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Development of Educational Software for Learning Equation Solving

A thesis
submitted in partial fulfilment
of the requirements for the Degree of
Master of Applied Science

at
Lincoln University

by
D. E. Robson

Lincoln University
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Research into the contribution made by educational software to student learning has shown mixed results. More needs to be known about how to include learning theories in the design of educational software, particularly in the area of mathematics. This study investigates the effects of interactive software on students’ learning of equation solving.

When solving equations, students need to understand overall equation solving strategies as well as the procedures for individual steps. There is a need for more emphasis to be placed on helping students understand the concepts of equation solving strategies.

In this study, software was developed to encourage students to develop strategies for solving equations. Strategies were emphasised by allowing students to focus on making decisions about equation solving strategies while the software carried out the procedures. Students were able to search for multiple solution strategies. In addition, the feedback given to students emphasised strategies and included equations which showed the consequences of students’ strategic decisions.

The software was tested, and data were collected on students’ equation solving performance, strategies students used and students’ attitudes to the software. Results showed that most students found the software and its emphasis on strategies helped them learn. The activity in which students searched for multiple solution strategies for each equation appeared to be suited to students who could already solve an equation using one strategy. More improvement in performance was shown by students studying in a lower level course.

Analysis focussed on the students who showed the most improvement in equation solving performance, and this led to suggestions for additional learning activities to suit the other students. These suggestions have been summarised as a framework of learning goals for helping students learn to solve equations. The proposed framework, which includes practical guidelines for emphasising the concepts of equation solving strategies, will help teachers to structure lessons and software developers to plan learning activities.

**Keywords:** learning equation solving; educational software design; online math
Preface

Ever since computers were introduced into New Zealand classrooms, I have been interested in their potential to help students learn. Yet, many years later, I use educational mathematics software to help students learn in only a limited way.

This study has provided me with the opportunity to investigate the potential of computers to enhance learning. I was able to explore the design of software that is based on learning theories that are not practical to use in standard classroom environments. I am grateful to Christchurch Polytechnic Institute of Technology and Lincoln University for this opportunity and for their financial support.

I would like to thank my supervisors who provided guidance, shared in my journey, and made this study such a wonderful experience. Walt Abell, the principal supervisor and computing specialist, provided leadership and expertise. He provoked many lively discussions with his questions and challenges. Dr Therese Boustedt, a specialist in mathematics education, shared her knowledge, offered helpful advice and encouragement, and hosted our regular meetings. Dr Keith Unsworth was expected to take a supporting role, but he became actively involved and contributed useful ideas and suggestions.

I have appreciated the comments, interest and support from my colleagues and past colleagues, Karl Dodds, Mark Hutchinson, Lydia McKinnon and Dr Joe Dobinson. I would also like to thank the students who so willingly participated in the study.

A special thank you to my partner, Gavin Keats, for his comments, love and encouragement and to our daughter Jane and son Luther for their continued interest.
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1 Introduction

Adult students undertaking tertiary study in applied fields such as engineering, science or computing need to be able to use and apply mathematical formulae relevant to their field. To use and transpose formulae, students need to be able to solve equations. Hence a thorough understanding of equation solving is important when applying mathematics to technical subjects.

Adult students enter mathematics courses with a range of prior knowledge of equation solving and therefore have a variety of learning needs. Students who have never solved equations before need a thorough introduction to basic concepts, whereas students with prior knowledge of equation solving can progress much faster.

For many years, the potential for computers to improve learning has been recognised (Gayeski, 2005), but this potential has often not been achieved (Kirkup & Kirkwood, 2005; Hokanson & Hooper, 2000). However, computers are widely used in education, and their recent increase in use for online learning is being driven by students' needs to study at flexible times and locations (Inglis, Ling & Joosten, 1999).

Software designed to help students learn to solve equations needs to be based on relevant learning theories, but more needs to be known about how to apply learning theories to learning activities that use technology (Tallent-Runnels et al., 2006). There is also a need for more research into using computers for learning that concentrates on mathematics, particularly in topics other than geometry (Atkinson, 2005). This study makes a contribution to these needs by investigating the application of learning theories to the design of educational software for helping students learn equation solving.
Solving an equation involves being able to change back and forth between a conceptual understanding of an overall equation solving strategy and a procedural understanding of the algebraic manipulation for individual steps. Thomas and Tall (1988) have described this combination of conceptual and procedural understanding as “versatile thinking”.

Conceptual understanding of an equation requires each side of an equation to be viewed as an object, and the equals sign to be seen as showing the equivalence of these objects (Sfard & Linchevski, 1994). An equation must be understood in this way in order to understand an equation solving strategy as a series of strategic decisions which maintain the equivalence of both sides of an equation. Conceptual understanding of equation solving also includes understanding the relationships between the strategic decisions of individual steps and the goal of the overall strategy (Skemp, 1971).

Procedural understanding involves being able to accurately carry out the procedure for each step. It is possible to learn how to carry out procedures by rote or repetitive learning, but this shows a limited form of understanding (Star & Seifert, 2006). Students generally gain a procedural understanding first and from this develop conceptual understanding, although the latter is much harder to achieve (Sfard & Linchevski, 1994).

Many textbooks and teachers emphasise procedures (Beeson, 2002), but there is a need to place more emphasis on conceptual understanding with its concepts, overview of strategies and mental representations (Kieran, 1992). Graphical representations can help students visualise concepts (Mayer & Gallini, 1990). Therefore educational software, which includes visual elements such as images or graphics, may be suited to helping students visualise the strategies of equation solving. A need for visualisation such as this is required for graphics to contribute to learning (Rieber, 1994).

Thus, there is potential to use graphics and to emphasise strategies in the design of interactive software for helping students learn equation solving. The main research question for this study is “What effect does visual interactive software which emphasises equation solving strategies have on performance and equation solving strategies of students?”

In this study, software that emphasises equation solving strategies was designed and developed. Software trials were conducted so that the effects of specific features of the software on student learning could be investigated. Features investigated were separation of strategies from procedures, availability of multiple strategies and visual features of the
interface. In addition, the characteristics of students who gained the most learning benefit from the software were investigated.

The first stage of this study was to review the literature relevant to the design of educational software and its effects on learning. There are two main topics, learning theories and principles of interface design, and these are reviewed in Chapter 2. Examples of existing software for helping students learn to solve equations are also presented.

The proposed research is described in Chapter 3. Included are the research questions, an overview of the software design, and an outline of the software trials conducted with students so that the research questions could be investigated.

After considering learning theories and interface design guidelines, a learning activity was designed and the software developed. The design decisions and software development process are described in Chapter 4.

Trials were conducted in which students used the software, and data about the effects of features of the software on learning were collected. The procedure followed in the software trials is described in Chapter 5. The results and analysis are presented in Chapter 6.

The main findings of this study are summarised in Chapter 7, and their contribution to answering the research questions is considered. Issues arising from this study that need further investigation are also described.
2 Literature Review

2.1 Introduction
With the focus of this study being the design of visual interactive software for helping students learn to solve equations, this literature review first considers the use of computers for helping students learn and then concentrates on two main areas: theories of learning and principles of software design relevant to educational software. Types of learning theories are described, as well as the mathematical understanding needed for equation solving. In addition, principles for the design and development of educational software are considered with particular emphasis on interactivity, feedback, visual interface design and instructional design.

Examples of existing software for helping students learn to solve equations are shown to illustrate how learning theories and software design principles can be combined in the design of educational software.

2.2 Computers in education
2.2.1 Computers for learning
The potential for using computers to improve learning has been recognised for many years, so there is much interest in using computers for this purpose. For example, the government in New Zealand encourages the innovative use of computers as teaching and learning tools (Ministry of Education, 2003).

Despite the expectations that using computers could improve learning, in practice this has been difficult to achieve (Kirkup & Kirkwood, 2005; Rieber, 2005). Many studies have investigated the contribution computers made to learning by comparing learning outcomes of
traditional face to face courses with those delivered by media. Russell (2001) searched for studies which identified cases where technology resulted in more learning than traditional courses and was surprised to find that there were very few. However, he found and documented 355 studies that showed no significant difference when learning outcomes were compared. Reeves (2005) agreed that learning outcomes were usually similar whether computers were used or not.

The promised benefits of learning with computers have long been challenged. More than 20 years ago, Clark (1983) claimed that “media do not affect learning” and after much debate he continues to point out that instructional methods are more likely to encourage learning than delivery media (Clark & Feldon, 2005). This view is supported by Berge (2005) who tries to “take the distance out of distance education” because the education is important for learning rather than the media.

Although learning benefits are unclear, computers can allow education to be more flexible in terms of time, place, pace, entry and exit points. Indeed, it is this flexibility that is driving the increased use of computers for learning (Inglis et al., 1999) with adults in particular wanting to study at their own choice of time and location (Szabo, 2000). It is also becoming increasingly practical to use computers in education because they are now widely available and affordable (Romiszowski, 2005) as well as being more reliable with technical support available (Kearsley, 2000).

The current trend for increased use of computers should not be followed without ensuring that learning needs are being met (Romiszowski, 2005). New technologies have been introduced to education in the past and lessons from these should be learned. For example, large amounts of money were spent on many interactive videodisc developments in the 1980s but most did not live up to expectations and 80% of big initiatives were abandoned before completion (Gayeski, 2005). Educational television and programmed instruction also showed promise but fell out of favour, not because of lack of learning benefits, but because of how they were implemented into educational systems.

Computers, however, are widely used and successfully implemented into tertiary education systems, so now the potential for computers to improve learning needs to be investigated. Education is a major field of research and much is already known about how students learn. Established learning theories should continue to be followed when any change in education occurs such as an advance in technology (Inglis et al., 1999). Successful learning is the result
of the alignment of learning goals, learning activities, feedback and evaluation (Berge, 2005). As in face to face delivery, these need to be designed for the learning needs of students and should take into account their learning styles and existing knowledge (Romiszowski, 2005).

Throughout history, the individual learning needs of students have been met by teaching students individually (Bork, 2005). There is a need to investigate the contribution computers can make to the individual learning needs of students (Szabo, 2000) as there is potential for software to focus on specific problems and to interact with students individually. This use of computers has contributed to a shift towards student centred learning (Seidel, 2005).

2.2.2 Student attitudes to computers for learning
Students generally have positive attitudes to using computers for learning (Garland & Noyes, 2005; Sanders & Morrison-Shetlar, 2001). These attitudes can improve further after students use computers as part of a course (Abbot & Faris, 2000) with students being more willing to undertake online courses in 2003 than in 1999 (Brinkerhoff & Koroghlanian, 2005). It has also been found that students with prior experience of computers were more satisfied with online courses (Tallent-Runnels et al., 2006).

Computers are seen to have the potential to motivate students to learn, but the evidence shows only that they motivate students to choose courses which use computers and that there is no evidence of additional learning (Clark & Feldon, 2005). After initial interest in using a computer for learning, students are often bored by the quality of the interactivity (Gayeski, 2005).

The widespread use of computers in tertiary education may be because adults, with their mature attitudes, tend to be motivated and capable of independent study (Kearsley, 2000). As students tend to have positive attitudes to using computers for learning and much potential is seen for them to improve learning, many different types of educational software have been developed. In this study, educational software will be considered according to the type of learning theory on which it is based.

2.3 Learning theories
There are three broad categories of learning theories: behaviourist, cognitivist and constructivist. In this section each is described.
2.3.1 Behaviourism

Behaviourist learning theories are based on stimuli and responses. An activity (stimulus) is provided which causes a response from the student (Burton, Moore & Magliaro, 2004). Feedback is given to provide consequences or reinforcement. Behaviourist learning theories lead to a very structured learning environment and should be used when the content is suited to this, for example learning multiplication tables. Behaviourist learning theories have been criticised for encouraging passive learning in which the student is a recipient (Grabowski, 1996). However behaviourism was not developed as a passive theory; it is based on stimuli and responses and the student is expected to take an active part (Burton et al., 2004). An example of a behaviourist learning activity is students practising to factorise quadratic expressions using a specific procedure.

2.3.2 Cognitivism

Cognitive perspectives were added to behaviourism to try to explain situations where a stimulus resulted in unpredictable responses. Cognitive learning theories emphasise thought processes that cause behaviour rather than the behaviour itself. Mental representations of knowledge are considered, e.g. visual images, verbal processes and mental models (Winn, 2004). An example of a cognitive learning activity for factorising quadratic expressions is a discussion about a given procedure and why it works, followed by students using the procedure and checking their solutions by expanding. A follow up discussion about why the procedure worked could complete the learning activity.

2.3.3 Constructivism

Constructivist learning theories add a further dimension. Students are encouraged to construct their own understanding as a result of experiences and activities, while the role of an instructor is to support this process (Duffy & Cunningham, 2004). As part of his constructivist learning theories, Vygotsky (1978, p84-91) proposed that "an essential feature of learning is that it creates the Zone of Proximal Development". In this zone, students are able to solve problems which they are unable to solve on their own. The provision of appropriate guidance and support, called scaffolding, enables the higher level problem solving. The scaffolding is usually provided through interaction with a teacher or a peer and the social contribution to learning is acknowledged. An example of a constructivist learning activity is for students to expand expressions such as \((x - 3)(x + 4)\) and then to explore ideas for a procedure for factorising the answer. A class discussion of student procedures would provide feedback and scaffolding so that students could modify their procedures and try them on new examples.
Theories of adult learning are generally constructivist with the emphasis being on the student (Huang, 2002). The most important feature of adult learning is that adults need to have active input into their own learning. Adults also need a supportive learning environment tailored to their prior knowledge and learning needs (Knowles, 1984). For example, adults appreciate useful feedback and being able to work at their own speed (Rieber, 1994).

2.3.4 Comparison of learning theories
All three types of learning theories have similarities. As well as the common goal of helping students learn, importance is placed on encouraging students to take an active part in their learning (Deubel, 2003) and providing feedback to students.

The three main types of learning theories differ in the amount of direction given to students. Behaviourist learning activities are the most prescriptive, providing clear steps for students, whereas constructivist learning activities are the least prescriptive, encouraging the student to take control of exploring their ideas and building their own understanding.

2.4 Learning mathematics and equation solving
In order to choose appropriate learning theories for mathematics, and in particular algebra and equation solving, the cognitive processes of mathematics are examined.

2.4.1 Conceptual and procedural understanding
Mathematical thinking, or mathematical understanding, is often considered to be of two main types, and these have been discussed in depth and summarised by various terms. One type of mathematical understanding involves concepts whereas the other involves procedures, and in this study the terms conceptual and procedural are used, as suggested by Hiebert and Lefevre (1986). Conceptual understanding involves concepts and how they relate to each other whereas procedural understanding involves carrying out procedures. Conceptual understanding requires a global overall view whereas procedural understanding requires systematic attention to detail. Descriptions and terms used by different researchers for these two types of mathematical understanding are summarised in Table 2-1.
## Table 2-1 Conceptual and procedural understanding

<table>
<thead>
<tr>
<th></th>
<th>Conceptual</th>
<th>Procedural</th>
</tr>
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<tbody>
<tr>
<td>Hiebert and Lefevre</td>
<td>Conceptual knowledge.</td>
<td>Procedural knowledge.</td>
</tr>
<tr>
<td>(1986)</td>
<td>Relationships between mathematical ideas and concepts.</td>
<td>Algorithms (or steps or procedures) for solving mathematical problems.</td>
</tr>
<tr>
<td></td>
<td>A network of relationships.</td>
<td>A series of steps.</td>
</tr>
<tr>
<td>Skemp (1971)</td>
<td>Relational understanding.</td>
<td>Instrumental understanding.</td>
</tr>
<tr>
<td></td>
<td>Knowing what to do and why.</td>
<td>Being able to use rules.</td>
</tr>
<tr>
<td></td>
<td>Goal is general, e.g. solve any linear equation.</td>
<td>Goal is specific, e.g. solve a particular type of linear equation.</td>
</tr>
<tr>
<td></td>
<td>Analogy: like a map which can be used to navigate from any point A to any point B.</td>
<td>Analogy: like a route, e.g. turn left, second right to get from a particular point C to a particular point D.</td>
</tr>
<tr>
<td>Tall and Thomas (1991)</td>
<td>An overall view that allows appropriate paths to be selected and errors to be sensed.</td>
<td>A succession of mechanistic steps.</td>
</tr>
<tr>
<td></td>
<td>Mathematical objects.</td>
<td>Mathematical processes.</td>
</tr>
<tr>
<td></td>
<td>Supported by visual imagery.</td>
<td>Supported by verbal representations.</td>
</tr>
</tbody>
</table>

Much discussion has occurred about the nature and relative importance of conceptual and procedural understanding. Conceptual understanding with its emphasis on relationships between concepts and strategies is generally acknowledged to be higher level thinking and to be essential for a thorough understanding of mathematics. For example, Skemp (1971) stated that for meaningful learning to take place, relationships must be created or become understood by the learner. Students often make poor strategic decisions and lack the overall view of an equation that is needed when deciding how to approach each step (Kieran, 1992). On the other hand, many teachers and textbooks often emphasise procedures, and Kieran
states that although procedural understanding is important, more emphasis needs to be placed on conceptual understanding, including strategies. Beeson (1998) agreed that many textbooks concentrate on procedures and rules. He encourages the teaching of concepts and strategies, but also points out that procedural understanding should not be ignored.

There are different theories on the order that concepts and procedures should be learnt (Carpenter, 1986). Students generally start with procedures and gradually build conceptual understanding but this reification can be difficult to achieve (Sfard and Linchevski, 1994). Skemp suggests that the order depends on whether a student is trying to understand conceptually or procedurally and recommends that the initial teaching style should match the student’s learning style.

2.4.2 Versatile thinking
Thomas and Tall (1988) have explained the need for both conceptual and procedural understanding and describe a combination of both as “versatile thinking”. They quote Scott-Hodgetts’ definition of versatile thinking in which students need to frequently switch between a local analytical view (procedural) and a global overview (conceptual) in order to help them understand how the procedures fit into the whole structure. (Scott-Hodgetts, 1986 as cited in Thomas & Tall, 1988). It is often the lack of conceptual thinking that causes students to be unable to think with versatility (Thomas, 1995).

2.4.3 Mental representations and visualisation
According to Paivio (1990), information recorded by the mind can be of three types: real events and objects, structures and processes, or theoretical models of structures and processes. The ways that information is stored in the mind are called mental representations. These are generally considered to be stored in two modes: verbal descriptions and visual images. Paivio’s Dual Coding Theory assumes that these modes are processed by separate cognitive channels. Paivio suggests that the use of both channels leads to more powerful learning and improved problem solving ability.

Mental representations have been linked to conceptual understanding. In a study to investigate mathematical mental representations, a talented mathematics student was studied as he developed his understanding of conceptual relationships in mathematics (Pinto and Tall, 2002). He formed mental representations that were a combination of descriptions and visual images, and he gradually modified these as his conceptual understanding developed. He used both verbal and visual channels for his representations.
In the context of algebra, Thomas and Tall (1988) have linked visual mental images to the development of conceptual thinking. They suggest that the relationship occurs because imagery and conceptual thinking are both associated with the thinking that occurs in the right hemisphere of the brain. Mayer and Gallini (1990) describe how illustrations helped students build mental representations that were helpful for learning and understanding concepts. Sfard (1991) also describes the connection between visual imagery and conceptual understanding.

Thomas and Tall explored ways of improving conceptual thinking. They concluded that using a computer program that encouraged the formation of mental models for concepts could improve conceptual thinking and in turn encourage more versatile thinking. Sfard and Linchevski (1994) agreed that there is potential for computers to help with conceptual understanding.

In an attempt to help students develop conceptual understanding of equation solving, various models have been proposed. For example, a visual model such as balance scales emphasises the concept of equivalence of both sides of an equation (Vlassis, 2002). In this model, the equation itself is viewed as an object, and the equivalence or balance is maintained visually when appropriate operations are performed.

2.4.4 Equation solving

2.4.4.1 Problem solving theory

VanLehn (1989) has described equation solving as an example of a multi-step problem. Such problems consist of three components: an initial state, operators which change the problem to a new state, and a test for whether a problem state is a solution. The search for a solution starts with the development of an understanding of the objects in the problem and the relationships between them. Suitable operators are chosen and applied to create a new state which needs to be reconciled with the rest of the problem. VanLehn also described schemas which experienced problem solvers develop for a class of problems. A schema consists of information about the type of problem and information about the solution. Before using a schema, the problem solver must recognise the class of problem, select an appropriate schema and adapt it for the current problem.

VanLehn’s analysis of solving multi-step problems such as equations includes descriptions of conceptual understanding of relationships and procedural understanding of the application of operators. VanLehn also described the need to combine the procedures for each step with an overview of the relationship of each step to the problem as a whole.
2.4.4.2 Flexibility in equation solving

Star (2005) observed that researchers have placed much importance on students’ development of deep conceptual understanding, whereas procedural understanding is often considered to be a shallow form of understanding in which rules are learnt and applied in a repetitive fashion. Star’s investigations led to his description of a deep procedural understanding, in which students plan procedures taking into account the goals of each step, the order of steps and the characteristics of the particular equation being considered (Star, 2000). This deep procedural understanding was later described as “flexibility” in equation solving (Star and Madnani, 2004). According to Star and Madnani, students with a sophisticated view of efficient strategies for solving equations also showed conceptual understanding, flexibility in their thinking and an ability to think strategically. These students were also better at solving “transfer equations” (i.e. equations with unfamiliar features) than students with naïve views of efficient strategies.

The flexibility in equation solving described by Star and Madnani is similar to the versatile thinking described by Tall and Thomas (1991) in that both involve considering equation solving from different points of view. Another similarity is that encouraging students to think with either versatility or flexibility is seen as a way of helping students develop conceptual understanding. However, they are different in that versatile thinking is a combination of conceptual and procedural understanding, whereas flexibility in equation solving is a deeper form of procedural understanding which involves solving an equation using several different strategies.

Star and Madnani have pointed out that it is difficult to define the most efficient equation solving strategy because criteria such as number of steps, time taken, amount of mental effort or aesthetics can yield conflicting results, even amongst mathematicians. In a later study, Star and Seifert (2006) define a standard strategy for solving equations, including those with brackets but not fractions, and describe a strategy as inefficient if it uses more steps than their standard strategy. Star and Madnani also state that although it is well recognised that analysis of strategies for efficiency is important in equation solving, there is very little research into how students learn to do this analysis.
2.4.4.3 Multiple strategies

Learning activities which help students think with the flexibility needed for equations solving include finding and comparing multiple strategies (Star & Seifert, 2006) and thinking about which strategy is best (Star & Madnani, 2004). In order to compare strategies and develop an opinion about which is best, students must first develop knowledge of multiple strategies. The ability to use and select from multiple strategies is widely associated with mathematical expertise. For example, versatile thinking includes being able to use a number of different strategies (Pegg & Tall, 2002). Another example is the New Zealand Numeracy Project which describes eight levels of numeracy. The higher levels are characterised by the ability of students to use a range of different strategies and apply these appropriately according to the nature of the problem (Ministry of Education, 2005; Hughes, 2002).

Encouraging students to develop multiple strategies is not a common theme in the literature but several researchers do suggest this. Jonassen (2000) has suggested that as research shows that experts are able to view problems in multiple ways, this needs to be specifically taught. Beeson (1998) recommends that students need to be taught how to recognise which of several strategies would be most appropriate. In a study on multiple strategies, Gucler and Star (2005) found that students needed direct instruction to initiate the most efficient strategies as they did not discover them on their own.

Thus, expert equation solvers choose from a range of strategies, but students tend to learn only one strategy unless they are specifically taught others. This suggests that there may be potential for encouraging students to explore multiple strategies.

2.4.4.4 Required prior knowledge

Students find it difficult to solve equations if prior concepts are not understood (Kuchemann, 1981; Kieran, 1992). The importance of required prior knowledge is highlighted by Beeson (1998) who found that 80% of errors in a calculus exam were due to mistakes in algebra and trigonometry. These were required prior knowledge for the calculus course. Like calculus, equation solving depends on prior concepts being understood.

Kuchemann described six levels of understanding of a letter in an algebraic expression or equation. Solving equations requires at least a level four understanding of a letter, in which it is seen as standing for one particular value that is not yet known. However, in order to apply equations and formulae to practical situations, a level six understanding is required, in which letters are seen as variables.
Several concepts related to algebraic expressions need to be understood when solving equations. Kieran described a common error called the deletion error in which students simplify an expression such as $13x - 3$ to $10x$. Tall and Thomas (1991) observed that equivalent mathematical expressions can describe different processes. For example, in $3(a + b) = 3a + 3b$ both sides are equivalent, but the left side describes the process of adding $a$ and $b$ then multiplying the result by 3, whereas the right side describes the process of multiplying each of $a$ and $b$ by 3 then adding the results. They also pointed out that expressions are not necessarily processed from left to right, for example in both $2 + 3x$ and $3x + 2$ multiplication takes precedence over addition, so in both expressions the $x$ should be multiplied by 3 before the 2 is added.

Kieran described how the concept of an equals sign causes difficulties for some students as it can represent a different concept in algebra from that learnt in arithmetic. In arithmetic, students often expect the number on the right hand side of an equals sign to be the answer to the process on the left hand side, e.g. $5 + 4 = 9$. For algebraic equations, it also needs to be understood that an equals sign indicates the equivalence of both sides, e.g $3x + 2 = 2x - 4$.

### 2.5 Software development

When designing educational software for mathematics, learning theories should be chosen that are suited to both the content and its delivery by computer. The incorporation of learning theories and content into the design of the software can be co-ordinated by following the principles of instructional design.

The term, multimedia learning is used in this study according to the definition of Mayer (2001) and refers to learning activities which include both text and images. On a cognitive level, this definition relates to the verbal and visual channels described by Paivio (1990). The term, visual interactive interface is used in this study to refer to a software interface which includes both graphics and interactivity.

#### 2.5.1 Software design and learning theories

There is potential for software to implement learning theories which are difficult for teachers to apply in classroom situations. Early examples of this were programmed instruction and computer based training (CBT). Although based on sound behaviourist principles with empirical evidence of student performance, their use has not been as popular as expected. Kearsley (2000) suggests that this is because the interactivity is limited.
When designing learning activities for problem solving, the way students think must be considered (Jonassen, 2000). Equation solving is therefore suited to learning activities that are based on cognitive learning theories. It is important that educational software does not cognitively overload students (Sweller, 2005), and the level of the learner must also be taken into account (Kalyuga, 2005).

2.5.1.1 Scaffolding
There is potential for visual interactive interfaces to provide scaffolding to support students as they learn. Scaffolding could be provided during the interaction of a student with another person, for example in emails or online discussion. It could also be provided during the interaction of a student with the software. An example of scaffolding being provided by software is when calculations are carried out by computer. Students can focus on conceptual understanding by reflecting on both the calculations and feedback provided by the software (Healy, Pozzi & Sutherland, 2001). Another example of scaffolding is software which does not allow students to make incorrect steps and provides guidance when students need it (Beeson, 2002).

2.5.1.2 Cognitive load
The term cognitive load refers to the amount of a student’s working memory being used. This is limited, so students need to process information in a way that allows them to construct schemas which can be stored in their long term memory. Instructional design should encourage students to develop these schemas and hence minimise the cognitive load (Sweller, 2005).

2.5.1.3 Learner level
As for any learning environment, multimedia learning environments should be tailored to the level of the learner (Kalyuga, 2005). Scaffolding recommended for multimedia instruction includes complementary words and pictures, highlighted elements and worked examples. These generally help low level learners, but Kalyuga has also described studies in which these same features may hinder students who have more experience in the domain. He recommends more research be done on how to tailor activities to different learner levels and that consideration be given to gradually reducing scaffolding as the learner progresses.
2.5.1.4 Motivation
Motivation can be considered to be of two types: intrinsic motivation which arises from within a student as a result of their own feelings, and extrinsic motivation which is a result of external rewards or deterrents (Lee & Boling, 1999).

In the ARCS model of motivation (Keller, 1987), four components are identified: Attention, Relevance, Confidence and Satisfaction. After students’ attention is attracted, it needs to be retained. For this to happen, students need to recognise the relevance of the activity to their own learning goals. They also need to experience success which leads to confidence in their ability and a feeling of achievement. A simplified version of Keller’s ARCS model of motivation has been validated for use in e-learning environments (Keller and Suzuki, 2004).

2.5.2 Instructional design
Instructional design is a systematic process for incorporating learning theories into the design of lessons and learning systems. Rieber (1994) recommends that standard principles of instructional design should be followed when developing educational software.

Early systems of instructional design were based on behaviourist learning theories and these still form the basis of most instructional design systems, although some cognitive and constructivist principles have been incorporated as for example in the popular Dick and Carey Systems Approach Model (Dick, Carey & Carey, 2001, p5). The generally behaviourist and systematic nature of instructional design systems ensure all components of instructional design are completed. However, the type of learning theories chosen as the basis of the lessons or learning activities should be appropriate to the content and may be behaviourist, cognitivist or constructivist as appropriate.

A generic model for instructional design systems, ADDIE, has five components: Analysis, Design, Development, Implementation and Evaluation (Gustafson & Branch, 1997). Although the process has been criticised for being too linear (Kearsley, 2000), the order of the components can be modified, with components being revisited as often as required (Crawford, 2004). This makes the ADDIE model flexible and easy to apply as well as ensuring all instructional design aspects are considered. Appropriate use of this model is an effective means of ensuring quality learning activities (Hannum, 2005) and is used to provide a framework for the instructional design in this study. In the following sections, each component is considered in turn with Design being considered in the most depth because it is an important focus in this study.
2.5.3 Analysis
The purpose of the first phase, Analysis, is to ensure that the eventual design of a learning activity meets the goal for the learning and is appropriate to the target learners. An understanding of the characteristics of both the students and the content needs to be gained. This information is used to develop objectives and select appropriate learning theories. Relevant characteristics of students are their age, gender, prior knowledge and context for the learning. Required prior knowledge for learning the content also needs to be known. The prior knowledge of the students needs to be compared to that needed for the content so that objectives can be chosen to bridge any gaps (Dick et al., 2001, p95).

2.5.4 Design
In the Design phase, an instructional strategy is developed by choosing relevant learning theories. The choice of learning theories is important and should be made on the basis of the relevance and suitability to both the content and the learner (Mergel, 1998). Learning activities include student actions for which they receive feedback. This interactivity between the student and the software must be designed so that it helps students learn. The contribution of graphics must also be considered if a visual interface is used. In this section, the relationship between learning and the following features of a visual interactive interface are considered: interactivity, feedback, graphics and user interface design.

2.5.4.1 Interactivity
Interactivity between students and software allows students to take an active part in their learning. It also allows scaffolding to be provided to guide and support student learning (Gillani & Relan, 1997). Interactivity has been shown to contribute to motivation, positive student attitudes, more meaningful learning, faster learning and improved achievement (e.g. Sutton, 2001; Najjar, 1998).

With the benefits of interactivity for learning being so clear, Prensky (2003) looked at the intense interactivity of computer games. He compared principles of game design with principles of learning and found that many overlap. He also noted that whereas learning theories require students to take an active part and engage with their learning, the main goal of computer games is to engage the user for as long as possible. To improve student motivation, Prensky explored the idea of harnessing the strong engagement of users with games and applying it to learning activities. His analysis led to the following suggestions for the application of game design principles to educational software:
2.5.4.2 Feedback

Feedback makes a valuable contribution to learning as it provides communication to students after they have performed a learning task. Most feedback is intended to either acknowledge a correct response or provide information about an incorrect response allowing students to learn from both their successes and their errors.

Although feedback is recognised as an important aid to learning, studies which investigated specific features of feedback such as amount, timing and frequency are inconsistent (Mory, 1996; Mason & Bruning, 2001). It is however generally agreed that feedback improves learning if it encourages students to think actively. For example, Lyster and Ranta (1997) found that immersion language students were more likely to correct their errors after receiving feedback that required more than a yes/no response or just repeating the correct answer.

When reviewing the recommended amount of feedback found to help learning, Mory (1996) observed that some studies found more information helped learners develop their understanding whereas others found this increased the cognitive load or distracted the learner. To balance these conflicting findings, the option for students to request additional feedback is suggested, although once again results are inconsistent (Mason & Bruning, 2001).

Timing and frequency of feedback are related and there are many recommendations. When a learning activity involves a step which depends on a previous one, immediate feedback is usually more effective than delayed feedback (Fleming, 1987). Students should not be able to look at the answer before giving a response (Mory, 1996). The opportunity to try again after receiving feedback about an error has been found to be helpful (Dihoff, Brosvic, Epstein & Cook, 2005). When choosing frequency of feedback the student's stage in the learning process needs to be considered. Frequent feedback may be desirable in early stages of learning to reduce accumulation of errors (Wlodkowski, 1999), but learning benefits can be gained by gradually removing feedback as the student progresses (Sedighian & Klawe, 1996).

Feedback can be private to the learner without any personal judgement from a teacher (Schulmeister, 1997). It can be presented in different ways, such as in the current window or
by opening a new window (Dalgarno, 1998), and features such as amount and timing can be controlled by either the learner or the software. Different feedback can even be provided according to the response of the learner; Narciss and Huth (2004) found this improved learning.

Thus, recommendations from studies of feedback are varied and sometimes conflicting. As the studies were done in a wide variety of contexts, it is relevant to consider a study of feedback for algebra. Nguyen-Xuan, Nicaud and Gelis (1997) tested different types of feedback and found that it should be short, include consequences of errors, give enough information for students to see why their response was incorrect, but allow them to work out the next step themselves.

Mory (2004) has stated that feedback has long been acknowledged as very important for learning, but the nature of feedback suited to different learning environments is a complex issue in need of further research. Mory recommends that feedback in computer-based learning environments should be investigated in combination with learning theories.

2.5.4.3 Visual interfaces

Developments in computer technology continue rapidly and each new feature offers fresh promises of benefits for learning. It is currently common for educational software to include video clips, graphics, animation, audio, simulations and interactivity. The expression “visual interface” is used in this study to refer to software interfaces in which graphics are an integral part. The graphics may be static (e.g. diagrams) or dynamic (animations).

Visual interfaces have the potential to help students learn by helping them form mental representations. Graphics can allow students to use both visual and verbal channels. Students can analyse visual images, observe patterns and spot relationships (Pfitzner, Hobbs & Powers, 2001). This can help students develop their understanding of relationships, including those in algebra (Hewitt, 1996).

Research on whether graphics contribute to learning is varied. Some studies found that both static and dynamic graphics helped learning more than text only (Harrison, 1995; ChanLin, 2000). Many studies found no significant difference between graphics and text only (e.g. Rieber, 1994; Morrison & Tversky, 2001). Other studies found that in some cases, visual interfaces decreased learning (Najjar, 1998). One study found that although students preferred animations, they tended not to read the associated text, and the text only group retained their learning longer (Palmiter and Elkerton, 1991).
Rieber (1994) claimed that an important criterion for the use of a visual interface is a need for visualisation, i.e. the need to form a mental picture. This need may be with either the content or the student and many researchers support this (e.g. Betrancourt 2000; Iskander & Curtis, 2005).

The type of content should determine the type of media to be included. In other words, any graphics must be integral to the topic and the learning, and should be included for a specific reason (Rieber, 1994). The Congruence Principle states that the structure and content of the visual interface must correspond to the structure and content of the mental representation (Tversky, Morrison & Betrancourt 2002). This congruence of the visual interface and the content may be with either the representation of physical objects or the structure of abstract concepts. (Betrancourt, 2005).

The spatial ability of students may influence the helpfulness of a visual interface. In some studies, visual interfaces were of more benefit to students with a high spatial ability (Mayer, 2001; Klein, 2004). In other cases, visual interfaces were of more benefit to students with a low spatial ability (Morrison and Tversky, 2001). A visual interface that helps students with high spatial ability, may cognitively overload students with low spatial ability (ChanLin, 2000). In contrast, a simple visual interface may simplify concepts and assist low spatial ability students to visualise them (Hewitt, 1996), but may be unhelpful to students with high spatial ability. Therefore the design of a visual interface needs to be suited to the spatial ability of students.

Visual interfaces may be suited to students with low prior knowledge (Mayer and Gallini, 1990; Rieber, 1996), but interfaces that help beginners learn may be unhelpful to students at later stages of learning. Visual interfaces need to be suited to the level of understanding of the student and more needs to be known about how interfaces should be designed to do this (Kalyuga, 2005).

There does not appear to have been much empirical research about the use of visual interfaces in mathematics when compared to other domains, including science. (Atkinson, 2005). Furthermore, Atkinson states that it is critical that sub-domains other than geometry be investigated. After reviewing the modest number of studies available that investigated visual interfaces for mathematics, Atkinson recommends a number of design principles which have been shown to help students learn. When both graphics and equations are included these should support each other, as the need to locate and match graphics with equations adds to the
students' cognitive load. Visual interfaces should encourage students to build mental representations. Dynamic graphics with text offer more benefit to high achieving and high spatial ability students. Atkinson recommends that more research is needed to help understand how to design animation so it helps students learn mathematics. He also recommends that more research be done to investigate how the design of graphics and animation can contribute to students' learning and understanding of mathematics.

According to Najjar (1998), many design principles for visual interfaces for learning are based on a limited number of empirical studies in specific situations. More research is needed to further understand when and why visual interfaces help students learn (Najjar, 1998) and how they can be designed to help students learn (Betancourt, 2005).

2.5.4.4 User interface design

When designing a user interface, the needs of the user and the purpose of the interface need to be considered as the interface should empower the user to achieve the goal of the software (Nielsen, 2005). User interfaces that are difficult to use can interfere with the instructional value of the program (Frye & Soloway, 1987). Poor screen design can lead to students taking longer and being less likely to complete lessons, although their achievement was not affected (Szabo, 2000). Poor interface design affected student motivation as students lost interest when they became confused (Vonderwell & Zachariah, 2005).

Eight "golden rules" of interface design have been described by Shneiderman (1987) and are well established principles. Interfaces should be consistent, suit different types of users, provide informative feedback, indicate completion of actions, prevent user errors, permit actions to be reversed, minimise cognitive load and allow the user to feel in control.

According to Norman (1995), enough information about the current state of the system must be provided with possible actions clearly visible so that users can predict the effect of their actions (Norman, 1995).

In addition to these broad guidelines for interface design, Watzman (2003) has described specific recommendations for screen design. The overall layout should be balanced and elements should work well together with related elements in close proximity. Typefaces should be easy to read and text should be placed so that it is easy to find. Graphics should be simple, consistent and appropriate to the content. Colours should be chosen taking into account their purpose and effect. The appearance of colours on different monitors and the needs of colour-deficient users must be considered. In addition, Nielsen (1993) recommends
that the visibility of objects should relate to the needs of the user. These guidelines contribute to students' motivation and help prevent them from losing interest (Lee & Boling, 1999).

Nielsen (1993) has recommended that cognitive load should be minimised. To help with this, the user should need only a few rules to use the interface. Also, as every element on the screen adds to the cognitive load, fewer elements make an interface easier to use. Nielsen suggests putting only the most important information on the main screen with additional information on other screens. For example, he suggests dividing messages into two levels so that a short message is displayed and a longer message available when requested.

Metaphors can be used to assist learning as well as provide structure to the interface design (Nielsen, 2000). For example, a "home page", with its icon of a house is the main starting point for a web site. The metaphor suggests that the user is being welcomed, and that it is a safe place to return to if the user becomes confused by the web site. An effective metaphor also allows users to apply knowledge of the metaphor to the activity. It must be easily understood but free of meanings which could mislead the user (Erickson, 1995).

There are many interface design guidelines which need to be taken into account, and Nielsen points out that a designer must often decide which of two considerations is more important.

2.5.5 Development, Implementation and Evaluation

In the Development phase, software is produced on the basis of the preceding Analysis and Design phases. Rapid prototyping can improve the efficiency of this process. The Implementation phase ensures the delivery of the software to students, and the Evaluation phase consists of two types: usability testing of the interface and trials of the software in context. Rapid prototyping and usability testing are considered here and software trials are described in Chapter 5.

2.5.5.1 Rapid prototyping

Instructional design principles can be applied to the planning of any learning experience but a problem which arose when it was applied to the design of educational software was the time and cost of making changes during the development process. To address this issue, a rapid prototyping model was adapted from software engineering and proposed for use in the development of educational software (Bichelmayer, 2004).

In rapid prototyping, a brief analysis is done to determine initial objectives. A series of quick software prototypes are designed and developed. After each prototype is evaluated by
designers and students, objectives may be modified before the next quick prototype is
designed and developed. In this way the design can be modified as required with the changes
causing minimum waste of resources.

2.5.5.2 Usability testing
A usable system is easy to learn, easy to remember, pleasant and efficient to use, and prevents
user errors (Nielsen, 1993). The interface should not distract from the main purpose of the
software which in this case is to help students learn. To achieve this usability, students should
be involved in usability testing of the interface at an early stage and such testing should
continue throughout the development process (Gould, 1995). Ellender (2003) agrees and
points out that the greatest cost benefits come from the earliest usability testing.

Nielsen (1993) has suggested that the most effective form of usability testing is the “think
aloud” method with real users. Users are observed one at a time and are asked to think aloud
while doing specific tasks with the software. The observer takes notes which can be used to
work out the reasons for users’ actions. One variation of the think aloud method is to include
two users, with the advantage being that people find it easier to discuss an interface than to
think aloud and therefore more comments are made. Another variation is the “coaching”
method in which a coach answers a user’s questions. The purpose of this coaching is to find
out what additional information users need to use the software.

Questionnaires are described by Nielsen as being useful for finding out about features that
users like or dislike. They are good for recording attitudes that are difficult to measure
objectively with the most useful answers obtained when the questionnaires are completed
immediately after using the software. However, Nielsen also noted that there can be low
correlation between users’ predictions about a new feature and their later satisfaction with it.

Data can be logged automatically by software and this can provide additional detailed data
about how software is used. The easiest way to collect this data is to modify the software
being evaluated (Nielsen, 1993).

2.5.5.3 Software trials
A single group, pre- and post-test design is suited to an investigation which seeks to
understand how a learning activity such as software is being used (Bamberger, Rugh &
Mabry, 2006). As well as ensuring the usability of educational software, the effect on learning
also needs to be determined. Learning is often measured by recording student performance on
specific learning outcomes before and after using the software (e.g. Nguyen-Xuan et al., 1997; Iskander & Curtis, 2005).

Trials of educational software should be conducted in real situations so that data collected, about the effects on student learning, are based on realistic student behaviour (Mitrovic, Martin & Mayo, 2002). However, there are problems to overcome in real situations as the timing of the trials is determined by the class programme and students may not complete all activities in the trial.

2.6 Review of equation solving software

There are many types of computer programs designed to help students learn to solve equations and for this review, they are grouped into categories. Each category provides learning activities based mainly on one of the major types of learning theories: behaviourist, cognitive or constructivist. In this section, five types of interactive software are considered: drill-and-practice, cognitive software, microworlds, intelligent tutoring systems and computer algebra systems.

2.6.1 Behaviourist software

This type of software is characterised by its prescriptive approach in which students are directed to perform structured tasks that concentrate on procedural understanding and the achievement of narrow learning objectives. Examples are, tutorials, instructional games and drill-and-practice activities (Misanchuk & Schwier, 1993).

For equation solving, equations are usually presented one at time, a student works out the answer using pen and paper, and then enters their answer. Feedback on correctness is provided by the software, hints may be available, and student progress may be recorded. The immediate verification feedback is an advantage compared to textbook exercises and students should also find it less daunting to see one equation at a time.

Tutorials use a repeated pattern of presentation of content, practice and feedback. Instructional games have a high level of interactivity and provide goals which are extrinsic to the content (Parrish, 1996).

Drill-and-practice software is considered here as an example of behaviourist software. It consists of a series of questions and answers and is usually used for reviewing or practising rather than for learning new material (Misanchuk & Schwier, 1993). Drill-and-practice software is suited to helping students become more fluent rather than helping them develop
conceptual understanding (Norman & Spohrer, 1996). An example of drill-and-practice software for equation solving is shown in Figure 2-1.

In this software, students are presented with an equation (stimulus) for which they type their answer (response). Feedback about the correctness of the answer is provided, and students and may also request a hint.

### 2.6.2 Cognitive software

The term cognitive software is used in this study to describe interactive software based on cognitive learning theories. Cognitive software emphasises conceptual understanding either in addition to or instead of procedural understanding. Vlassis (2002) investigated the visualisation of concepts and formation of mental representations with the use of concrete models, e.g. geometric models, algebraic models, puzzles or images. Software which simulates concrete models is an example of cognitive software. For example, in Figure 2-2, an image of balance scales is used to indicate the equivalence of both sides of an equation.

---

**Figure 2-1** Algebra linear equations quiz (Syvum, 1999-2006)

**Figure 2-2** Algebra Balance Scales (NLVM, 1999)
At each step, students decide on an action to be applied to both sides of the equation, the software applies it to both the balance scales and the algebraic equation, and the student can reflect on the result.

According to Vlassis (2002), studies of balance scale models show inconsistent results in their contribution to students’ learning. After studying the balance model in detail, Vlassis concluded that such models have limitations (e.g. for equations with negative and decimal numbers) but could help students develop mental representations for equation solving. In one of his studies, the model was correctly retained by students eight months later. Aczel (1998) found that students’ equation solving strategies improved after using his balance scale software and suggested that software was more suited to presenting this model than a textbook.

Another example of software which uses a visual model to represent concepts is Grid Algebra (Hewitt, 1996) in which opposite directions represent inverse operations. Left and right shifts represent subtraction and addition, whereas up and down shifts represent division and multiplication. A shift from the first to the second row doubles both the expression entered and the effect of left and right shifts (see Figure 2-3).

Hewitt’s purpose of using opposite directions to represent inverse operations is to help students build a mental representation of the relationship between an operation and its inverse in a way which helps conceptual understanding. Although this model is applied to algebraic expressions in Grid Algebra, there is potential to apply it to equations as these have algebraic expressions on each side of the equals sign.
2.6.3 Constructivist software

Constructivist software is characterised by the opportunities it offers for students to build their own understanding (Huang, 2002). Examples considered here are microworlds, intelligent tutoring systems and computer algebra systems.

2.6.3.1 Microworlds

Microworlds were proposed during research into artificial intelligence as simplified models of real or abstract systems (Copeland, 2000). They are small coherent computer environments of mathematics or science in which students can learn by exploring and solving problems (Sarama and Clements, 2002). Students are able to develop their own conceptual understanding by applying operators to objects and relationships in order to create new relationships (Bouhineau, Nicaud, Pavard & Sander, 2002). Examples of microworlds for learning equation solving are Aplusix (Nicaud, Bouhineau, Chaachoua, Huguet & Bronner, 2003) and MathXpert (Beeson, 2002). In both of these microworlds, students enter their own equations using standard algebraic notation and are able to explore their own equation solving strategies. All strategies which are mathematically correct are accepted by the software regardless of whether they are efficient or not. Aplusix is shown in Figure 2-4.

![Aplusix](image)

**Figure 2-4 Aplusix (Nicaud et al., 2003)**

Educational software which provides a practice activity is often most helpful as a supplement rather than a replacement for teacher directed learning (Cotton, 1991). This is supported by both Beeson and Nicaud. Beeson intended MathXpert to provide practice activities as part of a learning programme and his studies confirmed the learning benefits of MathXpert in this role. Nicaud found that Aplusix was suited to use in class situations as it corrected and
reinforced students' existing knowledge but that it was not suitable as an introduction to a topic.

The exploratory nature of microworlds means that students must be intrinsically motivated to explore. Specific learning outcomes may not occur, so Balacheff and Kaput (1996) recommended that teachers should be able to customise microworlds to fit their teaching programme. Beeson (2002) observed that most students and teachers using MathXpert did not want to type in their own equations but did want to use equations similar to those studied in class. Both Aplusix and MathXpert address this issue by providing optional sets of exercises which teachers can customise for their classes.

2.6.3.2 Intelligent tutoring systems

Another development from artificial intelligence research is “intelligent tutoring systems” (ITS). These provide instruction which adapts to the individual learning needs of the student. Beal (2004) has described the three components of ITS: content model, student model and instructional model. When a student is presented with a question, their answer is compared with that in the content model. The instructional model is used to decide on the feedback for the student, to update the student model and to decide the next question to be presented to the student. As successive repetitions of this process occur, the ITS continually adapts to the learning needs of the student.

The use of ITS was not as widespread as predicted because of technical difficulties of design (Reeves, 1999), but interest in these has increased with the general availability of more powerful computers (Inglis et al., 1999).

In both microworlds and intelligent tutoring systems, students are free to make errors. In order to provide relevant feedback, the type of error is determined by the software and feedback chosen according to a set of heuristics.

2.6.3.3 Computer algebra systems

A “computer algebra system” (CAS) can perform calculations and simplifications for mathematical problems with symbolic, numerical or graphical functions. CAS, which is available for computers and calculators, uses symbolic notation which allows operators, exponents, and functions to be entered from the keyboard. Examples are Mathematica (Mathematica, 2006), Maple (Maple, 2006) and CAS calculators. Although they have a wide variety of applications in performing mathematical calculations, they can also be used in...
learning activities where they have been found to allow students to develop deeper understanding and more versatile thinking (Tideswell, 2006).

The use of CAS in mathematics education is increasing and curricula are being modified in recognition of this (Stacey, 2005). Studies have found that students show similar performance on procedures done by hand regardless of whether or not they use CAS while learning. However, in some cases students who use CAS have a better conceptual understanding (Heid et al., 2002). CAS has the potential to relieve students of the boredom of routine calculations and allow more time for developing conceptual understanding (Leinbach, Pountney & Etchells, 2002). Thus, CAS can provide scaffolding for students as they learn. In practice, however, this requires carefully designed learning activities and there is also a lot of potential to use CAS inappropriately. The pedagogy is important and CAS should be regarded as a tool which needs to be used appropriately (Leinbach et al., 2002). Research is still needed to find the best mix of manual calculation and CAS calculation for learning (Stacey, 2005).

2.6.4 Guided exploration

Microworlds, intelligent tutoring systems and computer algebra systems provide many opportunities for learning activities based on constructivist learning theories in which students are encouraged to build their own conceptual understanding. However, their exploratory nature also means that students may not achieve specific learning outcomes (Balacheff & Kaput, 1996).

Mayer (2004) analysed the use of exploratory learning activities promoted in the 1960s, 70s and 80s. In each decade, the evidence showed that guided exploration was more helpful to learning than unguided exploration. Mayer described learning as requiring students to select information, organise it and relate it to existing knowledge. He suggested that students need guidance with selecting information relevant to the learning objective or they will not have appropriate information with which to build their own understanding.

2.7 Research needed

There is research interest in technology for the support it may be able to offer to cognitive aspects of the learning process (Mills, 2004), but more needs to be known about how software can be designed so that it helps students learn (Mayer, 2005) and what effects specific software features have on learning. Research needs to concentrate on learning theories and their application to learning activities (Tallent-Runnels et al., 2006). This is expected to provide more useful information and guidelines for the design of educational software than
direct comparisons of learning outcomes between software and conventional classroom situations.

Reeves (1999) describes the need for research to explore inventive ideas for learning activities which use interactive software, with the goal being to improve the way that software contributes to learning. The research may require several iterations and a mixture of quantitative, qualitative and analytical research methods. Instead of trying to isolate single variables, multiple variables should be examined (Hay, Kim & Roy, 2005). For example, Nicaud, Bouhineau & Huguet (2002) analysed data from pre- and post-tests and a questionnaire, as well as logged data of student-software interactions and time spent on equations. Reeves (1999) summarises by stating that research needs to focus on improving rather than proving.

Atkinson (2005) describes the amount of research into the design of educational software which concentrates on mathematics as limited, when compared to other subjects. Investigations are needed into the use of static and dynamic graphics for helping students understand mathematical concepts. Information from existing research about visual interactive software needs to be tested for its relevance to learning mathematics. Atkinson also describes the need for mathematical topics other than geometry to be investigated.

### 2.8 Summary

It is important for student learning that educational software is based on established learning theories. Cognitive learning theories of versatile thinking, mental representations and visualisation are applicable to the design of learning activities for solving equations. Constructivist learning theories, which suggest students should be encouraged to build their own understanding in the Zone of Proximal Development while being supported by scaffolding, are also relevant. However, guided exploration rather than unguided exploration is more likely to ensure learning outcomes are met.

When learning to solve equations, students need to develop versatile thinking. This requires a combination of conceptual understanding and procedural understanding, but there is a need for more emphasis to be placed on helping students with conceptual understanding. Conceptual understanding involves an understanding of the relationship between individual steps and overall equation solving strategies.
There is potential to design a visual interface that emphasises strategies as this could be suited to helping students visualise mental representations of concepts such as strategies. Further emphasis on strategies rather than procedures may be achieved if software is designed to carry out the procedures. Exploration of multiple strategies may also help students learn.

Interactivity is an important part of implementing learning theories. Scaffolding can be provided with a combination of interactivity and feedback. Individual needs and different levels of students can also be catered for with different types of interactive learning activities and feedback.

Thus, there is potential to design software to help students learn to solve equations with an innovative combination of learning theories, interactivity, feedback and a visual interface. The effects of specific features of the software design on learning could then be investigated.
As shown in the literature review, there is a need to investigate how the design of educational software can help students learn, particularly in mathematics. Educational software should be based on established learning theories. For equation solving, there is a need to help students develop the combination of conceptual understanding and procedural understanding that form versatile thinking. As it is recommended that more emphasis be placed on conceptual understanding, learning activities that emphasise equation solving strategies are needed. This study investigates a software design that emphasises strategies. There are two main parts to this study, the design of the software and the investigation of its effects on student learning.

When designing the software, interactivity and graphics were combined with learning theories to provide learning activities which students can explore with guidance. Standard guidelines for interface design were followed and the ADDIE instructional design system was used to structure the software development process.

Effects on learning were investigated by conducting software trials with students. A variety of data was collected from students about their equation solving performance, strategies and attitudes. The data were analysed in a search for answers to the main research question for this study.

3.1 Research question

The main research question is:

What effect does visual interactive software which emphasises equation solving strategies have on performance and equation solving strategies of students?
This question was explored by investigating the following sub-questions:

- What effects do the following software features have on performance and equation solving strategies?
  - Separating strategic decisions from algebraic procedures
  - Exploring multiple strategies
  - Feedback which emphasises strategies
- What effects do the following have on student learning?
  - Interface design
  - Usability of software
  - How students used the software
- What are the characteristics of students who gain the most learning benefit from software which emphasises equation solving strategies?

3.2 Research overview
In this section, the design of the software, the software trials and the setting for the study are briefly described.

3.2.1 Software design
The software design combines learning theories, interactivity and graphics. The main feature is the emphasis on strategies and this is achieved in several ways:

- Strategies are separated from algebraic procedures so that students can make all strategic decisions for solving an equation without being hindered by procedural errors.
- The software accepts multiple solution strategies so that students can construct a deeper conceptual understanding by exploring various solution strategies.
- Informative feedback is given to students at each step and refers to strategic decisions rather than algebraic procedures.
- A visual interface encourages students to visualise strategies and form mental representations.
3.2.2 Software trials

Software trials with students were conducted. These had ethics approval from both Lincoln University and CPIT. Quantitative and qualitative data were collected for an investigation of any effect of the software design on learning as measured by student performance and equation solving strategies. Data collection included the use of:

- Pre- and post-tests and a marking scheme that allowed both performance and equation solving strategies to be recorded.
- Questionnaires that surveyed students for characteristics and attitudes relevant to using the software.
- Logged data that recorded students' actions while they used the software.

The various types of data were summarised with descriptive statistics and analysed for relationships between design features of the software and student learning.

3.2.3 Study Setting

This study was set at Christchurch Polytechnic Institute of Technology (CPIT), a tertiary institution in New Zealand which provides vocational study for adults. Qualifications are offered in many fields at several levels, as well as pre-entry courses. Mathematics courses form part of programmes in engineering, science, computing and medical imaging. Equation solving is an important topic because students need to be able to solve equations in order to work with and transpose the mathematical formulae of their main field of study.

Equation solving is included in three different mathematics courses at NZQA Level 1 and Level 2. The Level 1 courses are Algebra 1 and Mathematics for Computing; the Level 2 course is Algebra 2. The age of students at CPIT is at least 16 with the average age being 31. The prior knowledge of equation solving varies widely. Some students have very little prior knowledge and find algebra and equation solving very new or difficult. Other students have a good basic knowledge and quickly learn how to solve equations.

The types of equations studied include those with fractions, brackets, or unknown terms on both sides of the equation. In a typical class lesson on equation solving, applications of these types of equations are considered and principles of equation solving introduced. Worked examples follow in which the teacher leads the class to solve a number of equations. Students then practise solving equations, usually by doing exercises from a textbook or a worksheet.

The software in this study is designed to make a contribution to these student exercises.
The software was designed for students such as those in the Level 1 and 2 mathematics courses at CPIT. Its design therefore needed to incorporate theories of adult learning and cater for students with varying levels of prior knowledge and experience in equation solving.
4 Software Development

4.1 Introduction

The software developed for this study was based on both cognitive and constructivist theories of learning. This is because equation solving is a type of problem solving which is suited to cognitive learning theories (Jonassen, 2000), and adults are suited to constructivist learning theories (Huang, 2002). Learning theories that relate to learning in general were used as well as those which apply specifically to mathematics. Learning theories were combined with principles of software design to create a visual interactive interface for solving equations. The main feature of the design of the software, Equations2go (Robson, 2004), was an emphasis on strategies.

In this study, the term “strategic decision” refers to the strategic decision a student makes at each step of solving an equation, and the term “strategy” refers to a sequence of strategic decisions that together solve an equation. The term “efficient” is used to describe a strategy that does not require too many steps and in which each step makes clear progress towards the solution. For most equations, there are several different strategies that could be described as efficient. The term “efficient” is also applied to strategic decisions that form part of an efficient strategy. The terms “accepted” or “available” are used to refer to the strategies and strategic decisions that are accepted by or available in Equations2go. The term “dynamic graphic” refers to a graphic in which there is a change in position, colour, brightness or visibility. The visual interface includes both static and dynamic graphics.

In Equations2go, students make a strategic decision at each step by clicking the mouse on “hot spots” on the equation and choosing options from visual menus. The interface includes a stepping stones metaphor in which each step occurs on a stone and a successful step causes
the next stone to appear. Guidance is provided to students with several types of feedback, and a visual record of strategic decisions is provided by "trails". In Figure 4-1, a partially solved equation in Equations2go is shown.

![Figure 4-1 Partially solved equation in Equations2go](image)

In this chapter, the development of Equations2go is described with particular emphasis on its design.

### 4.2 Software development process

The development of Equations2go took place in several phases and followed the standard ADDIE instructional design model with its five components: Analysis, Design, Development, Implementation and Evaluation. The software development process included rapid prototyping in which a simplified version of the main part of the software was developed. Throughout the software development process usability testing and associated software modifications were done as recommended by Gould (1995).

In this way, all five components of ADDIE were addressed but rather than being done sequentially, each component was revisited as required. The first four components of the ADDIE model are described in the following sections of this chapter. The Design phase is considered in most detail as it is most closely related to the purpose of this study. The final Evaluation phase was done by conducting software trials and these are described in Chapter 5.
4.3 Analysis

The first phase of the ADDIE model is Analysis. The purpose of this is to define the context for the software as it is important that the software provides a learning activity that is suited to the content and the learning needs of the students.

4.3.1 Students

The target audience for Equations2go is adult students studying algebra at NZQA Level 1 or Level 2. These students have diverse attitudes to mathematics and various levels of prior knowledge of equation solving.

Level 1 students have no prior qualification in algebra, and they may not have studied algebra before. Level 2 students have previously solved Level 1 equations, either recently or many years ago. Some students in both Level 1 and Level 2 classes have more knowledge than their qualifications in algebra suggest. This may have been gained in a mathematics course that was not passed or in a related course such as physics. Equations2go must therefore be designed to cater for students with varying levels of prior knowledge by providing appropriate types of equations and feedback.

Many students have little confidence in their mathematical ability and some have negative attitudes to algebra. On the other hand, a few students are confident in their mathematical ability but have not had the opportunity to gain a qualification in algebra. Equations2go needs to cater for these different levels of mathematical confidence. Therefore, the software must be easy to use so that students who are low in confidence don’t perceive yet another obstacle, and it should be interesting so that students are attracted to using it.

4.3.2 Content

4.3.2.1 Types of equations

Formulae used by students in their main field of study may range from very simple linear equations to those with several terms. Equation solving is introduced in Level 1 and extended to include more complicated equations in Level 2.

In Level 1, equations may include variables on both sides of the equation, brackets and simple fractions. Examples of Level 1 equations are:

\[
2a - 3 = 5 \quad 3B - 3 = B + 2 \quad 3(t - 2) = 12
\]

\[
5 + \frac{x}{3} = 8 \quad \frac{F}{2} + \frac{F}{3} = 2
\]
In Level 2, time is spent consolidating the equations from Level 1. These are then extended to those which require more steps to solve, include more terms, or more complicated fractions. Examples of Level 2 equations are:

\[
\frac{v + 2}{3} + 2 = \frac{5}{2} \quad \frac{5}{2c} - \frac{4}{3c} = 1 \quad 2(s - 3) = 7s - 2s
\]

In addition, students may be challenged to transfer their knowledge to equations which require many steps to solve, for example:

\[
\frac{K - 3}{6} + \frac{3K}{4} = 2 \quad 4 - 3(2T - 3) = 2T + 8
\]

4.3.2.2 Equations in Equations2go

As the results of this study will be used to inform the design of a future version of Equations2go, as many types of equations as possible were included so that associated design issues could be investigated. Key features are brackets, fractions, and terms which include the unknown being on both sides of the equation. As the time available for software development allowed only four equations to be developed, all were chosen from Level 1 so that all equations would be suitable for all participants in the trials. The following equations were selected:

\[
3x - 4 = 5 \\
3F - 2 = 5F + 1 \\
3(4a - 5) = 6 \\
d + d = 1 \\
3 \\
5
\]

Solving these equations requires a variety of techniques. The equations include various letters, in both upper and lower case, because formulae in the students’ main field of study use letters appropriate to the field, e.g. \( F \) for force, \( a \) for acceleration.

4.3.2.3 Instructional model

When teaching students to solve equations, an instructional model can be used to help students learn to plan an equation solving strategy. There are many variations of these models but all lead to efficient strategies. The instructional model chosen as the basis of Equations2go encourages students to solve equations using the following sequence of goals:
1 Deal with fractions
2 Remove brackets
3 Collect unknown terms together
4 Isolate the unknown term
5 Isolate the unknown

This model provides more structure than some other models but was chosen for this reason as it provides a clear framework for the design of feedback in Equations2go. It also accommodates a wider range of strategies than some other models as it does not prescribe how the goals should be achieved.

Consider this instructional model applied to the following equation:

\[ 4 - (2T - 3) = 2T + 8 \]

Expand the brackets:
\[ 4 - 2T + 3 = 2T + 8 \]
Collect T terms on one side:
\[ 4 + 3 = 4T + 8 \]
Isolate the T term:
\[ 4 + 3 - 8 = 4T \]
\[ -1 = 4T \]
Isolate T:
\[ \frac{-1}{4} = T \]
\[ T = -0.25 \]

In the first step, the brackets were expanded. Next, the \( T \) terms were collected on the right hand side. The \( 4T \) term was then isolated and finally \( T \) was isolated. In this way the instructional model guided the equation solving decisions. The steps in the solution shown are not the only ones that fit the instructional model, for example in the second step, the \( T \) terms could equally well have been collected onto the left hand side.

### 4.4 Design of software

The design of Equations2go formed a major part of this study. The learning activity in the software was based on learning theories relevant to equation solving combined with interactivity, feedback and a visual interface.

In this section, the issues and decisions which contributed to the design of Equations2go are described. The main considerations are relevant learning theories, emphasis on strategies, type of feedback, and nature of the visual interactive interface.
4.4.1 Learning theories

The main learning theory underpinning the design of Equations2go is that of versatile thinking (Thomas & Tall, 1988) in which conceptual understanding of equation solving is combined with the step by step nature of carrying out procedures. As researchers have identified the need to help students with conceptual understanding, (e.g. Kieran, 1992; Skemp, 1971), the main principle of the design of Equations2go is an emphasis on an aspect of conceptual understanding – that of strategies.

Asking students to consider the best strategy for solving an equation has been found to improve flexibility in thinking and conceptual understanding (Star & Madnani, 2004). In order to consider which strategy is best, students must first consider multiple strategies for solving an equation. Therefore, Equations2go accepts multiple solution strategies.

4.4.2 Strategies

The emphasis on strategies in Equations2go is achieved with three main features: strategic decisions are separated from algebraic procedures, multiple solution strategies are available, and feedback is related to students’ strategic decisions.

4.4.2.1 Strategic decisions separated

In Equations2go, students make strategic decisions at each step but the software performs the algebraic procedures. With the software carrying out the algebraic procedures, students are able to concentrate on strategies so that their progress through the equation solving process is not hindered by their procedural errors. This principle of separating strategies from procedures is similar to using a computer algebra system (CAS) for learning mathematics, as students make strategic decisions and the CAS software does the calculations.

If a strategic decision is accepted by Equations2go, the student progresses to the next step, and a new stone appears with a simplified equation which is the result of the algebraic procedure for that strategic decision.

If a student’s strategic decision is not accepted by Equations2go, the student is given feedback about their decision. The feedback has sufficient information to allow a student to rethink their strategic decision and to try again. In this way, feedback forms part of the separation of strategic decisions from procedures. The feedback also contributes to the emphasis on strategies by referring to strategic decisions rather than procedures.
4.4.2.2 **Multiple solution strategies**

Equations can usually be solved in several ways and Equations2go allows several different strategies to be used for most equations. Students first solve an equation using one strategy and can then search for the other strategies available. The purpose of exploring different strategies and searching for all strategies accepted by Equations2go is to help students construct their conceptual understanding of equation solving from their own experiences.

4.4.2.3 **Strategies accepted by Equations2go**

All strategies accepted by Equations2go are efficient strategies, and they were carefully chosen so that they contribute to students' learning. Criteria were established to help decide which strategies would be accepted by Equations2go. These criteria require that a strategy must not have too many steps, must be distinct, and must adhere to the instructional model (see Section 4.3.2.3).

The reason for the first criterion, of not too many steps, is so that students learn to solve equations efficiently and directly with each step making progress towards the solution.

The reason for the second criterion, of each accepted strategy being distinct, and not just a minor variation, is that students need to learn from their search for all accepted strategies. Minor variations between strategies may annoy students if they waste time hunting for something trivial. For example, in the equation \(-2x = 5\) students who divide both sides by \(-2\) should not then be required to find the less efficient strategy in which both sides are divided first by 2 and then by \(-1\).

The purpose of the final criterion, of following the instructional model, is to structure the way feedback is written so that students receive consistent guidance. This criterion requires accepted strategies to conform to the instructional model in which fractions and brackets are removed first, unknown terms are collected together and isolated, and finally the unknown is isolated.

However, a problem arose during usability testing for the following equation:

\[ 3F - 2 = 5F + 1 \]

A solution based on the instructional model involves collecting the unknown terms on either the left or the right side of the equation and leads to the following two accepted solution strategies:
However, some students preferred to collect the numbers together first instead of the unknown terms. In this strategy, the first step involves collecting the numbers onto either the left or the right side as follows:

\[
\begin{align*}
3F - 2 &= 5F + 1 \\
-2F - 2 &= 1 \\ -2F &= 3 \\ F &= -1.5 \\
\end{align*}
\]

or

\[
\begin{align*}
3F - 2 &= 5F + 1 \\
3F - 3 &= 5F \\
-3 &= 2F \\ -1.5 &= F \\
\end{align*}
\]

These two strategies are clearly efficient strategies and need to be accepted by Equations2go. They meet the first two criteria of not using too many steps and of being distinct, but they do not meet the third criterion of fitting the instructional model. Modification of the instructional model so that it includes all four strategies was considered. Although this would help, other modifications to the instructional model may be necessary for other equations. Rather than making continual changes to the instructional model, it was decided that additional strategies would be accepted by Equations2go on a case by case basis as determined by usability testing. Data from such usability testing would provide a basis for reconsidering the instructional model for a future version of the software.

Another issue related to the number of accepted strategies was that usability testers wanted to know how many different strategies to search for, so this information was displayed for each equation.

4.4.3 Feedback

Feedback for equation solving should be short, show consequences of errors, and allow students to see why they are incorrect in a way that allows them to work out the next step themselves (Nguyen-Xuan et al., 1997). To achieve this, design issues related to the amount, nature and timing of feedback are considered.

4.4.3.1 Amount of feedback

The amount of feedback needs to be a balance between short feedback recommended for equation solving and sufficient information for guiding students in their next decision. In Equations2go, the balance was achieved by displaying a brief “tip” inside a flag with a more
informative explanation being available at the request of the student. This conforms to the recommendation that screen design should be kept as simple as possible to minimise cognitive load but that more information should be available on request (Nielsen, 1993). It also allows students to request more feedback when required, for example in the early stages of learning (Mason & Bruning, 2001).

The flag, which only appears when strategic decisions are not accepted, contains a short phrase about the type of strategic decision needed. The explanations, which are available for all strategic decisions, consist of an equation and one or two sentences of information. They provide students with more information than the flag, but do not require students to read large amounts of text, as recommended by Prensky (2003). In this way, short feedback is always provided by the flag, whereas the student controls the display of the more informative explanations. Both types of feedback are shown in Figure 4-2 in which the strategic decision chosen was to divide both sides by 2.

![Figure 4-2 Feedback for a strategic decision not accepted by Equations2go](image)

### 4.4.3.2 Nature of feedback

As well as providing information about students’ strategic decisions, the feedback in Equations2go also shows students the consequences of their decisions.

When a strategic decision is not accepted, the information in the explanation reminds students of what they should be trying to achieve, and the equation in the explanation shows the consequences of carrying out the procedure for the strategic decision chosen. Students can then see the lack of progress towards their goal.
For example, in Figure 4-2, dividing both sides by 2 was not accepted by Equations2go and the consequences are shown, thus allowing students to see that this strategic decision makes the equation more complicated and difficult to solve. The feedback does not tell students what action to take, but the equation and information in the explanation provide students with enough information to work out why their strategy was not accepted. They are expected to use this information, analyse their own responses and work out what to try next, thus constructing their own learning.

When a student’s strategic decision is accepted, the equation in the explanation shows the algebraic manipulation for the associated procedure and the information in the explanation further verifies the student’s decision. Although the emphasis of the feedback is on strategies, the equation in each explanation also provides some information about algebraic procedures. See Figure 4-3.

Visual changes provide additional feedback to verify each accepted strategic decision. A new stone appears ready for the next step, and a trail appears between the two stones recording the student’s strategic decision. If necessary, all the stones move to a new position on the screen so that the new stone is near the centre with room for the menus to be displayed in the next step. The scoring panel on the tree also changes colour. The score is described in more detail in Section 4.4.4.8. A quick animation (a spinning star) is another visual change and this occurs when the equation is solved. Thus, visual changes contribute to the feedback that verifies student decisions. See Figure 4-4.
4.4.3.3 Timing

The timing of feedback in Equations2go follows the principle that feedback should not be available until after the student has made an attempt (Mory, 1996). The feedback flag is not visible and the explanation cannot be requested until the student has chosen a strategy. Another timing principle is that students should be able to try again after an incorrect response (Dihoff et al., 2005). When a strategic decision is not accepted, a trail recording the strategic decision is displayed along with the feedback flag. The explanation is also displayed if requested by the student. Students can consider their strategic decision with its feedback and try again. See Figure 4-5.
4.4.4 Visual interactive interface

Interactivity and graphics contribute to the emphasis on strategies in Equations2go and their contribution to the interface design is described in this section.

4.4.4.1 Interactivity

The interactivity of Equations2go emphasises equation solving strategies by allowing students to make strategic decisions after which the software responds with feedback. This interaction between students and software allows students to concentrate on strategies. Interactivity also allows students to control the amount and frequency of feedback as well as navigate between the equations.

4.4.4.2 Metaphor

In Equations2go, metaphors are used to help students form mental representations in which equation solving strategies are seen as a series of steps or strategic decisions and different strategies can be used to solve an equation.

The step by step nature of equation solving is represented by a stepping stones metaphor of “one step at a time”. The equation to be solved appears on the first stone, and as each step is completed a new stone appears with the simplified equation ready for the next step. In this way, a stone visually identifies each step and allows students to consider each step in turn.

Another interpretation of the stone metaphor is “leave no stone unturned” which reflects the students’ search for different strategies. The concept of different strategies leading to the same solution is supported by the metaphor of tracks in a forest in which several routes lead to the same place and signposts provide guidance. In Equations2go, the tree image suggests a forest, trails record the route taken, and feedback flags act as signposts.

4.4.4.3 Graphics

For equation solving, there is a need for students to visualise mental representations of equation solving strategies, and therefore graphics are included in the design of Equations2go (Reiber, 1994). Another reason for including graphics is to support and implement the interactivity.

The aim has been to use graphics appropriately, and in a simple way, so that they do not distract the student. The graphical elements of stones and trails combine to build up images of strategies used by students to solve equations. Multiple solutions are reinforced by different visual representations for each strategy.
The visual stepping stones metaphor is supported by colour and visibility changes of stones, menus, trails, scoring panel and the spinning star. Interactivity is supported by visibility changes to buttons and feedback, thus allowing the interactive learning activity to take place.

Opposing menu items visually represent inverse operations. This principle was inspired by Grid Algebra (Hewitt, 1996), and in Equations2go the operations on the menus are positioned so that up and down directions represent addition and subtraction, whereas left and right directions represent division and multiplication. The symbolism of a direction being assigned to each operation is reinforced by the arrow head shape of each menu option. See Figure 4-6.

![Figure 4-6 Opposing directions for inverse operations](image)

The same directions are used to position each new stone, above, below, to the left or right of the current stone, depending on the operation performed. As the steps are completed, trails record the strategic decisions. In this way, the completed trails, including their direction, contribute to a picture of the strategy used to solve an equation.

### 4.4.4.4 Hot spots and menus

Hot spots and menus allow students to make strategic decisions. Students make a strategic decision by clicking on a hot spot followed by a menu option.

Hot spots are located on equation elements to draw attention to the equation element and the strategic decisions available. Hot spots become visible when the mouse hovers over them but disappear when the mouse moves off them. When a hot spot is visible, a menu preview is also visible.
When a hot spot is clicked, the menu preview brightens and becomes clickable so that options can be chosen (see Figure 4-7). Menu previews were included because usability testers wanted to know what options would be available before they clicked a hot spot.

The menus differ from standard menus in that menu items are positioned around stones rather than in a list. However, the menus use the standard principle of menu items appearing faded when they not available. With the menus positioned around stones, the distance the mouse must be moved between clicking a hot spot and clicking a menu option is minimised.

Most hot spots are placed on numbers, unknown terms or unknowns, and these allow students to add, subtract, multiply or divide using the equation element marked by the hot spot. Strategies which cannot be made available from hot spots on terms can be accessed from hot spots on symbols such as “+”, “−”, “(” or “)”. The menus associated with hot spots on symbols also surround the stones, but their shapes point in diagonal directions and are coloured differently to distinguish them from the menus with four operations in which the directions have a meaning. These menus include options such as “Expand brackets”, “Multiply all terms by a number”, “Add terms” or “Add fractions”. In the example in Figure 4-8, the menu on the equals sign has only one option but students must also select the number they wish to multiply all terms by from a secondary menu.
The location of hot spots was chosen so that all strategic decisions that are accepted by Equations2go are available. These and other hot spots also allow strategic decisions that are not accepted by Equations2go to be available. Design decisions about the location of hot spots were made so that strategic decisions that are suitable for helping students learn are available. For example, consider the following equation:

\[ 3F - 2 = 5F + 1 \]

A hot spot on the first term could allow students to add, subtract, multiply or divide both sides of the equation using \(3F\), \(3\) or \(F\). In this case, the hot spot was located on \(3F\) to allow students to subtract \(3F\) from both sides. If students try to add, multiply or divide both sides by \(3F\), feedback is provided and students can try again. Consider another case:

\[ -3 = 2F \]

Here, the hot spot was located on \(2\) rather than \(2F\) so students can divide both sides by \(2\).

However, the decisions on the placement of the hot spots were not always straightforward. For example, consider:

\[ -2F - 2 = 1 \]

The hot spot on the first term could be on \(-2F\), \(2F\) or \(2\). In this case, the hot spot was located on \(2F\) so students can add this to both sides. It is not on \(-2F\) because students may find it confusing to subtract this negative term when an easier way of achieving the same result is to add \(2F\). The hot spot could have been placed on \(2\) to allow students to divide both sides by \(2\) but two hot spots on the same term would make it difficult for students to find them with the mouse. In this particular equation, the hot spot on the second term allows students to use the number \(2\) in their strategic decision and the problem is resolved. However, the problem is not yet fully resolved in general. For example a hot spot would be needed on both \(3\) and \(3F\) in the following equation:

\[ 3F - 6 = 9 + 6F \]

Hot spots on each element are as large as possible without overlapping neighbouring elements. This is because usability testing showed that hot spots on small equation elements such as brackets were not easily found. In some cases, equation elements were spread out to give more room for hot spots. Also, where practical, a hot spot becomes visible when the mouse is either over or close to it. These features were included to make it easier for students to find hot spots with the mouse.
4.4.4.5 Buttons
As well as the interactivity provided by hot spots and menus, there are buttons which allow
students to change their mind and to control the display of the explanations. In addition, there
is a set of buttons at the top of the screen which allows students to switch the score on or off
and to navigate between the equations.

The Undo button was created because usability testers were concerned that they could not
change their mind after clicking a hot spot. The Undo button becomes visible after a hot spot
is clicked, but before an operation is chosen from the menu. Thus, it is only visible when it
has a use, so its visibility relates to the users’ needs as recommended by Nielsen (1993).

The button for turning the explanation on or off was renamed from “Explanation ON” and
“Explanation OFF” to “Show explanation” and “Hide explanation” to better indicate its
purpose.

A button for turning the score on or off allows students to explore strategic decisions without
seeing the score. This issue was raised by a usability tester who was unwilling to try other
strategic decisions because she didn’t want a low score.

With only four equations in this version of Equations2go a simple navigation system allows
students to move between equations. Buttons are available in consistent locations on a
navigation bar at the top of the screen for “Next”, “Previous” and “Show again”. However, the
design of the navigation is outside the scope of this study.

4.4.4.6 Stones and Trails
Steps are separated visually with a stone which frames the equation for each step. Trails
display links between steps and record strategic decisions. See Figure 4-4.

4.4.4.6.1 Stones
When designing the image for the stone metaphor, a round shape was initially chosen for the
stone. However, this took up too much screen space, limiting the number of stones, and hence
steps, that would fit vertically on the screen. The change to an oval stone suits the shape of an
equation and allows more steps to fit on the screen. The stones were made to look more
realistic by making their shape more irregular and adding a shadow.

The colour of the stone for a current step is grey. When a new stone appears, the colour of the
stone for the previous step darkens and takes on a green tint indicating that it now forms part
of the trail showing the record of completed strategies. The reason for the darker colour on previous stones is to allow colour-deficient students to detect the change.

**4.4.4.6.2 Trails**

Trails display strategic decisions that have been accepted by Equations2go. These are coloured green to indicate that the way is clear to proceed to the next step. Each trail consists of the arrow head from the menu and an arrow that links to the next stone. In this way, all strategies accepted so far are visible to the student, giving a context for the next step.

Trails also display strategic decisions that are not accepted by Equations2go (see Figure 4-2). They are coloured brown and show the equation element and the menu option chosen. These trails remain on screen only until the student has tried again.

**4.4.4.7 Screen layout**

The screen layout was designed to be as simple as possible to minimise the cognitive load associated with learning to use the software. Graphics and typefaces are simple and used consistently. To keep the number of elements on the screen to a minimum, each element is included for a specific reason related to the learning activity. For example, the stones are related to the stone metaphor used to help students visualise the step by step nature of equation solving. The reason for the Show/Hide Explanation button is to allow students to control the amount of feedback displayed. It also helps simplify the screen layout as the explanation is not visible until a student requests it.

The explanations are in a consistent location at the bottom of the screen so that they do not obscure the main equation. However, the location of the Show/Hide Explanation button was changed as result of usability testing as some users did not notice this button until it was pointed out to them. This button was moved from a consistent position on the left of the screen to a variable position that was close to the stone and other screen elements for the current step. The decision about the location of this button is an example of an interface design decision in which two guidelines had to be prioritised (Nielsen, 1993). In this case, the location of a button was changed so that it was no longer consistent but instead varied according to the location of elements to which it was related.

**4.4.4.8 Scoring system**

Equations2go calculates a score to indicate student progress through an equation. Students can choose whether to display the score or not. The main purpose of the score is to help motivate students by providing short term goals, as recommended by Prensky (2003). The score also
provides feedback to students by quantifying the number of strategies tried and accepted. In Equations2go the scoring system is very simple and was included in order to find out about students’ reaction to a scoring system.

If the scoring is turned on, the current score is displayed as well as the target score for the equation. The score starts at zero and is increased each time a strategic decision is accepted by Equations2go. Most accepted strategic decisions result in a gain of 10 points but some steps gain 20 points so that the target score for an equation is the same regardless of which strategy is used. Two points are deducted when a strategic decision is not accepted and one point is deducted each time the “Undo” button is used. The two points lost for a strategic decision that is not accepted reflect the two mouse clicks required to choose it. The Undo button is available after the first click and therefore only one point is lost. This deduction of points may discourage students from clicking randomly.

In Figure 4-9, the strategic decision for the first step has been accepted and seven points gained. This score is the result of three attempts at this step. Two points were lost for trying a strategic decision that was not accepted, and one point was lost for using the Undo button. In the third attempt, ten points were gained for choosing a strategic decision that was accepted.

**4.4.4.9 Instructions**

Each of the thirteen pages of instructions deals with just one topic. Several of the pages are interactive as shown in the example in Figure 4-10.
4.5 Development

Because a rapid prototyping process was used, the Development phase of the ADDIE model was interwoven with the Design and Evaluation phases.

Initially, six different designs were proposed and simple prototypes developed. Consideration of these first designs, as well as the reactions of students, teachers and researchers, led to the design of a further prototype that became the basis of Equations2go. The software for one equation was completed followed by two phases of usability testing by students, during which recommendations by Nielsen (1993) were followed. Individual and pairs of student volunteers were asked to “think aloud” as they used the software as well as comment on any features of the software that they found helpful or annoying. Their comments were noted and modifications were made to the interface after each phase.

Once the design and development of the first equation was complete, three more equations were developed as well as a simple navigation system to allow users to move from one equation to another. Further usability testing and modifications took place with several minor changes made.

The researcher conducting this study used Flash MX 2004 (Macromedia, 2004) to develop Equations2go. This was chosen because it results in small files suitable for online delivery. Graphics tools in Flash MX 2004 were used to create the graphics objects and Actionscript was used to program the interactivity and animation.
4.6 Implementation

In the Implementation phase of the ADDIE instructional design system, Equations2go was installed on a web server at CPIT. For the period of the trials, Equations2go was protected by password security to ensure that it was only accessed during controlled trials. To assist the Evaluation phase of the ADDIE system, equation solving decisions made by students as they used the software were logged in a text file on the web server. This was achieved with a program written in PHP (PHP, n.d.), as this programming language is suited to transferring data between a user's computer and a web server.

4.7 Software design and learning benefits

In this section, the design of Equations2go is compared to the design of other learning activities in which students can practise equation solving. The purpose of this comparison with classroom situations and microworld software is to show the nature of the learning activity provided by Equations2go.

In a classroom situation, time is usually allocated for students to practise equation solving by doing exercises from a textbook, worksheet or whiteboard. In these exercises, students find out whether they have made errors when they check the answers. While students work on the equation solving exercises, the teacher helps as many students as possible, but this is limited by the number of students in the class and the time allocated to the exercise. By comparison, Equations2go provides feedback that is more frequent and more detailed as the feedback is provided after each strategic decision and includes information as well an equation showing consequences of each decision made. This level of feedback could not be provided to each student by a teacher supervising a classroom activity in which students solve exercises from a textbook.

When solving exercises from a textbook, it is possible for a small procedural error in an early step to lead to a very different and possibly difficult strategy. Any planned graduation in the difficulty of equations could thus be lost. By comparison, students using Equations2go are not able to make procedural errors, are alerted immediately a strategic error occurs and are given feedback about how to improve. In this way, students are guided to choose appropriate strategies without interference from procedural errors.

Microworlds such as Aplusix (Nicaud et al., 2003) and MathXpert (Beeson, 2002) are based on constructivist learning theories and allow students to freely explore any strategy. All
strategies that are mathematically correct are accepted regardless of whether they make progress towards a solution. By comparison, Equations2go allows a more limited form of exploration in which students have fewer choices. They are also given guidance to try a range of efficient strategies which are suitable for building their own understanding, as recommended by Mayer (2004). This guidance also helps students experience the success which is one of the factors needed for motivating students to learn (Keller, 1987).

Another difference between microworlds and Equations2go is that students can enter their own equations into microworlds. However, Beeson observed that most students and teachers did not want to enter their own equations into MathXpert and instead requested equations similar to those being studied. For this reason, both MathXpert and Aplusix provide sets of equations. By comparison, Equations2go only provides predefined equations. It therefore contributes directly to learning objectives that are more specific than those of microworlds.

Feedback must also be designed differently for microworlds because it must be provided for strategic decisions for any equation that students choose to enter. To achieve this, feedback is chosen from a standard set. The feedback is chosen according to the type of error and does not refer to the context of the error. By comparison, feedback in Equations2go is specifically written for each strategic decision that is available for each equation. The advantage of this is that feedback relates directly to each strategic decision. The disadvantage is the number of individual feedback statements and equations showing consequences which must be written.

In summary, Equations2go is designed to provide more frequent, detailed and informative feedback than a classroom exercise done from a textbook. Compared to a microworld, Equations2go is designed to provide students with a more limited form of exploration but to give them more guidance towards achieving specific learning outcomes. It may be that once specific learning outcomes have been achieved, students may have enough knowledge about equation solving to extend and direct their own learning while exploring a microworld.

### 4.8 Summary

The development process for Equations2go included all components of the ADDIE instructional design system: Analysis, Design, Development, Implementation and Evaluation.

The software design provides a learning activity which is based on a combination of cognitive and constructivist learning theories. Students receive guidance as they explore multiple equation solving strategies. The main feature of the design is an emphasis on strategies with
the purpose of this being to develop the conceptual understanding students need for versatile thinking.

The emphasis on strategies is achieved in two ways. Firstly, strategies are separated from procedures so that students can concentrate on making strategic decisions while the software carries out the algebraic procedures. Secondly, multiple solution strategies are accepted by Equations2go so that students can gain a better overall view of equation solving by solving an equation in several different ways.

The emphasis on strategies is supported by the feedback given to students. Students must make an attempt before receiving feedback and they can try again after an unsuccessful attempt. Brief feedback is displayed for each strategic decision but students can also request further information and this includes an equation showing the consequences of their strategic decision.

The emphasis on strategies is also supported by the visual interactive interface. Interactive elements include buttons, hot spots and menus which allow student actions. Graphics include stones which support the stepping stones metaphor, trails which record students’ successful actions, and arrow-shaped buttons pointing in opposite directions which represent inverse operations. The static and dynamic graphics are designed to help students visualise equation solving strategies.

The software development process included rapid prototyping and usability testing. The software was implemented onto the CPIT computer system, ready for evaluation by students for its contribution to learning. The software trials conducted for this purpose are described in the next chapter.
5 Trials

There are two main parts to this study, the software design and the software trials. The purpose of the software, whose design was described in Chapter 4, is to help students learn to solve equations. The purpose of the software trials was to collect information about the effects of the software on student learning. The trials are described in this chapter.

Prior to the main trials, a pilot study was undertaken to try out the procedure for the trials and to help finalise the focus for the study. The main trials, which were carried out during mathematics classes, used a pre- and post-test design with the intervention being the use of Equations2go during which student actions were logged by the software. Data were collected on students' performance, strategies and attitudes. A marking scheme for the tests was written which led to the distinction between strategies and procedures being further defined. Finally, the data were analysed.

This chapter describes the pilot study, ethical issues, procedure for the trials, data collection, marking scheme for the tests and data analysis.

5.1 Pilot study

In order to try out the planned procedure for the trials, a pilot study was conducted. Three students trialled the software and three students trialled a worksheet alternative. Students in the worksheet pilot trial had to complete a pre-test, worksheet and post-test as well as the questionnaires. These students showed signs of boredom during the post-test, probably because the tests and worksheet were very similar activities. This suggested that some students might not complete the post-test and thus the worksheet trial might not give valid data for a comparison with the software trial.
The students in the software pilot trial, however, were happy to complete all activities, including the post-test. Several types of data were collected and the computer logged data were particularly useful for determining the equation solving strategies used by students.

It was decided to continue with only the software trials as the pilot study showed that the variety of types of data collected would allow a thorough qualitative analysis of student strategies. This plan also fitted with recommendations in the literature to investigate student learning thoroughly during software use, rather than compare the software to other learning activities (e.g. Reeves, 1999; Hay et al., 2005).

This shift in focus for the study was the main change resulting from the pilot study, although minor changes were also made to the procedure for the software trials. These included modifying the coding system to ensure student code numbers appeared on all data and wording changes to the questionnaires.

5.2 Sample

To maximise the sample size, all Level 1 and Level 2 mathematics classes in Semester Two, 2004 and Semester One, 2005 were chosen to take part in the trials. During this time there were four Level 1 classes (two classes each of Algebra 1 and Mathematics for Computing) and four Level 2 classes (Algebra 2). The eight classes were taught by five teachers, none of whom was the researcher. In order to investigate realistic effects of Equations2go on learning, the trials were conducted by real users as recommended by Nielsen (1993) and Mitrovic et al. (2002).

5.3 Ethics

Ethics approval was obtained from the Human Ethics Committee at Lincoln University as well as the Academic Research Committee at CPIT. Approval was obtained from both committees because the study was based at Lincoln University but the participants were students at CPIT. Procedures that were followed to ensure voluntary participation and anonymity are described in this section.

As the trials were conducted during class time, voluntary participation was important. To ensure this, the researcher provided information to the students about the trials in one class session and the trials took place in the next session. Trials were conducted in the last hour of a class session and participants moved from the classroom to a computer room. Students not
taking part in the trial could remain in class with their teacher. A consent form which included information about the data that would be collected was signed by all participants.

Anonymity of participants was preserved by allocating a code to each student. Although student names were recorded on the questionnaires, they were removed after all data were matched to student codes.

5.4 Trials

The trials were conducted after students had spent class time learning to solve equations, so they were ready to do an exercise in which they practised on their own.

Pre- and post-tests were conducted before and after a period of 20 minutes during which Equations2go was used. As a control group was not used, the trials were designed so that the only activity between the two tests was the use of Equations2go. This was to ensure that any observed changes could be attributed to the use of Equations2go. Information about students' attitudes, characteristics and reactions to Equations2go was gathered by pre- and post-questionnaires as well as during the group discussions which followed the post-tests.

5.4.1 Trial procedure

The trials consisted of a series of student activities as shown in Table 5-1. The tests, questionnaires and discussion questions used in the trials are in the Appendices.
<table>
<thead>
<tr>
<th>Activity type</th>
<th>Description of student activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-questionnaire</td>
<td>Answer questions about attitudes to learning algebra.</td>
</tr>
<tr>
<td>Pre-test</td>
<td>Solve seven equations.</td>
</tr>
<tr>
<td>Use of Equations2go (20 minutes)</td>
<td>Exercise 1 Solve all four equations. Research purpose: To allow students to answer questions about Equations2go.</td>
</tr>
<tr>
<td></td>
<td>Exercise 2 Turn the score on. Solve the equations again, aiming for maximum scores. Research purpose: To allow students to answer questions about whether they preferred the score on or off.</td>
</tr>
<tr>
<td></td>
<td>Exercise 3 For each equation, look for other strategies that lead to a correct solution. Research purpose: To allow students to answer questions about multiple solutions.</td>
</tr>
<tr>
<td></td>
<td>Exercise 4 Turn score off. Explore inefficient strategies and the associated feedback. Research purpose: To allow students to answer questions about feedback.</td>
</tr>
<tr>
<td>Post-questionnaire</td>
<td>Answer questions about attitudes to using Equations2go.</td>
</tr>
<tr>
<td>Post-test</td>
<td>Solve seven comparable equations.</td>
</tr>
<tr>
<td>Discussion groups</td>
<td>Discuss Equations2go in groups of two or more students and record group comments.</td>
</tr>
</tbody>
</table>

In the first activity, the pre-questionnaire, students were asked about their confidence with equation solving and their attitude to using a computer specifically for this. They were also asked their age group, gender, level and main field of study. The questions used either a multiple choice format or a Likert scale and space was provided for additional comments.

In the next activity, the pre-test, the students were asked to solve seven equations. The equations gradually increased in difficulty from single-step equations to those with brackets or fractions. Equations 1 to 6 were suitable for both Level 1 and Level 2 courses but Equation 7 was a Level 2 equation.

Just prior to using Equations2go, students were advised to read three of the thirteen instruction pages: the purpose of the software, the goal to follow while using the software, and
an instructional model of equation solving strategies. Students could then either go directly to Exercise 1 or they could read the other instructions first. While students worked with the equations, the software logged their actions.

In the post-questionnaire, students were asked about their equation solving confidence and their attitude to using Equations2go as a learning activity. They were also asked to identify features of the software which helped or hindered their learning of equation solving. Space was provided for additional comments.

In the post-test, students were asked to solve another set of seven equations. Each equation in the post-test corresponded to a similar equation in the pre-test that could be solved using the same strategy. This allowed any changes in student strategy to be seen.

In the discussion groups of two or more students, each group was provided with a discussion paper with three questions and space for comments. The students were asked to discuss what they liked about Equations2go, what they found annoying and what they thought about the separation of strategies from algebraic procedures. In the later trials, an extra question was added in which students were asked to discuss their attitudes to searching for multiple strategies. Groups discussed these questions and recorded their comments.

5.4.2 Data collection
The sequence of student activities in the trials allowed information to be collected about students’ equation solving strategies and performance, as well as their characteristics and their attitudes to Equations2go and equation solving. Students’ performance and strategies were recorded in pre- and post-tests. Students’ strategies were also logged while they used the software. Students’ attitudes were recorded in pre- and post-questionnaires as well as during group discussions. The various types of data collected and the methods of collection are summarised in Table 5-2.
Table 5-2  Data collection

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Data collection methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance in solving equations</td>
<td>Pre-test and post-test</td>
</tr>
<tr>
<td>Strategies used to solve equations</td>
<td>Pre-test</td>
</tr>
<tr>
<td></td>
<td>Logged data</td>
</tr>
<tr>
<td></td>
<td>Post-test</td>
</tr>
<tr>
<td>Attitudes to equation solving and using computers to learn</td>
<td>Pre-questionnaire</td>
</tr>
<tr>
<td></td>
<td>Post-questionnaire</td>
</tr>
<tr>
<td>Attitudes to software in this study</td>
<td>Post-questionnaire</td>
</tr>
<tr>
<td></td>
<td>Discussion groups</td>
</tr>
<tr>
<td>Use of software features such as score and feedback</td>
<td>Logged data</td>
</tr>
<tr>
<td></td>
<td>Post-questionnaire</td>
</tr>
<tr>
<td>Student characteristics</td>
<td>Pre-questionnaire</td>
</tr>
</tbody>
</table>

The data collection methods are consistent with recommendations to use pre- and post-tests (Bamberger et al., 2006) and questionnaires and logged data (Nielsen, 1993) for collecting data to investigate how the software was used.

5.5  Marking scheme

In order to assess student performance and equation solving strategies in the pre- and post-tests, a marking scheme was devised to record strategies separately from procedures.

In the marking scheme, marks are awarded for strategic decisions that form part of an efficient strategy. One mark is allocated to the strategic decision for each step and hence the number of marks for each question is determined by the number of steps needed to solve the equation. A total of 18 marks are allocated in each of the pre- and post-tests.

In making the distinction between strategic decisions and algebraic procedures, a complicating factor is that performing algebraic procedures requires strategic decisions from prior learning. For example, when students are learning to solve equations, they may make a strategic decision to add $4x$ to both sides of an equation. Having decided to do this, they must then carry out the procedure for adding algebraic terms. However, the procedure for adding algebraic terms includes making further strategic decisions about how to add terms. In the New Zealand Numeracy Project, procedural understanding has been called knowledge and is described as strategies which are so familiar that the slower process of strategic thinking is no
longer needed (Hughes, 2002). A similar principle is used in this study, so that an algebraic procedure includes any embedded strategies and strategic decisions as these are considered to be prior knowledge for the procedure.

5.5.1 Marking scheme applied to a correct solution

In the example in Table 5-3, the marking scheme is applied to an equation which has been solved correctly with a strategy that requires two steps and hence two strategic decisions.

<table>
<thead>
<tr>
<th>Strategic decision</th>
<th>Equation and solution</th>
<th>Strategy marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>$4x - 7 = 3$</td>
<td></td>
</tr>
<tr>
<td>Step 1</td>
<td>Isolate $4x$ by adding 7 to both sides.</td>
<td>$4x - 7 + 7 = 3 + 7$</td>
</tr>
<tr>
<td>Step 2</td>
<td>Isolate $x$ by dividing both sides by 4.</td>
<td>$\frac{4x}{4} = \frac{10}{4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x = \frac{5}{2}$ or $x = 2.5$</td>
</tr>
</tbody>
</table>

The marking scheme provides a measure of a student’s performance in terms of their choice of strategy. It does not measure performance according to a student’s ability to perform the relevant algebraic procedures or according to the correctness of the solution for $x$.

5.5.2 Marking scheme applied to incorrect solutions

Incorrect solutions contain errors which may be either strategic or procedural in nature. Strategic decisions which do not form part of an efficient strategy are not awarded a strategy mark, whereas procedural errors are recorded separately. The marking scheme is applied to two incorrect solutions in the examples in Table 5-4 and Table 5-5.

Table 5-4 Marking scheme: Example A
In Example B, two strategy marks out of a possible two marks are awarded. Both strategic decisions are appropriate but an error occurs in the division procedure for the second step. This is recorded separately as a procedural error.

In Example C, one mark is awarded to the first strategic decision, but no mark is given for the second strategic decision as subtracting four from both sides does not make progress towards a solution and could be considered inefficient.

This example illustrates the difficulty of deciding whether an error is strategic or procedural. The second error has been recorded as a strategic error because subtracting four from both sides does not make progress towards a solution. However, it could also be interpreted as a procedural error of subtracting from four from $4x$ to get $x$. Furthermore, it could be considered
as a combination of both these errors. After experimenting with the marking of student solutions, three principles for applying the marking scheme were established:

- A strategic error is recorded when the strategic decision made for a step does not contribute to an efficient equation solving strategy.

- Strategic errors embedded in carrying out a procedure are considered to be procedural errors. This means that strategies and concepts from prior knowledge are considered to be part of a student’s procedural understanding, whereas strategies for the current topic of solving equations are considered to be strategic in nature and hence part of a student’s conceptual understanding.

- Procedural errors are only allocated when an efficient strategic decision for solving the equation has been made. Therefore, if a mark is lost because of a poor or missing strategic decision, procedural errors for that decision are not recorded.

These principles, in which strategic errors refer to strategic decisions for solving equations, procedural errors include embedded strategic errors that are considered prior knowledge, and strategic errors take priority over procedural errors, were able to be applied consistently when marking the pre-and post-tests.

### 5.5.3 Marking scheme and Equations2go score

The marking scheme for the tests is similar to the scoring system in Equations2go in that both allocate marks or points for appropriate strategic decisions. In the marking scheme one mark is allocated per step and in Equations2go, ten points are allocated per step. In some equations, one efficient solution strategy requires one more step than another. In these cases, one of the steps in the shorter strategy is allocated double marks or points.

Although the marking scheme and the Equations2go score are similar in their allocation of marks or points to strategic decisions, they are different in their treatment of errors. Equations2go deducts two points for each unsuccessful strategic decision attempted. This is to discourage students from randomly clicking different options till their selection is accepted. In the marking scheme, this is not necessary as students write out their solutions. Another difference is that procedural errors are recorded in the marking scheme but not in Equations2go because the algebraic procedures are done by the software.
5.6 Data analysis

The variety of types of data collected allowed several types of analysis to be undertaken in order to investigate how Equations2go affected students' learning.

Quantitative data of students' performance in the pre- and post-tests, as determined by applying the marking schedule, was used to organise the data into groups. Statistical tests (two sample z-test for means or paired two sample t-test for means, depending on the sample size) were used to determine whether differences between groups were significant. The data were analysed to find out which students appeared to gain the most benefit from using Equations2go.

Qualitative data of student attitudes, characteristics and comments were summarised using descriptive statistics such as tables, charts, averages and percentages. These data were analysed to find out common characteristics of students who gained the most benefit from Equations2go. Characteristics such as age, gender, main field of study, prior knowledge and level of study were considered, as well as student attitudes to equation solving and interest in using a computer for a learning activity.

To find out how Equations2go affected students' equation solving strategies, the logged data were examined to determine which strategies students used and whether there was any relationship to their equation solving strategies in the post-test. Although this study focuses on strategies, procedural errors in the pre- and post-tests were also analysed for any changes or relationships to other data.

Effects of the visual interactive interface on learning were investigated by examining post-questionnaire answers, and student comments recorded on the post-questionnaire and during group discussions.

5.7 Summary

This chapter has described the software trials during which data were collected for analysis. Ethics approval was obtained and an initial pilot study was conducted. As a result, it was decided to focus the trials on the effects of Equations2go on learning, rather than doing a comparison with a standard classroom activity.

Both quantitative and qualitative data were collected during trials, which were conducted during mathematics classes. Pre- and post-tests were used to gather quantitative data on
student performance. When the marking scheme was applied to these tests, the difference between strategic errors and procedural errors was not always clear, so the definition of strategies and procedures for this study was refined. An equation solving strategy consists of all strategic decisions that lead to the equation being solved. Procedures, associated with each strategic decision, include embedded strategies that are considered prior knowledge for equation solving. The marking scheme allowed errors of both types to be recorded.

Pre- and post-questionnaires recorded qualitative data about student attitudes as well as student characteristics such as age and level of study. Data, which were logged while students used the software, recorded student actions and from these student strategies were determined.

Thus, several types of data were collected for determining changes in student performance and equation solving strategies as well as student impressions of their learning. These data were analysed to investigate the effects on student learning of the separation of strategies, multiple strategies and the visual interactive interface of Equations2go. The data were also analysed to find out about the characteristics of students who benefit most from Equations2go. In the next chapter, the results are presented and discussed.
6 Results and Discussion

6.1 Introduction
In this chapter, the results of the software trials are recorded and discussed. The sample is described and quantitative and qualitative data are summarised. Aspects of the results are discussed including reasons for a focus on particular data, possible causes of results and links between results.

The data include two types of student comments: individual student comments written by students in the pre- or post-questionnaires, and discussion group comments recorded in the final activity of the trials, during which groups of two to four students discussed the effects of the software on their learning. If a similar comment is recorded by both an individual student and the discussion group they were in, this is stated.

When recording student performance, the term “marks” refers to the pre- and post-tests whereas the terms “points” and “score” refer to Equations2go.

The logged data are missing for five students because a software failure occurred. In addition, logged data of student use of the explanations and the Undo button were collected for only the last five of the eight class trials.

6.2 Sample
6.2.1 Participants
A total of 62 students completed the trials conducted in eight classes. There were a further 13 students who took part in the trials but who were not included in this study because their data were unusable or incomplete. Three of these students were in a Level 2 class in Semester One 2005 but had already participated in the trial in their Level 1 class in the
previous semester, one withdrew after answering the first questionnaire, two left early for appointments and seven did not complete the post-test.

The 62 valid participants were studying mathematics to support their main field of study in medical imaging, engineering, computing or science as shown in Table 6-1.

Table 6-1  Main field of study of participants

<table>
<thead>
<tr>
<th>Field of study</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical Imaging</td>
<td>21</td>
</tr>
<tr>
<td>Engineering</td>
<td>15</td>
</tr>
<tr>
<td>Computing</td>
<td>11</td>
</tr>
<tr>
<td>Science</td>
<td>8</td>
</tr>
<tr>
<td>Other</td>
<td>7</td>
</tr>
</tbody>
</table>

Twenty nine participants were male and 33 were female. Twenty five participants were studying equation solving in Level 1 mathematics courses and 37 were in Level 2 courses. The age groups of participants are shown in Table 6-2.

Table 6-2  Age groups of participants

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;20</td>
<td>16</td>
</tr>
<tr>
<td>20-29</td>
<td>31</td>
</tr>
<tr>
<td>30-39</td>
<td>9</td>
</tr>
<tr>
<td>40+</td>
<td>6</td>
</tr>
</tbody>
</table>

6.2.2  Discussion groups

At the end of each class trial, students discussed specific questions in pairs or small groups. There were 29 discussion groups in total, most of which had two students but some groups had three or four students. The discussion groups included some of the students whose data were invalid, but the comments of these students were included because it was not possible to identify and exclude their contribution to the discussion groups.
The first two questions were about what students liked and disliked in Equations2go, and these prompted discussion and many written comments. The third question was about separation of strategies. The last 5 of the 8 classes were also asked a fourth question about multiple strategies. However, many of the written comments for these last two more specific questions about strategies did not give direct answers to these questions.

6.3 Student performance
The purpose of this section is to investigate possible links between the use of Equations2go and improvement in student performance. As well as students’ overall and per question performance in the pre- and post-tests, characteristics of students whose performance improved are considered.

6.3.1 Student performance in pre-test and post-test
As described in Section 5.5, the pre- and post-tests were marked out of 18 with marks based on strategic decisions only. Because this study focuses on strategies, marks were not deducted for procedural errors. However, these errors were recorded separately and are analysed in Section 6.4.4.2.

The average number of marks, out of 18, was 10.3 in the pre-test and 12.1 in the post-test. This increase is small and not statistically significant according to the two sample z-test for means \( (n = 62 \text{ and } z = -1.89 \text{ at } p = 0.05) \). To investigate further, student performance in each question was considered.

6.3.2 Student performance in each question
For each question, the percentage increase between the average number of marks in the pre- and post-tests was calculated (see Table 6-3). Note that only one mark was allocated to Question 5 because it required only the first step to be done whereas all other questions required the equation to be solved.
Table 6-3  Average strategy marks per question in pre-test and post-test

<table>
<thead>
<tr>
<th>Question</th>
<th>Type of equation</th>
<th>Pre-test question</th>
<th>Possible marks</th>
<th>Pre-test average$^1$</th>
<th>Post-test average$^1$</th>
<th>Percentage increase$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>One step equation</td>
<td>$-2y = 7$</td>
<td>1</td>
<td>0.7</td>
<td>0.8</td>
<td>11%</td>
</tr>
<tr>
<td>2</td>
<td>Two step equation</td>
<td>$\frac{5b}{8} = 2$</td>
<td>2</td>
<td>1.6</td>
<td>1.7</td>
<td>6%</td>
</tr>
<tr>
<td>3</td>
<td>Two step equation</td>
<td>$4x - 7 = 3$</td>
<td>2</td>
<td>1.8</td>
<td>1.9</td>
<td>4%</td>
</tr>
<tr>
<td>4</td>
<td>Unknowns on both sides of equation</td>
<td>$3-2a = 3a + 4a - 4$</td>
<td>3</td>
<td>2.0</td>
<td>2.3</td>
<td>14%</td>
</tr>
<tr>
<td>5</td>
<td>Equation with brackets</td>
<td>$4(2P - 3) = 8$</td>
<td>1</td>
<td>0.8</td>
<td>0.9</td>
<td>10%</td>
</tr>
<tr>
<td>6</td>
<td>Equation with fractions</td>
<td>$\frac{I}{2} - \frac{I}{7} = 1$</td>
<td>4</td>
<td>1.7</td>
<td>2.6</td>
<td>53%</td>
</tr>
<tr>
<td>7</td>
<td>Harder equation with fractions</td>
<td>$\frac{V + 2}{3} - 1 = \frac{V}{4}$</td>
<td>5</td>
<td>1.6</td>
<td>1.9</td>
<td>18%</td>
</tr>
</tbody>
</table>

$^1$ Rounded to one decimal place
$^2$ Calculated from averages rounded to three decimal places

Question 6 stands out from the others with its 53% increase in average marks. It is also the only question in which the increase is statistically significant ($n = 62$ and $z = -2.75$ at $p = 0.05$, two sample z-test for means).

To further investigate student performance in each question, the percentage of students who completed each question with full marks for strategy is shown in Table 6-4.
Table 6-4 Percentage of students who gained full marks for strategy

<table>
<thead>
<tr>
<th>Question</th>
<th>Pre-test (^1)</th>
<th>Post-test (^1)</th>
<th>Percentage Increase (^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74%</td>
<td>82%</td>
<td>11%</td>
</tr>
<tr>
<td>2</td>
<td>74%</td>
<td>81%</td>
<td>9%</td>
</tr>
<tr>
<td>3</td>
<td>90%</td>
<td>90%</td>
<td>0%</td>
</tr>
<tr>
<td>4</td>
<td>58%</td>
<td>69%</td>
<td>19%</td>
</tr>
<tr>
<td>5</td>
<td>82%</td>
<td>90%</td>
<td>10%</td>
</tr>
<tr>
<td>6</td>
<td>35%</td>
<td>58%</td>
<td>64%</td>
</tr>
<tr>
<td>7</td>
<td>26%</td>
<td>31%</td>
<td>19%</td>
</tr>
</tbody>
</table>

\(^1\) Rounded to nearest percent
\(^2\) Calculated from percentages rounded to 1 decimal place

Once again, Question 6 stands out from the others. The seven questions were presented in approximate order of increasing difficulty. From the pre-test results, it is clear that the majority of students could already solve Questions 1 to 5 when they did the pre-test, therefore there was little potential for student performance to improve in these questions. In Question 6, however, only 35% of the students were able to solve the equation in the pre-test and this increased to 58% after students used Equations2go. In addition to the students who improved and gained full marks in Question 6 of the post-test, another 11% of students showed some improvement over their pre-test performance in this question.

The final question in the pre- and post-tests, Question 7, was beyond the level required in Level 1 and was a “transfer” question at Level 2 as it could be solved by applying Level 2 skills to a new situation. Only 26% of students were able to solve Question 7 in the pre-test and this showed just a small increase to 31% in the post-test. Equations2go did not include an equation comparable to Question 7.

Only Question 6 stands out from the others. It shows the largest increase in average score and has the highest number of students who improved between doing the pre- and post-tests. Because of these improvements, the fractions equation in Question 6 was investigated further.
6.3.3 Performance groups

Participants were classified into four groups according to their performance in Question 6 of the pre- and post-tests. These groups were labelled Good, Improved, Poor and Worse, and are described in Table 6-5.

Table 6-5 Performance groups

<table>
<thead>
<tr>
<th>Group name</th>
<th>Performance in pre- and post-tests</th>
<th>No of students in group</th>
<th>Pre-test average marks</th>
<th>Post-test average marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>Scored full marks in both tests</td>
<td>20</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Improved</td>
<td>Improved between pre- and post-tests</td>
<td>20</td>
<td>0.5</td>
<td>3.6</td>
</tr>
<tr>
<td>Poor</td>
<td>Scored 0 or 1 in both tests</td>
<td>20</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Worse</td>
<td>Solved correctly in pre-test but not in post-test</td>
<td>2</td>
<td>4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Hence, the students who showed the most improvement in Question 6 were labelled the Improved group. The average marks in Question 6 for students in this group increased from 0.5 to 3.6 and this improvement is statistically significant according to a paired two sample t-test for means (n = 20 and t = -12.4 at p = 0.05). Only the three main groups (Good, Improved, Poor) were considered when comparing groups further as the last group (Worse) had only two students. These two students gained high marks in all other questions in both the pre- and post-tests (including Question 7) but appeared to become confused when doing Question 6 in the post-test. They may have lost concentration at this stage of the trial. For the three main groups, the average performance in the tests is shown in Table 6-6.
Table 6-6  Average marks in pre-test and post-test for each group of students

<table>
<thead>
<tr>
<th>Group</th>
<th>Pre-test (out of 18)(^1)</th>
<th>Post-test (out of 18)(^1)</th>
<th>Percentage increase(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>16.2</td>
<td>16.4</td>
<td>1%</td>
</tr>
<tr>
<td>Improved</td>
<td>8.8</td>
<td>13.3</td>
<td>52%</td>
</tr>
<tr>
<td>Poor</td>
<td>5.4</td>
<td>6.4</td>
<td>19%</td>
</tr>
</tbody>
</table>

\(^1\) Rounded to one decimal place
\(^2\) Rounded to nearest percent and calculated from averages rounded to two decimal places

As in Question 6, the Improved group stands out. The 52% increase in average overall mark is statistically significant according to a paired two sample t-test for means (\(n = 20\) and \(t = -4.54\) at \(p = 0.05\)). Thus, the students in the Improved group showed a significant improvement in both the fractions equation and in the whole test.

Sixty of the 62 participants can be categorised into three equal sized groups. Students in the Good group were already able to solve the fractions equation as well as most of the other equations in the pre-test. All the students in the Improved and Poor groups were unable to solve the fractions equation on their own in the pre-test but could solve it when supported by Equations2go. Therefore, the fractions equation was in the Zone of Proximal Development for students in the Improved and Poor groups. Students in the Improved group were also able to transfer their learning to the post-test where they solved this type of equation on their own.

Further tests at a later date were not conducted as it was decided not to investigate retention. This was because the study focused on the effects of Equations2go on learning, and retention data would be confounded by other learning activities.

6.3.4  Characteristics of Improved group

The characteristics of the group of students who gained most benefit from using Equations2go (Improved) are compared to those of the whole sample in Figure 6-1.
The spread of age and main field of study for the Improved group is similar to the rest of the sample. There appear to be small differences for gender and level of study, so these are investigated further.

### 6.3.4.1 Gender

The percentage of females in the Improved group is a little higher than in the whole sample. To investigate whether Equations2go was of more benefit to female students, the performance of all students is broken down by gender in Table 6-7. Performance data are shown for the whole test and for Question 6.

Table 6-7 Performance and gender of students

<table>
<thead>
<tr>
<th></th>
<th>Female students</th>
<th>Male students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>33 students</td>
<td>29 students</td>
</tr>
<tr>
<td>Pre-test</td>
<td>Post-test</td>
<td>Difference</td>
</tr>
<tr>
<td>Average test mark (out of 18)</td>
<td>11.4</td>
<td>13.1</td>
</tr>
<tr>
<td>Average mark in Question 6 (out of 4)</td>
<td>2.1</td>
<td>3.0</td>
</tr>
<tr>
<td>Percentage of students who gained full marks in Q6</td>
<td>45%</td>
<td>70%</td>
</tr>
</tbody>
</table>

$^\dagger$ Significant according to the paired two sample $t$-test for means
Female students performed better than male students in the pre-test. However, the average increase in performance in the post-test for both female and male students was similar, so Equations2go appears to have had a similar effect on performance for both genders.

The higher pre-test performance for females could either be due to more concentrated learning in the class sessions prior to using Equations2go or to learning prior to the current course. Students' learning during prior class sessions may be related to motivation from their career goals and main field of study as the gender balance differs in each field as shown in Table 6-8.

**Table 6-8 Main field of study and gender**

<table>
<thead>
<tr>
<th>Main field of study</th>
<th>Number of female students</th>
<th>Number of male students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical Imaging</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>Computing</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Engineering</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Science</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Medical imaging students, who were mainly female, may have been more motivated to learn in class as they were preparing to apply for a limited number of places in the Medical Imaging degree. This may explain the higher pre-test performance of females as students in the other fields had met entry requirements and were studying mathematics as part of their qualification.

Medical Imaging students may have had more knowledge of equation solving from a prior course. The Medical Imaging qualification is a degree course, whereas the qualifications in the other fields are certificates or diplomas. It could be that students intending to undertake a degree course have more prior knowledge than those studying for a certificate or diploma.

Thus, Equations2go appears to have led to a similar increase in performance for both female and male students, and the higher pre-test performance of female students may be due to their motivation to study the content of their pre-trial classes or to learning from a prior course.
6.3.4.2  **Level of study**

To help determine whether Equations2go was of more benefit to Level 1 students, the performance of all students in Question 6 is broken down by level in Table 6-9.

**Table 6-9  Performance of Level 1 and Level 2 students in Question 6**

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th></th>
<th></th>
<th>Level 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>Post-test</td>
<td>Increase</td>
<td>Pre-test</td>
<td>Post-test</td>
<td>Increase</td>
</tr>
<tr>
<td>Average test mark</td>
<td>8.2</td>
<td>10.6</td>
<td>2.4(^1)</td>
<td>11.7</td>
<td>13.1</td>
<td>1.4(^1)</td>
</tr>
<tr>
<td>(out of 18)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average mark in Q6</td>
<td>1.1</td>
<td>2.5</td>
<td>1.4(^1)</td>
<td>2.1</td>
<td>2.7</td>
<td>0.6(^1)</td>
</tr>
<tr>
<td>(out of 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage of students who gained full marks in Q6</td>
<td>24%</td>
<td>60%</td>
<td>36%</td>
<td>43%</td>
<td>57%</td>
<td>14%</td>
</tr>
</tbody>
</table>

\(^1\) Significant according to the paired two sample t-test for means

Level 1 students gained fewer marks than Level 2 students in the pre-test and this was to be expected as the students were in a lower level course. However, the Level 1 students showed a larger increase in performance in the post-test suggesting that Equations2go was of more benefit to these students. This may be because lower pre-test scores mean there is more potential for Level 1 students to improve. The fractions equation in Equations2go may be better suited to the Zone of Proximal Development of Level 1 students.

Further evidence that Equations2go may have been of more benefit to Level 1 students is that the percentage of Level 1 students in the Improved group (50%) was a little higher than the whole sample (40%). Also, the percentage of Level 1 students who were in the Improved group (40%) was higher than for Level 2 (27%).

6.3.5  **Student attitudes**

In the pre-questionnaire, students were asked about their attitudes to equation solving. Students rated these on a Likert scale from 1 (low/dislike/negative) to 5 (high/enjoy/positive). For the three main performance groups defined in Table 6-5, the average student rating of equation solving attitudes is shown in Figure 6-2.
The Improved group were less confident about solving equations than the Good group as their average ratings were lower for all attitudes. This was reflected in their pre-test performance.

From the differences between the Good and Improved groups, it might be expected that the Poor group would have lower attitude ratings than the Improved group. However, apart from the attitude to equation solving, this was not the case as the Poor group rated their prior knowledge of equation solving and interest in using a computer higher than both the other groups. The confidence of the Poor group, compared to the other groups, was not reflected in their pre-test performance.

The Improved group were the least interested of all groups in using a computer for a learning activity and four of the 20 students in this group indicated negative attitudes to using a computer for learning, in the pre-questionnaire. Yet, the Improved group showed the most learning after using Equations2go.

6.4 Student strategies
Equation solving strategies of students are considered in this section. The main focus is the fractions equation, and examples of strategies used by students are shown. Two important features of Equations2go, multiple strategies and separation of strategies from procedures, are
investigated. Student comments in post-questionnaires and group discussions are used to illustrate particular issues.

6.4.1 Strategies for equations which include fractions
In order to find out more about how Equations2go contributed to student learning, the strategies used by selected students are investigated in this section. As most improvement in performance was shown in Question 6, these equations from the pre- and post-tests and the similar one in Equations2go are considered. These equations have two fractions with different denominators and are shown in Table 6-10.

Table 6-10 Fractions equation in pre-test, post-test and Equations2go

<table>
<thead>
<tr>
<th>Question 6 in pre-test</th>
<th>Equation in Equations2go</th>
<th>Question 6 in post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{t}{2} - \frac{t}{7} = 1$</td>
<td>$\frac{d}{3} + \frac{d}{5} = 1$</td>
<td>$\frac{m}{3} + \frac{m}{7} = 1$</td>
</tr>
</tbody>
</table>

Equations2go accepts three different strategies for solving the fractions equation and these are shown in Table 6-11.

Table 6-11 Strategies accepted by Equations2go for the equation with fractions

<table>
<thead>
<tr>
<th>Strategy 1</th>
<th>Strategy 2</th>
<th>Strategy 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply all terms by 15, simplify each term, and then add terms.</td>
<td>Write each fraction as an equivalent fraction with the same denominator, add the fractions, and then multiply both sides by 15.</td>
<td>Write the fractions over a single common denominator, multiply both sides by 15, and then add terms.</td>
</tr>
</tbody>
</table>
| $\frac{d}{3} + \frac{d}{5} = 1$ \[ \frac{15 \times d}{3} + \frac{15 \times d}{5} = 15 \times 1 \]
| $5d + 3d = 15$ \[ 8d = 15 \]
| $d = 1.875$ | $\frac{d}{3} + \frac{d}{5} = 1$ \[ \frac{5d}{15} + \frac{3d}{15} = 1 \]
| $8d = 15$ \[ 8d = 15 \]
| $d = 1.875$ | $\frac{d}{3} + \frac{d}{5} = 1$ \[ \frac{5d + 3d}{15} = 1 \]
| $5d + 3d = 15$ \[ 8d = 15 \]
| $d = 1.875$ |

Each strategy comprises several steps. At each step, students made strategic decisions and in Equations2go their choices logged by the software. In the later trials, the logged data also
showed the number of explanations requested and the number of times the Undo button was used.

6.4.2 **Examples of student strategies**

In this section, the logged data for two students in the Improved group are summarised and compared with the strategies they used in the pre- and post-tests. In addition, the strategies used by all students in the Improved group are summarised.

6.4.2.1 **Strategies used by Student A**

The logged data for Student A is summarised in Table 6-12. Student A solved the fractions equation in Equations2go seven times. She explored Strategy 1 twice and Strategy 2 five times. She clicked the Show Explanation button a total of 26 times to see explanations of her strategic decisions.

<table>
<thead>
<tr>
<th>Attempt</th>
<th>Strategy used</th>
<th>Number of explanations requested</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>Solved correctly.</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>Solved correctly.</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>Multiplied all terms by three but Equations2go recommended looking for a better number to multiply by. Multiplied all terms by 15 and solved correctly.</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>Repeated Attempt 3 with the same result.</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4</td>
<td>Solved correctly.</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>4</td>
<td>Solved correctly.</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>First strategic decision correct and then trial ended.</td>
</tr>
</tbody>
</table>

Student A’s strategies in the fractions equations can be examined further by considering her answers to Question 6 in the pre- and post-tests. In the pre-test, Student A solved Questions 1 to 5 correctly but did not attempt Question 6. She did attempt Question 7, but all steps were incorrect. After using Equations2go, she correctly solved Question 6 in the post-test using Strategy 2 as shown in Table 6-13. Her attempt at Question 7 in the post-test was similar to her pre-test attempt and was again incorrect.
Table 6-13 Student A: Solutions to Question 6 in pre-test and post-test

<table>
<thead>
<tr>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{t \times \frac{1}{2}}{7} = 1 ]</td>
<td>[ \frac{m + \frac{m}{3}}{2} = 1 ]</td>
</tr>
<tr>
<td>[ \frac{3m + 3m}{2} = \frac{10m}{21} = 1 \times 21 ]</td>
<td>[ \frac{10m}{10} = \frac{21}{10} ]</td>
</tr>
<tr>
<td>[ m = 2.1 ]</td>
<td>[ m = 2.1 ]</td>
</tr>
</tbody>
</table>

In summary, Student A did not attempt Question 6 in the pre-test, explored Strategies 1 and 2 in Equations2go, and solved Question 6 correctly in the post-test using Strategy 2. Thus, she successfully applied one of the strategies she used repeatedly in Equations2go to solving a similar equation using pen and paper in the post-test.

6.4.2.2 Strategies used by Student B

The logged data for Student B is summarised in Table 6-14. This student attempted the fractions equation three times using Equations2go. In the first two attempts, she explored the use of Strategy 1 and Strategy 2 unsuccessfully but on the third attempt solved the equation correctly with Strategy 2. She clicked the Show Explanation button to see nine explanations and also made use of the Undo button.
### Table 6-14  Student B: Strategies explored in fractions equation of Equations2go

<table>
<thead>
<tr>
<th>Attempt</th>
<th>Strategy used</th>
<th>Number of explanations requested</th>
<th>Comments</th>
</tr>
</thead>
</table>
| 1       | 1             | 3                                | Multiplied all terms by 3 but Equations2go recommended looking for a better number to multiply by.  
Multiplied all terms by 5 and received the same feedback as above.  
Divided all terms by 3.  
(The order in which these strategic decisions were made was not recorded.) |
| 2       | 2             | 1                                | Correctly made the first strategic decision to add fractions using a common denominator of 15 then decided to start this equation again. |
| 3       | 2             | 5                                | Solved correctly.  
Used Undo button 3 times. |

In the pre-test, Student B solved Questions 1 to 5 correctly, and although she attempted Questions 6 and 7, all strategic decisions were incorrect. In the post-test, this student solved Questions 1 to 6 correctly and used Strategy 2 for Question 6. She started Question 7 incorrectly in a similar way to the pre-test but then wrote "I don’t know". This suggests that in the post-test, she recognised her initial strategic decision was not going to lead to a solution, whereas in the pre-test she had continued with incorrect strategic decisions.

In summary, Student B clearly did not know how to solve Question 6 in the pre-test, but she solved it correctly in the post-test by applying the strategy she explored successfully in Equations2go.

### 6.4.2.3 Strategies of Improved group

Of the 20 students in the Improved group, 17 of them explored their successful or improved post-test strategy in Equations2go. Equations2go appears to have made a direct contribution to learning for these students. Of the other three students, one used a strategy in the post-test that she did not explore in Equations2go, one did not get as far as the fractions equation in Equations2go, and for the last student the logged data were not available.
6.4.3 Multiple solution strategies

For most equations, multiple solution strategies are accepted by Equations2go. Students’ use of these multiple strategies is considered in this section, as well as the design of this feature in the light of students’ reactions and suggestions for improvements.

6.4.3.1 Use of multiple strategies

Some students made more use of multiple strategies than others. The number of different strategies explored by students from the three main performance groups is shown in Table 6-15.

<table>
<thead>
<tr>
<th>Number of strategies explored by students in each performance group for fractions equation of Equations2go</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Good</td>
</tr>
<tr>
<td>Improved</td>
</tr>
<tr>
<td>Poor</td>
</tr>
</tbody>
</table>

Students in the Good group explored more strategies than students in the other groups. All three strategies were explored by half of the Good group compared to less than a quarter of the Improved or Poor groups. It may be that students who did not have a thorough understanding of one strategy were cognitively overloaded by multiple strategies. It may also be that these students needed more guidance to find other strategies.

6.4.3.2 Student attitudes to multiple strategies

Fifty one students (86%) indicated that it helped their learning to look for multiple solution strategies. Of the other students, five found it unhelpful, five indicated they didn’t look for other strategies and one didn’t answer the question. Explanations for students’ reactions to multiple strategies were found in their written comments. Comments from students who found multiple strategies helpful are considered first followed by comments from those who found them unhelpful.
6.4.3.2.1 Multiple strategies helpful

Ten of the 29 discussion groups commented on the multiple strategies available in Equations2go. Six discussion groups liked these multiple strategies. For example:

"Interesting to see the different ways that the equation could be solved."

An individual student (in the Good group) elaborated:

"I liked the way it gave choices in how you solved the equation. I didn't realise you could get rid of the brackets 2 ways!!"

These comments indicate that some students were learning to look at an equation from several points of view. They were developing their overview of possible strategies and hence developing their conceptual understanding.

6.4.3.2.2 Multiple strategies unhelpful

Four discussion groups found the multiple strategies unhelpful to their learning. For example:

"Made it confusing. Prefer simplest way/method."

These four discussion groups included three of the five students who indicated in the post-questionnaire that multiple strategies were unhelpful.

Of the five students who found multiple strategies unhelpful, one was in the Poor group and four were in the Improved group. Thus, 20% of the Improved group did not like exploring multiple strategies, so it may have been other features of Equations2go that helped these students learn. As none of the students in the Good group found multiple strategies unhelpful, this feature may have been suited to students who could already use one strategy to solve the equation with fractions.

The feature which all these five students did find helpful was the explanations. They had mixed opinions about the quick tip and the separation of strategies, so it appears that the explanations helped them the most. Only one of these students commented on the feedback and it was the immediacy that he liked:

"It was helpful to see if you were on the correct path right away."

As these five students were all unable to solve Question 6 in the pre-test, they were at an early stage of learning to solve equations with fractions. They all found multiple strategies unhelpful but liked the explanations. They may have preferred to concentrate on just one strategy while being supported by informative and timely feedback. This is consistent with the earlier suggestion that students who cannot solve an equation with one strategy may be cognitively overloaded by multiple strategies (see Section 6.4.3.1).
6.4.3.3 Strategies accepted by Equations2go

Some students thought that more strategies should be available in Equations2go. Three discussion groups indicated that their preferred strategy was not available. For example:

"The computer program has its own way of solving problems which is different from my way of solving."

The students did not give examples of which strategies they wanted to use, but the logged data provided some clues. For the equation with fractions, one strategy accepted by Equations2go is to multiply all terms by 15, the lowest common multiple of 3 and 5. If students chose to multiply all terms by just one of the denominators, either 3 or 5, they were asked to look for a better number and their score was not penalised. There is evidence that students tried to use this strategy in Equations2go, for example 47% of the Improved group. Yet, only three of the 62 students attempted to use this strategy in the pre-test and all three used it incorrectly. In the post-tests, nobody used this strategy. Perhaps this strategy was attempted when students were exploring Equations2go in Exercise 3 of the trials (see Table 5-1), rather than when students were using their first choice of strategy. Perhaps the hot spots on 3 and 5 led students to try this strategy.

Subsequent informal enquiries to several mathematics teachers found that some teachers use this strategy, particularly with lower level students. Although an extra step is required, the strategy may be conceptually easier for students to understand.

There is other evidence of a student using an efficient strategy which is not available in Equations2go. When solving the fractions equation, one step requires the equation \( \frac{8x}{15} = 1 \) to be solved. The strategy accepted by Equations2go is to multiply by 15 then divide by 8, but in the pre-test one student first divided by 8 and then multiplied by 15.

6.4.3.4 Interface design for multiple strategies

Three discussion groups and three individual students (two of whom were in one of the three discussion groups) indicated that although the number of available strategies was displayed, the number found by the student was not. One group suggested a ghost path could be shown for each solution found. Another group suggested the displayed number of available strategies could decrease as each is found. In most of the class trials, this issue was also raised verbally with students wanting to know which strategies they had found.

The suggestions made here highlight the need for usability testing throughout the software development process, as the number of available strategies displayed with each equation was
included in the design as a result of usability testing early in the development process. Once this feature was included in the software, students in the trials could then envisage improvements to it.

6.4.4 Strategies and procedures
A key feature of Equations2go is that strategies are separated from procedures, so students make strategic decisions and the algebraic manipulation of procedures is done by the software. This separation is considered in this section as well as the strategic and procedural errors made by students.

6.4.4.1 Strategies separated
The majority of students found the separation of strategies helpful with 76% rating it very helpful, helpful or fine while the remaining students rated it sometimes unhelpful or unhelpful. It is this feature that allowed all students to successfully solve equations and experience success, which is one of the components of motivation (Keller, 1987).

Students’ reactions to the separation of strategies were reported by 19 of the 29 discussion groups. The comments are summarised in Table 6-16.

Table 6-16 Discussion group comments about strategies being separated

<table>
<thead>
<tr>
<th>Type of comment</th>
<th>Number of discussion groups</th>
<th>Example of comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>11</td>
<td>“Great for learning order of ops etc, without getting bogged down by numbers (ie doing the calculations). Would love a copy.”</td>
</tr>
<tr>
<td>Qualified positive</td>
<td>4</td>
<td>“Good for people just learning rules. Further on we may need a bit of practice of actual calculation.”</td>
</tr>
<tr>
<td>Negative</td>
<td>4</td>
<td>“It made it too easy.”</td>
</tr>
</tbody>
</table>

Some students felt Equations2go made it too easy and four discussion groups noted this. One student (in the Good group) explained to the researcher that she made mistakes in procedures rather than strategies, and so she didn’t like the algebraic procedures being done for her.

Another individual student commented that:

“The program was very helpful but I don’t know if I will be able to do them on paper because the program made it easy to do them on the computer”.
However, she completed the post-test and then went back to the post-questionnaire and added: 

"Now that I've just done the post-test I think the program helped a lot".

Data from the pre- and post-tests for this student (Student C) show that she was not in the Improved group as her strategies in Question 6 showed no change. However, her marks improved from 4 out of 18 in the pre-test to 9 out of 18 in the post-test. This was a result of her improved strategies in Questions 3 and 4 which she solved correctly in the post-test. Her solutions to Question 3 and 4 in the pre- and post-tests are shown in Table 6-17.

Table 6-17  Student C’s solutions to Questions 3 and 4 in pre-test and post-test

<table>
<thead>
<tr>
<th>Question 3</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{4x-7}{4}=3 \div 4$</td>
<td>$2D-9=3+9$</td>
<td></td>
</tr>
<tr>
<td>$x=\frac{3}{4}+\frac{9}{4}$</td>
<td>$2D=12$</td>
<td></td>
</tr>
<tr>
<td>$x=\frac{0.75}{-7}$</td>
<td>$0=6$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question 4</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3-2a}{b}=3a+4a-4 \div a$</td>
<td>$4-3y=2+7y-2y+3y \div 3y$</td>
<td></td>
</tr>
<tr>
<td>$3-2=3a+4a-4 \div a$</td>
<td>$2-4=2+8y \div -2$</td>
<td></td>
</tr>
<tr>
<td>$3-2+4=\frac{7a-4}{a}$</td>
<td>$2=8y \div \frac{8}{8}$</td>
<td></td>
</tr>
<tr>
<td>$6=\frac{7a}{a} \div \frac{b}{b}$</td>
<td>$4y=\frac{1}{4}$</td>
<td></td>
</tr>
</tbody>
</table>

In the pre-test, this student attempted to use an inefficient strategy for both these equations by isolating the unknown before isolating the unknown term. This inefficient strategy required procedures involving fractions and she carried these out incorrectly. In the post-test, however, she chose an efficient strategy with simple procedures which she carried out correctly.
In the Equations2go equation that is similar to Question 3, Student C's first attempt was similar to her pre-test attempt but this was not accepted. She went on to solve the equation successfully three times in Equations2go and it was this strategy that she used successfully in the post-test.

In the Equations2go equation that is similar to Question 4, Student C attempted to use her pre-test strategy three times, solved it correctly five times, and successfully used this strategy in the post-test. While using Equations2go, this student requested 68 explanations. Thus, when using Equations2go Student C explored equations several times and looked at many explanations. She then applied strategies she used successfully in Equations2go to the post-test.

Although this student felt Equations2go made it too easy, she was pleasantly surprised to find that she was able to successfully transfer her learning from Equations2go to the post-test. Equations2go appears to have helped this student learn to solve equations, and this is supported by evidence from the logged data, pre- and post-test performance, and student comments.

The design intention for Equations2go was for the separation of strategies to provide scaffolding in Vygotsky's Zone of Proximal Development, thus helping students solve equations which they could not solve on their own. This appeared to be the case as many students found this feature helpful. Students such as Student C and most of the students in the Improved group were able to complete a strategy with the support of Equations2go and were then able to successfully transfer this strategy to the post-test. Two discussion groups recognised that making only the strategic decisions, without doing the algebraic procedures, would be most helpful at an early stage of the learning process.

### 6.4.4.2 Strategic and procedural errors

In this section, the numbers of strategic and procedural errors are considered as well as the types of errors made by each of the performance groups.

A comparison of the numbers of students who made procedural and strategic errors in the pre-test is shown in Table 6-18.
Table 6-18 Strategic and procedural errors in pre-test

<table>
<thead>
<tr>
<th>Question</th>
<th>Percentage of students with strategic errors</th>
<th>Percentage of students with procedural errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23%</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>23%</td>
<td>5%</td>
</tr>
<tr>
<td>3</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>4</td>
<td>40%</td>
<td>3%</td>
</tr>
<tr>
<td>5</td>
<td>15%</td>
<td>7%</td>
</tr>
<tr>
<td>6</td>
<td>63%</td>
<td>3%</td>
</tr>
<tr>
<td>7</td>
<td>73%</td>
<td>7%</td>
</tr>
</tbody>
</table>

As more students made strategic errors than procedural errors, students appear to have needed more help with these, and this justifies the emphasis that the design of Equations2go placed on strategies.

However, the larger percentage of students with strategic errors is partly explained by the method for recording errors. For any step or equation that was not attempted, it was assumed that the student did not know what strategic decision to make (i.e. it was treated as a strategic error). Furthermore, a student who did not attempt the first strategic decision for an equation could not attempt strategic decisions for subsequent steps. Procedural errors could only occur after an appropriate strategic decision was made, so when fewer strategic decisions were successfully made, there was less potential to make procedural errors.

Strategic errors suggest that a student was unable to solve an equation on their own. The high percentage of students with strategic errors in Question 6 of the pre-test (see Table 6-18) combined with the fact that most students successfully solved the similar equation in Equations2go suggests these equations were in the Zone of Proximal Development for these students. Question 7 may also be in this zone, but there is no evidence of students solving this type of equation with guidance because Equations2go did not have an equation similar to Question 7.

Students in the Good group made the most strategic decisions in the pre-test and therefore had the most potential to make procedural errors. This is supported by the data on procedural
errors as a greater percentage of students in the Good group made procedural errors than students in the other groups (see Table 6-19).

Table 6-19 Percentage of students with procedural errors in each group

<table>
<thead>
<tr>
<th></th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>45%</td>
<td>60%</td>
</tr>
<tr>
<td>Improved</td>
<td>20%</td>
<td>40%</td>
</tr>
<tr>
<td>Poor</td>
<td>20%</td>
<td>10%</td>
</tr>
</tbody>
</table>

The number of procedural errors made by students in the Good group also increased in the post-test. These errors are difficult to explain as most of the new procedural errors occurred in questions in which students gained full marks for strategy in both the pre- and post-tests. Perhaps these students were losing concentration in the post-test.

More students in the Improved group made procedural errors in the post-test than in the pre-test. This is consistent with other data for the Improved group as it is reasonable to expect these students to make more procedural errors once their strategic decisions improved.

Compared to the pre-test, the percentage of students in the Poor group with procedural errors in the post-test decreased. However, these percentages were low, probably because students in this group made only a limited number of efficient strategic decisions in both tests, and therefore these students had few opportunities to make procedural errors. Students in the Poor group, with an average mark of only 5.4 out of 18 in the pre-test and 6.4 out of 18 in the post-test, probably needed more assistance with strategic decisions before concentrating on procedural errors.

6.5 Other software features

The main design feature of Equations2go is the emphasis on strategies, and this is supported by the design of several features of the interface. These features, which include quick tips, explanations, graphics, buttons, instructions and a score, are considered in this section.

In the post-questionnaire, students were asked whether specific features of the interface helped them learn. The responses are summarised in Table 6-20 and referred to in the following sections.
6. Results and Discussion

Table 6-20  Helpfulness to learning of features in Equations2go

<table>
<thead>
<tr>
<th>Feature</th>
<th>Percentage of responses to questions in post-questionnaire</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Helpful</td>
</tr>
<tr>
<td>Quick tips</td>
<td>81%</td>
</tr>
<tr>
<td>Explanations</td>
<td>84%</td>
</tr>
<tr>
<td>Undo button</td>
<td>58%</td>
</tr>
<tr>
<td>Instructions</td>
<td>73%</td>
</tr>
<tr>
<td>Score</td>
<td>79%</td>
</tr>
</tbody>
</table>

6.5.1  Quick tip and explanations

Most students found both the quick tip in the flag and the more detailed feedback in the explanations helped them learn. Although logged data for the use of the explanations were recorded for only the last five of the eight class trials, they showed that students made good use of the explanations. Students requested them an average of 57 times per student during the 20 minutes of the trial. This is an average of three explanations per minute.

In the last exercise of the trials, students were asked to explore strategies they thought would not work and to look at the consequences of their decisions, as shown by the equations that are included with the explanations. This exercise probably caused students to view more explanations than they would have otherwise and may also have contributed to the helpfulness of the explanations.

Comments about the explanations were made by nine discussion groups and two individual students (one of whom was in one of the nine discussion groups). Students particularly liked the explanations and several reasons were given:

"It told you when you got it wrong and showed you what your answer would be if you continued on that track."

"Explanations clear and helpful."

"Language was simple to understand."

The explanations were only displayed when requested by a student so that the main screen was kept as simple as possible (Nielsen, 1993), with more informative feedback provided when students needed it (Mason & Bruning, 2001). Students used the explanations very
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frequently, so in future it may be worth displaying the explanations every time a student makes a strategic decision rather than only when requested. This would allow students to read an explanation for every strategic decision without having to take any action.

However, displaying the explanations after every strategic decision would reduce student contribution to the interactivity. Clicking a button to request an explanation means students are taking an active part in their interaction with the software. Students may also be less likely to read an explanation that is displayed after every attempt. They may be more likely to read an explanation if they request it.

6.5.2 Graphics

The only student who commented about the graphics mentioned the tree which formed a background for the score and the colour scheme. Verification feedback is provided by the score, which is displayed on the tree. When a strategic decision is accepted, the score increases and the leaves on the tree that form a background to the score change colour. The tree image was chosen as it relates to the metaphor of tracks in the forest and gives context to the stones. During the design process, there was an element of doubt about this choice as the tree was not integral to equation solving and only provided a hint about the metaphor. Therefore, the tree image did not appear to follow the recommendation of Rieber (1994) that graphics should only be used when there is a need for visualisation. However, the student who commented on the graphics, and who also disliked the colour scheme, saw a metaphor for positive feedback in the colour change in the tree:

"The tree that glows like a light bulb ... genius."

As no other comments were made about the metaphors, they probably met the recommendation that they should not be misleading (Erickson, 1995). However, there is no direct evidence that the stone metaphor, the forest metaphor or the opposite directions that represent inverse operations made any contribution to learning.

6.5.3 Undo button

In the post-questionnaire, just over half the students indicated the Undo button was helpful whereas 38% indicated they did not use it. Thus, the Undo button was helpful to most of the students who used it. The logged data for this button, which were only available for the last five out of eight trials, are consistent with the post-questionnaire data as they show that 35% of students did not use the Undo button. The 65% of students who did use the Undo button used it an average of four times.
One discussion group and one individual student in this discussion group liked the Undo button:

"...if I made a mistake I could always undo it, keeping the equation neat, whereas on paper you have to scribble on it."

The only other comment about the Undo button was made by a group which included the only student who indicated that it was unhelpful in the post-questionnaire:

"Didn’t like the Undo button because couldn’t understand."

6.5.4 Other buttons

Although most students found it easy to use Equations2go, three individual students commented that active spots were not where they expected to find them or that the associated buttons did not do what they expected, for example:

"It confused me knowing which button to push."

It is important that students are able to predict the effect of their actions (Nielsen, 1993), and this problem may increase if Equations2go is extended to accept more strategies.

6.5.5 Location of buttons

When using Equations2go, students’ main focus should be the equation on the current stone and this is always positioned near the centre of the screen. The Undo button and the Show/Hide Explanation button were originally placed near the edge of the screen so that their location was consistent (Shneiderman, 1987). During the software development process, usability testing showed that some students did not notice the Show/Hide Explanation button. This button was relocated closer to wherever the current stone is positioned, where it follows the recommendation that related elements should be close to each other (Watzman, 2003).

6.5.6 Instructions

The logged data showed that most students explored the instructions. Three students looked at only the three screens that were recommended as a minimum during the trials. Three students looked at an average of seven screens and the rest looked at all 13 screens.

As shown in Table 6-20, the instructions were helpful to many students, but during the trials it was observed by the researcher that a few students spent a lot of time exploring the instructions and didn’t have much time left for solving the equations. The logged data verify this observation as three students did not complete the first exercise in the trial which was to solve each of the four equations, and they did not start the other exercises. One of these
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students commented to the researcher that she was “computer illiterate”. Another student, whose logged data were not available, wrote in the post-questionnaire:

“Did not have time to become familiar with how the programme worked.”

The only student comment about the instructions was a suggestion:

“Perhaps a demonstration of what’s going to happen (working demo) in the instructions part.”

A demonstration of an equation being solved may encourage a few students, who were reluctant computer users or who spent much of the trial time on the instructions, to get started. This may also make it quicker and easier for other students to learn how to use Equations2go.

6.5.7 Score

The score was initially switched off in the trial, and in the second exercise students were asked to switch it on. Many students (79%) liked the Equations2go score switched on, so there is potential to develop the scoring system further to provide more short and long term goals as recommended by Prensky (2003). Such goals could motivate students to use Equations2go in particular ways in order to achieve high scores: for example, to solve an equation without making errors, to find all available solution strategies, or to explore feedback.

As other students (18%) preferred the score switched off, and early usability testing showed that the score could inhibit student exploration, a balance needs to be found between encouraging students to explore Equations2go in specific ways with a score and encouraging students to explore without the consequences of a poor score.

6.6 Student attitudes

Student attitudes as well as student suggestions for improving Equations2go were recorded in questionnaire responses, and individual and group comments.

Most students found Equations2go easy to use with 92% indicating that it was very easy, easy or OK to use. In the group discussions, ease of use was mentioned by seven groups. This ease of use is important because learning can be affected if students find an interface difficult to use (Frye & Soloway, 1987).

The majority (70%) of students felt Equations2go helped with their learning. The correlation between this and students’ pre-questionnaire interest in using a computer is low (r = 0.14).
This supports Nielsen’s (1993) finding that the correlation between students’ predictions about software and their later satisfaction with it was low.

Most students (86%) would recommend Equations2go to others and some described specific benefits of Equations2go. For example, one student, who found Equations2go helped him only a little, wrote:

“The program would be helpful though if I felt like I could never ask my tutor how to do questions I didn’t understand.”

This comment supports Schulmeister’s (1997) observation that software can allow students to learn in private without personal judgement.

Another student, who was in the Improved group, appreciated being able to work slowly:

“Lovely to go at my own pace. I work quite slow so this enables me to fully understand what is being said.”

Students recognised the limitation of solving only four equations in Equations2go and 73% of students would like to use an extended version of the software. Requests for more equations and harder equations were made by two discussion groups and two individual students, for example:

“Would have liked to have tried harder equations or more than just 4 but it was a good way to learn though.”

Two students, both of whom were in the Poor group wanted to be able to enter their own equations, for example:

“You should be able to add custom equations.”

These students may have wanted to explore specific equations they have had difficulty with, but it may be that a wider range of equations would meet their learning needs.

Two discussion groups and one individual student thought it was possible to use Equations2go without learning anything:

“Could just push any random button to get there without realising the method.”

It is true that this is possible, but there are up to 16 options for each step of each equation so it may be a frustrating activity and the low score would indicate the low level of success. To discourage random clicking, the second exercise of the trial in which students aimed for a top score could be incorporated into the interface.
6.7 Summary

The fractions equation in Question 6 of the pre- and post-tests was analysed in detail because it stood out with its significant improvement in average student performance. Approximately one third of students (Improved group) showed a significant improvement in this question as well as in their total test performance. Level 1 students appeared to gain more benefit from Equations2go than Level 2 students. Male and female students showed similar benefits.

In this chapter, information from the logged data about equation solving strategies was illustrated for two students and compared to their strategies in the pre- and post-tests. Similar analysis for other students in the Improved group showed that most had used their improved post-test strategy successfully in Equations2go. Although the analysis focused on the equation with fractions and the students in the Improved group, analysis of the data for another student (Student C) illustrated a similar result for two other equations.

The majority of students found that being able to concentrate on the strategies in Equations2go without having to carry out the procedures helped them learn, although some students were concerned that this made it too easy. The number of students in the Improved group who made procedural errors doubled in the post-test, perhaps because they successfully made more strategic decisions.

Searching for multiple solution strategies was helpful for the majority of students and more use of this feature was made by students who were able to solve the Question 6 equation in the pre-test (Good group). Some students found multiple strategies confusing, and may have been cognitively overloaded by several strategies.

Of the other features of the interface, the explanations helped many students learn and the score was also helpful. Few comments were made about the graphics, suggesting students found them an integral part of the interface.

In the following chapter, implications of these results are considered.
The aim of this study was to investigate the effects of using interactive software that places a strong emphasis on equation solving strategies. The reason for emphasising strategies in the design of the software that was developed for this study was to help students develop the versatile thinking needed for equation solving (Tall & Thomas, 1991). Equations2go was trialled by students, and data were collected on students’ equation solving performances, strategies and attitudes, as well as student characteristics.

This chapter presents the main findings, limitations of the study, a proposed framework of learning goals for equation solving, and suggestions for future research and development.

7.1 Main findings
The following sections describe the main effects of the software, its emphasis on strategies, its usability and the way it was used. Characteristics of students who gained the most learning benefit are also considered.

7.1.1 Effects of software on student performance
As students showed most improvement for an equation with fractions, analysis focussed on this equation. Students were grouped according to their pre- and post-test performance in this equation. The groups were labelled the Good group (students who solved the equation in both tests), the Improved group (students who improved in the post-test) and the Poor group (students who were unable to solve the equation in both tests).
One third of the students (Good group) could already solve a fractions equation in the pre-test, and their performance showed little change after using Equations2go. The equations in Equations2go were not in the Zone of Proximal Development (Vygotsky, 1978) for these students as they could already solve them using an efficient strategy.

Another third of the students (Improved group) were unable to solve the fractions equation in the pre-test, but did solve this type of equation in Equations2go and in the post-test. Students in the Improved group showed the most improvement in performance after using Equations2go, so much of the analysis focussed on this group.

The remaining third of students (Poor group) were unable to solve the fractions equation in both the pre- and post-tests, even though they were able to solve this type of equation in Equations2go.

Thus, for students in the Improved and Poor groups, the fractions equation in Equations2go was in the Zone of Proximal Development. Equations2go appeared to provide scaffolding that guided these students to solve an equation they had been unable to solve on their own. Students in the Improved group were able to transfer their learning in Equations2go to a similar equation in the post-test, but students in the Poor group were unable to do this. There may be a number of reasons for this.

Some students in the Poor group may need to practise by solving more equations of a similar level of difficulty. Others may need help with transferring their learning in Equations2go to pen and paper, perhaps by seeing the steps displayed on the screen in the same way they would be written on paper. It may also be that some of these students did not have sufficient conceptual understanding of the prior algebraic concepts that Kieran (1992) found to be necessary for equation solving.

### 7.1.2 Characteristics of students who benefited

The students who gained the most learning benefit from Equations2go were those in the Improved group. Characteristics and attitudes of these students are compared to those of the other participants in this section.

In the Improved group, there was a higher proportion of students from Level 1 classes than in the sample as a whole. Therefore, Equations2go appeared to be of more benefit to students studying at a lower level. However, there did not appear to be a difference in learning for students of different age or gender.
The average rating for prior knowledge of equation solving by students in the Improved group was the lowest of all groups, whereas students in the Poor group rated this the highest of all groups. Students in the Improved group appeared to under-rate their skills compared to students in the Poor group. Perhaps the relative over-confidence of students in the Poor group interfered with their willingness to learn.

Before using Equations2go, students in the Improved group were the least interested in using a computer, yet they gained the most benefit. In contrast, students in the Poor group were the most interested in using a computer for learning, but their performance did not improve. Thus, for students in the Poor and Improved groups, their interest in using a computer for learning tended not to match their subsequent learning. Nielsen (1993) also found poor correlation between users “predicted” and “actual” satisfaction when investigating attitudes to software features.

In summary, students who benefited from using Equations2go tended to be studying at a lower level, be less confident about their prior knowledge of solving equations and be less interested in using a computer for learning.

7.1.3 Effects of emphasising strategies
The features of Equations2go which emphasise strategies are the separation of strategies from procedures, the multiple strategies that can be explored, and the feedback. The effects of each of these on students’ learning are considered.

7.1.3.1 Separation of strategies from procedures
The majority of students found the separation of strategies from procedures helpful to learning and found it easier to learn equation solving strategies when they didn’t have to carry out the procedures. By carrying out the procedures for the students, the software appeared to make it easier for students to concentrate on structuring their strategic decisions into a logical order to form an equation solving strategy.

For equation solving, conceptual understanding includes an understanding of equation solving strategies (Tall & Thomas, 1991) and is associated with visual imagery (Sfard, 1991). As the separation of strategies in Equations2go is combined with visual imagery, this feature may have helped students visualise equation solving strategies and hence helped them improve their conceptual understanding.
Although most students found that making only the strategic decisions helped them learn, some students felt this made it too easy. However, the detailed analysis of the data for one student (Section 6.4.4.1) illustrated how a student could learn more than she realised from a learning activity that she considered easy.

Once students' strategic decisions improved, there were more procedures to be carried out and procedural errors increased. Therefore, it would still be important to help students improve their procedural skills.

### 7.1.3.2 Multiple solution strategies

Most students found that exploring multiple strategies helped them learn. Some students were surprised and interested to find that equations could be solved in several ways and that different strategies led to the same solution. By comprehending this, students' conceptual understanding may have improved. A similar conclusion was recorded by Star and Seifert (2006) who found that finding and comparing multiple solution strategies could help students develop the flexible thinking needed for equation solving.

Multiple strategies appeared to be of more benefit to students in the Good group, who were at a later stage of learning, as they could already solve the fractions equation using one strategy. These students also made more use of this feature by exploring more of the strategies available.

Although most students found it helpful to explore multiple strategies, some students found it confusing. Their comments indicated that they may have been cognitively overloaded by several strategies. They may have been at an earlier stage of learning in which they needed to concentrate on learning just one strategy.

### 7.1.3.3 Feedback

Students made frequent use of the explanations which emphasised equation solving strategies and showed consequences of students' strategic decisions. Most students found the explanations helpful, including the two students whose performance improved but who did not find that either the separation of strategies or the multiple strategies helped them learn.

Many students commented on the helpfulness of the explanations. Features which students liked were their immediacy, clarity, simple language and being able to see the consequences of their strategic decisions.
As the feedback in Equations2go followed recommendations of Nguyen-Xuan et al. (1997) that it should be short, include consequences of errors and allow students to work out the next step themselves, this study supports these principles of feedback for helping students learn to solve equations.

7.1.4 Effects of the way the software was used
During the trials, students searched for all the strategies available in Equations2go. Although this search was of more benefit to students at a later stage of learning, most students found that it helped them learn. Without this exercise students may not have explored the multiple strategies available.

Students were also asked to explore strategic decisions that they thought would be wrong or inefficient and to look at the explanations for these. This exploration appears to have contributed to the frequent use students made of the explanations. Most students found the explanations helpful, so this exploration also probably helped them learn.

7.1.5 Effects of usability
This section summarises findings about the main usability issues and the nature and timing of usability testing.

It is important that the interface is easy to use, as poor usability could affect students’ learning (Frye & Soloway, 1987). Usability issues were raised, such as difficulty in finding active spots, unpredictability of the function of the Undo button, and nature of the online instructions. Also, although the total number of strategies available was displayed, students could not see which strategies they had found or how many were still to be found. The design of these features needs to be improved so that students find it easier to learn how to use Equations2go, easier to predict the effect of their actions, and easier to see their progress towards finding all the strategies available.

Nielsen (1993) found that conflicting usability guidelines need to be prioritised. This situation occurred when designing the location of the Show/Hide Explanation button, which was originally located at the left of the screen in a consistent position (Shneiderman, 1987). However, usability testing showed that some students did not notice this button, so it was moved as close as possible to the current stone, but this meant that its location varied for each step. In this case, the guideline that related elements should be close (Watzman, 2003) took priority over placing the button in a consistent location.
Conclusions

Usability issues arose throughout this study. During software development, several phases of usability testing led to subsequent modifications to the software. These made it easier for students to explore the strategies and explanations which they found helped them learn. Further usability data were gathered during the trials, although the main purpose of the trials had been to investigate the effects of the software on learning. Thus, usability testing that began in the early stages of the design phase of the ADDIE instructional design system, continued into the final stages of data collection for this study. This supports the recommendation that usability testing should be done throughout the development process (Gould, 1995).

7.2 Limitations of the study

There were limitations to this study caused by aspects of the prototype software, the sampling, the type of study, and the data analysis.

A limitation of Equations2go was that it was developed as a prototype and therefore had only a small number of equations. This limited its potential to contribute to learning, and was noted by students who requested more equations and harder equations.

The sample size was limited by the number of students attending classes at the level targeted by Equations2go. To increase the sample size, trials were conducted during classes in two different semesters. Although the sample size of 62 was too small for a thorough quantitative analysis of subgroups, it was suitable for the analysis of the effects on learning in this study.

Another limitation of the sample was that it may not be representative as a few students chose not to participate and others were absent from the class sessions in which the trials were conducted. In addition, data for 13 students had to be discarded because they were invalid.

By focussing this study on the effects of software on learning, useful information to inform the design of educational software was gained. Another approach would have been to look at differences between learning with software and learning in a standard classroom situation. If the sample had been larger, a comparative investigation between Equations2go and a control could have been included in this study.

Another limitation of this study is that effects of specific visual elements, for example the stone metaphor or that opposite directions represent inverse operations, were not investigated. The study provided little evidence that graphics either contributed to or interfered with learning, as only one student commented on the graphics by mentioning the colours.
With regard to the data analysis, although student performance was analysed for all participants, student strategies were only analysed in detail for students who showed the most change between the pre- and post-tests. The reason was that this data showed the most potential for yielding insights into effects of Equations2go on students’ learning. Also the analysis focuses on one type of equation only. The reason for focusing on the equation with fractions was that many of the students could already solve the other types of equations in the prototype software.

7.3 Proposed framework of learning goals

In this section, the aspects of equation solving considered in this study are summarised as a set of learning goals and are proposed as a framework for teaching equation solving.

Using Equations2go provided students with two learning goals: first to find a strategy for solving each equation, and second to find multiple strategies for solving each equation. These learning goals appeared to be most helpful for the learning stage of students in the Improved group.

Analysis of the results of this study has led to suggestions for learning goals suited to students at other stages of learning. Students at a very early stage of learning to solve equations may have needed a better understanding of prior algebraic concepts. Other students, at a later stage of learning, were able to choose appropriate equation solving strategies but needed to work at eliminating procedural errors.

Another learning goal which may help students with equation solving is described by Star and Madnani (2004), who found that identifying a “best” strategy helped students think with flexibility. Once students have explored multiple strategies for solving an equation, as in Equations2go, it would be an appropriate time to learn about which of these strategies was most efficient.

The flexible thinking described by Star and Madnani is similar to the versatile thinking described by Thomas and Tall (1988) in that students must think about an equation in different ways and be able to combine these views. One of the goals of the software in this study was to help students develop versatile thinking. Once students are able to both choose efficient strategies and carry out procedures for each step, they may also need practice in combining these to think with versatility.
It is proposed that the learning goals described above be combined into a framework for helping students learning to solve equations. Each learning goal in the framework is suited to a different stage of learning and contributes to the overall goal of developing versatile thinking for equation solving. The proposed framework of learning goals follows:

1. Understand prior algebraic concepts needed for equation solving
2. Solve an equation using one strategy
3. Solve an equation using multiple strategies
4. Identify an efficient strategy
5. Decide on an efficient strategy before attempting to solve an equation
6. Reduce and eliminate procedural errors
7. Combine strategies and procedures to think with versatility

As this study focussed on the second and third of these learning goals, there is much potential for developing learning activities and investigating the effects of the other goals in this framework on student learning.

7.4 Future research and development

This study is the first of several iterations of research, as recommended when developing educational software (Reeves, 1999). The prototype software developed for this study included only four equations so that the effects on learning could be investigated, and the software could be modified before developing more equations. Information from this study will be used to modify and extend Equations2go ready for the next iteration of research. A comparison study between a future version of Equations2go and the version in this study could also provide useful information about the effects of specific features on learning.

In the following sections, two types of future research are considered: research related to further development and use of the software, and research related to the framework of learning goals.

7.4.1 Development and further uses of the software

With Equations2go currently providing learning activities for two of the goals in the proposed framework of learning goals, there is potential to develop learning activities for the other goals. In this way, different learning activities would be available for students at various stages of learning, as recommended by Kalyuga (2005). There is also potential to use Equations2go in ways other than the individual student exercise investigated in this study.
When modifying and extending Equations2go according to the findings of this study, usability issues need to be considered carefully, as problems may compound when the software is developed further to include a wider range of learning activities, equation types and strategies. There is a need to further investigate the design of features that encourage students to use the software in specific ways, the design of the feedback and its effects on learning. Research related to the development and use of Equations2go is considered in the following sections.

7.4.1.1 Providing goals for students
During the trials, students were asked to use the software in specific ways. As both the search for multiple strategies and the exploration of explanations helped many students learn, ways of encouraging students to use the software in these ways need investigating.

There is potential to apply principles of game design, as the brief investigation of the scoring system in this study found it was helpful for many students. However, other types of rewards should also be investigated as some students preferred the score switched off, while the score appeared to discourage some students from exploring.

7.4.1.2 Feedback
Including more equations and accepting more strategies in Equations2go will require a larger number of explanations to be written, so a simpler system for writing these needs to be investigated. It may be possible to partially apply the principle used for creating microworld feedback, in which frequently used phrases are chosen from a list. The standard phrases could be combined with phrases written specifically for each strategic decision so that the feedback in Equations2go would still be tailored individually to strategic decisions, but development time would be reduced.

There is potential to investigate the contribution feedback makes to learning according to whether it is specific or general in nature. The contribution of the equations which showed consequences in the explanations also needs to be investigated.

7.4.1.3 Further uses of Equations2go
Equations2go was designed as an individual student exercise but it could also be used in other class activities. For example, teachers could use Equations2go to demonstrate multiple strategies to a class and hence stimulate discussion about the efficiency of different strategies. Teachers could also use Equations2go to demonstrate the consequences of inefficient strategies as the equations in the explanations allow this to be shown very quickly.
Furthermore, there is potential for teachers to show students how to think with versatility by combining conceptual and procedural thinking. This could be demonstrated in Equations2go by alternating between the strategic decision for a step and the equation in the explanation in which the associated procedure can be seen. Equations2go could also form the basis of group activities, for example, two or three students could work together to find all the strategies available and then discuss and justify their opinions about which strategy is best.

In addition, as this study involved adult learners, there is potential to investigate the use of Equations2go for school students.

7.4.1.4 Other uses of the main principle of Equations2go
The main principle of the design of Equations2go is to emphasise strategies, with the purpose of this being to help students develop the versatile thinking needed for solving equations. This principle could also be applied to many other topics in mathematics which require versatile thinking, for example solving quadratic equations, simplifying algebraic expressions or finding lengths using trigonometry.

7.4.2 Future research related to framework of learning goals
As activities are developed for each of the learning goals in the proposed framework, the effects of each activity on learning will need to be investigated. In addition, there is a need to investigate the effectiveness of the set of learning goals in the proposed framework in achieving their intended purpose of helping students think with the versatility needed for equation solving. There is also potential to use the framework of learning goals and the principles on which these are based in other ways and for other applications.

Proposed research related to the learning goals in the framework is described in the following sections.

7.4.2.1 Understanding prior algebraic concepts
The first learning goal in the proposed framework is for students to understand the prior algebraic concepts that Kieran (1992) found to be necessary for equation solving such as variables, expressions, the equals sign and maintaining equivalence. A lack of understanding of these concepts may have contributed to the difficulties of students in the Poor group, so research is needed to identify gaps in prior knowledge. There may also be other reasons for the difficulties these students had with equation solving, so investigation of other learning needs of students like those in the Poor group is needed.
Compared to the other groups, students in the Poor group were the most interested in using a computer for learning. There must be potential to build on this positive attitude with software designed for their learning needs.

7.4.2.2 Learning equation solving strategies
The investigations in this study, which focussed on the second and third learning goals in the framework by helping students find strategies for solving equations, found that more strategies needed to be available in Equations2go. Research is needed to find out which equation solving strategies contribute to learning and which strategies are minor variations of each other. This information is important because the software needs to accept as many efficient strategies as possible. However, it should only require students to search for strategies which contribute to conceptual understanding, as students may lose motivation if required to search for strategies which are only minor variations of each other.

7.4.2.3 Using efficient strategies
Once students have practised finding multiple strategies, the next goal in the framework is to learn to choose strategies which are efficient. This learning goal may need to be separated into two stages so that students first learn to choose which of several strategies found in Equations2go is most efficient, and then learn to think about the efficiency of possible strategies before attempting to solve an equation. The findings of Star and Madnani (2004), who investigated the use of “best” strategies, should be considered in the design of learning activities which encourage students to choose efficient strategies. There is potential for software to be designed to provide these activities.

7.4.2.4 Reducing procedural errors
There were students in this study who needed to focus on their procedural errors, the penultimate goal in the framework. This was the case for approximately half of the students in the Good group as well as students in the Improved group after they had used Equations2go. Therefore, there is a need to investigate a learning activity that emphasises procedures. Software could be designed to provide this, or it could be as simple as asking students to solve each equation using pen and paper after they have solved it in Equations2go.

7.4.2.5 Learning to think with versatility
Once students understand equation solving strategies and procedures as described by the first six learning goals of the framework, they may need specific assistance to combine the overall view of an equation solving strategy with the algebraic procedures required in each step. To
think with versatility, students need to know how to concentrate on an individual procedure while keeping its relationship to the associated strategic decision and the overall equation solving strategy in mind. Interactive software that requires students to reflect on the relationship of different procedures to associated strategic decisions could assist students to achieve this versatile thinking.

7.4.2.6 Validating the framework
The purpose of proposing the framework of learning goals is to help students develop the versatile thinking needed for equation solving. To determine the validity of the framework for this purpose, learning activities for each learning goal need to be developed so that their contribution to versatile thinking can be investigated. In addition, the completeness of the seven goals in the proposed framework needs to be considered.

7.4.2.7 Further uses of the framework of learning goals
As well as providing a structure for designing learning activities, the learning goals in the proposed framework could also be used by teachers to structure their lessons. The learning goals may assist teachers to follow recommendations in the literature to place more emphasis on conceptual understanding (Kieran, 1992) and versatile thinking (Thomas & Tall, 1988). Teachers could also use the framework of learning goals to help diagnose students’ learning needs by determining which learning goals students have achieved and therefore which they need help with.

7.4.2.8 Applications of the main principle of the framework of learning goals
The main principle of the framework of learning goals is to help students concentrate on a global conceptual style of thinking, separately from a logical procedural style of thinking, before combining the two thinking styles. Interactive software is particularly suited to providing learning activities based on this principle, because as found in this study, it can be designed to perform the actions determined by one style of thinking while students concentrate on learning the other style of thinking. This principle could also be applied to helping students learn other subjects which require a combination of different thinking styles.

7.5 Contribution of this study
By focussing on software for helping students learning to solve equations, this study has made a contribution to the need identified by Atkinson (2005) for more research into the design and effects of educational software for mathematics.
This investigation has shown how software could help students learn equation solving, by providing individual scaffolding in the Zone of Proximal Development in ways that are difficult to provide in a standard classroom situation. The scaffolding was provided by an innovative design for the interactive interface which allowed students to concentrate on equation solving strategies and receive individual feedback showing consequences of their decisions while the software carried out the procedures. In this way, emphasis was placed on strategies, which form part of conceptual understanding.

With the design of Equations2go being based on theories of conceptual understanding and versatile thinking, this investigation demonstrated the potential for software based on educational theories to improve learning.

As a result of investigating the effects of Equations2go, a framework has been proposed which describes the learning of versatile thinking for equation solving as a set of learning goals. The framework is based on a combination of the findings of this study and existing learning theories. By helping teachers structure their lessons and diagnose student learning needs, the framework could contribute to the guidance Thomas and Tall (1988) claimed teachers need to help students develop conceptual understanding. The framework could also help software developers design learning activities for students at different stages of learning to solve equations.

The practical investigation of prototype software in this study resulted in the development of a promising theoretical framework for the teaching and learning of equation solving. This combination of practical and theoretical considerations provides a sound basis for future research into educational software designed for helping students who are learning to solve equations.


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9 Appendices

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Appendix A: Pre- and post-questionnaires

Pre-questionnaire

Please tick the appropriate circles to fill in this questionnaire.

1. Which mathematics class(es) are you attending this semester?
   - Math221
   - Math222
   - Math223
   - Math224
   - Math111
   - Math112
   - Math 113
   - CCBS230
   - Other ______

2. What is your major area of study or goal at CPIT?
   - Engineering
   - Medical Imaging
   - Science/Lab technician
   - Nursing/Midwifery
   - Computing
   - Other ______________________

3. Indicate your age group.
   - under 20
   - 20 -29
   - 30 -39
   - 40+

4. Indicate your gender.
   - Female
   - Male

Questions 5 to 7 show a five point scale with the two end points labelled. Please tick the circle which best describes your position on this scale.

5. What is your attitude to algebra and in particular equation solving?
   - Dislike
   - Enjoy

6. How would you rate your knowledge of solving equations at this level when you started this course?
   - None
   - Excellent

7. How confident do you feel about solving equations?
   - Not confident, hardly ever solve correctly.
   - Confident, usually solve.

8. How much do you think you would enjoy using a computer program to supplement your learning of equation solving?
   - Would prefer pen and paper only.
   - Am very keen to use a computer program.

9. Do you have any other comments about equation solving?
Post-questionnaire

Questions 1 to 3 show a five point scale with the two end points labelled. Please tick the circle which best describes your position on this scale.

1. How confident do you feel about solving equations?
   - Not confident, hardly ever solve correctly.
   - Confident, usually solve.

2. Did you like using the computer program or would you prefer to have used a written worksheet for this exercise?
   - Prefer pen and paper only.
   - Enjoyed using the computer program.

3. How much did the computer program help you learn to solve equations?
   - Not at all
   - A little
   - Some help
   - Helpful
   - Very helpful

4. Would you like to use an extended version of the computer program with more equations for your study?
   - No
   - Yes

5. Would you recommend other students use an extended version of this program to help them learn equation solving?
   - Wouldn’t recommend
   - Recommend
   - Strongly recommend

6. The computer program allowed you to decide what step to do next and then carried out the step for you. How did you find this?
   - Unhelpful
   - Sometimes unhelpful
   - Fine
   - Helpful
   - Very helpful

7. How easy was it to learn to use the computer program?
   - Very hard
   - Hard
   - OK
   - Easy
   - Very easy
8 Did you find the following features helpful?

<table>
<thead>
<tr>
<th>Feature</th>
<th>Yes</th>
<th>No</th>
<th>Didn’t use</th>
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<tbody>
<tr>
<td>Undo button</td>
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<tr>
<td>Flag with a quick tip</td>
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<tr>
<td>Explanation at bottom of screen</td>
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<td>Instructions</td>
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<tr>
<td>Being able to find different correct solutions</td>
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9 Did you prefer to have the score

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<th>On</th>
<th>Off</th>
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10 Please write any other comments you have about the computer program. Eg, Why did you like or dislike any of the above features?

11 Do you have any other general comments?

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12
Appendix B: Pre- and post-tests

Pre-test

Please solve the following equations, showing all your working.

1. $-2y = 7$
2. $\frac{5b}{8} = 2$
3. $4x - 7 = 3$
4. $3 - 2a = 3a + 4a - 4$
5. For this equation, only do the first step. $4(2P - 3) = 8$
6. $\frac{t}{2} - \frac{t}{7} = 1$
7. $\frac{V + 2}{3} - 1 = \frac{V}{4}$
Post-test

Please solve the following equations, showing all your working.

1. $-2H = 9$

5. For this equation, do the first step only.
   $2(3a - 1) = 8$

6. $\frac{m}{7} + \frac{m}{3} = 1$

2. $\frac{5S}{7} = 4$

3. $2D - 9 = 3$

7. $\frac{K - 3}{2} + 3 = \frac{2K}{5}$

4. $4 - 3y = 2 + 7y - 2y$
Appendix C: Discussion group questions

Discussion Questions

1. What did you like about the program?

2. What did you find annoying about it?

3. In the program, you decided on the next step and then the computer carried out the step for you.
   
   How did you find this? Did it help you learn? In what way?

4. Do you have any comments about being able to solve the equations in different ways?