Agricultural Economics Research Unit

The Effect of Taxation Method on Post-Tax Income Variability

By
A.T.G. McArthur

Technical Paper No. 13
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THE EFFECT OF TAXATION METHOD
ON
POST-TAX INCOME VARIABILITY

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A.T.G. McArthur

Agricultural Economics Research Unit Technical Paper No. 13
December 1970
In this paper Mr McArthur analyses the effect of various income tax assessment methods on year-to-year variation in post-tax income of self-employed people like farmers. Assessment methods involving adjustment factors for previous tax periods tend to increase the variation in post-tax income compared with pre-tax income, and methods involving P.A.Y.E. based on current income tend to reduce such variation. Studies of this sort should be of considerable interest to farmers, farmer organisations, other associations concerned with the self-employed, and the Inland Revenue Department itself.
THE EFFECT OF TAXATION METHOD ON POST-TAX INCOME VARIABILITY

INTRODUCTION

Income variability is one of the serious disadvantages of farming. It makes it difficult to organize farm development wisely and upsets the farm family's standard of living. In the past there have been boom years when farmers have spent wastefully to prevent windfall gains being lost to the farm in taxation. In years of low farm income it may be difficult to carry on a development plan started in better times and thus not exploit past investment. Moreover few farming families have the liquid reserves to see them over bad seasons and it is customary for banks and stock and station agents to carry their clients over bad seasons. Farmers have to "draw their horns in" too. Development ceases, holidays are foregone, and teenage children may be brought home from boarding school following a difficult season.

In the past New Zealand agricultural policy has aimed to reduce the fluctuations in farm income. The guaranteed price for butterfat and the floor price for wool are examples of policy aimed at reducing price fluctuations and the drought relief scheme is an example ameliorating the results of technical uncertainty.

The Income Tax Assessment Act of 1957 which introduced the pay-as-you-earn method of paying income tax encourages a system which increases rather than decreases the variation in post-tax income -
a reversal of previous policy. The Act provided for provisional and terminal taxation. Terminal taxation is the difference between the provisional tax paid in the previous year and the tax which should have been paid. This annual square up, which can be a refund of an overpayment or a demand for underpayment, adds extra variation to a farmer's post-tax income compared with the method of tax assessment before 1957. The Tax Department and accountants encourage farmers to base provisional tax on the previous year's income and this, in conjunction with terminal tax, results in three years' income having an influence on post-tax income as will be explained later.

While farmers and other businessmen are well aware of the adverse effects on post-tax income variation of provisional and terminal tax, the author has not found any quantitative investigational work done in this field.

Consequently this paper describes the provisions for paying tax under the existing taxation legislation in New Zealand and outlines some alternative methods. The use of the standard deviation as a measure of income variability is described. Then follows a case study in which the implications of these taxation methods are evaluated using pre-tax incomes from Lincoln College's Ashley Dene farm over a 16 year period. Finally, general analytical methods are developed for calculating the variance of post-tax income. These methods treat pre-tax income as a random variable. This makes it possible to predict the standard deviation of post-tax income under a wide range of conditions and to draw more general conclusions than is possible from a case study.
TAXATION ASSESSMENT METHODS

There are two methods which farmers can use as a basis for paying tax under the existing legislation which are referred to in this paper as the Normal and the PAYE methods. Four other tax assessment methods are then described.

Existing Methods. Provisional taxation is paid on an estimate of income for the current year, one third being paid in September and the rest being due in March. For those whose balance date is between April and the end of September and who regularly receive more than a half of their gross cash income after the 7th of February, it is possible to pay provisional tax in three equal amounts; in September, March and June.

There are two ways in which the estimate for the year is made. With the Normal method, provisional taxation is based solely on the income in the previous year. In other words the previous year's income is used as an estimate for the current year's income. Because pre-tax income varies there is always an adjustment by way of an annual squaring up called terminal tax.

Alternatively a farmer can choose the PAYE method of paying provisional tax. Here he submits an estimate of his expected income to the Tax Department. He may estimate and re-estimate his income up to the date of payment of the last tax instalment.

In making tax calculations under the PAYE method, I have assumed that the farmer and his accountant are able to estimate income for the current year with such precision that when he pays his
last tax instalment, the provisional tax paid equals the tax due for the year. Hence there is no terminal tax payable.

However, in practice, if a farmer chooses to estimate his income for provisional tax he runs the risk that should his provisional tax payment be less than 80 percent of the tax due for the year, there can be a penalty amounting to a tenth of the difference between the tax due and the provisional tax paid. Officially this penalty applies to any system of estimating income except where the estimate is based on last year's income.

Some examples will help clarify the position. Throughout this paper the income tax schedule announced in the 1970 budget which will come into operation in 1971-72 has been used. These rates are shown in the Appendix. The surcharge on income tax imposed by the recent "mini-budget" of October 1970 has been ignored throughout this paper. For these examples it has been assumed that total taxation exemptions amount to $2000. For the purposes of the examples I have selected hypothetical pre-tax incomes which decline. In the first example, the years have been numbered from year 3 in order that extra years can be added before this for other examples. Table 1 shows an example of the PAYE method of calculating post-tax income. This is the simplest method to describe but needs accurate forecasting and budgeting.
### Table 1

**AN EXAMPLE OF THE CALCULATION OF POST-TAX INCOME USING THE PAYE METHOD (TAX EXEMPTIONS ARE $2000)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre-Tax Income</th>
<th>Tax Due and Provisional Tax</th>
<th>Post-Tax Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) $</td>
<td>(2) $</td>
<td>(1)-(2) $</td>
</tr>
<tr>
<td>3</td>
<td>6000</td>
<td>990</td>
<td>5010</td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td>635</td>
<td>4365</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>345</td>
<td>3655</td>
</tr>
</tbody>
</table>

Post-tax income under the PAYE method is the difference between pre-tax income and provisional tax. Because we assume that income estimation for each year is perfect, tax due and provisional tax are the same amount. Hence no terminal tax is included in the calculation.

In Table 1 pre-tax income has a range of $2000 (from $4000 to $6000) while post-tax income has a range of only $1355. Thus the tax system buffers the post-tax income against a fluctuating pre-tax income. This reduces post-tax income variability compared with pre-tax income as will be demonstrated more conclusively later on.

Table 2 shows an example of the Normal method of calculating provisional and terminal tax. Hypothetical pre-tax incomes for years 1 and 2 are needed for this example.


Table 2 shows provisional tax separately from tax due, because provisional tax equals the tax due in the previous year. In this example an extra column for terminal tax has been added. This was zero under the PAYE method and did not appear in Table 1. The negative values in the terminal tax column, -$323 and -$355 mean that the Tax Department paid a rebate back to the farmer in years 3 and 5. Terminal tax equals provisional tax in the previous year less tax due in the previous year which is now known. In year 3, $482 was paid as provisional tax. In fact it turned out afterwards that $990 should have been paid. Hence the terminal tax for year 4 is $990-$482 which is $508, the figure shown in the terminal tax column of Table 2.

The post-tax income in Table 2 shows a range of $2339.
This is a wider range than the corresponding pre-tax income range of $2000. The Normal tax method disturbs the post-tax income increasing its year-to-year variability.

Other Possible Methods. Four other possible taxation methods will now be described, termed the Old, the 3 Year Moving Average (without terminal tax), the 3 Year Moving Average plus Terminal Tax, and the Exponential Average.

The Old method refers to the method of paying tax used before 1957 when tax was paid in the following year. This is similar to provisional taxation without a terminal tax, when provisional tax is based on the income earned the previous year. The Old method has been included for comparison because it substantiates the claim that since the new tax legislation was introduced, the taxation system has increased the variability of post-tax income. Table 3 gives an example of the Old method.

**TABLE 3**

**AN EXAMPLE OF THE CALCULATION OF POST-TAX INCOME USING THE OLD METHOD (TAXATION EXEMPTIONS ARE $2000)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre-Tax Income</th>
<th>Tax Due</th>
<th>Tax Paid</th>
<th>Post-Tax Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) $</td>
<td>$</td>
<td>(2) $</td>
<td>(1)-(2) $</td>
</tr>
<tr>
<td>2</td>
<td>4500</td>
<td>482</td>
<td>482</td>
<td>5518</td>
</tr>
<tr>
<td>3</td>
<td>6000</td>
<td>990</td>
<td>482</td>
<td>4010</td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td>635</td>
<td>990</td>
<td>3365</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>345</td>
<td>635</td>
<td></td>
</tr>
</tbody>
</table>
Pre-tax income has a range of $2000 while post-tax income has a range of $2153, which is less than the range of $2339 shown in post-tax income under the Normal method in Table 2.

Instead of basing tax on a single year's income as in Table 3, it would be possible to base tax on the average of a number of years, if the tax legislation were changed. This would have the advantage of reducing the extra tax which farmers pay because their pre-tax income fluctuates. (McArthur 1969) A moving average without terminal tax also has the advantage to the developer that he can develop away from his tax bill. Low incomes in the early years of a farming career reflect in a moving average on which tax is based, thus reducing the taxation demand in years of prosperity earnt by the developed farm. Thus it could be a tax incentive for development as well as a post-tax income stabilizer as will be shown later on. A 3 Year Moving Average was chosen as a compromise between possible political acceptance on the one hand and stability of income on the other.
Table 4 shows an example of this method.

TABLE 4

AN EXAMPLE OF THE CALCULATION OF POST-TAX INCOME USING THE 3 YEAR MOVING AVERAGE METHOD (WITHOUT TERMINAL TAX)

(TAXATION EXEMPTIONS ARE $2000)

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre-Tax Income</th>
<th>Estimated Income Based on a 3 Year Moving Average</th>
<th>Tax Paid and Provisional Tax</th>
<th>Post-Tax Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5500</td>
<td>$</td>
<td>$(2)</td>
<td>$(1)-(2)</td>
</tr>
<tr>
<td>2</td>
<td>4500</td>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>3</td>
<td>6000</td>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td>5333</td>
<td>737</td>
<td>4263</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>5167</td>
<td>686</td>
<td>3314</td>
</tr>
</tbody>
</table>

The 3 Year Moving Average of $5333 in year 4 is calculated by summing the pre-tax incomes of the three previous years of $6000 plus $4500 plus $5500 and then averaging them by dividing by 3. This provides the basis for the tax paid of $737 in year 4. Table 4 shows only two complete years for the sake of simplicity and so comparison between the post-tax incomes in this table and the previous tables is not possible. This applies to Tables 5 and 6 as well.

The 3 Year Moving Average plus Terminal Tax is the fifth possible method. It would not require any significant change in the legislation except that the Tax Department would have to class a
moving average as a fair basis for paying provisional tax. The effect of this would be that the Department would not penalize farmers if the estimate for provisional tax based on the moving average was less than 80 percent of the tax actually due for that year. Table 5 shows the calculation.

**TABLE 5**

An example of the calculation of post-tax income using the 3 year moving average plus terminal tax method

(Taxation exemptions are $2000)

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre-Tax Income</th>
<th>Tax Due</th>
<th>Estimated Income Based on a 3 Year Moving Average</th>
<th>Provisional Tax</th>
<th>Terminal Tax</th>
<th>Post-Tax Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5500</td>
<td>$805</td>
<td>$</td>
<td></td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>2</td>
<td>$4500</td>
<td>$482</td>
<td>$</td>
<td></td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>3</td>
<td>$6000</td>
<td>$990</td>
<td>$</td>
<td></td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>4</td>
<td>$5000</td>
<td>$635</td>
<td>$5333</td>
<td>$737</td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>5</td>
<td>$4000</td>
<td>$345</td>
<td>$5167</td>
<td>$686</td>
<td>$-102</td>
<td>$3416</td>
</tr>
</tbody>
</table>

This method is almost identical to the previous method shown in Table 4 except the addition of terminal tax. There is a rebate of $102 in the terminal tax column. This is because only $635 was due in year 4 but in fact the three year moving average produced a provisional tax of $737 - an overpayment of $102 which is refunded as terminal tax the following year.
The final method is the **Exponential Average**. This method has no terminal tax. This average was constructed so that it produces the same variance in post-tax income as the **3 Year Moving Average** method. It is comparable in almost every way with the **3 Year Moving Average** (without terminal tax) method except that this method keeps up with income trends better than a simple moving average.

The **Exponential Average** used in this paper has the effect of giving a weight of \( \frac{1}{2} \) to this year's income, a weight of \( \frac{1}{4} \) to the previous year, \( \frac{1}{8} \) to the year before that, and \( \frac{1}{16} \) to the year before that again and so on ad infinitum. A **3 Year Moving Average** puts \( \frac{1}{3} \) of the weight on this year's income, \( \frac{1}{3} \) on the previous year, and \( \frac{1}{3} \) on the year before that, but goes no further. If a downward trend occurs with a disastrous year, this will be reflected in the **Exponential Average** (a weight of \( \frac{1}{2} \) in the first year) to a greater degree than in the moving average (a weight of \( \frac{1}{3} \) in the first year).

The particular **Exponential Average** used here is found by making the estimated pre-tax income for next year equal to half this year's actual income plus half last year's estimate.

Imagine we are at the end of year 2. Pre-tax income was $4500 in year 2. Assume that $5500 was the estimate made in year 1 for year 2. Then the estimate for year 3 is

\[
\left( \frac{1}{2} \times 4500 \right) + \left( \frac{1}{2} \times 5500 \right) = 5000.
\]

$5000 becomes the estimate on which to base tax in year 3.

Moving onto the end of year 3 and making an estimate for year 4 on which to base tax, we take half the actual income of $6000 for year 3 and half the estimate for year 3 of $5000 made the previous year.
This adds to $5500 and is the estimate for year 4. The result of this series of calculations together with post-tax incomes are shown in Table 6.

TABLE 6

AN EXAMPLE OF THE CALCULATION OF
POST-TAX INCOME USING THE EXPONENTIAL AVERAGE METHOD

<table>
<thead>
<tr>
<th>Year</th>
<th>Pre-Tax Income</th>
<th>Estimated Income Based on Exponential Average</th>
<th>Tax Paid</th>
<th>Post-Tax Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5500</td>
<td>$</td>
<td>$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6000</td>
<td>5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td>5500</td>
<td>805</td>
<td>4195</td>
</tr>
<tr>
<td>5</td>
<td>4000</td>
<td>5250</td>
<td>711</td>
<td>3289</td>
</tr>
</tbody>
</table>

Like the 3 Year Moving Average, the Exponential Average reduces extra tax payments because of income fluctuation and also acts as an incentive. However it would need legislative changes before it could be introduced.

Having described six methods of taxation, the next section deals with the measurement of income variability.

MEASUREMENT OF INCOME VARIABILITY

Statisticians measure year-to-year variability with an index of
dispersion called the standard deviation. The standard deviation is defined as the square root of the average of "deviations-from-the-mean squared".

Taking the hypothetical pre-tax incomes in Table 2, we find their mean is $5000. \[ (5500 + 4500 + 6000 + 5000 + 4000)/5 \].

The deviations from this mean of $5000 are +500, -500, +1000, 0, and -1000. One way of measuring variation is to find the average absolute deviation from the mean. Ignoring the signs we calculate the average absolute deviation as \( (500 + 500 + 1000 + 0 + 1000)/5 \) which is $600.

Squaring the deviations has the advantage of automatically turning the negative deviations into positive numbers. Minus 500 squared becomes plus 250,000. Minus 1000 squared becomes 1,000,000.

Having added up and averaged these squares of deviations, the square root is found, bringing the resulting standard deviation to a numerical value not far removed from the average absolute deviation just calculated.

The calculation of the standard deviation is shown in Table 7.
### TABLE 7

**AN EXAMPLE OF A STANDARD DEVIATION CALCULATION**

<table>
<thead>
<tr>
<th>Pre-Tax Income</th>
<th>Deviation</th>
<th>Deviation Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>5500</td>
<td>+500</td>
<td>250,000</td>
</tr>
<tr>
<td>4500</td>
<td>+500</td>
<td>250,000</td>
</tr>
<tr>
<td>6000</td>
<td>+1000</td>
<td>1,000,000</td>
</tr>
<tr>
<td>5000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4000</td>
<td>-1000</td>
<td>1,000,000</td>
</tr>
</tbody>
</table>

**SUM = 2,500,000**

VARIANCE = $2,500,000/4 = $625,000

STANDARD DEVIATION = VARIANCE = 625,000 = $791

In Table 7 the division of the "sum of deviations squared" by 4 rather than 5 is the statistically correct procedure. The divisor is always one less the number of observations.

With an estimate of standard deviation of pre-tax income, it is possible to deduce the standard deviation of post-tax income. This mathematical advantage is the reason that the standard deviation was selected as a measure of income variation.

**A CASE HISTORY**

The six methods of taxation were tried out on a case history. A series of incomes from one farm which had experienced wide
fluctuation in income due to varying seasons and prices were readily available. Ashley Dene is an 800 acre light-land sheep farm, belonging to Lincoln College, which is farmed on a normal commercial basis. The soils have four to six inches of silt loam over gravel and with an average rainfall over the last 15 years of 27 inches. The property is drought prone. This leads to variation in wool weights, lambing percentages, and lamb slaughter weights. This technical uncertainty is compounded with the price uncertainty associated with wide variations in the prices for wool and lamb. The story of Ashley Dene has been well documented by Flay (1965) and more recently by Stewart (1970).

A series of 19 years data was available from Lincoln College's Farm Accounts. The equivalent to a farmer's pre-tax income is the figure in these accounts termed "surplus for year". This consists of total revenue less costs. However these costs include the cost of a farm manager but on a normal farm the owner is the manager. On the other hand "surplus for year" excludes interest, a charge which farmers normally pay. However any constant error is of no significance in this analysis because we are interested in income variation rather than average level of pre-tax income. "Surplus for the year" in the Ashley Dene accounts was taken as being equivalent to a pre-tax income on a normal farm.

Pre-tax income for Ashley Dene over 19 years is shown in Table 8.
In the application of the six methods of tax assessment to this series, the initial years 1952, 1953 and 1954 were used to provide the basis for provisional and terminal tax payments in the years 1955 onwards. These first three years were used to warm the system up. The 16 relevant years lie between 1955 and 1970. Pre-tax income varied from $12168 to $1310, averaging $4870 with a standard deviation of $3796. The coefficient of variation (standard deviation expressed as a percentage of the mean) amounts to 78%. This figure indicates the high degree of uncertainty faced by sheep farmers on light land.

A FORTRAN computer program was written to execute the calculations needed to find post-tax income from Ashley Dene pre-tax incomes under the six taxation methods which have just been described. It assumed that total tax exemptions amounted to $1000. The post-tax income standard deviation was computed and compared with the pre-tax...
income standard deviation by an index termed the relative variability.

\[
\text{Relative Variability} = \frac{\text{S.D. of Post-Tax Income}}{\text{S.D. of Pre-Tax Income}}
\]

This relative index measures the disturbing or buffering effect of each taxation method. The relative variability is in fact the factor by which the standard deviation of pre-tax income is increased by the taxation system in order to find the post-tax income standard deviation.

The results in terms of standard deviation and relative variability are shown in Table 9.

**TABLE 9**

**STANDARD DEVIATION AND RELATIVE VARIABILITY OF POST-TAX INCOME UNDER SIX TAXATION METHODS (ASHLEY DENE DATA)**

<table>
<thead>
<tr>
<th>Method of Taxation</th>
<th>Standard Deviation</th>
<th>Relative Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>4429</td>
<td>1.17</td>
</tr>
<tr>
<td>PAYE</td>
<td>2726</td>
<td>0.72</td>
</tr>
<tr>
<td>Old</td>
<td>3854</td>
<td>1.01</td>
</tr>
<tr>
<td>3 Year Moving Average</td>
<td>3911</td>
<td>1.03</td>
</tr>
<tr>
<td>3 Year Moving Average plus Terminal Tax</td>
<td>3973</td>
<td>1.05</td>
</tr>
<tr>
<td>Exponential Average</td>
<td>3844</td>
<td>1.01</td>
</tr>
<tr>
<td>Pre-Tax Income</td>
<td>3796</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 9 brings out the point that the Normal method would have had a very disturbing influence on post-tax income if it has been applied to a farmer owning Ashley Dene. Compared with pre-tax income variation the method raises the standard deviation of post-tax by 17 percent, the relative variability being 1.17.

This is further amplified by the graph which compares pre-tax and post-tax income using the Normal method over 16 years. The troughs in years 6 and 14 are good examples of the un stabilizing effect of this method of tax assessment.

If a farmer owning Ashley Dene had used the PAYE method by accurately predicting his income in each year, his post-tax income variability would have been buffered by the tax system, reducing the post-tax income standard deviation to 0.72 of the pre-tax income standard deviation.

The relative variability of the Old method of paying tax of 1.01 compared with the Normal method of 1.17 shows that the legislative changes which introduced the pay-as-you-earn system increased post-tax income variation under Ashley Dene conditions.

The 3 Year Moving Average plus Terminal Tax method would have resulted in a relative variability of only 1.05 which is a distinct improvement on the Normal figure of 1.17.

The 3 Year Moving Average and the Exponential Average both have low relative variability values of 1.03 and 1.01 respectively. This indicates that if these methods were introduced to provide an incentive for developers, post-tax income variation would be less than it is at present.
GRAPH

VARIATION IN INCOME AT ASHLEY DENE

PRE-TAX

POST-TAX (NORMAL)

YEAR

DOLLARS
These results are confirmed by a mathematical analysis in the next section and generalized to cover a wider set of circumstances rather than being limited to one case history at Ashley Dene. Those who are not familiar with the operations involved with expected values and variances can proceed to the discussion at the end of this paper after the inspection of Table 12. Table 12 summarizes the results from calculating the relative variability by formulae developed for five tax assessment methods given various levels of expected pre-tax income.

AN ANALYTICAL APPROACH

Analytical methods have been developed in order to calculate relative variability of the six taxation methods for a pre-tax income with any mean and standard deviation. This involved formulating a mathematical model for each method, deriving a simplified tax function, developing formulae for the variance of post-tax income for each method, and checking the result of these formulae against Monte Carlo simulated results.

Models of the Taxation Methods.
The symbol list is as follows:

\[ C_t \] is post-tax income in the \( t \)th year,

\[ \sigma_C^2 \] is the variance of post-tax income,

\[ I_t \] is the pre-tax income in the \( t \)th year,

\[ \hat{I}_t \] is the estimate for pre-tax income for the \( (t+1) \)th year made in the \( t \)th year,
\( \bar{I} \) is mean pre-tax income,
\( \sigma^2 \) is the variance of pre-tax income,
\( X_t \) is taxable income in the \( t^{th} \) year,
\( \bar{X} \) is mean taxable income,
\( X \) is taxable income,
\( T_t \) is tax due in the \( t^{th} \) year,
\( T \) is tax due,
\( N \) is the number of years in a moving average,
\( \alpha \) is the exponential average smoothing constant,
\( M \) is total taxation exemptions,
\( F \) is the first derivative of the tax function,
\( R_i \) is the \( i^{th} \) sample from a uniform distribution,
\( R_t^* \) is a random normal number for the \( t^{th} \) year,
\( S \) is the standard deviation of the log of pre-tax income,
\( \mu \) is the mean of the log of pre-tax incomes.

For tax due in the \( t^{th} \) year

\[
T_t = f(I_t, M) \tag{1}
\]

a tax function which will be explained in the next section.

For the Normal method, post-tax income for the \( t^{th} \) year can be calculated as:

\[
C_t = I_t - f(I_{t-1}, M) - (f(I_{t-1}, M) - f(I_{t-2}, M)),
\]
Equation 2 shows that with the normal method, three years of pre-tax income contribute to post-tax income and how the contribution from the income in the \((t-1)\)th year is doubled.

For the PAYE method, post-tax income can be calculated as

\[ C_t = I_t - f(I_t, M) \]  

only one year's pre-tax income contributing to post-tax income.

For the Old method, post-tax income can be calculated as

\[ C_t = I_t - f(I_{t-1}, M) \]

provisional tax being based on the income in the previous year with no terminal tax.

For an \textit{N Year Moving Average} method, post-tax income can be calculated as

\[ C_t = I_t - f\left(\frac{1}{N} \sum_{i=1}^{N} I_{t-i}, M\right) \]  

there being no term for provisional tax.

For the \textit{N Year Moving Average plus Terminal Tax}, post-tax income can be calculated as

\[ C_t = I_t - f\left(\frac{1}{N} \sum_{i=1}^{N} I_{t-i}, M\right) - \left[ f(I_{t-1}, M) - f\left(\frac{1}{N} \sum_{i=1}^{N} I_{t-i-1}, M\right) \right] \]

the second term on the R.H.S. is provisional tax and the third is terminal tax.
It is convenient at this point to explain that in the analytical method, the assumption is made that each year's pre-tax income is a random variable, each year's income being independent of any other. Further it is assumed that the mean of these random variables are equal as are their variances.

$\bar{I}$ is the mean of pre-tax incomes and

$\sigma^2$ is the variance of pre-tax income.

Hence the variance of the moving average of $N$ years will have a variance of $\sigma^2/N$.

For the Exponential Average the post-tax income in the $t^{th}$ year can be calculated as

$$C_t = I_t - f[(\alpha I_{t-1} - (1-\alpha) \hat{I}_{t-2}), M]$$

(7)

provisional tax being based on an exponentially weighted moving average in which greatest weight is placed on the most recent incomes. This weighting is achieved through the smoothing constant $\alpha$ which lies between zero and one. $\hat{I}_{t-2}$ is the estimated pre-tax income, an estimate made in the $(t-2)^{th}$ year, forecasting pre-tax income for the $(t-1)^{th}$ year.

In the $t^{th}$ year, the forecast for the income in the next year $I_t$ is,

$$\hat{I}_t = \alpha I_t + (1-\alpha) \hat{I}_{t-1},$$

(8)

which is this year's actual pre-tax income weighted by $\alpha$, plus the forecast for this year made in the previous year weighted by $(1-\alpha)$. In the case of $\alpha=1$, the exponential average merely uses last year's
income to predict this year's income. When $\alpha = 1$ we have the Old method.

As with the $N$ Year Moving Average it is convenient to find the variance of the Exponential Average. Developing equation 8, we have,

$$I_t^* = \alpha I_t + (1-\alpha) [\alpha I_{t-1} + (1-\alpha) I_{t-2}]$$

$$= \alpha I_t + (1-\alpha) \alpha I_{t-1} + (1-\alpha)^2 I_{t-2}$$

$$= \alpha I_t + (1-\alpha) \alpha I_{t-1} + (1-\alpha)^2 \alpha I_{t-2} + (1-\alpha)^3 \alpha I_{t-3} \ldots$$

The variance of the exponential average is,

$$\text{Var}(I^*) = \sigma^2 \alpha^2 + [(1-\alpha)\alpha]^2 \sigma^2 + [(1-\alpha)^2 \alpha]^2 \sigma^2 + \ldots$$

$$= \sigma^2 [\alpha^2 + (1-\alpha)^2 \alpha^2 + (1-\alpha)^4 \alpha^2 + (1-\alpha)^6 \alpha^2 + \ldots]$$

$$= \sigma^2 \alpha^2 / (1-(1-\alpha)^2) \quad (9)$$

The variance of the exponential average will equal the variance of a moving average of $N$ years when,

$$\sigma^2 / N = \sigma^2 \alpha^2 / (1-(1-\alpha)^2)$$

Hence,

$$N = 2/\alpha - 1 \quad \text{and} \quad \alpha = 2/(N+1) \quad \text{for equal variance of the two averages given} \ \alpha \ \text{or} \ N \ \text{respectively. A moving average of 3 years was used in the Ashley Dene Case history. This has the same variance as an Exponential Average with a smoothing constant of} \ 2/(3+1) = 0.5, \ \text{the smoothing constant used in this paper.}$$

The Tax Function.

The schedule of taxation rates set out in the 1970 Budget for
operation in the years from 1971-72 onwards is a function of pre-tax income and exemptions. Taxable income $X_t$ is pre-tax income less exemptions.

$$X_t = I_t - M$$ (10)

Within the range of taxable income between zero and $12000$ a quadratic function,

$$T_t = aX_t^2 + bX_t + c$$ (11)

fits the tax schedule reasonably well.

A least squares computer routine for fitting polynomial equations (Anon. 1967) was used to find values for the tax function constants $a$, $b$, and $c$ such that the squares of the differences between actual tax and estimated tax (by equation 11) were minimized when values for taxable income were used at $500$ intervals in the range $0$ to $12000$.

Values found for the constants were $a = 1.42 \times 10^{-5}$, $b = 0.24$ and $c = -139$. Subsequently the following simplified tax function was used to determine tax due.

$$T_t = 1.42 \times 10^{-5} X_t^2 + 0.24 X_t - 139$$ (12)

Table 10 shows the comparison between actual tax and estimated tax using the simplified tax function given by equation 12.
TABLE 10

A COMPARISON BETWEEN ACTUAL TAX AND ESTIMATED TAX

<table>
<thead>
<tr>
<th>Taxable Income</th>
<th>Actual Tax</th>
<th>Estimated Tax</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-139</td>
<td>139</td>
</tr>
<tr>
<td>1000</td>
<td>124</td>
<td>116</td>
<td>8</td>
</tr>
<tr>
<td>2000</td>
<td>345</td>
<td>399</td>
<td>54</td>
</tr>
<tr>
<td>3000</td>
<td>635</td>
<td>711</td>
<td>76</td>
</tr>
<tr>
<td>4000</td>
<td>990</td>
<td>1052</td>
<td>-62</td>
</tr>
<tr>
<td>5000</td>
<td>1390</td>
<td>1420</td>
<td>-30</td>
</tr>
<tr>
<td>9000</td>
<td>3240</td>
<td>3180</td>
<td>60</td>
</tr>
<tr>
<td>12000</td>
<td>4700</td>
<td>4797</td>
<td>-97</td>
</tr>
</tbody>
</table>

Equation 12 fits the actual tax schedule quite well as shown by the reasonably small residuals in Table 10.

The variance of tax payments can be approximated by the use of Taylor's Theorem. Equation 13 is a simplified version of equation 11.

\[ T = aX^2 + bX + c \]  

(13)

Expanding this as a Taylor series about the mean of taxable income \( \bar{X} \) we have,

\[ T = f(\bar{X}) + \left[ \frac{dT}{dX} \right] \frac{1}{X}(X-\bar{X}) + \frac{1}{2} \left[ \frac{d^2T}{dX^2} \right] \frac{1}{X^2} (X-\bar{X})^2 + \frac{1}{2\cdot3} \left[ \frac{d^3T}{dX^3} \right] \frac{1}{X^3} (X-\bar{X})^3 + \ldots \]  

(14)
The values for the derivatives of equation 13 are,

\[
\left( \frac{dT}{dx} \right)_x = 2a \bar{x} + b = 2 \times 1.42 \times 10^{-5} \bar{x} + 0.24 \quad (15)
\]

\[
\frac{d^2 T}{dx^2} = 2a = 2 \times 1.42 \times 10^{-5}
\]

\[
\frac{d^3 T}{dx^3} = 0
\]

The fourth term of equation 14 will be zero because the third derivative is zero and all those that follow it. As the second derivative is very small we can approximate tax by

\[
T \approx f(\bar{x}) - \left( \frac{dT}{dx} \right)_x (X-\bar{x}),
\]

and by re-arranging the terms we have,

\[
T - f(\bar{x}) \approx \left( \frac{dT}{dx} \right)_x (X-\bar{x})
\]

Squaring both sides and taking expected values,

\[
E(T-f(\bar{x}))^2 \approx \left( \frac{dT}{dx} \right)^2 E(X-\bar{x})^2
\]

Therefore the variance of tax payment, \( \text{Var}(T) \), is

\[
\text{Var}(T) \approx \left( \frac{dT}{dx} \right)^2 E(X-\bar{x})^2
\]

Because exemptions (M) are constant, the variance of pre-tax income, \( \sigma^2 \), equals the variance of taxable income.

\[
E(X-\bar{x})^2 = \sigma^2.
\]
Thus,

$$\text{Var}(T) = \frac{d^2 T}{dX^2} \sigma^2$$

$$= \left( \frac{dT}{dX} \right)^2 \sigma^2$$

$$= \left( \frac{dT}{dX} \right)^2 \sigma^2$$

(16)

Example: Let $\sigma^2 = 1000^2$

$I = 6500$

$M = 1500$

Then $X = 5000$ by equation 10.

The first derivative of equation 13 by equation 15 is,

$$\frac{dT}{dX} = 2 \times 1.42 \times 10^{-5} \times 5000 + 0.24 = 0.382$$

The variance of tax paid by equation 16 is,

$$\text{Var}(T) = 0.382^2 \times 1000^2 = 146,000$$

The Variance of Post-Tax Income.

The position has now been reached where we can derive the formulae for the variance of post-tax income given the mean and standard deviation of pre-tax income. To simplify the notation let

$$F = \left( \frac{dT}{dX} \right) \frac{I}{I-M}$$

and also let $\sigma^2_c$ be the variance of post tax income.

For the Normal method (see equation 2), the variance of post-tax income is,

$$\sigma^2_c = \sigma^2 + 2^2 F^2 \sigma^2 + F^2 \sigma^2$$

$$= \sigma^2 (1 + 5 F^2)$$.  

(17)
For the PAYE method (see equation 3) the variance of post-tax income is,
\[ \sigma_c^2 = \sigma^2 (1-F)^2. \] (18)

For the Old method (see equation 4), the variance of post-tax income is,
\[ \sigma_c^2 = \sigma^2 (1+F)^2. \] (19)

For the N Year Moving Average method (see equation 5), the variance of post-tax income is,
\[ \sigma_c^2 = \sigma^2 + F^2 \frac{\sigma^2}{N} \]
\[ = \sigma^2 (1 + F^2/N). \] (20)

For the Exponential Average method (see equations 7 and 9) the variance of post tax income is,
\[ \sigma_c^2 = \sigma^2 + F^2 \frac{\sigma^2 \alpha^2}{(1 - (1-\alpha)^2)} \]
\[ = \sigma^2 (1 + F^2\alpha^2/(1 - (1-\alpha)^2)). \] (21)

For the N Year Moving Average with Terminal Tax, it is necessary to derive two covariance terms. Repeating the model for post-tax income we have,
\[ C_t = I_t - f(\frac{1}{N}\sum_{i=1}^{N} I_{t-i,M})-[f(I_{t-1,M})-f(\frac{1}{N}\sum_{i=1}^{N} I_{t-1-i,M})] \] (6)
\[ = I_t - f(\frac{1}{N}\sum_{i=1}^{N} I_{t-i,M})-f(I_{t-1,M})+f(\frac{1}{N}\sum_{i=1}^{N} I_{t-i-1,M}). \]
\[ \text{(term 2)} \quad \text{(term 3)} \quad \text{(term 4)} \]
\[ \sigma_c^2 = \sigma^2 + 2\sigma^2 + 2\sigma^2/\nu + 2\sigma^2 + 2\sigma^2/\nu \]

\[- 2\sigma^2 \text{COV (term 2, term 4)} + 2\sigma^2 \text{COV (term 2, term 3)}. \quad (22)\]

Because exemptions, \( M \), are constant we can consider the covariance between taxable incomes instead of between pre-tax incomes. The covariance between term 2 and term 3, will first be derived. This is the covariance between the moving average containing last year's taxable income \( \frac{1}{N} \sum_{i} X_{t-1} \) (term 2) and last year's taxable income \( X_{t-1} \) (term 3).

\[
\text{COV (term 3, term 2)} = \frac{1}{N} \sum_{i} X_{t-1} \frac{X_{t-1}-\bar{X}}{N} + \frac{X_{t-2}-\bar{X}}{N} + \ldots + \frac{X_{t-N}-\bar{X}}{N}.
\]

For the covariance between term 2 and term 4, the two moving averages \( \frac{1}{N} \sum_{i} X_{t-1} \) and \( \frac{1}{N} \sum_{i} X_{t-1-1} \), we have,

\[
\text{COV (term 2, term 4)} = \frac{X_{t-1}-\bar{X}}{N} + \frac{X_{t-2}-\bar{X}}{N} + \ldots + \frac{X_{t-N}-\bar{X}}{N} + \frac{X_{t-2}-\bar{X}}{N} + \frac{X_{t-3}-\bar{X}}{N} + \ldots + \frac{X_{t-N}-\bar{X}}{N}.
\]

Multiplying we obtain,

\[
\text{COV (term 2, term 4)} = \frac{X_{t-1}-\bar{X}}{N} + \frac{X_{t-2}-\bar{X}}{N} + \ldots + \frac{X_{t-N}-\bar{X}}{N} + \frac{X_{t-2}-\bar{X}}{N} + \frac{X_{t-3}-\bar{X}}{N} + \ldots + \frac{X_{t-N}-\bar{X}}{N}.
\]
There are $N-1$ terms of the form $E[(\frac{-X}{N})^2]$ which equal $\sigma^2/N$. The remaining terms are zero. Thus,

$$\text{COV (term 2, term 4)} = (N-1)\sigma^2/N.$$ 

Completing equation 22 we have,

$$\sigma^2_c = \sigma^2 + P^2 \sigma^2 + P^2 \sigma^2/N + F^2 \sigma^2 + F^2 \sigma^2/N$$
$$- 2P^2 (N-1)\sigma^2/N^2 + 2P^2 \sigma^2/N$$
$$= \sigma^2 (1 + P^2 + 4F^2/N + 2P^2 (N-1)/N^2). \tag{23}$$

Having developed formulae for calculating the post-tax variance from the pre-tax variance for each method of taxation, the next section deals with a Monte Carlo check.

A MONTE CARLO CHECK

The derivation of the formulae for calculating the variance of post-tax income required two simplifications so that the formulae are approximations only. The two simplifications were:

1. A least squares fitted quadratic tax function instead of the actual tax function (see Table 10).
2. A term was dropped from the Taylor series.

To check that these simplifications are not likely to invalidate conclusions drawn from the formulae for post-tax variance, they were checked against a Monte Carlo approach for determining post-tax variance.

It was decided to assume that pre-tax incomes are lognormally distributed as this tends to be the distribution that fits economic
data. It has the advantage that it does not have any negative values in its domain. (See Aitchison and Brown 1957).

A thousand synthetic pre-tax incomes (Lognormally distributed) were generated by Monte Carlo methods using a FORTRAN computer program.

The Lehmer (1951) method of power residues was used to generate samples from a uniform distribution. These samples were transformed into random normal numbers by,

\[ R_t^* = \sum_{i=1}^{12} R_i - 6 \]

where \( R_i \) is the \( i \)th sample drawn from a uniform distribution and \( R_t^* \) is a random normal deviate.

Samples from a lognormal distribution were generated by finding the standard deviation of the log of pre-tax incomes, \( S \).

\[ S = \log_e \left( (\sigma / \bar{I})^2 + 1 \right) \]

where \( \bar{I} \) is the mean of pre-tax income and \( \sigma \) is the standard deviation as before. The mean of the log of pre-tax income, \( \mu \), is

\[ \mu = \log_e (\bar{I}) - S^2/2 \]

The generated pre-tax income in the \( t \)th year is

\[ I_t = e^{(\mu + R_t^* S)} \]

A thousand random normal numbers were first generated and then these were transformed into three pre-tax income series, one with a mean of \$3000 another with a mean \$5000 and a third with a mean of \$7000. The standard deviation was made equal to the mean in each case.

As before a total tax exemption of \$1000 was assumed. The six methods of taxation were evaluated over the three pre-tax income
series. Table 11 shows the comparison of the standard deviation of post-tax income calculated from these Monte Carlo generated pre-tax incomes with the standard deviations calculated by the formulae developed in the last section.

**TABLE 11**

A COMPARISON BETWEEN THE STANDARD DEVIATION OF POST-TAX INCOME FOUND BY MONTE CARLO METHODS AND FOUND BY THE FORMULAE (IN PARENTHESIS) FOR THE SIX METHODS OF TAXATION.

The Deviations between the two results have a sign

<table>
<thead>
<tr>
<th>Pre-Tax Income Mean</th>
<th>Pre-Tax Income SD</th>
<th>Post-Tax Income Mean</th>
<th>Post-Tax Income SD</th>
<th>Standard Deviation</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000 3000</td>
<td>(2102)</td>
<td>1809 3065</td>
<td>2882 3065</td>
<td>3201</td>
<td>3745</td>
</tr>
<tr>
<td>3000 3000</td>
<td>(3130)</td>
<td>(3044)</td>
<td>(3349)</td>
<td>(3604)</td>
<td></td>
</tr>
<tr>
<td>+293</td>
<td>+65</td>
<td>+162</td>
<td>+148</td>
<td>+141</td>
<td></td>
</tr>
<tr>
<td>5000 5000</td>
<td>(3226)</td>
<td>2702 5207</td>
<td>4853 5530</td>
<td>5530</td>
<td>6627</td>
</tr>
<tr>
<td>5000 5000</td>
<td>(5305)</td>
<td>(5103)</td>
<td>(5807)</td>
<td>(6381)</td>
<td></td>
</tr>
<tr>
<td>+524</td>
<td>+98</td>
<td>+250</td>
<td>+277</td>
<td>-246</td>
<td></td>
</tr>
<tr>
<td>7000 7000</td>
<td>(4120)</td>
<td>3586 7356</td>
<td>6832 7877</td>
<td>7877</td>
<td>9536</td>
</tr>
<tr>
<td>7000 7000</td>
<td>(7569)</td>
<td>(7194)</td>
<td>(8487)</td>
<td>(9511)</td>
<td></td>
</tr>
<tr>
<td>+534</td>
<td>+213</td>
<td>+362</td>
<td>+610</td>
<td>-25</td>
<td></td>
</tr>
</tbody>
</table>

There are small but appreciable differences between the post-tax standard deviation calculated by the two methods. Part of the difference is due to the Monte Carlo method using sampling procedures.
to estimate post-tax standard deviation. Repeated samplings of runs of 1000 pre-tax incomes indicate that this source makes only a small contribution to the difference. Hence the approximation and simplifications used in deriving the six formulae is responsible for the major part of the difference in results from the two methods. The formulae appear to overestimate the post-tax income standard deviations. However differences are not large enough to invalidate any conclusions drawn from using the formulae.

**RELATIVE VARIABILITY OF POST-TAX INCOME**

Having checked the analytical method against the Monte Carlo results, it is now possible to generalize about the increased instability caused by the six methods of taxation. The relative variability measures the ratio of the resulting post-tax income standard deviation divided by the pre-tax income standard deviations. For the Normal method from equation 17 and the formula for relative variability,

\[
RV = \frac{1}{\sigma} \frac{\sigma^2 (1 + SF^2)}{\sigma} = 1 + 5F^2.
\]

The relative variability is a function of \( F \) which is the first derivative of the tax function evaluated at the mean taxable income. This is the mean pre-tax income less exemptions. Hence the relative variability is a function of mean taxable income and not a function of the standard deviation.

Again assuming that tax exemptions amount to $1000, the relative variabilities of post-tax income for five methods of taxation at mean
pre-tax incomes between $2000 and $12000 in steps of $2000 is shown in Table 12. The Exponential Average method has the same relative variability as the 3 Year Moving Average (without terminal tax) as the smoothing constant of 0.5 is used. This method has not been included in the table.

TABLE 12

<table>
<thead>
<tr>
<th>Mean Pre-Tax Income</th>
<th>Relative Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PAYE</td>
</tr>
<tr>
<td>$</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>0.730</td>
</tr>
<tr>
<td>4000</td>
<td>0.673</td>
</tr>
<tr>
<td>6000</td>
<td>0.616</td>
</tr>
<tr>
<td>8000</td>
<td>0.560</td>
</tr>
<tr>
<td>10000</td>
<td>0.503</td>
</tr>
<tr>
<td>12000</td>
<td>0.446</td>
</tr>
</tbody>
</table>

The relative variability figures in Table 12 show how effective the PAYE method is in reducing post-tax income variability. It also shows how seriously the Normal method disturbs post-tax income and how the Normal method has a much greater disturbing influence than the Old method which it replaced.

As the mean of pre-tax income rises, so the buffering effect of
the PAYE method and disturbing effects of all the other methods increases.

The relative variability figure can be used to measure the proportional reduction in the standard deviation of post-tax income by changing from one taxation method to another. For instance with a mean income of $6000, a shift from the Normal method to the PAYE method would result in a post-tax income of 0.468 (i.e. 0.616/1.316) of the standard deviation under the Normal method. In fact post-tax income variability could be more than halved.

DISCUSSION

The most interesting aspect of these results both with the derived formulae and the case study of Ashley Dene pre-tax incomes over 16 years is the large increase in post-tax income variation caused by the Normal method of paying provisional tax on last year's income plus an annual square up with terminal tax. This variability is much greater than when tax was paid under the Old system as can be seen by reference to Table 9 and Table 12.

Farmers and accountants have it in their own hands to reduce post-tax income variability by paying provisional tax on an estimate of income for the year rather than as simply on the income in the previous year. Some form of computerized continuous financial management system would facilitate this. If the income for the current year can be estimated precisely, the indicated reductions in income variability can be expected. However other considerations other than income variability may make a farmer decide to pay his provisional tax on the
basis of last year's income rather than on an estimate.

The Tax Department encourages farmers to use last year's income as the basis for provisional tax by threatening a penalty with the use of any other method of estimation if provisional tax based on the estimate is less than 80 percent of tax due for the year.

A moving average of N years as an estimate (as long as N is greater than 1) has a greater chance of being close to actual pre-tax income than an estimate based on the previous year. Hence it would seem logical for the Tax Department to encourage the use of a moving average as the basis of provisional tax rather than using the previous year's income by itself. In other words they should encourage an **3 Year Moving Average plus Terminal Tax** method rather than the **Normal** method. This encouragement should be given where farmers cannot estimate their income for the year and thus don't use the **PAYE** method. The **3 Year Moving Average plus Terminal Tax** method could then replace the **Normal** method which has such a disturbing effect on post-tax income.

The results with the two moving averages (3 Year Moving Average and Exponential Average) without a terminal tax indicates that these methods could be used as an incentive to developers without raising post-tax income variability above the level of the **Normal** method in use at present.

**ACKNOWLEDGEMENTS**

The author is grateful to Dr R.W.M. Johnson, Senior Research Officer, Agricultural Economics Research Unit, Lincoln College and
Mr D.M. Ross, Agricultural Economist, Department of Agriculture, Christchurch, for help and suggestions and to Miss Barbara Thompson, Statistician, Wool Research Organisation of New Zealand for assistance in the mathematical sections.

REFERENCES


## APPENDIX

Proposed taxation rate for the year ending 31.3.72

<table>
<thead>
<tr>
<th>Taxable Balance $</th>
<th>Basic Rate on each $</th>
<th>Total Tax on Final Amount of Taxable Balance Shown $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1- 650</td>
<td>7.85</td>
<td>51.02</td>
</tr>
<tr>
<td>651-1700</td>
<td>21.00</td>
<td>271.52</td>
</tr>
<tr>
<td>1701-2000</td>
<td>24.50</td>
<td>345.02</td>
</tr>
<tr>
<td>2001-2500</td>
<td>27.50</td>
<td>482.52</td>
</tr>
<tr>
<td>2501-3000</td>
<td>30.50</td>
<td>635.02</td>
</tr>
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<td>3001-3500</td>
<td>34.00</td>
<td>805.02</td>
</tr>
<tr>
<td>3501-4000</td>
<td>37.00</td>
<td>990.02</td>
</tr>
<tr>
<td>4001-4500</td>
<td>39.00</td>
<td>1185.02</td>
</tr>
<tr>
<td>4501-5000</td>
<td>41.00</td>
<td>1390.02</td>
</tr>
<tr>
<td>5001-5500</td>
<td>43.00</td>
<td>1605.02</td>
</tr>
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<td>45.00</td>
<td>1830.02</td>
</tr>
<tr>
<td>6001-7000</td>
<td>46.00</td>
<td>2290.02</td>
</tr>
<tr>
<td>7001-8000</td>
<td>47.00</td>
<td>2760.02</td>
</tr>
<tr>
<td>8001-10000</td>
<td>48.00</td>
<td>3720.02</td>
</tr>
<tr>
<td>10001-12000</td>
<td>49.00</td>
<td>4700.02</td>
</tr>
<tr>
<td>&gt; 12000</td>
<td>50.00</td>
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9. Estimation of Farm Production Functions Combining Time-Series & Cross-Section Date, A. C. Lewis.


12. The Economics of Retailing Fresh Fruit and Vegetables, with Special Reference to Supermarkets, G. W. Kitson.