endogenously at the beginning of the process will remain in circulation throughout the process if the original bank loans are repaid (Cottrell appropriately terms this “the Kaldor effect”, from Kaldor and Trevithick, 1981).
These three distinctions reveal the inadequacy of the orthodox IS-LM model for analysing the problem at hand. Although the IS-LM model separates real and monetary transactions in its two simultaneous equations, it does not easily allow equation (1) to be incorporated precisely because it does not distinguish between the demand for money to finance new investment expenditure and liquidity preference (although note the important article by Davidson, 1965, in which he explored the sort of adjustments that would have to be made if the IS-LM model was to be used in this way). More importantly, it is a comparative statics model, so that its treatment of time is superficial (a point which Moore reminds us was later acknowledged by the model's author; Hicks, 1976, p. 140). There is, however, an alternative methodology available, known as process analysis.

Process analysis involves tracing through logical time the economic processes initiated by some given event (such as an increase in investment expenditure financed by an increase in credit-money). Almost all writing on endogenous money theory implicitly involves some form of process analysis, usually in the form of a verbal exposition supplemented by appropriate mathematical equations (as in Paul Davidson's seminal work cited above, for example). Some authors, notably Victoria Chick (1977, Chapter 8, and 1983, Chapter 14) and Allin Cottrell (1986 and 1988), have also used the tabular analysis of Dennis Robertson (1936, repeated in 1940) to clarify their exposition. More recently, James Meade (1993) has explained that the original form of process analysis used by the Cambridge Circus to derive the multiplier concept was a diagrammatic one. This method of presentation is particularly clear, since a picture really can be worth a thousand words, but to the best of my knowledge has not been previously used to analyse the endogenous money flows arising out of an investment expenditure.

In this paper, I use an extension of Meade's (1993) diagram to analyse the processes initiated by investment expenditure financed by credit-money. The overall analysis confirms and extends the insights in Davidson (1968, 1978 and 1986) and in Kregel (1988), but I think it will contain some surprises for both Cottrell (1994) and Moore (1994). The following section presents the paper's basic model in the form of a diagrammatic process analysis of investment expenditure financed by credit-money. Subsequent sections then use that model to comment on the Keynesian multiplier theory, on Keynes's theory of the revolving fund of investment finance, and on the related debate involving liquidity preference initiated by Tom Asimakopulos (1983). These three sections (on the multiplier, endogenous money and liquidity preference respectively) then provide an appropriate foundation to explore the issues raised by Cottrell and Moore. The final section is a brief conclusion.
2. The Basic Process Analysis

The process analysis of a credit-money financed level of investment is shown in Figure 1. Before describing the flows in the diagram, two general comments may be helpful. First, the analysis takes place in logical time, rather than historical time. That is, the subscripts in the diagram refer to "rounds" of a process, rather than to "intervals" of time. Thus, $\Delta F_0$ in the first line, for example, refers to the change in finance-money in the initial round, and does not imply that the money supply will change by the same amount in any time interval (and so must not be confused with $\Delta M_t$ in equation 1). Second, the diagram contains real and monetary flows. The distinction is marked in the diagram by the use of equality signs. The transaction to the left of the equality sign in the first line is the monetary flow associated with the real flow of investment expenditure, while the transactions to the right of the equality signs in subsequent rounds refer to the monetary flows associated with saving decisions.

The analysis begins with the first line. For analytical convenience (although without loss of generality, as will be discussed in the next section), it is assumed that the whole of investment expenditure is financed by the creation of new credit-money by the banking system. This might be written as $I = \Delta M_0$, but in order to distinguish the demand for money to finance flows from the demand for money as a stock, the increased money supply to finance expenditure will be written as $\Delta F_0$, as shown in the first line of Figure 1. When the investment expenditure takes place, the factors of production involved in the capital goods sector receive income equal to $Y_0$, which increases their bank balances by $\Delta F_0$. The remainder of the analysis then involves tracing in logical time the expenditure and money flows that result from this initial transaction.

In the first round of the process, the factors of production who received the income arising out of the investment expenditure spend some proportion of that income on consumption goods and services, denoted $C_1$. The remainder is saved (by definition in this two-sector model), and this saving flow is denoted by $S_1$. Traditionally in Keynesian economics (following Keynes, 1936, pp. 114-5), the proportion between consumption and saving is modelled as a constant (determined by the marginal propensity to consume), but it is not necessary to make this assumption in the present model. The consumption expenditure creates further income (and the beginning of the multiplier effect), denoted by $Y_1$, at the end of the first round. The second decision that must be made is in what form the saving flow, $S_1$, will be held. In the model of this paper, there are two options.
First, savings can be used to purchase shares in the new capital stock created by the new investment. This is denoted in Figure 1 as $\Delta E_1$ (increased holdings of equities, which includes all financial instruments that give the holders an explicit or implicit share in the economy’s capital stock). The residual must result in increased money balances. Again this might be written as $\Delta M_t$, but in order to distinguish this demand for money as a stock (rather than a flow), it is denoted as $\Delta H_t$ (increased “hoarding”). Traditionally in Keynesian economics, the proportion
between money balances and equity has also been assumed constant, depending on "the marginal propensity to demand placements" (following Davidson, 1968, p. 314), but again this is not necessary in the current model. The sale of equities in the new capital stock provides funds to the investing firms that can be used to retire their original loans, and this reduces the stock of credit money by this amount (the "Kaldor effect", denoted here by $-\Delta F_r$).

In round 2 of the process, the receivers of income $Y_r$ in turn choose to spend a proportion of it on consumption goods and services, $C_r$, generating further income, $Y_{r+1}$, and the remainder is added to saving, $S_r$. The new saving must again be allocated between increased holdings of new equity in the investment projects, $\Delta E_r$, and increased money balances, $\Delta H_r$, and the supply of credit money supply falls by the former amount, denoted by $-\Delta F_r$. These real and monetary processes continue until a round occurs in which all new income (from the previous round’s consumption expenditure) is voluntarily saved (which may occur only asymptotically; for example, if the traditional assumption of a constant marginal propensity to consume is made). At this point, there is no new expenditure, and hence no new income, and so the processes stop.

3. The Keynesian Multiplier Analysis

The central columns in Figure 1 (that is, the real expenditure/income flows and the saving flows) demonstrate the process by which the Keynesian multiplier effect operates. Indeed, Meade’s recent paper contains a diagram (1993, p. 665) that presents a version of that process analysis on the assumption that saving in each round is a constant proportion of the previous round’s new income, and which Meade explains was how he first discovered the multiplier result that investment creates an equal amount of voluntary saving. That result can now be confirmed in this more general setting (see Dalziel and Harcourt, 1994). Consider the following equations, which are true for all rounds, $r > 0$.

$$I = Y_0$$

(4)

$$C_r = Y_r$$

(5)

$$Y_{r+1} = C_r + S_r$$

(6)

Note that these equations are in fact identities. Equations (4) and (5) record the identity that an act of expenditure for one agent must result in the same amount of income for other agents.
Equation (6) records that all income must be either consumed or saved. The three equations then imply that at the end of any round for $r > 0$:

$$I = \sum_{i=1}^{r} S_i + Y_i \quad (7)$$

Thus, at the end of the first round, some of the investment expenditure is held as voluntary saving, $S_1$, while the remainder is held as induced income, $Y_1$, in advance of the second round. At the end of that second round, the previous round's induced income has become further saving, $S_2$, and further induced income, $Y_2$, so that $I = S_1 + S_2 + Y_2$. This pattern continues throughout the process, until eventually (or perhaps asymptotically) a round occurs in which all of the additional income is voluntarily held as saving (so that in this terminal round, denoted $R$, $Y_R = 0$). At this point, equation (7) records that the multiplier process concludes with exactly sufficient voluntary saving to match the increased investment; that is:

$$I = S = \sum_{i=1}^{R} S_i \quad (8)$$

This result is very important, but so is the way in which it is obtained, so that it is not surprising that Post Keynesian textbooks have often used tabular process analysis to explain the multiplier theory; see, for example, Harcourt et al. (1967, Chapter 10), Chick (1983, Chapter 14) and Davidson (1994, Chapter 3). Process analysis makes clear that the result is not some quirk of the underlying mathematics, nor a matter of choice about assumed equilibrating mechanisms (interest rates or real income), but is the inevitable outcome of two very simple economic identities: expenditure equals income and income equals consumption plus saving. Adding money flows does not interfere with these identities, nor with the process connecting them, so that it must be stated as clearly as possible that Basil Moore was wrong to announce the "knock-out" of the multiplier (which is not to deny, of course, Moore's other substantial contributions to Post Keynesian monetary economics), and that Allin Cottrell has done us a service in pointing this out.

4. Keynes's Revolving Fund of Investment Finance

Figure 1 can also be used to illustrate Keynes's theory of the revolving fund of investment finance, which he developed after The General Theory in a series of articles in the Economic Journal.
Consider the money flows accompanying the real flows in the diagram. By construction, the following equalities hold for all \( r > 0 \):

\[
\Delta F_0 = Y_0 \quad (9)
\]

\[
S_i = \Delta E_i + \Delta H_r \quad (10)
\]

\[
\Delta E_r = -\Delta F_r \quad (11)
\]

These equations imply that:

\[
\Delta F_0 = I = S = \sum_{i=1}^{R} \Delta H_i + \sum_{i=1}^{R} -\Delta F_i \quad (12)
\]

This simply records that the credit money originally demanded to finance investment expenditure comes to be either willingly held by economic agents in the form of increased money balances (which will be written as \( \Delta H_r \), defined as the sum of \( \Delta H_i \) over the full process), or is destroyed again by the repayment of the original bank loans.

The next step in the analysis involves moving from the logical time used in Figure 1 to real time made up of a succession of time intervals. In any empirical application, this is very difficult, since there is no reason for thinking that the "rounds" in Figure 1 will take any particular or fixed length of time, regardless of the unit of time used, and indeed it should be noted that this problem led Keynes (1937a) himself to doubt the usefulness of the process analysis method. In further theoretical analysis, however, the normal practice has been to assume that the multiplier is instantaneous (see, for example, Meade's comment to this effect; 1993, p. 665), so that the process analysis in Figure 1 takes place over two time intervals - the interval in which the investment takes place and the next interval in which the equal amount of voluntary saving is generated. This practice has been challenged by Asimakopoulos (1983), generating an intense debate that will be considered in the following section, so that it is worth recording this assumption formally.

**Assumption 1:** Assume that the multiplier is instantaneous, so that \( S_i = I_{r,F} \).
Note the change in subscripts from $r$ to $t$ to emphasise the move from logical time “rounds” to real time “time intervals”. Two other assumptions that will be relaxed in due course can also be formally recorded here.

Assumption 2: Assume that there is no increase in desired money balances over time, so that 
\[ \Delta H_t = 0. \]

Assumption 3: Assume that there is no economic growth over time, so that \( \Delta \bar{I}_t = 0 \).

Given these assumptions, consider any representative time period. In the time period, two events are occurring simultaneously. First, firms are obtaining credit from the banking system to finance the current interval’s investment projects. Second, the instantaneous multiplier process initiated by the previous interval’s investment expenditure is generating sufficient saving to retire the bank loans arranged in the previous interval. Assumption 2 ensures that all saving is used for this purpose, and Assumption 3 ensures that the new credit being granted and the loans being retired are equal in value. In other words, these assumptions provide sufficient conditions to create a revolving fund of investment finance, as analysed by Keynes (1937c, pp. 219-20):

I return to the point that finance is a revolving fund. In the main the flow of new finance required by current ex ante investment is provided by the finance released by current ex post investment. When the flow of investment is at a steady rate, so that the flow of ex ante investment is equal to the flow of ex post investment, the whole of it can be provided in this way without any change in the liquidity position.

Of course, once the underlying processes are understood, it is no longer necessary to assume that all the investment finance is provided by the banks, nor that all loans made in one interval are retired in the next. Instead, any number of more realistic institutional details might be introduced; for example, Kaldor (1939) suggested that specialist speculators might act as intermediaries between firms and banks, and between savers and firms, while Davidson (1986) has provided a particularly good description of modern arrangements in the United States. Indeed, every country is likely to have variations in the procedures actually followed for financing investment and then converting subsequent saving into equity. These details, however, should not obscure the macroeconomic relationships that must hold, and which can be represented without distortion or loss of generality in the stylised approach of Figure 1.

Consider now what happens if there is an increase in planned investment as a matter of either public policy or increased private sector confidence (so that Assumption 3 does not hold). In this case, the amount of credit-money required to finance the interval’s investment is greater than the
amount of credit-money being retired by debt repayment, and the difference just equals the increase in investment (assuming Assumptions 1 and 2 remain valid). This gives rise to Davidson’s (1968, p. 314) relation in equation (1) above that $\Delta M_i = \Delta F_i = \Delta I_i$ (and indeed it should be said that the whole of the section is little more than a confirmation of Davidson’s analysis in that paper and his subsequent 1978 textbook, pp. 269-72). If this extra credit-money is not forthcoming, then the extra investment cannot take place, leading to Keynes’s famous dictum (1937c, p. 222):

> The investment market can become congested through shortage of cash. It can never become congested through shortage of saving. This is the most fundamental of my conclusions within this field.

### 5. The Asimakopulos Critique

Nearly fifty years after the above quote was written, Tom Asimakopulos drew on earlier disputes by Dennis Robertson (1938) and Nicholas Kaldor (1939, pp. 20-24) to claim that “there may be limits, related in some way to the propensity to save, to the extent to which firms are in a position to increase their rate of investment even if short-term credit is available to finance such an increase” (1983, p. 232), contrary to Keynes’s “most fundamental conclusion”. Asimakopulos argued, in particular, that the finance sector could suffer a short-term shortage of liquidity after an increase in investment because the Keynesian multiplier process is not instantaneous (that is, Assumption 1 of the previous section does not hold), and so at least for a time there is insufficient saving to restore liquidity. Secondly, Asimakopulos argued that Keynes and his followers had paid insufficient attention to the confidence firms must have about obtaining long-term finance on reasonable terms before they will borrow short-term funds to finance increased investment. This cannot be taken for granted, since the holders of cash subsequent to its initial expenditure will not necessarily return their deposits to the investing firms, perhaps because of an increase in liquidity preference (so that Assumption 2 of the previous section does not hold), and hence long-term interest rates might have to rise to induce them to do so, unless saving rates increased.

Asimakopulos (1985a, 1985b, 1986a, 1986b and 1986c) replied to all of the earlier respondents (until illness took its toll) and remained unswayed by their arguments, reasserting in his final book after a lengthy discussion on the question that "the independence of investment from saving ... does not hold under all circumstances" (1991, p. 116; see also his 1990 journal article). In turn, a forthcoming volume in memory of Asimakopulos includes three essays by Davidson (1995), Harcourt (1995) and Kregel (1995) that again defend the Keynesian orthodoxy. To the best of my knowledge, Trevithick (1994, p. 88) is the only person who has said that "Asimakopulos has got it about right".

The process analysis in Figure 1, and the discussion of the previous section, untangle the confusion by clearly distinguishing between the real and money flows initiated by the original transaction. Consider first the impact if Assumption 1 does not hold, so that the multiplier is not instantaneous. In particular, assume that the interval of time in which investment flows are recorded is shorter than the interval of time required for the process analysis of Figure 1 to be completed. As Kaldor (1939, p. 21) first analysed, all this does is increase the size of the revolving fund needed to support a given volume of investment. This is because for any particular interval of time, the fund must finance not only the interval's investment, but also the money balances being held to finance future consumption and saving flows (arising out of previous investment projects) that have not yet had time to occur. To illustrate this result, consider Kaldor's original example, where the investment time interval is assumed to equal the (constant) time involved in each round of the multiplier process (so that the units of rounds \( t \) and time intervals \( t \) are identical), and where there is a constant propensity to save out of income, denoted by \( s \). Table 1 then shows how large a fund is required to support a permanent increase in investment expenditure equal to \( \Delta I \).

The first column of Table 1 records the number of the time interval, which is assumed to equal a round of the multiplier process. The second column shows the increase in the finance required to fund the higher level of investment in each period. This is just \( \Delta I \). Column 3 records the volume of funds being returned to the fund as a result of saving flows generated by investment expenditure in earlier rounds. This steadily increases over time, until asymptotically it reaches the value of \( \Delta I \). The final column is the difference between column 2 and column 3, and shows the net increase in the finance fund each interval. The sum of this column gives the increased funds required to finance the permanent increase in investment; that is, \( \Delta I/s \).
### Table 1

**Kaldor's Revolving Fund Model**

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>New Funds</th>
<th>Returning Funds</th>
<th>Net Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Delta I$</td>
<td>0</td>
<td>$\Delta I$</td>
</tr>
<tr>
<td>2</td>
<td>$\Delta I$</td>
<td>$s\Delta I$</td>
<td>$(1-s)\Delta I$</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta I$</td>
<td>$s(1-s)\Delta I + s\Delta I$</td>
<td>$(1-s)^2\Delta I$</td>
</tr>
<tr>
<td>4</td>
<td>$\Delta I$</td>
<td>$s(1-s)^2\Delta I + s(1-s)\Delta I + s\Delta I$</td>
<td>$(1-s)^3\Delta I$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$\Delta I$</td>
<td>$\Delta I$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Total Increase in Funds Required:** $\Delta t/s$

Asimakopulos was well aware of Kaldor’s model (1983, p. 229, and 1991, pp. 115-6). His argument was that a low propensity to save, $s$, increases the size of the additional funds needed to finance an increase in investment, as Table 1 confirms. Assuming that a larger fund puts pressure on interest rates to rise, this discourages investment, and hence there is a link not recognised by Keynes between saving and investment. In particular, Asimakopulos (1983, p. 230) argued that an increase in the propensity to save, $s$, could relieve congestion in the investment market (contrary to Keynes’s “fundamental conclusion”) by reducing the size of the revolving fund needed to finance a given increase in investment. This chain of logic is sound as far as it goes, but the process analysis allows two crucial points to be added to give a very different policy conclusion.

First, note that the congestion both before and after the change in $s$ is caused by the liquidity constraint (the limit on the size of the investment fund), and not by a lack of prior saving. Thus the true villain of the piece is the liquidity constraint (as Keynes argued) and the authentic Keynesian response is to call on the central bank to provide more funds, not to call for greater saving. Second, note also that any increase in investment expenditure achieved by increasing the propensity to save occurs at the expense of an equivalent reduction in consumption expenditure. To see this, denote the constraint on increasing the size of the investment fund by $\Delta F$. From Table 1, the volume of new investment that can be undertaken is then given by $\Delta I = s\Delta F$, but the level of additional aggregate income is calculated from $\Delta Y = \Delta t/s$, whence we obtain the result that $\Delta Y =$
\( \Delta F \) (see, also, Wells, 1981, for a similar derivation within a slightly more general model). Introducing policies to affect \( s \) cannot change \( \Delta F \) in this model; again, the only way to increase income growth is by relaxing the liquidity constraint, \( \Delta F \).

Suppose now that Assumption \( 2 \) of the previous section does not hold, so that agents choose to use some of their savings to increase their money balances (that is, suppose there is an increase in liquidity preference, and so \( \Delta H > 0 \)). This is the case first analysed by Davidson (1968, p. 314, and 1978, p. 255), in which the marginal propensity to purchase placements out of saving is less than unity. This has the potential to seriously affect the discussion so far, since it implies permanent leakages from Keynes's revolving fund of investment finance. Hence, even if there are no ongoing increases in investment, the finance sector must continuously increase the money supply to replace these leakages. Asimakopulos (1991, p. 113) acknowledged in a footnote that the banks might be willing to do so, since they grow and prosper by increasing their loans. Further, it is clear it would be sound banking practice to do so, since the new loans would be backed by adequate collateral (the value of investment not sold as equity).

The major event that might intervene is if the finance sector did not have sufficient liquid assets to support increasing levels of bank deposits (perhaps because of a refusal by the central bank to accommodate the monetary expansion). In this case, banks would have to increase the rate of interest to reflect their illiquid position, and to the extent this was foreseen, firms might reduce their investment expenditure. This is the common element in Asimakopulos's two criticisms, and indeed it can be recognised as a standard concern of post Keynesian endogenous money theorists. Where Asimakopulos went fundamentally wrong, however, was in attributing this problem to a shortage of saving relative to investment, and in suggesting that increased saving could alleviate the problem.

Looking only at the right-hand-side of the process analysis of Figure 1, it is easy to see how the error can be made. At first sight, it does appear that higher values of saving in each round will increase the purchases of new equities, \textit{ceteris paribus}. If this led to a higher value of \( S \) in equation (12), it might be thought that there could be room for a positive value of \( \Delta H \) and there still be sufficient equity sales to replenish the investment fund. But the essence of Keynes's \textit{General Theory} was to recognise that the \textit{ceteris paribus} assumption is not valid, and that instead the level of income will necessarily adjust so that the value of \( S \) will always equal \( I \), regardless of the value of the propensity to save in each round (as shown in the central columns of the process analysis). The problem can never be one of inadequate saving, but is always one of inadequate liquidity, just as Keynes argued.
6. The Cottrell-Moore Debate

The framework presented in the previous three sections now allows a discussion of the issues raised in the debate between Cottrell (1994) and Moore (1994). Recall from the introduction above that the basic issue concerns the different predictions made by the constant-propensity-to-save multiplier theory (equation 2) and the constant-income-velocity-of-money equation of exchange (equation 3) after an increase in investment expenditure financed by an increase in credit-money (equation 1). The relevant equations from the introduction are repeated here for convenience, but with some slight adjustments in keeping with the discussion so far. Hence, the change in credit money in equation (1) is denoted as \( \Delta F \), in equation (13), the marginal propensity to consume in equation (2) is replaced by the marginal propensity to save, \( s \), in equation (14), and the change in the stock demand for money in equation (3) is denoted as \( \Delta H \), in equation (15).

\[
\Delta I = \Delta F \quad \text{(13)}
\]
\[
\Delta Y = \Delta I / s \quad \text{(14)}
\]
\[
\Delta Y = \Delta H / h \quad \text{(15)}
\]

The final step in the argument is to add a fourth equation bringing together the increase in the demand for money to finance flows and the increase in the demand for money as a stock. This is done in equation (16).

\[
\Delta M = \Delta F + \Delta H \quad \text{(16)}
\]

There is now no contradiction in the mathematics, and the economic interpretation is clear-cut. If there is a permanent increase in investment by \( \Delta I \), then the size of the revolving fund in Section 3 above must increase by the same amount. Statistically, therefore, an increase in investment will be reflected in an increase in the money supply by the amount \( \Delta M \), \textit{ceteris paribus}. But note carefully that this increase is fully absorbed by the revolving fund, \( \Delta F \), so that there is no need to inquire what will lead economic agents to voluntarily choose to hold this increased money supply. This is the mistake made by Moore (1994, p. 129). Moore argues that the change in the money supply is given by equation (1), and that to induce agents to hold the extra money income must increase by the amount given in equation (3), and that therefore the multiplier relationship in equation (2) is irrelevant. Instead, the analysis of Figure 1 reveals that the new credit money is fully taken up by the need to finance income-induced consumption expenditure (Moore’s
As noted in the introduction above, this distinction has been emphasised recently by Randall Wray (1990, p. 20 and pp. 162-70; see also his 1992 article). Wray defines “money demand” as “a willingness to expand one’s balance sheet in order to spend on goods, services, or assets”, and distinguishes this from “liquidity preference”, which is “a preference to exchange illiquid items on a balance sheet for more liquid items, or even to decrease the size of a balance sheet by retiring debt”. Wray’s “money demand” is precisely the sense in which it is argued here that investment expenditure produces a demand for finance that is met by an endogenous increase in the money supply (equation 13), and his “liquidity preference” is a more sophisticated version of the stock demand for money in equation (15). Because the former increase in the money supply matches a pre-existing money demand (the increase in the size of the revolving fund), there is no need for any increase in “liquidity preference” to absorb it, contrary to Moore’s argument.

Now consider equation (14), which summarises the Keynesian multiplier theory on the assumption that the propensity to save is constant throughout the multiplier process. Recall from Section 2 above, however, that the logic of the multiplier process in Figure 1 does not require a constant propensity to save. Rather, it depends on two identities relating expenditure to income, and income to consumption and saving. Hence it is possible to reject as an empirical matter any behavioural hypothesis about saving decisions without affecting the validity of the multiplier theory. It might be proposed, for example, that agents base their consumption expenditure decisions not on their income, but on the level of excess money balances that they hold (as both Cottrell and Moore seem to do; 1994, p. 115 and p. 129 respectively); that is:

\[ \Delta C_t = \Delta M_t - h\Delta Y_t \]  

Since every monetary flow is also a real flow in this model (because the price level is assumed constant), beginning with the initial increase in investment expenditure financed by new credit money, it follows that \( \Delta M_t = \Delta Y_t \) in every period. Further, the changing in saving is given by \( (\Delta Y_t - \Delta C_t) \), so that equation (17) implies that:

\[ \Delta S_t = h\Delta Y_t \]
This looks remarkably like the standard Keynesian behavioural assumption (although of course the microfoundations for \( h \) are radically different from those for \( s \)), so that the standard mathematical formula for the summation of a geometric series then produces a familiar form for the equilibrium condition:

\[
\Delta Y_t = \Delta I/h
\]  

(19)

This, of course, is simply equation (15), but has been properly derived here as the outcome of the multiplier process under a certain behavioural assumption rather than as the outcome of the equation of exchange identity. Thus, Moore can reasonably argue that equation (19) could be adopted as an alternative for the traditional equation (14), but this does not mean that the multiplier theory is discredited. There is also a serious difficulty with this formulation, since it allows no mechanism by which agents can hold equity in the new capital being produced by the investment projects, and consequently Keynes’s concept of the revolving fund of investment disappears. This is because the underlying logic requires all saving to be in the form of money balances. Many will form the judgment, I suspect, that the standard Keynesian behavioural assumption about consumption is more realistic than this one.

Finally, consider equation (15) itself. To introduce a better understanding of what this equation involves, assume to begin with that there is no change in liquidity preference as a result of the increased level of investment expenditure, in the sense that there is no desire for increased nominal money balances and hence all increased saving is converted into the purchase of equities. Note carefully that a statistician would then record an increase in the economy’s money supply as given by equation (13), where at any moment in time the money is being held as convenience saving or by firms in advance of investment expenditure. The statistician would also record an increase in national income that, on the constant propensity to save assumption, would be given by equation (14). Under these circumstances (again holding the price level constant) the published income velocity of money, \( V \), would be calculated by substituting (13) into (14) to produce:

\[
V = I/h = 1/s
\]  

(20)

In other words, the underlying logic of Keynes’s revolving fund of finance (when liquidity preference does not change) produces a situation where the measured ex post \( h \) will equal the propensity to save, contrary to Cottrell’s (1994, p. 115) view that this "could only be coincidental" and Moore’s (1994, p. 126) view that “there can surely be no logical reason” for this.

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The assumption of unchanged liquidity preference, however, is unlikely to be valid once increased investment produces higher real incomes. Instead, it is reasonable to suggest that agents will increase their desired level of money balances, for example as a precaution against unexpected expenditure in the future. Both Cottrell and Moore assume that this can be achieved out of the increased money supply created to finance the rise in investment, but the analysis of the Asimakopulos critique in Section 4 above shows that this would produce a leakage in the revolving fund, which must be topped up if investment is to continue at its higher level. In other words, the aggregate money supply must be increased to accommodate both the higher financing needs and the higher demand for money as a stock, as recorded in equation (16). If this does not occur, then interest rates will almost certainly rise. This is necessary to reduce the desired level of money balances as a stock and/or to reduce the need for money finance by diminishing the level of planned investment.

7. Conclusion

The above discussions illustrate well what Victoria Chick (1985, p. 80) has called the “quite powerful” results that can be obtained by using process analysis. In particular, this paper has demonstrated how process analysis can be used to distinguish the different concepts of the Keynesian multiplier, of liquidity preference and of endogenous money without losing sight of their significant interconnections. If Randall Wray (1992, p. 88) is correct to say that these concepts “are ‘three sides of the same coin’, in the sense that they may be combined into a single theory of the adjustment processes which determine flow and stock equilibrium points” (and I think he is), then a process analysis such as that contained in Figure 1 of this paper provides a suitable methodology for constructing and presenting such a single theory, and which might be more widely used by Post Keynesian theorists.
References


