

FLAT-FAN SPRAY ATOMIZATION MODEL

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ABSTRACT. *In pesticide application, the lack of a suitable theoretical atomization model for flat-fan spray nozzles forces a reliance on empirical data and correlations, even for computational simulations. There is considerable difficulty in the theoretical analysis of the liquid sheet emanating from flat-fan nozzles because no simplification to a two-dimensional analysis can be employed, as is done for cylindrical jets. Nonetheless, 50 years ago, Dombrowski and co-workers used linear stability analysis to analyze the breakup of flat-fan spray sheets into ligaments and from ligaments to droplets. Their correlations have not found use because they include parameters that are difficult, if not impossible, to measure. In this work, the Dombrowski model is simplified using dimensional analysis, resulting in a correlation to predict the volume median diameter of flat-fan sprays in terms of common user parameters, i.e., the nozzle size and operating pressure.*

Keywords. *Atomization, Droplet size, Nozzles, Pesticides, Sprayers.*

There are many theoretical and semi-theoretical atomization models for the breakup of cylindrical liquid jets issuing from circular orifices into air, as summarized by Lefebvre (1989), where the cylindrical symmetry helps to simplify the stability analysis. For flat-fan sprays of the type commonly used in agricultural application of pesticides from ground boom sprayers, the complex geometry makes it difficult to find simplifying assumptions. As a result, most analyses of droplet size spectra rely on empirical correlations or databases. Even computational simulations, such as those performed by the pesticide transport and deposition model AGDISP (Bilanin et al., 1989), require the user to input a measured droplet size distribution. Nonetheless, there have been some attempts at theoretical atomization models, although none have thus far found common use. There are two periods of research, an earlier period (1950-1975), largely the work of Dombrowski and co-workers (Dombrowski and Fraser, 1954; Dombrowski et al., 1960; Fraser et al., 1962; Dombrowski and Johns, 1963; Dombrowski and Munday, 1968; Clark and Dombrowski, 1972a, 1972b; Crapper and Dombrowski, 1984; Dombrowski and Foumeny, 1998), along with Dorman (1952), and the more recent period (2000-present), with the work of Altimira and co-workers (Rivas et al., 2006; Altimira et al., 2007, 2009, 2011, 2012; Butler Ellis et al., 1999, 2001; Brenn et al., 2002; Thompson and Rothstein, 2007; Cloeter et al., 2010; Negeed et al., 2011; Altieri et al., 2014). To date, none of these works has resulted in a suitable model to predict the droplet size for flat-fan nozzles. In the work

presented here, an atomization model is developed for simple (Newtonian) fluids that predicts the average spray droplet size (volume median diameter, $D_{v,0.5}$) in terms of easily measurable user parameters: operating pressure, nozzle size, nozzle spray angle, and formulation surface tension. This will be useful for nozzle designers and in simulation tools such as AGDISP.

DIMENSIONAL ANALYSIS

The approach starts with dimensional analysis, following Altieri et al. (2014). The relevant variables are listed in table 1. There are eight variables and three repeated units, so following the Buckingham Pi theorem (Post, 2009), there will be $8 - 3 = 5$ non-dimensional groups that characterize the problem. One group must be formed from the dependent variable d . Thus, a non-dimensional droplet size (d^*) is defined as:

$$d^* = \frac{d}{h} \quad (1)$$

Because d here is the mean droplet size formed through aerodynamic breakup, the Sauter mean diameter (D_{32}) is the appropriate mean diameter for characterizing the spray. The liquid viscosity (μ) can be non-dimensionalized using the Reynolds number (Re), which is the ratio of inertial to viscous forces:

Table 1. Variables for the dimensional analysis of droplet size produced by atomization from flat-fan nozzles sprays.

Variable	Symbol	Units
Liquid density	ρ_l	kg m ⁻³
Air density	ρ_g	kg m ⁻³
Liquid viscosity	μ	kg m ⁻¹ s ⁻¹
Liquid surface tension	σ	kg s ⁻²
Liquid sheet velocity	V	m s ⁻¹
Orifice height	h	m
Spray angle	θ	-
Mean droplet size	d	m

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$$Re = \frac{\rho_l V h}{\mu} \quad (2)$$

The liquid surface tension (σ) is non-dimensionalized in equation 3 using the Weber number (We), which is the ratio of the kinetic energy of the liquid stream to the surface energy of the liquid stream. The dynamic surface tension measured at a surface lifetime age of 20 ms is the appropriate value for atomization modeling:

$$We = \frac{\rho_l V^2 h}{\sigma} \quad (3)$$

The spray angle (θ) is already non-dimensional and thus can serve as its own non-dimensional group. The only variable not yet used is the gas density (ρ_g). The simplest way to form a non-dimensional density ratio (ρ^*) is:

$$\rho^* = \frac{\rho_l}{\rho_g} \quad (4)$$

Experience shows that, for fully turbulent sprays, the Reynolds number has only a weak effect compared to the Weber number, so common practice is to use Ohnesorge number instead of the Reynolds number to characterize the effects of liquid viscosity. The Ohnesorge (Oh) number is the ratio of viscous to surface tension forces:

$$Oh = \frac{\mu}{\sqrt{\rho_l \sigma h}} \quad (5)$$

The liquid Weber, Reynolds, and Ohnesorge numbers are related through:

$$Oh = \frac{\sqrt{We}}{Re} \quad (6)$$

Thus, the Buckingham Pi theorem states there exists a function of the form:

$$d^* = f(We, Oh, \rho^*, \theta) \quad (7)$$

The exact functional dependence of the non-dimensional droplet size on the Weber number and the other parameters must come from theory or experiments (empirical correlations). A physical atomization mechanism can provide this model.

ATOMIZATION REGIMES

In order to develop a model for atomization, a physical model of the atomization process must first be visualized. At different injection velocities, different physical processes can take place, as shown in figure 1 for cylindrical liquid jets issuing out a circular orifice.

In the Rayleigh regime, which takes place at very low injection velocities, the liquid cylinder breaks up into droplets that are larger in diameter than the original liquid stream. In the Rayleigh regime, the effects of the ambient air are negligible. As the injection velocity increases, the liquid stream

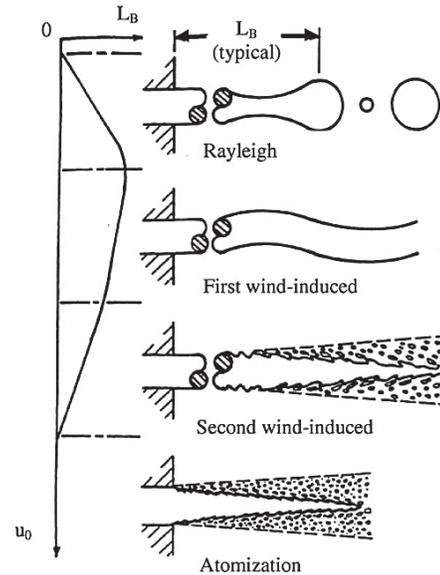


Figure 1. Sketch of different atomization regimes (Faeth et al., 1995).

enters the first wind-induced atomization regime. Here, the ambient air acts to enhance the breakup of the Rayleigh regime that is driven by surface tension. This results in droplets similar in size to the orifice diameter. As the injection velocity is further increased, the second wind-induced regime occurs, in which unstable short-wavelength waves grow on the liquid sheet surface and break it up into droplets smaller than the orifice. Finally, at high injection velocities, the aerodynamic forces are strong enough to strip droplets directly off the liquid sheet, resulting in droplets that are one or two orders of magnitude smaller than the orifice diameter. This breakup occurs immediately after the liquid exits the orifice.

The boundaries between these regimes are functions of the governing non-dimensional parameters (Weber, Reynolds, Ohnesorge numbers), as shown in figure 2. Table 2 shows the typical parameter ranges seen in agricultural ground boom spraying, based on the ranges used by the Spray Drift Task Force (Hewitt et al., 1996). From these values, the typical ranges of the Weber and Reynolds numbers

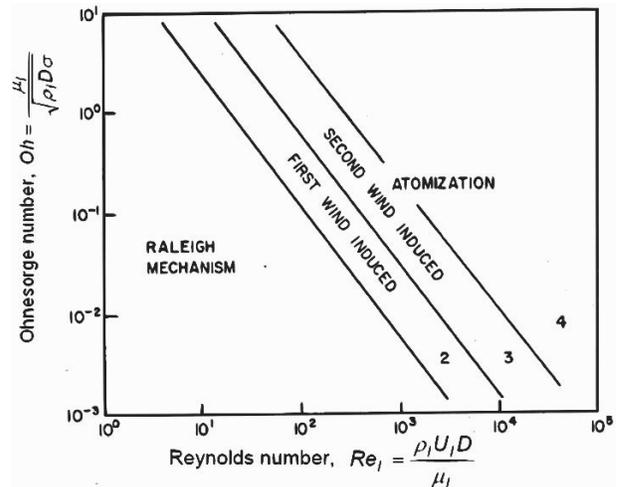


Figure 2. Plot of atomization regimes for cylindrical liquid jets as a function of non-dimensional parameters (Reitz, 1978).

Table 2. Parameters ranges typically seen with flat-fan nozzles in agricultural ground boom spraying.

Parameter	Minimum	Maximum	Units
Orifice size (height)	0.25	2.0	mm
Pressure	1.0	7.0	bar
Surface tension	0.025	0.075	N m ⁻¹
Viscosity	0.001	0.003	kg m ⁻¹ s ⁻¹
Liquid density	990	1020	kg m ⁻³
Air density	1.1	1.2	kg m ⁻³
Spray angle	80	130	degrees
Discharge coefficient	0.5	0.9	-
Velocity	7.0	35.0	m s ⁻¹
Reynolds number	2000	70,000	-
Weber number	700	30,000	-
Ohnesorge number	0.0025	0.0150	-

can be calculated; based on the criteria in figure 2, it is seen that agricultural spraying with flat-fan nozzles typically falls in the second wind-induced regime.

Based on the data in table 2, for the vast majority of nozzles, tank mixes, and operating conditions, agricultural ground boom flat-fan sprays will fall into one of the two wind-induced breakup regimes in figure 2. Lin and Reitz (1998) recommended a criterion for the full atomization regime as:

$$We_g = \frac{\rho_g V^2 h}{\sigma} > 40 \quad (8)$$

Using a gas density of 1.2 kg m⁻³ for the ambient air and the other parameter values in table 2 gives a maximum aerodynamic Weber number (We_g) of 36, so again it is safe to assume atomization via a wind-induced mechanism. Further, the image of flat-fan sheet breakup in figure 3 supports the assertion of a wind-induced breakup regime (Rivas et al., 2006). A long smooth intact liquid sheet can be seen, with the growth of spanwise surface waves that eventually rupture the sheet. Altieri et al. (2014) indicate that the ligament atomization mechanism is only valid for pressures greater than 1.4 bar (20 psi), which should be valid for all practical spray-

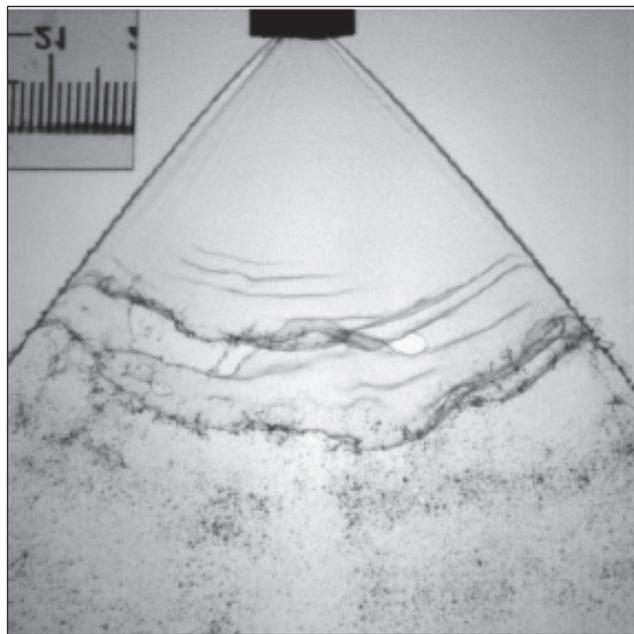


Figure 3. Flat-fan liquid sheet breakup showing formation of surface waves typical of a wind-induced atomization regime (Rivas et al., 2006).

ing applications, again confirming wind-induced atomization.

Now that the atomization regime has been identified, the next step is a detailed description of the breakup process that can lead to a mathematical model for droplet size. Figure 4 shows a schematic of the wind-induced sheet breakup mechanism originally proposed by Fraser et al. (1962). In this mechanism, a sinuous Kelvin-Helmholtz instability grows until the sheet necks down to a critical thickness value, at which point surface tension forces cause the sheet to rupture into spanwise ligaments, which assume a roughly cylindrical shape. These cylindrical ligaments then break down via the Rayleigh mechanism into spherical droplets. This mechanism is also consistent with the image in figure 3. The size of the droplets formed by this atomization mechanism may be calculated by the conservation of mass, if the critical wavelength (λ) causing the sheet to break up is known.

To calculate the critical wavelength of the fastest growing waves, a linear stability analysis is employed. In a linear stability analysis, the growth of small perturbations on an initially smooth liquid sheet is considered. A force balance is performed on the liquid surface, and the growth rate (ω) of all possible wavelengths (λ) is theoretically calculated. An optimization calculation is then performed to determine the wavelength that corresponds to the fastest growth rate. It is assumed that this fastest growing wavelength is ultimately responsible for the sheet breakup. For a flat liquid sheet, Squire (1953) showed that waves of sinuous shape (as sketched in fig. 4) cause disintegration of the sheet into cylindrical ligaments. For the cylindrical ligaments, the dominant waves form a varicose pattern, which eventually becomes large enough to cause the ligaments to break down into spherical droplets.

A few assumptions must be made as part of this stability analysis: (1) the velocity along the liquid sheet at the point of breakup is the same as the velocity of the sheet at the orifice exit, as confirmed by Dombrowski et al. (1960); (2) the liquid flow is fully turbulent and thus in a regime in which the discharge coefficient (C_d) can be treated as a constant for a given nozzle; (3) the turbulence in the liquid sheet does not significantly affect the atomization process, as confirmed by Fraser et al. (1962); and (4) the critical Reynolds number for fully turbulent flow is 9000, as given by Clark and Dombrowski (1972a), and this Reynolds number should be based on the hydraulic diameter (d_h) of the nozzle rather than the liquid sheet thickness:

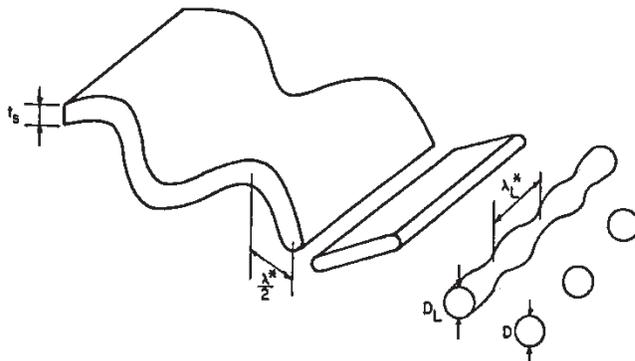


Figure 4. Schematic of flat-fan atomization (Fraser et al., 1962).

$$\text{Re} = \frac{\rho V d_h}{\mu} \quad (9)$$

where the hydraulic diameter (d_h) is defined as:

$$d_h = \frac{4A}{P} \quad (10)$$

where A is the cross-section of the orifice, and P is the perimeter of the orifice. For agricultural flat-fan nozzles that have an elliptically shaped orifice, the hydraulic diameter can be calculated by:

$$d_h = \frac{\pi ab}{\pi \left[3(a+b) - \sqrt{(3a+b)(a+3b)} \right]} \quad (11)$$

The denominator is the Ramanujan approximation to the perimeter of an ellipse, as there is no exact analytical formula. Here, a is the long axis of the ellipse, and b is short axis of the ellipse, where $b = h/2$. This will result in values of Reynolds numbers about 1.4 times higher than those based on the orifice slot height (h) based on the measurements of Guler et al. (2007). Thus, based on the data in table 2, for almost all practical spraying conditions, the liquid flow will be fully turbulent, as assumed.

INSTABILITY ANALYSIS

Fraser et al. (1962) showed that the fastest growing wavelength on the fan sheet is:

$$\lambda_{opt} = \frac{4\pi\sigma}{\rho_g V^2} \quad (12)$$

The diameter of the ligaments formed is calculated by assuming each ligament has the same volume as one half-wavelength of the liquid sheet:

$$d_{lig} = \sqrt{\frac{4\lambda_{opt} h_{crit}}{\pi}} \quad (13)$$

Substituting yields:

$$d_{lig} = \sqrt{\frac{16\sigma h_{crit}}{\rho_g V^2}} \quad (14)$$

To calculate the thickness of the liquid sheet at the breakup point (h_{crit}), Fraser et al. (1962) found:

$$h_{crit} = C_3 \sqrt[3]{\frac{K^2 \rho_g^2 V^2}{\rho_l \sigma}} \quad (15)$$

where K is a spray parameter related to the area of the orifice. Combining equations 14 and 15 yields:

$$d_{lig} = C_2 \sqrt[3]{\frac{\sigma K^2}{V^2 \rho_g \rho_l}} \quad (16)$$

Once the diameter of the ligaments is known, the further

breakdown of the ligaments into droplets proceeds by the Rayleigh mechanism, in which the diameter of the fastest growing varicose waves on the cylindrical ligaments is:

$$\lambda_{Rayleigh} = 4.51 d_{lig} \quad (17)$$

By conservation of mass then, the diameter of the droplets formed from the breakdown of the ligaments is:

$$d = 1.89 d_{lig} \quad (18)$$

The effects of viscosity on the atomization were considered by Dombrowski and Johns (1963). For the breakdown of the cylindrical ligaments into droplets, the viscosity affects the droplet size by:

$$d_{visc} = d_{inviscid} \left[1 + 3(\text{Oh}) \right]^{1/6} \quad (19)$$

This correlation is qualitatively consistent with the findings reported by Hewitt (2008) that increasing viscosity increases the droplet size. The theoretical results of Dombrowski and Johns (1963) indicate that the effects of viscosity are negligible below about 2 centipoise. It should be emphasized again in the current model that only Newtonian fluids (those of constant shear viscosity) are considered, as non-Newtonian fluids can change the atomization mechanism. Further, this model is not valid for emulsions, as it has been found that the presence of emulsion particles can affect the sheet breakup mechanism (Dexter, 1996). An attempt to analyze the atomization of non-Newtonian fluids was made by Altieri et al. (2014). Given the difficulty in finding data on the effects of viscosity for Newtonian fluids, and the near-impossibility of varying the viscosity independent of the surface tension, Oh will be treated as a constant for the remainder of the derivation.

Thus, from the models of Dombrowski and co-workers, the theoretical dependence should be:

$$d^* = k \times \text{We}^{-1/3} (\rho^*)^{1/6} f_1(\text{Oh}) f_2(\theta) \quad (20)$$

where k is a proportionality constant that must be fit to data. To put this into dimensional variables, substitute in variables for d^* and We, while f_1 and f_2 remain the as-yet unknown functions of viscosity and spray angle, respectively:

$$d = k \times h \left(\frac{\sigma}{\rho_l V^2 h} \right)^{1/3} \left(\frac{\rho_l}{\rho_g} \right)^{1/6} f_1(\text{Oh}) f_2(\theta) \quad (21)$$

The Bernoulli equation relates sheet velocity to applied injection pressure:

$$V = C_d \sqrt{\frac{2\Delta P}{\rho_l}} \quad (22)$$

Discharge coefficient (C_d) values are discussed by Post et al. (2017). They recommend $C_d \approx 0.9$ for standard flat-fan nozzles, $C_d \approx 0.7$ for pre-orifice nozzles, and $C_d \approx 0.5$ for air-induction nozzles. These values were obtained by measuring droplet velocities at sheet breakup with a phase-Doppler instrument and solving equation 22 for C_d as the unknown var-

iable. Dombrowski and Munday (1968) also used $C_d = 0.9$ for standard flat-fan nozzles. Substituting for velocity gives:

$$d = k \times h \left(\frac{\sigma}{C_d^2 \Delta P h} \right)^{1/3} \left(\frac{\rho_l}{\rho_g} \right)^{1/6} f_1(\text{Oh}) f_2(\theta) \quad (23)$$

The density ratio term $(\rho_l/\rho_g)^{1/6}$ will only vary by $\pm 1\%$ for the typical ranges of fluid properties shown in table 2, so it can be reasonably approximated as a constant and folded into the proportionality constant (k). We can further simplify equation 23 by combining the two terms that include the orifice size (h):

$$d = k' \times \left(\frac{h^2 \sigma}{C_d^2 \Delta P} \right)^{1/3} f_1(\text{Oh}) f_2(\theta) \quad (24)$$

The dependence of droplet size on surface tension was assessed experimentally by Butler Ellis et al. (2001) for standard flat-fan nozzles and pre-orifice flat-fan nozzles. They found that the effects of surface tension on droplet size were greater for surfactant solutions than for pure liquids. For surfactant solutions, which are relevant to agricultural spraying, they found power law exponents of 0.28 for standard flat-fan nozzles and 0.41 for pre-orifice nozzles, compared to the 0.33 exponent predicted by theory. They also noted that the surface tension should be measured at the surface age at breakup, about 2 ms.

For the effects of spray angle, various models have been proposed in which the droplet size is proportional to either the spray angle or the sine of the spray angle raised to a power. The primary physical effect of the spray angle is to spread the liquid sheet out, affecting the thickness of the sheet. The wider the spray angle, all else being constant, the thinner the liquid sheet will be at the point of breakup, and hence the smaller the droplets formed from atomization. From simple geometric arguments, the sheet thickness should be inversely proportional to the spray angle. Because the droplet size is predicted to be proportional to the initial sheet thickness (h) to the 2/3 power, the functional dependence on spray angle should be:

$$f_2(\theta) = \left(\frac{\theta}{110^\circ} \right)^{-2/3} \quad (25)$$

where 110° was arbitrarily chosen as a reference angle because it is a common angle used in agricultural flat-fan nozzles.

It is desirable to rearrange these relationships in terms of easily obtained operating parameters, i.e., nozzle size and operating pressure. Guler et al. (2007) showed a linear relationship between nozzle size and rated flow rate. Because the aspect ratio (height to width) of nozzles is reasonably constant, a function can be fit to Guler's data to predict the sheet thickness at the orifice (h) to the flow rating (FR) of the nozzle:

$$h = \sqrt{0.0875 \times (10 \times \text{FR})} \quad (26a)$$

for h in units of mm and FR in units of gallons per minute

(gpm) measured at 2.76 kPa (40 psi). For example, for a 110-03 nozzle, FR = 0.3, and the sheet thickness can be estimated as:

$$h = \sqrt{0.0875 \times (10 \times 0.3)} = 0.51 \text{ mm} \quad (26b)$$

Equation 26 is for standard flat-fan nozzles. For TeeJet air-induction nozzles the correlation is:

$$h = \sqrt{0.229 \times (10 \times \text{FR})} \quad (27)$$

On average, for the nozzles that Guler tested, the AI nozzles had an area 2.4 times larger than the XR nozzles of the same flow rating, a width 1.5 times larger than the XR nozzles, and an orifice height 1.6 times larger than the XR nozzles. Because AI nozzles have a lower discharge coefficient than XR nozzles, they must have a larger area to provide the same rated flow rate at the same pressure.

Note that most atomization models predict a value for D_{32} , while agricultural spray measurements are usually reported as volume mean diameter (VMD) or $D_{v0.5}$. The data from Nuyttens et al. (2007) show that the ratio of D_{32} to $D_{v0.5}$ is in a narrow range of 0.80 to 0.86, with the average ratio being:

$$D_{32} = 0.825 \times D_{v0.5} \quad (28)$$

With the knowledge that the orifice height (h) is proportional to the square root of the flow rating of the nozzle, equation 24 can be rewritten to give a general scaling law of droplet size with different nozzle and user parameters:

$$\begin{aligned} \text{VMD} = C_1 \times \left(\frac{\text{FR}}{01} \right)^{1/3} \left(\frac{P}{1 \text{ bar}} \right)^{-1/3} \left(\frac{\sigma}{\sigma_{\text{H}_2\text{O}}} \right)^{1/3} \\ \times \left(\frac{\theta}{110^\circ} \right)^{-2/3} C_d^{-2/3} f_1(\text{Oh}) \end{aligned} \quad (29)$$

The constant C_1 will have different values for different types of nozzles due to the different orifice sizes at the same flow ratings for standard, air-induction, and other types of nozzles.

RESULTS

The validation of the model presented here is hindered by the challenge of finding suitable comparison data. No published study analyzes the effects of pressure, nozzle size, nozzle type, nozzle angle, and spray formulation on droplet size. It is problematic to quantitatively compare measurements made by different authors with different nozzles and different measurement devices. It is also desirable to avoid laser diffraction measurements and use published data from more modern techniques for comparison. There are also issues of repeatability of measurements. Fritz et al. (2014) reported typical differences of 4% to 8% between laboratories for droplet size measurements with the same nozzle, day-to-day variance within each laboratory was found to be about 5%, and differences in $D_{v0.5}$ values as high as 14.9% were seen.

The first comparison (fig. 5) is for data provided by Tee-

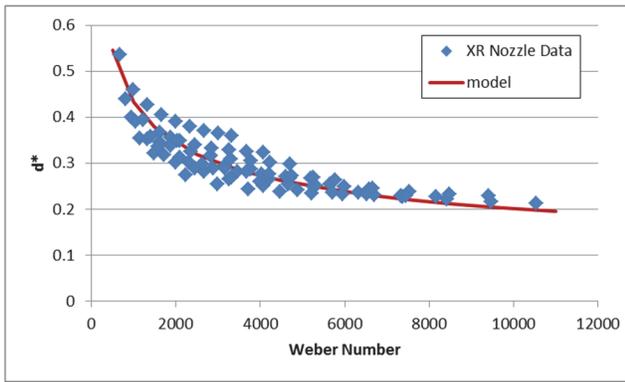


Figure 5. Non-dimensional droplet size (d^*) versus Weber number for TeeJet XR nozzles at varying pressure with water (data from TeeJet). The model function is equation 20 with a coefficient value of 4.33, so $d^* = 4.33We^{-1/3}$ ($R^2 = 0.795$).

Jet for a range of pressures with water as the fluid for XR 110 nozzles with VisiSizer (Oxford Lasers, Didcot, U.K.) measurements. Equation 28 was used to convert the reported $D_{v0.5}$ values to D_{32} .

SigmaPlot (Systat, 2014) was used to find the best-fit regression coefficient of 4.33. Because the experiments were conducted with tap water injected into laboratory air at standard conditions, the density ratio term in equation 21 can be calculated as:

$$\left(\frac{\rho_l}{\rho_g}\right)^{1/6} = \left(\frac{1000}{1.2}\right)^{1/6} = 3.07 \quad (30)$$

Thus, the constant k in equation 20 is $4.33/3.07 = 1.41$. Using this value of the constant curve fit to data from XR nozzles, the model was compared to measured droplet size data for AI nozzles, also provided by TeeJet (fig. 6). The largest discrepancies were at the lowest Weber numbers, which may indicate that a different atomization mechanism was dominant (in the first wind-induced regime instead of the second wind-induced regime).

Few authors have reported droplet size measurements for more than two or three pressures, making regression analysis problematic. Luck et al. (2015) report Malvern measurements on a custom 110° flat-fan nozzle for five different

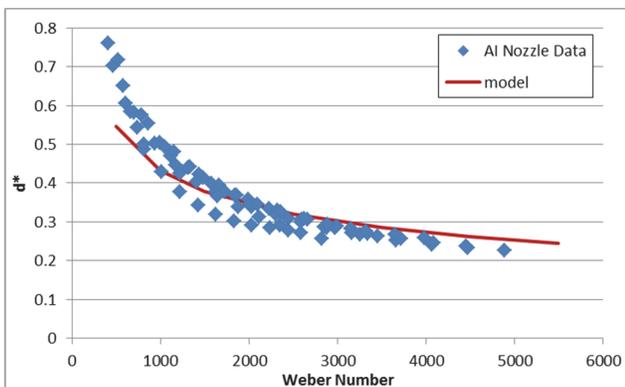


Figure 6. Non-dimensional droplet size (d^*) versus Weber number for TeeJet AI nozzles at varying pressure with water (data from TeeJet). The model function is the same as in figure 5 (eq. 20) with curve fit values of $k = 1.41$ and $d^* = 4.33We^{-1/3}$ ($R^2 = 0.814$).

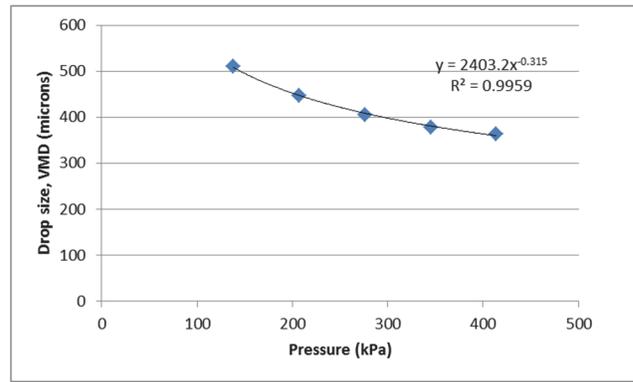


Figure 7. Droplet size versus pressure (data from Luck et al., 2015).

pressures. Because the orifice throat dimensions are not known, equation 23 cannot be used, but the power law curve fit exponent of -0.315, shown in figure 7, indicates that the $-1/3$ power dependence of droplet size on pressure is valid.

Few measurements have been reported in the literature on the effects of spray angle. Guler et al. (2012) measured droplet sizes from XR 110 and XR 80 nozzles using a VisiSizer. For -03 nozzles at 0.76 and 1.14 L min^{-1} and -06 nozzles at 2.27 L min^{-1} , their measurements show that, on average, the 80° nozzles produced droplets 23.6% larger than the 110° nozzles. The atomization model presented in this article predicted a change in droplet size of:

$$\left(\frac{80^\circ}{110^\circ}\right)^{-2/3} = 23.7\% \quad (31)$$

Thus, the model produced high conformity with experimental measurements of the effects of spray angle on droplet size. A final comparison of the model to the measurements of Guler et al. (2012) is presented for different nozzle sizes and spray angles, with a fixed pressure of 2.76 bar for XR nozzles (fig. 8). The accuracy of the model can be seen from the quality of fit.

CONCLUSIONS

The model presented here extends the previous work of Dombrowski and co-workers by incorporating a discharge coefficient, with the result that a single model can predict

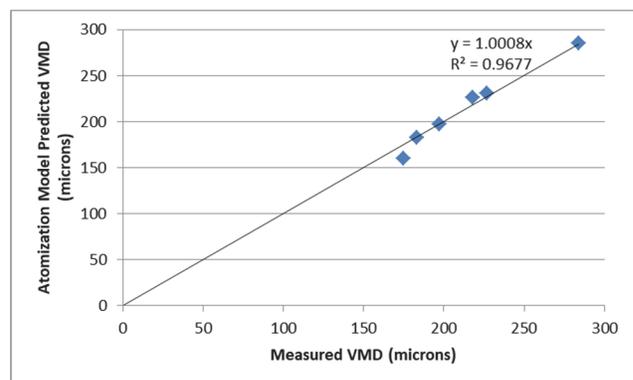


Figure 8. Comparison of model-predicted VMD with measurements of Guler et al. (2012) for standard flat-fan nozzles of varying rated flow rates and spray angles.

droplet sizes from all types of flat-fan nozzles with only a single empirical constant that remains a constant for all nozzles. Further, an improved model for spray angle dependence was introduced. This work also demonstrates the relationship between non-dimensionalization model parameters and end-user parameters (pressure and nozzle flow rating). The model can be used to predict how the droplet size from a nozzle will change when the pressure or formulation is changed.

Future work in this area can include effects of crosswind and three-dimensional effects on sheet breakup, which would require dedicated wind tunnel studies. For high-speed crosswind relevant to aerial applications, photographic examination of sheet breakup characteristics is also important. There may be a different atomization mechanism than the one proposed here. This model is only for Newtonian fluids; additional model development is required for non-Newtonian formulations, such as formulations with polymeric adjuvants.

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