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AN INTEGRATED REAL-TIME OPTIMAL FLOOD AND DROUGHT CONTROL OPERATION MODEL OF A MULTI-PURPOSE TWO-RESERVOIR SYSTEM: THE WAIAU RIVER SYSTEM.

A thesis
Submitted in partial fulfillment
Of
The requirements for the degree
Of
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at
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by
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Department of Natural Resources Engineering
Lincoln University
1997
The need to improve the operational performance of the Waiau River System, WRS, while avoiding dramatic failure by being risk-averse, became obvious after the 1991/1992 drought when the New Zealand hydro-system and the country energy production system as a whole experienced one of the worst long-duration, extreme hydrological events in its history.

This need is addressed in this study by developing a methodology and model for the real-time, flood and drought operation of the river system. This methodology is based on integrating the solutions of a deterministic risk aversion optimal control problem, using the min-max reservoir control approach, into a fuzzy logic controller. The min-max reservoir control approach's solutions are a range of possible and effective daily storages and releases that can guarantee optimum operation of the system under any hydrologic conditions. Using these solutions as a rule-base, the effective real-time operational policies were derived through the fuzzy logic controller. The integrated fuzzy logic control model using this methodology was tested for both the 1988 flood and the 1991/1992 drought.

The results of the test were satisfactory for both extreme hydrological events. In retrospect, if the model had been available and applied prior to those two events, it would have been possible to operate the system more effectively by:

- enhancing its performance in terms of water releases for environmental and non-environmental uses, and as fuel for energy generation, and yet,
- guaranteeing that the system states stayed within their acceptable limits under the 1988 and 1991/92 extreme hydrological circumstances and under any other hydrological circumstances not worse than the historically recorded, or system operator suggested, critical hydrological circumstances.

The results of the study also suggest that contrary to the concept of fixed flood mitigation and fixed water rationing trigger points adopted by the current mode of operating the system, the flood mitigation and water rationing trigger points for effective operation should be allowed to vary throughout the year depending on the day and season. It is a more flexible and improved way of operating the WRS and guaranteeing optimum results.

The other advantage of the proposed approach is that the developed fuzzy logic controller can, besides being an effective real-time operating tool, be used as a training tool helping senior operators to refine their techniques and providing guidance to novice operators.
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<td>d2</td>
<td>Flow of water diverted from Mararoa river into Manapourl (m³/s).</td>
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<td>i</td>
<td>Reservoir number.</td>
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<td>k</td>
<td>Planning horizon or year.</td>
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<td>r</td>
<td>Release.</td>
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<td>r&lt;sub&gt;all&lt;/sub&gt;</td>
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structure (m³/s).

$R_{i,max}$ Maximum allowable release from reservoir i.

$k_{Tr}^opt$ Optimum water flow through the Upper Waiau River during brown and rainbow trout spawning periods.

$R_{i,t}^k$ Water released during day t of year k from reservoir i to i+1 assuming the control structure's gates are kept permanently closed. $R_{i,t}^k$ is zero unless the reservoir capacity is exceeded.

$R_{i,t}^k$ Water released during day t of year k from reservoir i to i+1 assuming the control structure's gates are kept permanently wide open.

$R_{i,t}$ Reference releases or optimal target demands for different needs and seasons

$S_{i,max}$ Maximum allowable storage in reservoir i.

$S_{i,min}$ Minimum allowable storage in reservoir i.

$S_{i,t}$ Storage in reservoir i during day t.

$S_{i,0}$ Initial storage in reservoir i.

$S_{i,max}$ Maximum allowable storage in reservoir i during year k.

$S_{i,t}$ Storage in reservoir i at the beginning of day t of year k.

$S_{i,k}$ Storage in reservoir i at the end of year k.

$S_{i,0}$ Initial storage in reservoir i at the beginning of year k.

$S_{i,0,max}$ Maximum initial storage in reservoir i at the beginning of year k.

$S_{i,0,min}$ Minimum initial storage in reservoir i at the beginning of year k.

$S_{i,t,min}$ Minimum storage in reservoir i during day t below which water demand reduction release $\alpha_i R_{i,t}^-$ is triggered.

$S_{i,t,min}$ Minimum storage in reservoir i during day t below which water demand reduction release $\alpha_i^* R_{i,t}^-$ is triggered.

$S_{i,t,max}$ Maximum storage in reservoir i during day t above which water release for
flood protection storage $\beta_i S_{i,t}^+$ is triggered.

$S_{i,t,max}^-$ Maximum storage in reservoir $i$ during day $t$ above which water release for flood protection storage $\beta_i S_{i,t}^+$ is triggered.

$S_{i,0}^-$ Feasible initial storage, in reservoir $i$, solution to the problem of flood control satisfaction.

$s_1$ Describes a reservoir storage at Te Anau.

$s_3$ Describes a reservoir storage at Manapouri.

$s_2$ Dummy storage in Mararoa River.

$S_{i,t}^-$ Daily (day $t$) reference storage of reservoir $i$.

$t$ Represents a day.

$V_i$ Flood storage volume in reservoir $i$.

$W_{i,t}^k$ Water inflow into reservoir $i$ during day $t$ of year $k$.

$w_1$ Net water supply to Lake Te Anau.

$w_2$ Net stream flow through Cliffs gauging site.

$w_3$ Net water flowing into Manapouri from precipitation and local (Manapouri catchment) tributary streams.

$W_{(k-1)T}^{kT-1}$ Sequences of inflows each of the length $T$ where $T$ is the length of the planning horizon.

$W_{(t+1)}^{T}(T \geq t)$ Describes a set of finite sequences of inflows $W_{i,t}^T = (w_{i+1}, w_{i+2}, \ldots, w_T)$ within a planning horizon of length $T$.

$W_0^{kT-1}$ Sequences of inflows each of the length $T$ where $T$ is the length of the planning horizon.

$W_{i+1}^T$ Infinite sequence of inflows.

$\alpha_i$ Water shortage indication of reservoir $i$.

$\alpha_i^k$ Water shortage indication of reservoir $i$ over year $k$.

$\beta_i$ Flood control indication of reservoir $i$.

$\beta_i^k$ Flood control indication of reservoir $i$ over year $k$.

$\tau$ Represents a day.

$\xi$ Rate of energy generated per unit of water released through the power station.
CHAPTER 1

BACKGROUND

1.1. Introduction

Conflicts between groups interested in the preservation and protection of the unusual or the beautiful aspects of the natural environment, and those interested in economic development, have characterised the field of resource management in New Zealand for a number of years. The well-publicised arguments over building dams, diverting water for power generation, or flooding a valley, are only too well known to those interested in conservation and recreation. A case in point in recent years is the conflict in the South Island over the future of one of New Zealand’s largest and most beautiful and unusual fresh water bodies - the Waiau River System (WRS) in Southland. The diversion of water from Lakes Te Anau and Manapouri and the Mararoa River through the Manapouri power station is of particular concern. The power station is situated at the West Arm of Lake Manapouri. The water that goes through the power station is released into the sea at Deep Cove, and is therefore not returned into the Waiau River System. This then results in leaving a much lower flow than the natural flow for the downstream users.

The reservoirs in the WRS are operated by prescribing releases and reservoir levels in each time period according to the statutory guidelines for Lakes Te Anau and Manapouri, and the Upper Waiau River operation (Freestone and Eaton, 1993). The statutory guidelines map the current reservoir levels and river flows into releases, therefore specifying how water is to be apportioned temporally and spatially among purposes. The aim of this thesis is a review of the statutory guidelines for the purpose of finding effective operating rules that will minimise expected economic, environmental, aesthetic, fishery and recreational quality losses. The losses are expressed as a function of water release in periods of extreme hydrologic events such as floods and droughts. This thesis is topical because of the extreme long-duration “low-inflow crisis” of 1992, for which the Electricity Corporation of New Zealand (ECNZ) was publicly criticised by some parties.

The primary cause of the 1992 energy crisis was the prolonged drought sequence from November 1991 to June 1992 and in particular the extremely low inflows to the South Island hydro storage lakes during the months of March to June 1992. This hydrological event was further exacerbated by an unexpected increase in electricity demand. The questions often asked after such crisis periods are whether the negative effects of the crisis
could have been avoided and whether the operation and management of the reservoir systems were optimal during those periods? The answers often come after re-examination of the situations. Consequently, mindful of the erroneous blame, and conflicts arising from "drastic" hydrologic events, such as the flood and drought that plagued New Zealand in 1988 and 1992 respectively, it is evident that research that endeavours to understand the complexities of the causes of such events should be undertaken. The outcomes of such research would help to develop possible solutions that ensure effective use of the resources and equipment available in similar situations. In line with that thinking, this study is a first step towards developing a methodology for enhancing the operational performance of the WRS in periods of extreme hydrologic events while guaranteeing that the system states stay within their acceptable limits. The particular focus of this study is on the improvement of the effectiveness and consistency of electricity generation in the sub-region, while maintaining or enhancing environmental, aesthetic, recreational and ecological qualities.

1.2. The New Zealand electricity system

Electricity is the single most important energy source in New Zealand and accounts for 48% of the total, non-transport, energy market share (Centre for Advanced Engineering, 1993). Currently in an average year, between 66% to 83% of this energy is provided by hydro generation (Electricity Shortage Review Committee, 1992). The remainder is generated from thermal power stations that use geothermal energy, coal, gas, residual oil and distillate oil. This makes the management of water resources the dominant factor in energy management in New Zealand.

Prior to 1996, the Electricity Corporation of New Zealand, ECNZ, generated 95% of the total electrical energy available to New Zealand\(^1\). Small plants operated by various local authorities or industries and selling energy to the national system generated the 5% balance. One of the Corporation’s major electricity generation stations is the Manapouri hydropower station located in the Waiau catchment.

New Zealand is comprised of two main islands. Its economy is mainly based on farming, forestry, and light manufacturing, although there is an aluminium smelter at the southern tip of the South Island, a steel smelter south of Auckland (the largest city of New Zealand) and a significant North Island-based pulp and paper industry. The major types of

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\(^1\) ECNZ recently has transferred 8 of its stations to a new State-Owned Enterprise, Contact Energy, and in the future will be constrained to a market share of 45%.
electricity generation in the North Island are hydroelectric, geothermal and natural gas. The South Island contains only hydroelectric power stations and supplies the majority of the country's electricity (typically 70% in an average year). The Manapouri power scheme is one of the ECNZ's largest individual hydro power stations and represents a critical component of New Zealand's electricity system, as it produces, on average, 4300 GWh per year. This represents, approximately, 15% of the total New Zealand's electricity consumption (ECNZ, 1996).

New Zealand energy demands are high in winter and low in summer. However, the inflows in the South Island catchments, which supply the majority of electricity to the whole country, are inversely high in summer and low in winter. This poses a major challenge for the management of New Zealand water resources for electricity generation.

A particular feature of the New Zealand generating system is the allocation of distinct steps in operating cost for hydro, geothermal and thermal power stations. For example when inflows in the hydro-system are high (or low) the price of the unit water is low (or high) compared to that of thermal fuel. This attribution of distinct operating cost (marginal cost) is to enable the hydro and thermal resources to be managed effectively. This value varies in time and space, thereby reflecting the cost of shortage when water is not available. The geothermal and thermal stations have values determined by costs such as: fuel, fuel handling and maintenance.

Storage operating guidelines are derived weekly for the months ahead making use of: expected future inflows, outflow assumptions, demands, marginal cost of water, and other factors to define the water releases from storage and to determine the priority (merit order) in which power stations should generate. The marginal value of water in a reservoir is a function of the volume of water stored and of the expected future inflows. High inflows and high storage levels reduce the value of water. Therefore, the position of a power station in the merit order (i.e., the value of water in its upstream reservoir), can alter quickly as a result of changing hydrological conditions.

The two existing geothermal power stations have very low operating costs and, therefore, generate electricity on base load and come first in the thermal station merit order. Gas-fired stations are next in the merit order, followed by the coal-fired stations, the residual oil station and finally, the distillate oil gas-turbine stations. Operating costs are sufficiently separated from one another to make the scheduling of the thermal power stations highly predictable as demand for power varies from day to day. The only main uncertainty comes from the availability of water for hydropower stations. However this
does not affect the merit order of the thermal stations, but rather imposes a variation in the position of hydro stations in the merit order. Consequently, the operation guidelines of New Zealand electricity generation are fundamentally based on the policy that:

- if the value of water is less than the cost of generating from the least expensive thermal plant, then maximum use should be made of the appropriate hydro resource and minimum use of any thermal generation; or,

- If the value of water is greater than the cost of using some thermal plant(s) then, after uncontrolled and required minimum flows have been used for generation, maximum use should be made of those thermal stations before additional water is released from storage to meet demand.

The operation procedure described above is handled by using a stochastic Dynamic Programming model called SPECTRA (System Plant and Energy Co-ordination using a Two-Reservoir Approach). This model has the ability to handle stochastic inflows, correlation in time and space of these inflows, two reservoirs and catchment and unexpected power station and transmission dynamics. It does not however, handle all the requirements in detail for an individual river system. Only the main aspects of operation nation-wide in terms of power generation and consumption, i.e. the global picture of the power generation and knowledge of likely future demand, are dealt with. Therefore, the station operators are not able to decide individually whether or how much they should generate. The SPECTRA operation strategy is suggested as an operational security standard. This standard requires that the national grid electricity generation system be operated such that rationing of electricity due to hydro-storages will not be expected to occur unless inflows are below a 1:60 year level (Electricity Shortage Review Committee, 1992). The historical storages are plotted in relation to the present storage and an operational curve is determined to meet the operational security standard. The level of storage below which all-base load thermal plants must be committed to generate electricity is determined. The minimum level ensures that the worst of the observed historical inflow sequences would avoid causing the lakes to run below resource consent conditions (Halliburton and Truesdale, 1994). It is however worthwhile noting that, after recent restructuring of the New Zealand electricity generation industry this security standard is now governed by the Wholesale Electricity Market.

In the merit order of the hydro stations the Manapouri power station always comes last. This is because the Manapouri power station is, as a priority, to provide a continuous supply of electricity (543 MW) to the aluminium smelter at Tiwai Point, Bluff (ECNZ, 1996). The remainder of energy out of the peak station power output (600 MW) is used to
complement the national energy demand. Consequently, as mentioned earlier, the Manapouri power station with its high-energy production capacity is of huge importance to energy production and consumption in New Zealand. It is worthwhile noting that the actual installed capacity of the Manapouri power station is 700 MW, but it is limited by surge chamber and tailrace constraints. This limitation is expected to be alleviated by the utilisation of a second tailrace, which is now being constructed. Due to its high capacity energy production and its special relationship to the Bluff aluminium smelter, the Manapouri power station can be studied as a stand-alone power station that contributes to enhancing the performance of the national grid power production. Currently, the operation of the Waiau river system is primarily based on the guidelines and on the intuition and know-how of experienced operators who make decisions on the best use of the stored water given the current level of the lake and the MetService (Meteorological Service of New Zealand) warnings, rather than on simulation and optimisation. Their decision-making process is mainly based on their experience of the system without real consideration of possible similarities between future, current, recent past and historic hydrologic events and patterns. There is a possibility that the tacit, undocumented expertise that is required for operating the system might be lost when experienced personnel leave. Moreover, there is a need to know how the system will behave given ECNZ’s future plan of continuously releasing pre-defined minimum flows of water down the Lower Waiau river to fulfil environmental and downstream water use requirements. Therefore, an alternative approach to this restricted mode of operation is to develop, in collaboration with the managers and operators of the system, a daily operational plan that will:

1. refine and/or complement the judgement of the operators,
2. provide a training or decision-making aide for novice operators, and above all,
3. base the operation of the system on a realistic knowledge of past and future hydrologic events. This study proposes such a mode of operation for the WRS.

1.3. THESIS OBJECTIVES

The purpose of the study is to develop effective operating policies and techniques that will serve as a guide to, and an improvement of, the existing WRS operating techniques. These will allow the WRS’s managers and operators to make best use of the system water flow and head, as well as the installed capacity of the Manapour Power Scheme, to enhance its operational performance. This would include improving the effectiveness and consistency of electricity generation in the sub-region, while maintaining
or enhancing environmental, aesthetic, recreational and ecological quality. Within this aim there is a particular focus on performance through extreme events of flood and drought.

This study purpose therefore leads to the following objectives:

1. To develop a methodology for optimising the operational performance of the WRS during normal circumstances.
2. To develop a methodology for minimising the risks of not meeting catchment management requirements in terms of power generation, and environmental, ecological and recreational qualities, in extreme flood and drought events.
3. To use the results from the above methodologies to investigate compliance with, and review of, the statutory guidelines for Lakes Manapouri and Te Anau, and the Upper Waiau River operation.

1.4. Method of study and research approach

1.4.1. Reservoir system management and operation model

Marino and Loaiciga (1985) state that optimal operation of reservoir systems is of fundamental importance for the adequate functioning of regional economies as well as for the wellbeing of the population served by such reservoir development. Reservoirs provide wide varieties of indispensable services to society. These services include the provision of water supply for human consumption, agriculture and industrial activities, hydro-power generation, flood control protection, ecological and environmental enhancement, navigation and recreation. With the increasing expansion of human population and economic activities, the demand exerted on water stored by reservoirs has been increasing steadily. In addition, with the size of reservoirs at their maximum possible, due to water availability limitations, and with tighter budgetary constraints, it has become mandatory to operate reservoir systems in an efficient manner, so as to effectively and reliably provide their intended services to the public. In that context, problems of reservoir operation have become a topic of considerable practical and theoretical interest.

Systematic analyses of reservoir operation problems date from the late 1950s and significant advances have occurred over the last twenty years (McLaughlin and Velasco, 1990). Much of this work is reviewed in publications by Rosenthal (1980), Yakowitz (1982), Yeh (1982, 1985), Rogers and Fiering (1986) and Simonovic (1992), amongst others. The most researched approach is optimisation. In 1996 Ponnambalam and Adams
(1996) used a stochastic optimisation concept incorporating a heuristic algorithm to determine the optimal operation policies of a multi-reservoir system in India. Lund (1996) used a deterministic optimisation to develop operation rules for the Missouri River reservoir system. Napiorkowski and Terlikowski (1996) introduced a two-level optimisation concept for the real case operation of a complex multi-reservoir and multi-objective water reservoir. With their concept they managed to improve considerably the system's performance in comparison with standard operation rules for a 90-year long historical data record. In 1992, Mohan and Raipure developed and used successfully an optimisation model incorporating a linear multiobjective programming concept to derive the optimal releases for various purposes from a large-scale five-reservoir system in India. The objectives of their study were to maximise irrigation releases and hydropower production under normal, drought, and excess-flow conditions while ensuring that the constraints on physical limitations, environmental restrictions, and storage continuity were satisfied. Liu and Tedrew (1973) developed an optimisation model for establishing the operation rules in a five-reservoir system in New York State. The combination of dynamic programming and a search technique was used to obtain the solutions.

Efficient operation of reservoir systems, although desirable, is by no means a trivial task. Analysis of a complex water resources system could involve thousands of conflicting decision variables and constraints. A trade-off analysis is often performed when conflicting objectives are involved, especially when the conflicting objectives are expressed in non-commensurable units. Trade-off analysis has become a necessity for multiobjective planning (Mohan and Raipure, 1992). The analysis is based upon the fundamental concept of scarcity and substitution (Loucks et al., 1981). Cohon et al., (1979) used the concept of trade-offs to solve a two-objective, three-reservoir system. The objectives considered were maximisation of net national income and minimisation of reservoir capacity. Loganathan and Bhattacharya (1991) suggested five goal-programming techniques to solve the optimal operating problem of reservoir systems. The techniques: pre-emptive goal programming, weighted goal programming, min-max goal programming, fuzzy goal programming, and interval goal programming. They were used to solve management problems of the Green River catchment in Kentucky. Thampa Pillai and Sinden (1979) applied a multi-objective linear programming method incorporating the trade-off concept to successfully solve a two-objective maximisation of benefits and maintenance of environmental quality water resources problem.

With the growing need for the efficient management of reservoir systems, mathematical optimisation models for reservoir operation have become valuable tools for
improved planning and management of complex operational schedules. Modelling of reservoir management constitutes a classical example of the application of optimisation theory to resource allocation. Reservoir management modelling, however, cannot be accomplished without problems of its own, which are dominated by stochastic elements (streamflow, demand for water and power, etc.), the large size of the models (dimensionality), and the limitations of mathematical tools. This forces the modeller to compromise between accurate system modelling and complexity of the resulting optimisation models (Marino and Loaiciga, 1985). In an attempt to solve the problem of stochastic elements Wang and Adams (1988) introduced a two-stage optimisation framework that consists of a real-time model followed by a steady-state model for the monthly operation of multi-reservoir systems. Picardi and Soncini-Sessa (1991) applied successfully stochastic dynamic programming (SDP) with dense discretisation and inflow correlation assumption, to optimally control a reservoir system. They used the concept of parallel computation to solve the problem of huge computation associated with the solution of problems incorporating inflow correlation and a dense discretisation assumption. Liang, Johnson, and Mohan in 1996 presented a methodology of auto-regressive decision rules for an aggregated reservoir operation as a surrogate of a multireservoir system of the Upper Colorado catchment. The method incorporates a lag-1 correlation for the releases between consecutive periods with the optimal operating policy solved by stochastic dynamic programming. They then used the decision rules with and without incorporation of the auto-regressive correlation for the releases, to simulate the operation of the reservoir with historical inflow records. The results of the simulation showed that the auto-regressive decision rule yields more stable and higher reliability of annual water supply for the aggregated reservoir operations.

Some modellers tried to overcome the problem of dimensionality by introducing various decomposition (in time and space) and aggregation schemes, and successive approximation techniques. For example, Turgeon (1980), in an attempt at solving the dimensionality problem, developed and applied with success a dynamic programming successive approximation (DPSA) and a progressive optimality algorithm (POA) to a reservoir system.

Many successful applications of optimisation techniques have been made in reservoir studies. The choice of a method depends on the characteristics of the reservoir system being considered, the availability of data, and the objectives and constraints specified. However, the key to the success of these techniques for reservoir management is availability of good rainfall and streamflow data. In some circumstances and areas, reliable
rainfall or streamflow data may not be available, especially during periods of flood and drought. At such times, concern is not so much with optimising performance, but focusing attention and effort on avoiding actions that may endanger or damage the system. In other words, in most situations of extreme hydrological episodes (eg, flood or drought) the system manager and operator are risk-averse even if this entails a worse average performance of the system. In that context, several approaches have been developed and used successfully. Among them: the ‘deterministic min-max approach to reservoir management’ (Orlovski et al., 1984), the ‘set control approach’ (Georgakakos and Yao, 1993; a, b), and reliability programming in reservoir management encoding a risk-loss function (Simonovic and Mariño, 1981). The first two approaches demonstrated that in periods of extreme hydrological events, control actions could be derived guaranteeing the involved systems would meet constraints such as:

- water supply satisfaction,
- maintaining outflows below damaging levels,
- maintaining reservoirs’ storage within desired bounds, and
- reliable supply of the required energy.

The derived control actions can be valid for a given length of time and for all inflow sequences bounded by specific (desired) ranges suggested by the manager and operator. The inflow sequences may be real or synthetic inflow records or some hypothetical sequences of inflows which the manager and operator considers as particularly well-suited for testing the reliability of any operating rule (Orlovski et al., 1984).

Rapid advances in mathematical modelling algorithms and techniques of water resource systems and in computer technology over the past decade have favoured the development of decision support or computer-aided support programs. These decision support systems (DSS) are interactive computer graphics-based programs that have incorporated appropriate mathematical optimisation and simulation models, sometimes together with more qualitative rule-based or linguistic algorithms. They are designed to help decision-makers utilise data and models to solve unstructured or under-specified problems.

An ideal DSS is a system capable of increasing the effectiveness of its users as they perform their work, as they try to understand and synthesise solutions to specific problems, and as they prepare their reasons or arguments for making a particular decision or recommendation. Many successful applications of DSS applied to water resources planning and management have been made. Some examples are as follow:
AQUATOOL, a computer-assisted support system for water resources management (Andreu, Capilla and Sanchi, 1990).

HERMES (Hydroelectric Reservoir Management Evaluation System) designed for operations planning (Barritt-Flatt and Cormie, 1990).

WARFS, the Water Resource Forecasting System program developed by the National Oceanic and Atmosphere Administration (NOAA) is designed to lessen the impact of water crises. "WARFS’s goals include the implementation of hydrometeorologic/hydrologic modelling, dynamic stream flow modelling, extended streamflow prediction (ESP), reservoir operation and meteorological forecasting, and advanced products for water resources management" (Ingram, Welles, and Braatz, 1996).

The ACRES model (Sigvaldason, 1976), the RESER model (Simonovic, 1985), and the IRIS model (Resources Planning Association, 1990) are examples of a DSS model developed using a simulation concept. The above list is by no means an exhaustive one.

In 1997, Frevert et al. (1997) of the U.S.A. Bureau of Reclamation developed a data-centred DSS to provide a practical tool for multiple purpose water resources projects planners and managers. The data-centred DSS provides the means to utilise relational database and advanced modelling technologies for integrating historical, current, and forecasted water, power, and weather data (Frevert et al., 1997). Its satisfactory application to the Colorado River and its tributaries (Frevert et al., 1997) demonstrates its ability to ensure efficient and sustainable use of river-system resources, and to facilitate the postulation, testing, and analysis of alternative operational and planning scenarios that respect competing objectives. The above-cited DSS constitutes only a fraction of the large amount of existing DSS.

Simulation models for reservoir systems have also been developed and used widely to facilitate reservoir management. A simulation model is usually characterised as the mathematical representation of a physical system used to predict the response of systems under a given set of conditions. Simulation models allow very detailed and realistic representation of the complex physical and hydrological, hydraulics, economic and social characteristics of a reservoir system. They facilitate the postulation, testing, and analysis of alternative operational and planning scenarios that respect competing objectives. The concepts inherent in a simulation approach are easier to understand and communicate than other modelling concepts such as optimisation. For example, for the management of a reservoir system, it is relatively straightforward to derive, using a simulation model, an objective function which could effectively capture all the various water-based needs,
including the more subjective items such as recreation, water quality, fish and wildlife. Examples of applications of simulation date back to the early 1950s. The Harvard Water Program (Maass et al., 1962) produced the first publications documenting research in the development of reservoir system simulation. Examples of some of the earlier state-of-the-art models include HEC-3 and HEC-5. HEC-3 and -5 were developed by the Hydrologic Engineering Centre of the U.S. Army Engineering Corps and are described by Feldman (1981). Until today these models are still being used extensively in water resources planning and management. The Texas Water Development Board began developing a series of surface water simulation models in the late 1960s. In 1974 the Tennessee Valley Authority, TVA, started a project to develop a comprehensive water resources management method to cope with increasing complexity of multi-purpose reservoir system management (Yeh, 1985). The TVA model includes simulation of physical systems, evaluation of operating purposes, real-time operation, forecasting models for system inputs and demands, and input/output data management (Yeh, 1985). In 1982, Fiering used the simulation technique to develop resilience indices for several reservoirs. While his results were imperfect, they are encouraging and suggest areas for continuing work in the mapping of catchment characteristics and configuration into performance indices. Some other recent examples of the successful application of simulation models to water resources include:

- The application of simple water balance modelling to seven different catchments in two different climatic (humid, and semi-arid - semi-humid) regions of China for water resources assessment by Xu (1996). The proposed approach performed satisfactorily, and is believed by the author to be a valuable tool for planners and designers of water resources.

- Simulation modelling for water resources planning in Bangladesh. A complex network of river channels to a large extent interlinks the Bangladesh water resources system. The nature of the terrain, and the predominant hydrological and hydraulic processes are also very complex. These phenomena preclude the use of standard optimisation packaged models for assessing the country’s water resources (Wardlaw and Moore, 1996). It was in that line of thought that Wardlaw and Moore (1996) developed and applied satisfactorily a simulation model that appropriately: 1) took into account the particular geographic and hydrologic features of the water resources systems, and 2) used the available hydrologic, agricultural and water-use data on a scale that would not otherwise have been possible. The simulation concept is often incorporated in DSS. A typical example is the data-centred DSS described above.

Even though techniques for reservoir management and operations are increasing in
number, their adaptation to real-world systems still remains slow. A gap still exists between research studies and the application of the techniques developed in practice. Russell and Campbell (1996), Loucks (1992), Wurbs (1995), Simonovic (1992) and Yeh (1982) have discussed this issue. Loucks (1992) pointed out that the reasons for the existence of a gap are:

1) lack of communication between reservoir operators and modellers,
2) over-simplification of reservoir systems for research purposes, and
3) existence of institutional constraints on user-research interactions.

Yeh (1982) suggested that another important problem is the lack of good data, which can render even the best technique inadequate. Russell and Campbell (1996), and Shrestha, Duckstein and Stakhiv (1996) in their studies suggested that the reason for such gaps is that system operators are not comfortable with complex and often abstract optimisation and probability models, which procedures they cannot fully understand. They suggested the use of fuzzy logic programming to improve on the existing operation practices. Shepherd and Ortolano (1996) in her “critiquing expert-system approach” went a step further by instead evaluating, with the involvement of the operator, a plan proposed by the operator. The evaluation gives the operator information on the consequences of the proposed action, the alternatives to consider, and suggestions for improvement.

1.4.2. Problem formulation and the solution approach

Reservoirs are the most important elements of complex water resource systems. In the WRS they are used for spatial and temporal redistribution of water quantity and quality, and for enhancing water ability to generate hydropower. Power generation and its contribution to the regional and national grid electricity production is a priority in the operation of the Waiau River System. The attainment of this objective is, however, complicated by pressure from the public, Southland Regional Council and other agencies that are concerned about issues of environmental, aesthetic, fishery and recreational quality enhancement.

In the present study effective and improved methodologies are developed that will refine and complement the existing daily operating rules for the WRS. The rules were defined to provide releases of water primarily for power generation and secondarily for fish, wildlife, environmental quality enhancement, recreation, flood control, and existing water rights. The secondary water requirements impose constraints on the decisions regarding the production of electrical energy, and storage of water. Therefore, the problem
to be solved is to determine the effective daily controlled outflow sequences and the permissible lake levels able to guide the WRS managers to meet and improve the system’s objectives under any hydrological conditions. The determination is a function of the historically available data or some hypothetical sequences of inflows that the system operator considers as particularly well-suited for testing the reliability of any operating rule and any suggested future operation plan. However, the determination of the effective operating process can be seriously complicated by the fact that future system inputs are unknown. Indeed, New Zealand’s geographical isolation and its maritime climate are factors adding to difficulties and unpredictability of future hydrological events. The MetService does not provide routine weather forecasts longer than five days. Moreover, the weather forecasts are only localised ones. Therefore, the electric power production companies of New Zealand at the time of this study can only use these limited forecast values for its management purposes.

The traditional approach often suggested to deal with unreliability of future hydrologic inputs is the use of historical inflow data as a base for developing probabilistic input variables to optimise a system’s performance in some average sense. However, this is not often possible or sound due to lack of long and reliable historical data. For example, even though in the WRS, the available data records for Manapouri and Te Anau are long enough to establish probabilistic models, those for Mararoa River flows are short, missing and/or sparse at times. Another reason is that, the extent to which the future inputs will satisfy future demands is often unknown and it cannot be adequately handled by probability models, which only handle uncertainties lying in the values of the inputs and demands. Consequently, probabilistic input characterisations can be assessed as an inappropriate method of defining operating policies for the WRS even in the average sense. More importantly, during situations of extreme events, the WRS operators, being risk-averse, are not concerned with optimising the system’s performance. They only wish to avoid actions that may endanger or damage the system. In that line of thought, a control approach concept, that focuses on avoiding substantial failures of the system while endeavouring to accomplish an effective operation rather than just optimising system performance during severe hydrologic events, is proposed. The concept stipulates that future inputs, although unknown, are restricted to belong in certain sets called the reference inflow sequences (or reference inflow sets). The boundaries of these sets are to represent minimum and maximum input estimates or other historical extreme levels experienced or identified by the system managers as those conditions they wish to avoid in future operation. In this framework, the purpose of the control approach becomes the determination of admissible
control policies that would enable the system managers to operate the system within a desirable set of operational constraints in periods of extreme as well as normal hydrological inputs while enhancing its performance. Naturally, the solutions to the approach will be a function of both: initial storages in the reservoirs, and, the projected input data. The projected input data are assumed known as they are represented by pre-determined reference inflow sequences. Therefore, particular care must be taken, and quantitative and qualitative communication with the system operators maintained, when selecting the inflow sequences.

Other purposes were considered of minor importance since they were already satisfied, at least to a certain extent, by the constraints imposed on the operation of the system through the “operating guidelines”. For example, the avoidance of sudden propagation of flood waves downstream and the preserving of the natural lake shore ecology, is accomplished by releasing the same flood flow or low flow as would have occurred naturally when the lakes are higher than the defined maximum allowable levels or lower than the defined buffer levels respectively.

To achieve the purpose of this study, the min-max approach to reservoir management developed and successfully applied to the real-time daily operation of Lake Como in Italy by Orlovski et al., (1984) was considered. The major driving reasons behind the selection of the approach were:

1) The lack of predictability of inflows,
2) The fact that the adopted reservoir control rules specify a free release of water when the lakes' levels are between pre-defined storage ranges and must follow specified rules outside them.

The other driving reason is that both the Lake Como system and the Waiau river system problem consist of making tradeoffs between the conflicting objectives of minimising the risks of flood and water shortage, especially during periods of extreme hydrological conditions.

The central idea of the min-max approach is described in sub-section 1.4.1. It involves a hierarchy of three steps. The first step solves the problem of water demand satisfaction of downstream users during drought. The second step solves the problem of attenuation of storage peak or flood protection. The last step combines the two preceding steps to provide bounds on the effective solutions. These bounds have been incorporated in this study in a “fuzzy rule-based control model”, to determine effective daily operating rules for active and future management of the reservoir-system under any normal and/or
explicitly described or given worst possible pattern of supplies. It is believed that the adoption of the min-max concept and its application to reservoir control through fuzzy logic programming will result in effective operational strategies in any hydrologic condition. A fuller description of the min-max approach in its basic form may be found in Appendix A.

The effectiveness of any approach to solving a formulated problem will depend on the approach's ability to represent certain key factors such as the future nature of inflows. Therefore, given the nature of the WRS and the purpose of the study, the min-max approach to reservoir management model by Orlovski et al., (1984) proves useful because:

- It completely avoids statistical considerations on the inflows by assuming that inflows are unknown but bounded. It also minimises the worst possible operational failures of existing reservoir-systems when the inflows are equal, greater or lower than some prespecified critical values.
- It can be interpreted in terms of a classical decision-making procedure that considers the beginning-of-a-period t's storage and the total forecasted inflow for that period as parameters while seeking an optimum operating procedure for the period.
- It provides a sustained and effective management process of a reservoir system throughout all types of hydrological circumstances including those of drought and floods as opposed to processes that are abandoned as soon as the extreme hydrologic events are over.
- It permits the avoidance of excessive spill while maintaining the defined level of supply reliability. This enables better use to be made of available water in storage, the statutory water consent requirements, the environmental management goals, the security of supply standard, the forecasted inflows, and the historical hydrological events.
- It uses appropriately pre-defined or suggested reference inflow sequences to determine ranges of permissible and effective releases (instead of a single release value), ensuring the system states stay within their acceptable limits as far as the forecasted inflows remain within the pre-defined inflow sequences. Thus it introduces a mechanism for handling effectively the stochastic nature of inflows.
- It does not require complex algorithms and on-line optimisation (although this could be incorporated) because all the effective operating rules can simply be obtained by off-line repetitive simulations for different values of input data.
- From a training perspective, the approach is well suited to help operators refine their techniques as it can accommodate more than one operational style or water release
• It can contribute to developing a useful real-time operation tool by allowing the selection of optimum water releases during periods of flood and drought emergencies as well as during periods of normal hydrological events. It can contribute to minimising flooding and water shortage in reservoirs and yet contribute to maintaining an effective balance of flood control storage and low flow storage in reservoirs.

1.5. OUTLINE OF CHAPTER CONTENTS

The purpose of the study is to fulfil the need of enhancing the operational performance of the Manapouri power station while providing satisfactory water supply for power generation and other uses, and to protect the lakeshores and the sub-region against floods. The development, evaluation and implementation of a model that can achieve this purpose are tasks of considerable importance that will not be completed for some time. The intention in this study is to present, as guidance, a theoretical model along with some preliminary results from its application, to the system manager. The remainder of this section outlines the content of this thesis.

In Chapter Two, the Waiau River System catchment and its hydrologic features are described. The system is converted into a simple mathematical model for study.

In Chapter Three, the application of the ‘deterministic min-max approach to single-reservoir system management’ developed by Orlovski et al., (1984) has been extended to two-reservoir systems management. Also discussed is the method for determining quantitative flow values and reservoir state variables that trigger water demand reduction and storage peak attenuation during drought and flood respectively, or the likelihood of those events given reference releases, and storages. It is demonstrated in this chapter that the satisfaction of water demand and flood reduction in periods of extreme as well as normal hydrological events can be guaranteed.

In Chapter Four the characteristics of storage control problems are described. The state equation and control constraints are formulated. The theory behind how to select an appropriate inflow set for the formulation of control rules is described.

The reference releases, storage and the corresponding water rationing and flood alleviating trigger values are determined in Chapter Five.

In Chapter Six, a daily operational model incorporating the fuzzy logic controller concepts is developed and used to operate the WRS. The solutions of the min-max control
problem obtained in Chapter Five were used as rules for the fuzzy logic controller. An interpretation and analysis of the results of the simulation are given.

The general conclusions of the study and some recommendations are in Chapter Seven.
CHAPTER 2

THE WAIAU RIVER SYSTEM

2.1. Introduction

In this Chapter the Waiau River System catchment along with its hydrologic features is described. Simplification and conversion of the system into a mathematical model for study is also undertaken. The modified basic concept of the min-max approach as described in Chapter Three, can be adapted to suit the system’s mathematical modelling.

2.2. Waiau river catchment description

The Waiau River system (Figure 2.1) drains an 8134 km² catchment, the Waiau catchment, embedded in a large regional system. The Waiau catchment is a large and complex network of lakes (an average of 320 lakes dotted throughout the catchment) and rivers in a high rainfall area. It has nationally significant environmental values. Its lakes are part of the Fiordland National Park and a part of the area is even designated as a World Heritage Park (ECNZ, 1997). Prior to construction of the Manapouri Power Scheme and the control structure at Mararoa, the Waiau River was the second largest river in New Zealand in terms of volume of water (Riddell, Freestone and Nutting, 1992). The lakes represent considerable hydrologic features, which affect flow in the river-system and the sub-region. The hydro storage in the catchment alone represents 7.4% of the total hydro storage in New Zealand. Of the 320 lakes, only three have very significant value in terms of hydropower generation with readily available data. They are Te Anau, Manapouri and Monowai (Figure 2.1). The largest of the three lakes by far is Te Anau with a lake area of 352 km². It is also the largest South Island lake and New Zealand’s second largest. Lakes Manapouri and Monowai have surface areas of 142 km² and 31 km² respectively (Riddell, Freestone and Nutting, 1992). A smaller lake, North Mavora, is situated in the north-east of the catchment. Although it is minor compared to the other three larger lakes (Te Anau, Manapouri and Monowai), it remains a significant feature of the Mararoa river catchment, as it constitutes to some extent a water storage reservoir for the Mararoa River. Lake Manapouri with a maximum depth of 444 metres has the largest maximum depth of the
three major lakes. Lakes Te Anau and Monowai follow it with 417 metres and 31 metres respectively.

The rivers (including Upper, Lower Waiau and Mararoa) and lakes in the catchment are important for recreation and wildlife and have considerable potential for development, especially for the generation of power.

The catchment is sub-divided into six sub-catchments (Figure 2.2):
1. the Te Anau catchment with a lake of the same name and a surface area of 3095 km$^2$,
2. the Manapouri catchment with a lake of the same name and a surface area of 1388 km$^2$,
3. the Mararoa catchment with a river of the same name and contains North Mavora lake, and has a surface area of 1219 km$^2$,
4. the Monowai catchment containing Lake Monowai and having a surface area of 245 km$^2$,
5. the Sunnyside catchment with a surface area of 551 km$^2$, and
6. the Tuatapere catchment with a surface area of 1518 km$^2$.

The whole catchment is located in the south-west of the South Island on the eastern side of the Southern Alps (Figure 2.1), with the very wet Fiordland bordering it to the west and south sides. The Mararoa catchment is significantly affected by the very dry weather of the West Otago region situated to the east of the catchment (Riddell, Freestone and Nutting, 1992). The location of the whole catchment and its size produce a very significant hydrological contrast with the upper catchment streams being in very wet zones and the lower catchment streams in quite dry areas (Figure 2.3). The largest and dominant region is Fiordland.

The Fiordland region has a very high average annual precipitation that varies considerably spatially and temporally. In winter snow is added to rainfall. The region constitutes a block of high inaccessible mountains (Southern Alps) characterised by steep slopes and glacial U-shaped valley floors (fiord coastline). It incorporates a wide range of elevations varying from the sea level to over 3000 metres above sea level creating a rain shadow effect to the catchment (see Section 2.3).

Power generation at the Manapouri Power Station is an approximately linear function of water discharge and of storage head. The existing power station has a maximum permissible rated capacity of discharging 510 m$^3$/s flow of water through its penstocks.
Figure 2.1. Waiau River catchment (Reference: Riddell, freestone and Nutting, 1992)
With all turbines operating at full capacity for a day, the power station would release $41.5 \times 10^9$ litres of water through its turbines into the sea at Deep Cove.

Water right conditions stipulate that Lake Manapouri water levels must be kept between 176.8 and 178.6 metres above mean sea level (AMSL), Deep Cove datum. As the lake surface area is 142 km$^2$ at 178.6 metres AMSL, the volume of water stored in the lake between these levels is only about 255.6 Gt (approximately six days supply to the power station at maximum output flow with no inflows). This makes it beneficial to divert more water from the Mararoa River into Lake Manapouri. The diversion has resulted in normal flow to the Lower Waiau River being substantially reduced. The diversion of water from the Mararoa River into Lake Manapouri is an important source of "fuel" supply for the power station. However, undesirable sediment entry from the Mararoa River to the Manapouri lake, and turbidity of the diversion water (when the Mararoa River is in flood) are constraining factors in the power system operational rules and water right conditions. Sedimentation and turbidity have undesirable effects not only on the power station, but also on the lake appearance and the environment of bottom-dwelling fauna.

Another major complicating factor related to Waiau river system management is hydrologic uncertainty, especially during extreme events (e.g. flood and drought) for which data are sparse and/or inaccurate. Studies at the Climate Research Unit, of the Natural Resources Engineering Department of Lincoln University and elsewhere show that:

- Anticyclones leaving Australia are moving progressively further south with increasing centre pressures especially in summer, and (slightly) in autumn and spring (Pittoch, 1973; and Larsen, 1996).
- Rainfall appear to have been increasing over the past 50 to 60 years in the South Westland corresponding with an increase in the westerlies at $45^\circ$ south (Larsen).
- New Zealand as a whole shows a warming trend (Salinger, 1982).

All those facts plus the unavailability and uncertainty, or inaccuracy, of data make WRS operation decision-making difficult.

### 2.3. Hydrologic data

The Waiau hydrological regions have an average annual catchment precipitation of 1400 mm/yr. These precipitations are considerably heavier in the north, west and southwest parts of the catchment and range from 4800 mm/yr to 8000 mm/yr (see Figure 2.3). In the drier eastern portion of the catchment, the annual average rainfalls are low and range
Figure 2.2: Waiau sub-catchments.
between 1000 mm and 1600 mm. This is because the mountains (Southern Alps), bordering the Waiau catchment, stretch southwest to northeast along the 800 km length of the South Island of New Zealand. And with their elevation exceeding 2000 m, reaching over 3000 m in the central part, they impose a significant barrier to the Southern Hemisphere westerlies, which are consequently forced to rise. Thus, the enhanced uplift greatly increases precipitation, often by three to four times sea level values. This demonstrates, in accordance with Garr and Fitzharris (1994), why precipitations exceed 10,000 mm/year on the western flanks of the Southern Alps whereas the eastern flanks receive much less.

The three major sub-catchments of concern in this study are Te Anau, Manapouri, and Mararoa. These sub-catchments exhibited considerable variations in their mean monthly precipitation and thus in water inflows to the reservoirs (Figure 2.3). According to Riddell, Freestone and Nutting (1992) the water inflow into those sub-catchments presents a similar seasonal pattern to their rainfall and snow-melt. During winter, much of the precipitation falls as snow, and water is stored until it melts in spring and early summer (Riddell, Freestone and Nutting, 1992). Consequently, the catchment experiences its lowest mean monthly inflow in winter (June to August), and its highest in late spring (September to November) and summer when snow and glacier melt make significant contributions to inflows. Unfortunately, the low inflow period coincides with the high energy and water demand while the high inflow season coincides with low energy demand. Another important hydrological characteristic of the catchment is the substantial variability in mean annual flow. Over much shorter periods several flood flows of more than 5000 m$^3$/s have been recorded, and there have been flows as low as 11 m$^3$/s (Riddell, Freestone and Nutting, 1992, pp 3-8, 3-11).

Good knowledge of historical hydrological events of a reservoir-system is essential to developing its efficient management policies when adopting the min-max concept. The base data, used to obtain and analyse the WRS historical hydrological events, comes from the TIDEDA (TIme DEpendent DAta) flow data model maintained by Works Consultancy Services (now Opus International) since the years 1926 for Te Anau, Manapouri and 1963 for the Mararoa river. Although recorded from 1926, Lake Manapouri data contains a gap, and records are therefore, usually taken to commence in 1932 for Manapouri. The Mararoa river record stopped in 1967 and re-started in 1974.

The TIDEDA database incorporates river flows, power station discharge flows, lake inflows, outflows and levels. The lake inflows are calculated from lake level and outflow
Figure 2.3: Waiau river catchment distribution of mean annual rainfall (mm)
using a water balance approach. Lake storage, representing the available water for electricity generation and other uses, results from inflows less controlled outflows, and water loss through seepage and evaporation.

The available Mararoa river hydrological data series is both short and incomplete. A series of linear regression equations (see Appendix E) was used to obtain the missing data and extend the record where necessary. This was required since no single relationship could be used in all situations. The particular relationships used depend on the available data, the best $R^2$ value and the physical location of gauging stations.

Although some of the flow in the data base are in error and are missing - probably because of human factors in measurement, the geography of the area, and malfunctioning of the recording gauges at times - those data remain the only available data. And for the present, any management decision requiring historical lake inflow and level would have to rely on the available data.

2.4. System analysis

2.4.1. Simplification of the WRS for study

For in-depth understanding, there is a need to simplify a complex real-world system to an entity that can be analysed. Such an analysis will allow modifications to the simplified system to be synthesised in an attempt to predict behaviour of the real-world system if it were to be modified in an analogous way. The conceptual model of the system as in Figure 2.4 was therefore adopted. From it, simplified mathematical models were defined by using variables, parameters and structures suited to known techniques of analysis and the available data (Figure 2.5). In Figure 2.5, reservoirs 1 and 3 represent the storage in Lakes Te Anau and Manapouri respectively. Reservoir 2 represents a dummy reservoir to allow either diversion from, or return flow to, the Manapouri lake control structure between the Mararoa river and the Lower Waiau river. The dummy reservoir represents the Mararoa river at Cliffs gauging station situated approximately a kilometre upstream of the Manapouri lake control structure. Inflow into reservoir 3 is the sum of local inflows from adjacent streams, $w_3$, water released from reservoir 1, $r_1$, and of water diverted from reservoir 2, $d_2$. Water released into the lower Waiau River is the sum of water released or spilled from reservoir 3, $r_{3S}$ or $X_3$, and of that released into the Lower
Figure 2.4: Manapouri hydro-electric scheme. Lake system and contributing flows.

Figure 2.5: Waiau River System (Upper catchment) mathematical model.
Waiau River from reservoir 2 itself, \( r_2 \). Water released from reservoir 3 is the sum of water released for power generation, \( r_{3P} \), and that released or spilled into the Lower Waiau River, \( r_{3S} \). This release is important for the effective performance of the whole system. In Figure 2.5:

\[
\begin{align*}
    w_1 &= \text{net water supply to Lake Te Anau (m}^3/\text{s}), \\
    r_1 &= \text{controlled water released from Lake Te Anau (m}^3/\text{s}), \\
    w_3 &= \text{net water flowing into Manapouri from precipitation and local (Manapouri catchment) tributary streams (m}^3/\text{s}), \\
    r_{3P} &= \text{controlled water released through the power station (m}^3/\text{s}), \\
    w_2 &= \text{net stream flow through Cliffs gauging site (m}^3/\text{s}), \\
    d_2 &= \text{flow of water diverted from Mararoa river into Manapouri (m}^3/\text{s}), \\
    r_{3S} &= x_3 = \text{water flowing or spilled from Manapouri to the Manapouri lake control structure (m}^3/\text{s}), \\
    s_1 &= \text{reservoir storage at Te Anau (m}^3), \\
    s_3 &= \text{reservoir storage at Manapouri (m}^3), \\
    r_2 &= w_2 - d_2 \text{ (m}^3/\text{s}), \\
    r_3 &= r_{3P} + r_{3S}.
\end{align*}
\]

A water balance and continuity equation for the system can be written as:

\[
S_{i,t} = F(S_{i,t-1}, P_{i,t-1}, W_{i,t-1}, t-1) = A.S_{i,t-1} + B.W_{i,t-1} + M.d_{i,t-1}
+ D.r_{i,t-1} + C.x_{i,t-1} \tag{2.1}
\]

where,

\( r_{i,t-1}, S_{i,t-1}, d_{i,t-1}, x_{i,t-1}, \text{ and } W_{i,t-1} \) represent the release, the beginning storage, diverted water from reservoir \( i \) to \( i+1 \), spilled water from reservoir \( i \) to \( i+1 \) or to river, and the inflow to the \( i \)th reservoir respectively, at decision time \( t-1 \). Spills, whether controlled or not, represent loss of energy generation for the system.

\( S_{i,t} \) represents the reservoir \( i \) storage at time \( t \). is limited by the physical constraints of the reservoir.

\( F() \) = transformation operator.

\( P \) represents the control vector equal to the sum of water released, diverted, and spilled.
A, B, C, D, M are third-order matrices with:

- -1 in the position \( i \) when reservoir \( i \) diverts, releases, or spills water into reservoir \( i+1 \),
- +1 in the position \( i \) when water is entering reservoir \( i \) from upstream reservoirs and/or local streams or rivers, and,
- zeros elsewhere.

Consequently, Equation 2.1 can be re-written as:

\[
\begin{align*}
\mathbf{s}_1, t & = [1 0 0] \mathbf{s}_1, t-1 + [1 0 0] \mathbf{w}_1, t-1 + [0 0 0] \mathbf{d}_1, t-1 \\
\mathbf{s}_2, t & = [0 1 0] \mathbf{s}_2, t-1 + [0 1 0] \mathbf{w}_2, t-1 + [0 -1 0] \mathbf{d}_2, t-1 \\
\mathbf{s}_3, t & = [0 0 1] \mathbf{s}_3, t-1 + [0 0 1] \mathbf{w}_3, t-1 + [0 1 0] \mathbf{d}_3, t-1 \\
\end{align*}
\]

(2.2)

The various uses and purposes of the WRS are quantifiable via a set of constraints, eg., storage reservations for flood control, minimum storages for recreational use, mandatory water releases for fish and wildlife survival, contracted hydro-electric energy generation, and limitation on diversion water and channel flow. The aforementioned constraints affect the variables in Equation 2.2 and are described in the following subsections.

2.4.1.1. Constraints on storage

The constraints on the WRS storages are specified as guidelines for the acceptable operation of Lakes Te Anau and Manapouri which supply water to the power station. The guidelines were gazetted in 1981 and amended most recently. The purpose of the guidelines was for managing the lake level so as to safeguard the natural environmental features and stability of the vulnerable shoreline while optimising the hydroelectric potential of the water resources. The guidelines recognise three operating ranges for each lake: high, main, and low.

In the high operating range, problems are associated with death of woody vegetation when the roots are immersed above their tolerance limits by either flooding or prolonged high water tables. Two significant factors were derived:
1. the maximum duration at particular levels, and
2. the minimum interval between periods of high levels that would allow adequate drainage of water tables and root aeration. This is to satisfy the ecological requirements of the shorelines forest vegetation.

Values for both these factors (see Appendix B) were derived from what had been experienced naturally during the 3 wettest years from over 37 years of reliable and detailed records and study. When the water level in the reservoirs exceeds the upper limit of the high operating range, the operator must progressively open all the control structure gates in order to spill as much water as possible. This is to avoid excess flooding of the lakeshore and the release of damaging flow, down the river channel in the future. In other words, when the current storage level exceeds the high operating range, there is no freedom for the operator in making any release decision but to open wide the control structure gates. He or she must do this by abiding to the existing Lakes Manapouri and Te Anau flood rules described in the “hydraulic operating criteria HD 1265” (Freestone and Eaton, 1993). This is to avoid too much flooding of the lakeshores and the release of damaging flow down the river channel in the future. In other words, when the current storage level exceeds the high operating range, there is no freedom for the operator in making any release decision but to open the control structure gates according to the existing flood release rules.

In the main range the system operations are largely unconstrained. However, fluctuations in levels are required to prevent development of wave-cut platforms and associated mini-cliffing (ECNZ, 1997).

In the low operating range, problems are associated with the stability of shoreline sediment and possible impairment of recreational and amenity values. The guidelines provide:

1. maximum rates of drawdown to prevent slumping of fine glacial till underlying the limited areas of natural beaches, and,
2. maximum duration at particular levels to prevent combing down of beach sediments to levels where they could be irretrievably lost, and,
3. an absolute minimum level to protect the trout fishery of the lakes, the invertebrate fauna of the lakeshores and the trout feeding beds of submerged aquatic plants.

These provisions can have the effect of reducing the station output by up to 70%. However it is believed that they can result in the reservoir level not dropping below the absolute minimum level, while enabling the return of the storage to the main range. Above the
absolute minimum level $s_{t_{\text{min}}}$, the operator must release a flow equal to or lower than the minimum of the inflow and the maximum rates of drawdown. When the level of the reservoirs is however, equal to the absolute minimum storage level no release should be made. The aim of the guidelines is to simulate the normal fluctuations in lake levels as it was before construction of the lake control structures and of the power station. In each range, the system managers may operate within the levels set out. However, they should endeavour to operate the system such as to stay within the duration and intervals shown in the relevant tables in Appendix B.

(a) **Main operating range**
   
   (1) Lake Manapouri from 176.8 m to 178.6 m above mean sea level, amsl, Deep Cove datum;
   
   (2) Lake Te Anau from 201.5 m to 202.7 m amsl, Deep Cove datum.

(b) **High operating range**
   
   (1) Lake Manapouri from 178.6 m to 180.5 m amsl, Deep Cove datum;
   
   (2) Lake Te Anau from 202.7 to 204.3 m amsl, Deep Cove datum.

(c) **Low operating range**
   
   (1) Lake Manapouri from 175.86 m to 176.8 m amsl, Deep Cove datum with the absolute minimum level equals 175.86 m amsl

   (2) Lake Te Anau from 200.86 m to 201.5 m amsl, Deep Cove datum with the absolute minimum level equals 200.86 m amsl.

2.4.1.2. **Constraints on release of water**

Te Anau outflow is a limited function of the control structure gates wide-open (clear of water) release (see Appendix C), and of the seasonal water requirement for successful trout spawning. Release from Te Anau to Manapouri via the Upper Waiau River is to be kept constant during periods of peak spawning of brown trout (June) and rainbow trout (August). The Southland Fish and Game Council study estimates the period of incubation following spawning to continue for up to two months in each case. There is therefore a possibility of overlap between the two spawning runs and the possibility of conflicting priorities. Conflicting priorities occur in the sense that the extended period of
spawning sensitivity (possibly up to 6 months in duration) coincides with peak power generation requirements and also with times when low inflows are most likely. Hence, it is assumed that the preferred strategy would be to maintain flows in the Upper Waiau river at lower levels to encourage redd formation at levels which are unlikely to be subsequently dewatered during normal guidelines and operationally-dictated flows. Estimations are that flows above 180 cumecs cover the main river bed from bank to bank (ECNZ, 1991). Therefore, from ECNZ's perspective, the ideal flow during trout spawning would be somewhat less than 180 cumecs, and that after spawning, flows should ideally be maintained above the mean spawning flow until emergence is complete. The process would then be repeated during the later rainbow trout spawning and incubation. A sensitivity analysis was undertaken in this study to estimate the effect of enforcing trout spawning flow for a given fixed period. For this purpose, an assumption was made that the beginning of spawning periods, and the duration of their peaks for both brown and rainbow trout are those given in the "Waiau River Events Calendar" (ECNZ - Clyde hydro group, 1992). This is to determine with relative accuracy, and to reduce as much as possible the length of time for which the river flows must be maintained at low levels. This length of time is therefore assumed, for the purpose of the study, to be equal to the whole month of June and August each year for brown and rainbow trout spawning respectively. The selected length of time can of course be changed to satisfy the Southland Fish and Game Council requirements. The optimum water flow through the Upper Waiau River during brown and rainbow trout spawning periods, \( R_{opt}^{Tr} \), can then be computed as:

\[
R_{opt}^{Tr} = \min\{ r_{3,t}, r_{all} \}
\]

where:

\( r_{3,t} = r_{3P,t} + r_{3S,t} \) is the sum of probable volume of water to be discharged through the power station and/or spilled from Manapouri down the Lower Waiau river; and

\( r_{all} \) is the maximum possible flow of water allowed during spawning, and is an arbitrarily chosen value and can be changed to suit requirements. It is assumed less than or equal to 120 and 180 cumecs in the sensitivity analysis.

Manapouri outflow is the sum of the outflow through the Manapouri control structure and of water released through the power station. Therefore, it is a limited function of the maximum possible release through the power station and/or the control structure gates, assumed permanently wide open (free of water), and of the overflow capacity of the weir (see Appendix C). According to a study carried out by Jowett (1993) a constant minimum flow of 10 m³/s is to be kept in the Waiau river below the Mararoa weir.
at all times to satisfy minimum flow requirements for instream habitat. The installed maximum rated water discharging capacity of the power plant is 510 m³/s. However, because of the inefficiency of the existing tailrace to accommodate 510 m³/s, only 460 m³/s is often discharged through the power plant. The installation of the second tailrace is to alleviate that problem.

Moreover, although the operating guidelines have specified release policies for specific cases and for the low or high operating ranges (Sub-section 2.4.1.1), no specific rule was determined for release when the reservoirs are in the main operating range. Nevertheless, it can be postulated that after a short learning period, the operators are implicitly using an operating rule in the main operating range, and have varied it over the years each time they recognised the need of structural or hydrologic changes. In fact, it can be reasonably assumed that the operators decide the release of each day of the year as a function of electricity demand, reservoir inter-relationship and needs, environmental and non-environmental needs, and of the availability of water. Naturally, the release is periodic with respect to the periodicity of inflow and water requirement within a year. In that respect, ECNZ has decided, for its present and future management strategy, to adopt a release policy where its “preferred or reference” water releases are relatively fixed within a season but vary within the year in relation to the season (see Sub-section 5.2.1.1). ECNZ will endeavour to achieve its goal, but it can reasonably be assumed that, it will deviate whenever the lake levels enter the low or high operating range defined by the operating guidelines.

2.4.1.3. Constraints on power generation

The maximum energy to be produced, $E_{N_t}$, each day can be computed from:

$$ EN_t = \xi_t \min \{r_{3,t} - r_{3,\text{mand},t}, r_{3P,\text{max}}\} $$ (2.4)

where:

$r_{3P,\text{max}}$ is the ECNZ proposed maximum capacity of the Manapouri power plant in terms of water flow and is equal to 460 m³/s;

$\xi_t$ = the rate of energy generated per unit of water released through the power station;

$r_{3,t}$ = the total volume of water that can possibly be released from Manapouri at time $t$;

$r_{3,\text{mand},t}$ = the mandatory release from Manapouri into Lower Waiau river.

According to a fixed agreement between ECNZ and the aluminium smelter company, a continuous constant load of energy is to be supplied to the aluminium smelter company.
Therefore, if the energy supply is equivalent to a volume in cubic metres per second of water, $\Gamma_{\text{alum}}$, the energy to be produced at each instant $t$ can be constrained as followed:

$$\xi_t \Gamma_{\text{alum}} \leq EN_t \leq \xi_t \Gamma_{\text{P, max}}$$

(2.5)

### 2.4.1.4. Constraints on diverted water from Mararoa River into Lake Manapouri

$$d_{2,t} = \mathcal{G}(s_{3,t}, w_{2,t}) = \begin{cases} 
0, & \text{if } s_{3,t} \geq s_{3,\text{max}} \text{ and/or } w_{2,t} \geq W_g \\
\left| w_{2,t} - r_{2,t} \right|, & \text{if } s_{3,\text{min}} \leq s_{3,t} < s_{3,\text{max}}, w_{2,t} < W_g 
\end{cases}$$

(2.5)

where:

$\mathcal{G}(s_{3,t}, w_{2,t}) =$ diversion function; a function of storage in Manapouri and Mararoa stream flow in period $t$;

$s_{3,\text{max}}, s_{3,\text{min}} =$ maximum and minimum allowable storage in Manapouri;

$W_g =$ represents a water right condition based on expected sediment load in Mararoa river and is currently constrained to be \(30 \text{ net turbidity unit, NTU.}\) However at the time this study was initiated $W_g$ was \(40 \text{ m}^3/\text{s}\) and therefore this value will considered under objective 3 of the study.

### 2.4.2. Inter-reservoir zonal relationship

For an effective management of the WRS it was suggested that both reservoir storages should be maintained in balance as far as possible and that at all times the lakes should have the same “storage index level” in terms of their individual storage zone. This will introduce more flexibility in the control process and prevent occurrence of floods and water shortage in a reservoir when this can be avoided by releasing or not releasing water to or from its downstream or upstream reservoir. The operators of the WRS have adopted this mode of operation for years. They found that, it not only introduces flexibility in operation, but also increases efficiency in performance, especially during periods of flood and drought. The adoption of this mode of operation in this study imitates the real-life management of WRS and will provides the development of a meaningful operation strategy. Satisfaction of the purposes of the study, and the concept of balancing the storages in the reservoir advocated the adoption of the concept similar to that of reservoir zoning developed by the U.S. Army Corps of Engineers (1982) in defining the storage index level.
The concept of min-max reservoir control approach is to some extent similar to that of the reservoir-zoning concept. Releases are made depending on the reservoir storage and the available information. Using the min-max reservoir control approach as compared to the commonly used rule curve approach will provide better solutions to the problems of the study. The storage index levels were therefore represented by the release zones defined in Sub-section 3.4.

The concept of maintaining reservoir storage in balance stipulates that, the volume of water released from each reservoir be governed by the reservoir's index level such that the reservoir at the highest level at the end of the previous time period releases at its maximum possible. For instance, considering a situation in which Te Anau storage at the end of period t-1 is lower than that of Manapouri and that both lakes' storage levels are below the top of conservation storage (the upper limit $S_{t-1}^V$ of zone V in Figure 3.6). The releases from Te Anau during the current period t, will only be made to satisfy reduced water demand or minimum flow requirements and to, at least, bring Te Anau storage level up to that of Manapouri. However, if Te Anau’s release storage level at the end of the time t-1 is in a higher zone than that of Manapouri, say Te Anau is in zone V of Figure 3.6 while Manapouri is in zone III of Figure 3.6, then Te Anau will be operated to bring its storage level down to that of Manapouri during time t. It is necessary to use the previous time period’s storage index level because the current period reservoir releases at Manapouri would not be known until Te Anau releases have been made. It is also worthwhile stating that while balancing the storage in the reservoirs: 1) The release from Te Anau should be made such that it must not cause Manapouri to spill or waste water just for the sake of balancing storage levels. And 2) Manapouri must not be required to empty all of its stored water in meeting its requirements if there is still water in Te Anau. This second condition could occur without special routines due to the use of the storage at the end of current time period, assuming no release was made, for balancing storage level. It is necessary to make such assumptions because the water released from Manapouri, for the current time period, is not known when Te Anau release is being calculated. It is worthwhile noting that Although efforts should be made to balance the storage level in reservoirs, its achievement will not always be possible as releases are governed by physical and other constraints pre-imposed on the system. The maximum discharge capacity at the control structure gates could for example constrain Te Anau to be in the flood spilling zone while Manapouri is in
the conservation zone. Zonal variation could also arise if Upper or Lower Waiau river flows are outside their normal range.

2.4.3. Relation between reservoir storage and channel flow

Control structures are built at the outlets of the lakes to allow storage of water during periods of high inflow and controlled release during periods of high demand. They also have an important role to play in safety during periods of high inflow. River and/or channel bank and bed erosion can be kept to safe limits by reducing reservoir releases when inflows directly into river systems are high and can cause damage. This also reduces peak flows in the rivers, thereby minimising flooding and the subsequent damage and danger to downstream communities.

2.5. Conclusion

The Waiau River System, given the size of the catchment it drains and its geographic and hydrologic features represents a significant feature for electricity generation. Water inflows in the system vary dramatically in time and space. The variations not only occur seasonally, but also from year to year. This therefore, introduces a challenge in the management of water resources of the system in terms of power generation and other uses. Other significant challenges are introduced through: 1) the human-made operational constraints imposed on the system, 2) the physical constraints of the lakes and rivers constituting the system, and, 3) the fact that the low inflow season in the system coincides with high energy demand.

To develop a conceptual foundation for a real-time storage control approach of a multi-reservoir system, it is necessary at the outset to examine the operational relationship between reservoirs, and between reservoirs and rivers draining the system. Therefore the inter-relationships described in this Chapter constituted the mode of operation to be used in the study.
CHAPTER 3
DESCRIPTION OF THE ADOPTED STORAGE CONTROL APPROACH

3.1. Introduction

To achieve the aim of this study, the fundamental “if-then” principle concept is introduced as a basis for the model. The “if” is a vector of explanatory variables which define the present reservoir pool elevation, the net inflow, the preferred or reference release (assumed equal to the most probable aggregated water demand), and the time of the year. The “then” is a consequence in terms of the actual operating decision or release from the reservoir consistent with the prescribed operating policies.

In the past, it was difficult to simulate the response of multi-purpose, multi-reservoir river systems subjected to extreme and varying hydrologic inputs and conflicting operating procedures. Typically, models to simulate the behaviour of this type of river-system were developed by starting at the upstream end of the system and working systematically downstream, and as different constraints on channel flows and reservoir levels were encountered, progressive adjustments were repeatedly made to upstream levels and flows (Sigvaldason, 1976). This process of iteration was repeated systematically until the downstream end of the system was reached. Although this procedure was straightforward in concept, numerous programming difficulties occurred in practice, with corresponding tendencies to introduce bias into the solution procedure (ibid).

A major development in simulating complex water resources systems was the application of the deterministic min-max approach to reservoir management developed by Orlovski et al., (1984). In principle, quantitative flow values and reservoir state variables are identified that will trigger water demand reduction (change in release rates during period of drought) and storage peak control during drought, flood or the prediction of those events. These triggers can be determined using the min-max approach, guaranteeing that water demand and flood reduction in periods of extreme as well as normal hydrological events are satisfied.

In Sub-section 3.2, the framework for assessing the multi-reservoir operation is described. Sub-section 3.3 formulates the problem to be solved and describes the complete mathematical procedure to be used in solving it. Techniques for determining feasible, semi-efficient and efficient solutions to the problem formulated in Sub-section 3.3 are
described in Sub-sections 3.4 and 3.5. Sub-section 3.6 deals with inter-reservoir relationships.

### 3.2. Framework for assessing multi-reservoir operation

Prior to the development of a model which will deal with multi-reservoir system requirements, it is necessary to develop a framework within which various operating strategies can be examined. The framework should be designed to: 1) represent the ideal operation of the individual reservoirs and channels in the system, and provide a perception of alternative solutions under non-ideal conditions and 2) capture the operation of the system as a whole, particularly relationships between various reservoirs and between individual reservoirs and channels or rivers.

In the past, multi-reservoir system operation rules were examined through the development of several frameworks. Large numbers of these frameworks have used rule curves, i.e., most desirable storage versus time relationships for each reservoir (Sigvaldason, 1976). As commonly defined, the principle of the rule curves is that, if at any point in time the defined storages were not satisfied, then the operating policy determines how deviations from the individual rule curves should occur. For example, for an equal reservoir elevation policy, deviations from the rule curve for individual reservoirs occurred in a prescribed sequence, starting with the reservoirs in flood and progressing systematically to reservoirs with lower levels.

Numerous satisfactory solutions to multi-reservoir systems management problems have been obtained through studies carried out by utilising only the rule curve approach in conjunction with a prescribed operating policy (Sigvaldason, 1976). However, in the cases of systems designed with the objective of satisfying many and often conflicting water-based demands, the approach tends to be inadequate.

A significant development occurred with the concept of reservoir zoning developed and introduced by the US Army Corps of Engineers (1982). The concept stipulates division of reservoir storage into a number of zones (typically four to eight). This allows systems to be operated in such fashion that all reservoirs are balanced, that is, all reservoirs are maintained in a similar storage zone as far as possible. Moreover, it permits the derived operation rules to be based (if desired) on priority concepts or some prescribed inter-reservoir relationships (Sigvaldason, 1976). The concept was used successfully in several studies to provide solutions to multiple reservoir-system management problems. In the division of reservoir storage into zones, the uppermost zone is usually referred to as the "flood zone" (see Figure 3.1) and used as temporary storage for alleviating downstream
flood damage during periods of excessive inflows. Similarly, for periods of abnormally low inflows, a lower zone referred to as a “buffer zone” (Figure 3.1) is allocated. Within this zone, downstream discharge may be reduced or made equal to inflow to provide water for essential needs only. Immediately underneath the buffer zone is the undermost zone referred to as the “inactive zone”. This zone represents storage that cannot be regulated. It may, for example, have been designed for protection of the wildlife, the fauna, fish life and even sediment storage. It may as well represent storage below the lowest outlet. Several other zones can be defined between the flood and buffer zones depending on the users needs. However, the zone between the buffer and flood control zones is usually defined as the “conservation zone”. In some studies the uppermost zone is defined as the “spilling zone” and is underlain by the flood storage zone described above. The spilling zone is where the storage water cannot be kept. The water thus spilled is often estimated as lost for power generation. This zoning concept is the principle upon which the HEC-5 simulation decision support software was conceived.

In this study, a concept similar to some extent to the U.S. Army Corps of Engineers zoning concept will be used to estimate water releases. The zones will be defined as possible release zones (instead of storage zones) as a function of historically recorded daily initial storages and daily net inflows (see Sub-section 3.3).

Figure 3.1: A concept of zoning reservoirs.
3.3. Description of the mathematical model

3.3.1. Problem formulation and its solution

3.3.1.1. Problem formulation

Considering a multiple-reservoir-system operating on a daily basis, the problem to be solved is focused on the determination of operation rules or daily release policies of the form:

\[ r_{i,t}^k = r(t, S_{i,t}^k, W_{i,t}^k) \]  \hspace{1cm} (3.1)

where

- \( r_{i,t}^k \) is the release during day \( t \) of year \( k \) from reservoir \( i \)
- \( r(\cdot) \) represents the release function.
- \( S_{i,t}^k \) represents the water storage in reservoir \( i \) at the beginning of day \( t \) of year \( k \), and
- \( W_{i,t}^k \) represents water inflow into reservoir \( i \) during day \( t \) of year \( k \).

These operation rules must satisfy the following physical constraints:

\[ R_{i,t}^k < r(t, S_{i,t}^k, W_{i,t}^k) \leq R_{i,t}^k \]  \hspace{1cm} (3.2)

where \( R_{i,t}^k \) and \( R_{i,t}^k \) are the amounts of water that would be released in year \( k \) from reservoir \( i \) to \( i+1 \) if the control structures' gates were kept permanently closed and wide open (i.e., clear of water) respectively, during day \( t \). Thus, \( R_{i,t}^k \) is zero unless the reservoir capacity is exceeded, i.e., reservoir is in the spilling zone (see Figure 3.1). In that case \( R_{i,t}^k \) is equal to spillage. The upper bound \( R_{i,t}^k \) which includes spillage when the reservoir capacity is exceeded, is a function of the initial storage \( S_{i,t}^k \) and of the total projected net inflow, \( W_{i,t}^k \), during the day \( t \). The function specifies the instantaneous relationship between storage in cubic meters, and release in cubic meters per unit of time (e.g., day or second) in the form \( r = N(s) \).

In Sub-section 4.3 it is shown that for any day \( t \) \((t>0)\) and for positive supplies and initial storage it can be guaranteed that the storage at time \( t \), will be greater than or equal to
$0, \ s^k_{i,t} \geq 0$, and the function $N(s^k_{i,t}) \leq s^k_{i,t}$ with any daily release $r^k_{i,t} \leq N(s^k_{i,t})$ (Equation 4.3). Therefore, for the entire day $t$ the maximum volume of water to be released, i.e., $R^k_{i,t}$ is approximately equal to $N_i(s^k_{i,t})$ m$^3$. Consequently, the effective operation rules must satisfy the physical constraints:

$$\hat{R}^k_{i,t} \leq r(t, s^k_{i,t}, w^k_{i,t}) \leq N_i(s^k_{i,t}) \quad (3.3)$$

Generally, it is assumed that water demands of downstream users in day $t$, vary within a year (cycle), while remaining fixed from year to year i.e., stationary. In other words, water demands are often described as cyclo-stationaries. However, in the deterministic approach proposed in this study, such a definition is not appropriate. An assumption was therefore made that the aggregated daily water demands are a function of $t$, periodic over the planning horizon. That is, they vary according to the seasons (period), while remaining stationary from year to year i.e., they are periodic-stationaries. In the rest of the study the preferred water demands over any given period $t$, will be represented by $R^r_{i,t}$ and called the “reference releases”. In practice, the reference releases are chosen equal to the most probable aggregated and preferred water demands for any scenario of the aggregated water demands which seems particularly significant for evaluating the system performance over a given period $t$. For the WRS, $R^r_{i,t}$ is chosen equal to the optimal accumulated daily water demand for power generation, fish habitat and life, wildlife, downstream users, and maintaining of environmental quality needs. The problem to solve in this study, is one of typical water supply problems made complex by flood control requirements. Its solution can be found by using the so-called standard operating policy approach, also called the S-shaped curve of operation. The approach was introduced by Maass et al. in 1962. An illustration of it is given in Figure 3.2. The approach was developed for simulating the operation of a reservoir used for water supply only (Shih and Re Velle, 1994). It therefore is not necessarily a good suggestion for actual operation of a multi-purpose reservoir in real life. This is because the procedure suggests that:

- When there is a deficiency of water storage in meeting target releases, the reservoir releases all the available water, subject to the constraints imposed upon storage and release, to the extent of becoming empty;
- When however, the stored water is available in quantities larger than the demand, the reservoir fills up and spills its excess of water.
Mathematically, if the S-shaped curve of operation is applied to the case of this study, it can be expressed as:

\[
\begin{align*}
    r_{i,t}^k &= \begin{cases} 
    s_{i,t}^k + w_{i,t} - s_{i,t+1}^k & \text{if } s_{i,t}^k + w_{i,t} - s_{i,t+1}^k \leq R_{i,t}^- \\
    R_{i,t}^- & \text{if } R_{i,t}^- \leq s_{i,t}^k + w_{i,t} - s_{i,t+1}^k \leq s_{i,\text{max}}^k \\
    s_{i,t}^k + w_{i,t} - s_{i,\text{max}}^k & \text{if } s_{i,t}^k + w_{i,t} \geq s_{i,\text{max}}^k + R_{i,t}^- 
    \end{cases}
\end{align*}
\]  

(3.4)

Where \( s_{i,\text{max}}^k \) is the maximum allowable storage or capacity of reservoir \( i \) during year \( k \).

---

Fully utilised water resource systems, especially those that receive most of their inflow from naturally variable surface water sources such as streams and/or rivers, may be subject to frequent periods of water shortage and/or flood. During such periods, i.e., when streamflows are: 1) inadequate to supply established water uses or 2) capable of causing flooding hazards under a given management policy, reservoirs do sometimes fail to deliver their planned yields. In such situations, system operation becomes particularly critical. And the predominant operational objective of risk-averse system managers becomes, to minimise the overall damage to reservoirs, energy generation, fish habitat, wildlife, and human life to cite a few, from the reduced ability of the system to fulfil requirements. However, estimating damages caused by shortages or abundance of steamflow that have not yet occurred is not an easy task. Therefore, other easier to measure objectives relative to the underlying objectives of minimising damages, become the focus of attention. Some of those objectives can be to minimise the maximum expected water shortage or flood event.

---

Figure 3.2: S-shaped curve.
Common sense suggests that, in situations such as the one described above, system managers being risk averse, would prefer to incur a sequence of smaller water shortages and/or floods rather than one extreme water shortage and/or flood that would cause catastrophic damages. Therefore, lessening the consequences of potential catastrophic failures, becomes, as a consequence, the focus of attention of the manager. This implies:

- The institution of water demand restrictions or reduction for downstream uses as an effective means of reducing temporarily the level of demand supply. This is to preserve storage and inflows for future use during periods of drought or impending drought.
- The institution of controlled spillage as a better way of preserving storage for flood inflows thus preventing flood hazards.

The objective is therefore, to develop realistic release rules that allow reduction of demand during drought or impending drought while augmenting release during flood or impending flood thus preserving the system against any failure. As a consequence, it becomes critical to determine the quantitative values of the signals that are required to trigger water demand reduction and/or controlled spill to prevent larger shortage and/or extreme flooding in the future.

According to Hashimoto et al. (1982) the standard operating policy approach (Figure 3.2) is optimal only if one’s objective is to minimise the total release shortfall. They maintain that with the standard policy no water is released when the storage belongs to the AB segment of Figure 3.2. Only the portions of the S-shaped curve that allow deliverance of demand when it is available, and spill when the allowable capacity plus demand is exceeded, are used. Deriving from their studies, the standard operating policy can then be considered as inadequate for providing: 1) a mechanism for reducing or rationing supplies when there may be insufficient water, and 2) a mechanism for releasing more water when there is a surplus of water available or an upcoming flood. This reduces the approach to a rather unrealistic mode of operation particularly when sensible operations throughout drought and flood period are involved.

A min-max approach to reservoir management similar to that developed by Orlovski et al. (1984) can be used to determine an operating rule that caters for demand management during drought or impending drought, and storage control during flood or impending flood. With the min-max approach, once the reference releases, the storages, the demand reduction and flood protection storages have been defined, release becomes a function of the sum of reservoir storage at the beginning of a period t, plus the total projected net inflow of the t period. In this study, the trigger values consist of defined
values of storage in reservoirs. This demonstrates the appropriateness of using an approach similar to that of the min-max.

The WRS is a fully managed system with a functioning power station. Therefore, real-time management rules for demand satisfaction, and storage control are needed. The development of realistic rules for reducing outflow from the reservoir during drought and impending drought, or for augmenting spill during flood and impending flood, prompt at least three questions:

1. At what time and what storage levels should the reduction of demand and/or the conservation of storage begin in order to reduce later shortfalls?
2. How much should demand and hence controlled outflow be reduced during each of the intervals of the reduction or rationing period?
3. How much storage should be conserved during each of the intervals of the controlled spill period?

In order to provide effective answers to these questions, a mathematical simulation model incorporating the min-max approach is suggested.

The basic rules for the standard operating policy using the S-shaped curve are that:

- If the storage in a reservoir during a period $t$, is less than the reference release suggested for that period (line AB of Figure 3.2), then, to avoid negative storage, there will be no release. As a consequence, if such a condition occurred, the S-shaped curve model would declare the water release operation infeasible since release equal to the reference release would lead to negativity of storage volume - quite an unrealistic mode of operation. The infeasible region corresponds to line AB of Figure 3.2.

- If the storage is less than the sum of reference release and the reservoir capacity, and greater than the reference release, the reservoir will release exactly the reference release.

- If the storage is greater than the reference release plus the reservoir capacity (line CD of Figure 3.2), the outflow becomes the sum of the reference release and the spill.

From above it is obvious that, such a reservoir operation approach will be quite an inadequate mode of operating real-life river-systems during periods of: drought and/or flood, and impending drought and/or flood. This is because at those particular times, hedging or pack rules are common practices (Maass et al., 1962).

Contrarily to the S-shaped curve approach, the min-max approach to reservoir management is designed for drought, impending drought, flood and impending flood conditions. The approach proposes that:

- demand and consequently reference release should be manipulated to decline gradually as reservoir contents and projected inflow fall (line AB of Figure 3.2).
• release should be manipulated to increase gradually as reservoir content and projected
inflow increase (line CD of figure 3.2) and thus avoid proceeding blindly to spilling.

Clearly, the min-max operating approach is an adequate mode of operating real-life river-
systems. The advantage is in that, contrary to the standard operating approach, water
releases are subjected to gate opening, channel constraints, reference release of the current
period, and drought and flood constraints. Water can still be drafted even if the reservoir
storage belongs to the line AB of Figure 3.2.

3.3.1.2. Objective functions

3.3.1.2.1. Determination of $\alpha_i^k$

Assume that in the operation of a system, the managers have suggested reference
releases $R_{i,t}$ as the optimal target demands for different needs and seasons. Then, if an
actual daily release $r_{i,t}^k = N(s_{i,t}^k)$ is greater than the actual target demand $R_{i,t}$, common
sense would be to affirm that the release provides no surplus of benefit to the operation of
the system. On the other hand, if $r_{i,t}^k$ is less than $R_{i,t}$, then there certainly are shortfalls in
terms of downstream water uses due to low-flow. Consequently, when the system is
operated on a daily basis over a whole $k$ water year, the minimum yearly value of the ratio
between actual water released and actual target demand, i.e.,

$$\alpha_i^k = \min(\frac{r_{i,t}}{R_{i,t}}); \quad 1 \leq t \leq T$$

where

- $T$ is the length of the planning horizon;
- $k$ represents a planning horizon (or water year);
- $i$ an individual reservoir in the system;

will be considered a meaningful indicator of yearly damages suffered by the system users
(Orlovski et al., 1984). As a consequence, in order to mitigate the consequences of
potential failures due to low inflows, water restriction or rationing may be instituted. It
means preserving future storage and inflow use of water (recreation, hydropower,
environmental needs), by temporarily reducing the level of demand.

The WRS is designed to deliver volumes of water larger than that required for power
generation only. It is therefore, suggested to use the concept of $\alpha$ ratio to determine: i) the
quantitative value(s) of the signal(s) that should be used to trigger water reduction at the
approach of a drought and/or during a drought, and, ii) the quantity of reduced water
demand, to prevent larger shortages later. This means that, with the determination of the
meaningful indicator $\alpha_i^k$ out of the reference set of inflows $W_{0}^{(k-1)}$ (also written as $W_{(k-1)}^{(k-1)}$; see 4.4.1), and if the storage at time $t_i$ is less than the corresponding water reduction trigger value, then rationing will begin with a release of water less than or equal to the reduced demand $f(\alpha_i^k)R_{i,j}^\rightarrow$. However, if the storage at time $t_i$ is greater than the corresponding water reduction trigger value, then it may be possible to draft the full targeted outflow from the reservoir. It is assumed that, with this technique, the impact of different trigger values on water shortage can be assessed. In fact, the larger the quantitative value of the trigger signal, the smaller the maximum shortfall that will occur, but the more frequently water rationing will occur (Shih and ReVelle, 1994). Nevertheless, system managers, being risk-averse and eager to preserve their system against failure, are willing to incur smaller water shortages rather than one catastrophic shortage. The maximisation of the indicator $\alpha_i^k$, becomes therefore, the primary objective in the management of the WRS in terms of satisfying water demand. Consequently, from an optimisation point of view, a model incorporating a general rule similar to that of the S-shaped curve rule was developed. With the model, a set of rationing schedules and their associated $\alpha_i$ values that minimise the maximum daily shortfalls can be surveyed. The model is formulated as:

1) Define: 
   \[ S_{i,0}^{k,\text{min}} = \min S_{i,0} \quad k = 1, \ldots, m; \quad i = 1, \ldots, n \]  
   \[ (3.5a) \]
   where $k$ represents a planning horizon;
   $i$ represents an individual reservoir in the system;
   $m$ represents the number of planning horizons;
   $n$ represents the number of reservoirs in the system.

2) subject to:
   \[ S_{i,0}^{k} = S_{i,0} \quad k = 1, \ldots, m; \quad i = 1, \ldots, n \]  
   \[ (3.5b) \]
   \[ S_{i,t+1}^{k} = A_s S_{i,t}^{k} + B_s W_{i,t}^{k} + D_s R_{i,t}^{k} \]  
   \[ k = 1, \ldots, m; \quad t = 0, \ldots, T-1; \quad i = 1, \ldots, n \]  
   \[ (3.5c) \]
   where
   \[ r_{i,t}^{k} (t, S_{i,t}^{k}) = \min \{ N_{i,t}^{k} (S_{i,t}^{k}), R_{i,t}^{\rightarrow} \}, \]  
   \[ k = 1, \ldots, m; \quad t = 0, \ldots, T-1; \quad i = 1, \ldots, n \]  
   \[ (3.5d) \]
   where $T$ represents the length of a planning horizon.
   \[ S_{i,T}^{k} \geq S_{i,0} \quad k = 1, \ldots, m; \quad i = 1, \ldots, n \]  
   \[ (3.5e) \]

* For convenience the expression $\alpha_i^k R_{i,j}^\rightarrow$ instead of $f(\alpha_i^k)R_{i,j}^\rightarrow$ will be used throughout the rest of the thesis.
Equation (3.5a) is a definitional constraint and outlines the interest in finding the minimum initial storage that, together with the operating rule \( r^k_i(t, S^k_i) \), can lead to the determination of the optimum water rationing trigger value(s) and the associated reduced water demand. Equation (3.5b) specifies that the initial storage \( S_{i,0} \) must be the same for all planning horizons \( k \). Equation (3.5c) is the continuity constraint relating reservoir storages and controlled outflow in any period to the inflow volumes. Equation (3.5d) specifies that when the reservoir storage of the current period is less than the reference release, the controlled outflow is made equal to the outflow given by the control structure open-gate stage-discharge function. And when the reservoir storage is greater than or equal to the reference release, the draft is made equal to the reference release. Equation (3.5e) represents the optimum terminal (end-of-the-planning-horizon) constraint on storage. It does not allow for the borrowing of water from the initial storage during the interval of operation.

The model facilitates the determination of the ideal end of planning horizon storage (Equation 3.5a). Its solutions can be obtained by simulating the reservoir behaviour with an initial storage condition \( S_{i,0} \) and the operating rules \( r^k_i \) for all inflow sequences \( \{w^k_i\} \) of the reference set \( W^{* r-1}_0 \). Satisfaction of constraints (3.5d) and (3.5e) implies that \( S_{i,0,\min}^k \leq S_{i,0} \); otherwise \( S_{i,0,\min}^k \geq S_{i,0} \). Therefore, a very simple one-dimensional searching procedure can be used for the determination of the value \( S_{i,0,\min}^k \). The searching procedure used is described in Sub-section 5.3. \( S_{i,0,\min}^k \) represents the preferred or ideal end-of-the-planning-horizon target storage. Once the value of \( S_{i,0,\min}^k \) has been determined, the problem of determining the effective daily water releases \( r^k_{i,t,\min} \) that can ensure the satisfaction of all daily water demand constraints throughout the rest of the year, after a given day \( t=\tau \), can be formulated as described below. The storages associated with \( r^k_{i,t,\min} \) correspond to the quantitative values of the water reduction trigger signal.

**Problem \( \tau \)**

Determine:

\[
r^k_{i,\tau,\min} = \min \left( r^k_i(\tau, S^k_i) \right) \quad k = 1, \ldots, m; \quad i = 1, \ldots, n
\]

\[
\tau = 1, \ldots, T-1
\]  

Subject to:

\[
S^k_{i,\tau} = S_{i,\tau} \quad k = 1, \ldots, m; \quad i = 1, \ldots, n
\]  

(3.6a)
\[ S_{l+1}^k = A_k S_l^k + B_k W_l^k + D_k r_i^k (t, S_l^k) \]
\[ k = 1, \ldots, m; \quad t = \tau, \ldots, T-1; \quad i = 1, \ldots, n \]  
(3.6c)

\[ r_i^k (t, S_l^k) = \min \{ N_i^k (S_l^k), R_i^k \} \]
\[ k = 1, \ldots, m; \quad t = \tau, \ldots, T-1; \quad i = 1, \ldots, n \]  
(3.6d)

\[ s_{i, t}^k \geq s_{i, 0, \text{min}}^k \quad k = 1, \ldots, m; \quad i = 1, \ldots, n \]  
(3.6e)

Similarly, the concept used in finding \( s_{i, 0, \text{min}}^k \) can be used to find the solution \( r_i^k_{t, t, \text{min}} \).

It is worthwhile observing that, \( s_{i, 0, \text{min}}^k \) is used in Equation (3.6e) to constrain the end-of-planning-horizon storage. Equation (3.6e) imposes that daily storage, all year-through, should be properly constrained so that the end-of-the-year storage is greater than or equal to the minimum storage. That is to say, the end-of-the-planning horizon storage should always exceed or equal \( s_{i, 0, \text{min}}^k \). This is to ensure the satisfaction of: 1) the efficient water demand requirements year-through and 2) the optimum end-of-the-planning-horizon storage constraint. And to allow an effective solution to the problem of water demand requirement all year-through and under any hydrological event. Therefore, the problem of determining \( r_i^k_{t, t, \text{min}} \) should always succeed that of \( s_{i, 0, \text{min}}^k \). Furthermore, solving “problem \( \tau \)” can be carried out independently for each value of \( \tau \). The values of all the \( r_i^k_{t, t, \text{min}} \) will be used to determine the effective water rationing trigger parameter, \( \alpha_i^k \), solution, and its associated reduced water demand \( \alpha_i^k R_i^k \). In other words, the determination of the level of storage at which to start rationing, and the corresponding portion of water demand to be met.

3.3.1.2.1. Determination of the flood indicator \( \beta_i^k \)

The second objective is the attenuation of the storage peak or flood control. This second objective is in conflict with the first. For a quantitative description of this objective, it was assumed that the no-damage-causing maximum storage \( S_{i, t}^k \) (i.e., flood control storage) is known. The value \( S_{i, t}^k \) is referred to as the reference storage, and must, preferably, be much lower than the maximum capacity of the reservoir (Orlovski et al., 1984). It may for instance correspond, in the case where the reservoir-system is already in operation, to the lower level of the determined flood conservation storage zone or any other
given or determined value considered adequate to contain the worst of the worst experienced flood. Therefore, as above, the maximum-yearly-value of the ratio between actual and reference storage, i.e.,

$$\beta_i^k = \max(s_{i,t}^k/s_{i,t}^r)$$

$$0 \leq t \leq T-1$$

can be used as a meaningful indicator of flood damages suffered by the system users and as a parameter in the determination of: 1) the quantitative value(s) of the effective signal(s) that triggers storage conservation during flood or period of impending flood to prevent future flood damages, and 2) the associated quantity of controlled water to spill. Thus, with the determination of the effective ratio value $\beta_i^*$ (see Figure 3.3), if the storage at period $t$, is greater than the corresponding flood conservation storage trigger value, then controlled spills can begin. But, if the storage at period $t$, is less than the corresponding flood conservation storage trigger value, only the full targeted outflow can be released from the reservoir. As discussed earlier system manages would rather incur smaller floods than one catastrophic flood, the second goal in the management of the WRS becomes the minimisation of the indicator $\beta_i^k$.

The solution indicator $\beta_i^k$ can be determined through the formulation and solving of the problems stated below. As above, the first objective is to determine the ideal maximum initial storage $s_{i,0,max}^k$, such that during each daily operation (from day one to the last day of the planning horizon), daily flood storages are properly constrained such that the end-of-each-planning-horizon storage is always less than or equal to $s_{i,0,max}^k$ (Equation 3.8g). This is to ensure the determination of an efficient operating rule that would guarantee good performance of the system in any year and under any hydrological conditions similar to that of the reference set (see sub-section 4.4.1). Therefore, the problem consists of finding:

$$s_{i,0,max}^k = \max s_{i,0}^k \quad k = 1, \ldots, m \quad (3.7a)$$

Subject to:

$$s_{i,0}^k = s_{i,0} \quad k = 1, \ldots, m; \quad i = 1, \ldots, n \quad (3.7b)$$

$$s_{i,t+1}^k = A_i s_{i,t}^k + B_i w_{i,t}^k + D_i r_{i,t} \quad (3.7c)$$

$$r_{i,t} (s_{i,t}^k) = N(s_{i,t}^k) \quad k = 1, \ldots, m; \quad t = 0, \ldots, T-1; \quad i = 1, \ldots, n \quad (3.7d)$$
\[
0 \leq r_{ij}(S_{ij}) \leq R_{i,\text{max}} \quad k = 1, \ldots, m; \ t = 0, \ldots, T-1; \quad i = 1, \ldots, n \quad (3.7e)
\]
\[
S_{ij}^k \leq S_{i,\text{max}} \quad k = 1, \ldots, m; \ t = 0, \ldots, T-1; \quad i = 1, \ldots, n \quad (3.7f)
\]
\[
S_{ij}^k \leq S_{i,0} \quad k = 1, \ldots, m \quad i = 1, \ldots, n \quad (3.7g)
\]

Equation (3.7e) represents the physical limitation on water release. It imposes that, any volume of water released must be positive, and less than or equal to \( R_{i,\text{max}} \). \( R_{i,\text{max}} \) represents the allowable maximum amount of water that can be released in any day through the reservoir control structure gates without causing any damage downstream. Therefore, whenever the value \( N(s_{ij}) \), representing the open-gate stage-discharge (see Equation 4.3), is greater than \( R_{i,\text{max}} \), the draft is made equal to \( R_{i,\text{max}} \). Equation (3.7d) represents gate controlled release. Equation (3.7f) is the physical limitation on reservoir storage applied to each and all end-of-operating-day \( t \). \( S_{i,\text{max}} \) represents the maximum capacity of the reservoir. Equation (3.7f) ensures that storage does not exceed the maximum allowable level. Equation (3.7g) does not allows the lending of water to the initial storage during the interval of operation.

Similarly to the problem of reduced water demand satisfaction, once the value of \( S_{0,\text{max}}^k \) has been determined, the values \( S_{i,\tau,\text{max}}^k \) (of any day \( t=\tau \)), can be found by solving the following “problem \( \tau \)“. The values of \( S_{i,\tau,\text{max}}^k \) represent the level of maximum storage at which flood attenuation or controlled spill is triggered.

\[
S_{i,\tau,\text{max}}^k = \max(S_{i,\tau}) \quad k = 1, 2, \ldots, m; \quad i = 1, \ldots, n; \quad \tau = 1,\ldots,T-1 \quad (3.8a)
\]

Subject to:

\[
S_{i,0}^k = S_{i,0} \quad k = 1, 2, \ldots, m; \quad i = 1, \ldots, n \quad (3.8b)
\]
\[
S_{i,\tau+1}^k = A_S s_{ij}^k + B_W t_{ij}^k + D_r r_{ij}(S_{ij}) \quad k = 1, 2, \ldots, m \quad t = \tau, \ldots, T-1; \quad k = 1, \ldots, m \quad (3.8c)
\]
\[
r_{ij}(S_{ij}) = N(s_{ij}) \quad k = 1, \ldots, m; \ t = 0, \ldots, T-1; \quad i = 1, \ldots, n \quad (3.8d)
\]
\[
0 \leq r_{ij}(S_{ij}) \leq R_{i,\text{max}} \quad k = 1, \ldots, m; \ t = 0, \ldots, T-1; \quad i = 1, \ldots, n \quad (3.8e)
\]
\[
S_{ij}^k \leq S_{i,\text{max}} \quad i = 1, 2, \ldots, n \quad t = \tau, \ldots, T-1; \quad k = 1, \ldots, m \quad (3.8f)
\]
\[ S_{i,T}^k \leq S_{i,0,\text{max}}^k \quad i = 1, 2, \ldots, n; \quad k = 1, \ldots, m \quad (3.8g) \]

Since \( S_{i,0,\text{max}}^k \) is fundamental for solving "problem \( \tau \)", it should be defined prior to solving problem \( \tau \). \( S_{i,0,\text{max}}^k \) is used as a limiting factor in Equation 3.8g, to guarantee the satisfaction of flood protection requirements all year through. Equation (3.8g) is a definitional constraint and requires that peak flood storage in every period should lead to an end-of-planning-horizon storage less than or equal to the maximum beginning-of-planning-horizon storage \( S_{i,0,\text{max}}^k \). Solving problem \( \tau \) can be carried out independently for each value \( t = \tau \). The values of all the \( S_{i,r,\text{max}}^k \) with their corresponding \( r^k_i(t, S_{i,r,\text{max}}^k) \) can be used to determine the ideal flood indicator, i.e., trigger parameter \( \beta_i^k \). With the determination of \( \beta_i^k \), the maximum storage at which water spilling should be triggered can be determined.

Determination of an efficient operating rule that is appealing to a system manager requires comparison of all the feasible operating rules. It was therefore suggested that the indicators \( \alpha_i^k \) and \( \beta_i^k \) be determined using a set \( W_{(k-1)T}^{kT-1} \) (called the reference set, see Subsection 4.4.1) of k one-year-long daily inflow records or synthetic inflow sequences \( \{W_i^k\} \), i.e., \( W_{(k-1)T}^{kT-1} = \{\{W_i^k\} \mid t = 0, \ldots, T-1; \ k = 1,2,\ldots, m \} \); where \( k \) represents a water year.

The \( \{W_i^k\} \) are maximum and minimum input estimates or other synthesised or suggested particularly troublesome inflows of year \( k \) against which sound operational policies are to be developed. This is to ensure that the resulting operating rules will guarantee satisfactory performance of the system throughout any hydrological event. Therefore, it is preferable to select the most wet and most dry years experienced by the system manager, or any significant flow values suggested by the system manager, to formulate the reference set. Moreover, in so doing, the proposed operating rules can be compared to the performance the system manager was able to achieve in practice. Any optimum operating rules thus determined will certainly be appealing to system managers as they represent particularly robust solutions especially when the system is under stress. Therefore, the purpose in the formulation of an effective reservoir-system operating process, especially for periods of extreme hydrologic inputs, becomes the determination of admissible operating rules which minimise flood damages and water shortages in the worst possible case out of an adequately defined reference set \( W_{(k-1)T}^{kT-1} \). In so doing, focus must be on properly constraining the reservoirs, daily storages throughout the year so that the end-of-the-year storage satisfies the requirements specified in Equations (3.5e), (3.6e), (3.7g) and (3.8g).
Not doing so will result in an inadequate real-life operating process where good performance in one year could imply very poor performance during the next year. The operating rules determined using this approach are time invariant. This is because when solving the problem, constraints are formulated (Equations (3.5a), (3.6a), (3.7a) and (3.8a)) ensuring that the extreme values of the indicators, \(\min^k \alpha^k, \max^k \beta^k\) with \(k = 1, 2, \ldots, m\), that are determined guaranteeing satisfactory performance of the system in one year \(k\) out of the reference hydrologic input \(W^k_{(k-1)r}\) can also be guaranteed in any future year. More accurately, if it is assumed that:

- \(S_{i,0}\) is a set of the initial storages \(s_{i,0}\) of reservoir \(i\) (i.e., \(s_{i,0} \in S_{i,0}\)),
- \(s_{i,t} (s_{i,0}, r_i)\) the storage obtained at time \(t\) of year \(k\) \((k = 1, 2, \ldots, i, \ldots, j, \ldots, m)\) by applying the operating rule \(r_i\) to reservoir \(i\) with initial storage \(s_{i,0}\),
- the daily inflows \(w_{i,0}, w_{i,1}, \ldots, w_{i,T-1}\) belong to the set of reference inflow sequences \(W^k_{(k-1)r}\) (see Section 4.4.1), where \(k = 1, 2, \ldots, i, \ldots, j, \ldots, m; i = 1, \ldots, n\);
- and,
- \(\alpha^k_i (s_{i,0}, r_i)\) and \(\beta^k_i (s_{i,0}, r_i)\) are the corresponding values of water shortage and flood indicators during day \(t\) and year \(k\),

then the terminal (end-of-planning-horizon) constraints can be given in the following form:

\[
\alpha^I[S_{i,T}^I (s_{i,0}, r_i)] \geq \min_{S_{i,0} \in S_{i,0}} \min_{1 \leq k \leq m} \alpha^k(s_{i,0}, r_i) \quad (3.9.a)
\]

\[
\beta^J[S_{i,T}^J (s_{i,0}, r_i)] \leq \max_{S_{i,0} \in S_{i,0}} \max_{1 \leq k \leq m} \beta^k(s_{i,0}, r_i) \quad (3.9.b)
\]

where \(I\) and \(J\) represent any year out of the set \(k\) year.

The statement above is only true when the future hydrologic inputs are not worse than the worst in the reference set. Therefore, if the future hydrologic years are forecasted to be worse than the worst year in the reference year, a new set of reference input data set have to be defined and the process of defining Equations 3.9. a) and b) repeated.

From the above, it is possible to determine feasible and efficient solutions of the two-objective management problem of the WRS during normal, extreme or impending extreme hydrologic input periods. Feasible solutions to the problem are sets of initial storages \(s_{i,0}\), and operating rules \(r_i\) such that the storages and releases, computed by means of Equations (4.1) and (3.1) using the defined reference inflow sequence \(W^k_{(k-1)r}\), satisfy the
Figure 3.3: Efficient, semi-efficient, and dominated solutions in the space \((\alpha_i, \beta_i)\) of the indicators (Reference: Orlovski et al, 1984). The hatched area represents the feasible dominated solutions.

physical constraints (Equation 3.3) and the terminal constraints (Equations 3.9.a and b). An efficient solution on the other hand, is an optimum feasible solution. In other words, a feasible solution becomes an efficient solution when there are no other feasible solutions which can improve the indicators any further without making the optimal condition of the system worse (refer to curve BC of Figure 3.3). Feasible solutions with better values, of both indicators \(\alpha\) and \(\beta\), than that of other particular feasible solutions are called “dominated” (see the internal points of the shaded region of Figure 3.3). Feasible solutions that are neither “efficient” nor “dominated” as showed on segments AB and CD of Figure 3.3, are called “semi-efficient”. These solutions can in fact be improved by improving one of the two objectives without making the other worse. All these solutions are pointed out in the space \((\alpha, \beta)\) of the indicators in Figure 3.3. Also illustrated are the absolute maximum values \(\alpha_{i,\text{max}}\) and the absolute minimum value \(\beta_{i,\text{min}}\) of the indicators for which feasible solutions can be found. The solution at point U (with coordinates \(\alpha_{i,\text{max}}, \beta_{i,\text{min}}\)) is
infeasible because the goals are conflicting there. That point is therefore called the "utopia point". The optimum solution to the problem can be found on the segment BC of Figure 3.3. In order to ensure realistic solutions, the selection of the optimum indicators (say point X on BC segment of Figure 3.3) should be guided by the system operators' experience of the system.

The determination of efficient and semi-efficient solutions to the management problem can thus be achieved by considering two objectives. The first one, described in Sub-section 3.3.2, is called "water demand satisfaction". It consists of determining the solutions \((s_{i,0}^{\alpha}, r_i^{\alpha})\) which satisfy the physical constraints (Equation 3.3) and the terminal constraints (Equations 3.9a) for a selected value, say, \(\alpha_i\) of the indicator

\[
\min_{1 \leq k \leq m} \{\alpha_i'(s_{i,0}, r_i)\}.
\]

Similarly the second problem called "flood protection" and described in Sub-section 3.3.3, consists of determining the solutions \((s_{i,0}^{\beta}, r_i^{\beta})\) which satisfy the physical constraints (Equation 3.3) and the terminal constraints (Equations 3.9b) for a selected value \(\beta_i\) of the flood indicator.

3.3.2. Water demand satisfaction

The solution to the water demand problem is the determination of a set of initial storages and operating rules \((s_{i,0}^{\alpha}, r_i^{\alpha})\) which can guarantee that the determined yearly minimum value of the water reduction indicator \(\alpha_i^k\), will not be smaller than the assigned value \(\alpha_i\) for all inflow sequences of the reference set \(W_{kT-i}^{kT-1}\) of reservoir i. In other words, the operation process becomes the determination of admissible control policies \((s_{i,0}^{\alpha}, r_i^{\alpha})\) that would enable the system manager to operate the system guaranteeing the satisfaction of water demand for essential water needs during drought and impending drought whilst satisfying the preferred water supply during normal hydrological events.

The value of the assigned deficit indicator or water reduction trigger parameter \(\alpha_i\) needs to be sufficiently small to provide for the existence of an effective solution of the problem. One such solution, if it exists, corresponds to the so-called "minimum release policy" \(r_i^{\alpha_i}_{\min}\) which by definition means that the physical constraint specified in Equation (3.3) and described as:
\[ r_{i,t}^k = r_{i,t}^{\alpha, \min} (s_i, s_i^*) - \min \left \{ N_i(s_i^*) \alpha_i R_{i,t}^k \right \} \]  

(3.10)

is satisfied. Therefore the set \((S_{i,0}, r_{i,\min}^\alpha)\) can provide a solution to the problem of water demand satisfaction throughout drought or pre-drought and normal hydrological events. This is provided that the release \(r_{i,t}^k = r_{i,t}^{\alpha, \min} [t, s_i, s_i^*] \) never drops below the reduced water demand \(\alpha_i R_{i,t}^k\), and that the terminal constraint (Equation 3.4a) is satisfied for all inflow sequences out of the reference set \(W_{(k-1)T}^{kT-1}\).

Moreover, the function \(\alpha_i^k(s_i, r_i)\) does not decrease with \(s_{i,0}\), consequently, for any set \(S_{i,0} = [s_{i,0}^*, s_{i,0}]\), where \(s_{i,0}^*\) is the top of the inactive storage and \(S_{i,0}\) any initial storage, there is:

\[
\min_{1 \leq k \leq m} \left \{ \alpha_i^k (s_{i,0}, r_i) \right \} = \min_{s_{i,0} \in S_{i,0}} \left \{ \alpha_i^k (s_{i,0}, r_i) \right \}
\]

Therefore, the feasibility of a set \(S_{i,0} = [s_{i,0}, s_{i,0}]\) verifies the feasibility of the set \([s_{i,0}, \infty]\), and Equation (3.9.a) is equivalent to

\[ S_{i,T}^k(s_{i,0}, r_i) \geq S_{i,0} \]

\[ S_{i,T}^k(s_{i,0}, r_i) \geq S_{i,0} \]

\[ k = 1, \ldots, m; \quad s_{i,0} \in S_{i,0} \]

or

\[ s_{i,T}^k(s_{i,0}, r_i) \]

since \(s_{i,T}^k(s_{i,0}, r_i)\) is also a non-decreasing function of \(s_{i,0}\). Consequently, the objective is to find the minimum initial storage, \(s_{i,0, \min}\), which combined with the operating rule \(r_{i,\min}^\alpha\), can guarantee the satisfaction of the reduced water demand \(\alpha_i R_{i,t}^k\) throughout the year. In other words, the objective is the determination of the optimum minimum initial storage with its associated rationing schedules that would minimise the maximum of the daily shortfalls. The choice of minimising the maximum daily shortfall was made because losses are convex in shortfall (Shih and ReVelle, 1994), and as a consequence, minimisation of total shortfall is unlikely to minimise losses.

The minimum storage \(S_{i,0, \min}\) can be determined by solving the following simple mathematical programming problem. The concept of the mathematical program is an extension of that developed by Orlovski et al. (1984) and included in its general and original form in this thesis as Appendix A.
3.3.2.1. Determination of $s_{i,0,\min}^\alpha_i$

Define

$$s_{i,0,\min}^{\alpha_i} = \min s_{i,0}$$

$k = 1, \ldots, m; \ i = 1, \ldots, n$ \hspace{1cm} (3.11a)

Subject to:

$$s_{i,0}^k = s_{i,0}$$

$k = 1, \ldots, m; \ i = 1, 2, \ldots, n$ \hspace{1cm} (3.11b)

$$s_{i,t+1}^k = A_{i} s_{i,t}^k + B_{i} w_{i,t}^k + D_{i} r_{i,\min}^\alpha_i^k (t,s_{i,t}^k)$$

$k = 1, \ldots, m; \ t = 0, \ldots, T-1; \ i = 1, 2, \ldots, n$ \hspace{1cm} (3.11c)

$$r_{i,\min}^\alpha_i^k (t,s_{i,t}^k) = \alpha_i R_{i,t}$$

$k = 1, \ldots, m; \ t = 0, \ldots, T-1; \ i = 1, 2, \ldots, n$ \hspace{1cm} (3.11d)

where $s_{i,T}^k \geq s_{i,0}$ is the surrogate of the terminal constraint (Equation 3.9a). Equation (3.11e) does not allow the borrowing of water from initial storage during the interval of operation. The problem can be solved by simulating the behaviour of the reservoirs with the initial storage condition $s_{i,0}$, and the operating rule $r_{i,\min}^\alpha_i$ for all inflow sequences $\{w_{i,t}^k\}$ of the reference set $W_{(k-1)T}^{kT}$. Satisfaction of constraints (3.11d) and (3.11e) implies

$$s_{i,0,\min}^{\alpha_i} \leq s_{i,0}; \text{ Otherwise } s_{i,0,\min}^{\alpha_i} \geq s_{i,0}.$$  

Therefore, a very simple one-dimensional searching procedure can be used to determine the value $s_{i,0,\min}^{\alpha_i}$. The searching procedure used is described in Sub-section 5.3.

For WRS, the function $N(s_{i,t}^k)$ is non-linear and in the form of a look-up table. $N(s_{i,t}^k)$ represents the permissible releases through the control structure gates and is a function of available water storage. Therefore, determination of $s_{i,0,\min}^{\alpha_i}$ becomes a non-linear programming problem.

The problem of finding the daily minimum storages, $s_{i,0,\min}^{\alpha_i}$, where water rationing can be triggered guaranteeing - in combination with the minimum release policy stated in Equation (3.11) - the satisfaction of all water demand constraints throughout the rest of the year (after day $T$), can be formulated as below (Sub-section 3.3.2.2).
3.3.2.2. Determination of $S_{\alpha_i,r,min}$ ($\tau = 1,2,...,T-1$)

The determination of $S_{\alpha_i,r,min}$ can be formulated as follows:

$$S_{\alpha_i,r,min}^{\tau} = \min_i S_{i,\tau} \quad i = 1, ... , n; \quad \tau = 1, ... , T-1$$  (3.12a)

Subject to:

$$s_{i,\tau}^k = S_{i,\tau} \quad k = 1, ... , m; \quad i = 1, ... , n$$  (3.12b)

$$s_{i,\tau+1}^k = A_i s_{i,\tau}^k + B_i w_{i,\tau}^k + D_i \alpha_{i,\tau,min}(t, s_{i,\tau}^k)$$  

$$k = 1, ... , m; \quad t = \tau, ... , T-1; \quad i = 1, ... , n$$  (3.12c)

$$r_{i,\tau,min}^k (t, s_{i,\tau}^k = \alpha_i R_{i,\tau}^-$$

$$k = 1, ... , m; \quad t = \tau, ... , T-1; \quad i = 1, ... , n$$  (3.12d)

$$s_{i,\tau}^k \geq S_{i,0,min}^{\tau} \quad k = 1, ... , m; \quad i = 1, ... , n$$  (3.12e)

The technique used in the determination of $S_{\alpha_i,r,min}$ can also be used to find a solution to $S_{\alpha_i,r,min}$. It is worthwhile noticing that the determined value $S_{i,0,min}^{\alpha_i}$ of the previous problem is used in Equation (3.12e). This is to guarantee the satisfaction of the terminal constraint (Equation 3.9a) and the determination of an effective operating rule for a year-round efficient performance of the system. The problem of the determination of $S_{i,0,min}^{\alpha_i}$ must then be solved prior to that of $S_{i,r,min}^{\alpha_i}$. Problem of the determination of $S_{i,r,min}^{\alpha_i}$ can be solved independently for each value of $t = \tau$.

3.3.2.3. Summary

Derivation of solutions to the problem of water supply is a function of a solution to the problem of water demand satisfaction out of the reference hydrological conditions $W_{(k-1)T}^{\alpha_i,r-1}$. Such solution will ensure that a volume of water $r_{i,\tau}^k$ greater than

$$r_{i,\tau,min}^k (t, s_{i,\tau}^k = \alpha_i R_{i,\tau}^-$$

can be released without any concern about damage incurred by the system. This is true, provided the reservoir storage and/or the inflow are sufficiently high and greater than $S_{i,0,min}^{\alpha_i}$. Particularly, if during a day $t$ of a year $k$,

$$A_i s_{i,\tau}^k + B_i w_{i,\tau}^k \geq C_i s_{i,\tau+1,min}^{\alpha_i} + D_i \alpha_i R_{i,\tau}^-$$. Then any release $r_{i,\tau}^k$ between $\alpha_i R_{i,\tau}^-$ and the minimum of $(A_i s_{i,\tau}^k + B_i w_{i,\tau}^k - C_i s_{i,\tau+1,min}^{\alpha_i})$ and $N(S_{i,\tau}^k)$, as shown by the shaded area in Figure 3.4, will result in a storage $s_{i,\tau+1}^k$ greater than or equal to $S_{i,\tau+1,min}^{\alpha_i}$, This is a proof that the storage
$S_{i,0}^{\alpha_i, \min}$ is indeed the minimum value of the storage that can guarantee the satisfaction of all the constraints from time $t+1$ to the end of the planning horizon. However, it is only true, if the future hydrological conditions are not worse than those of the set $W_{(k-1)T}^{kT-1}$. In conclusion, the effective solutions to the problem of water demand satisfaction under any hydrological event, including extreme ones similar to or not worse than those of $W_{(k-1)T}^{kT-1}$, can be given by the pairs $(S_{i,0}^{\alpha_i}, R_i)$ such that:

$$S_{i,0}^{\alpha_i} = \{ s_{i,0} | s_{i,0} \geq S_{i,0}^{\alpha_i, \min} \}$$

$$\min\{N(s_{i,j}^k), \alpha_i, R^i_{j} \} \leq r_i^{\alpha_i}(t, s_{i,j}^k, W_{i,j}^k) \leq \min\{N(s_{i,j}^k), \max\{A_i s_{i,j}^k + B_i W_{i,j}^k - C_i S_{i,j}^{\alpha_i, \min}, \alpha_i, R^i_{j} \} \}$$

where $A$, $B$, $C$, and $D$ represent the matrix coefficients encoding the system layout and the interaction among its constituent elements.

Interpretation of Equation (3.13b) is given in Figure 3.4. The figure shows that, for sufficiently high values of stored water, there is a whole range of possible releases (shaded area). In concordance with the storage allocation zone, developed by the U.S. Army Corps of Engineers (see Sub-section 3.2), the storage axis of Figure 3.4 is divided into four storage allocation zones named I, II, III, and IV respectively. The first zone (I) depends only on $\alpha$ since its upper limit $s_{i,j}^f$ represents water rationing trigger signal. At that level, the stage discharge function value $N(s_{i,j}^f)$ equals the reduced water demand $\alpha_i R_{i,j}^-$. Zone I will never be entered if the inflow sequences $\{W_{i,j}^k\}$ are similar to those of the reference set $W_{(k-1)T}^{kT-1}$. That is, the zone I represents a kind of inactive storage similar to that described by the US Army Corps of Engineers (1982). This zone might nevertheless be entered during real-life operation if a drought more severe than those considered in the reference set occurs (Orlovski et al., 1984). In that case, the volume of water drafted will be equal to the minimum of the stage discharge function value $N(s_{i,j}^f)$ and the reduced water demand $\alpha_i R_{i,j}^-$. While water releases in zones III and IV can exceed $\alpha_i R_{i,j}^-$, those of zone II are equal to the reduced water demand $\alpha_i R_{i,j}^-$. In fact, in zone IV, not only a volume of water greater than the full reference water demand can be drafted, but the manager might even completely open the gates of the control structure without inducing any operational failure of the system in terms of water demand satisfaction (Orlovski et al., 1984).
From Figure 3.4 it is obvious that, while the line \( \alpha_i R_{i,t} \) and the curvilinear line \( N(S_{i,t}) \) are fixed in time and space, the straight line \( r_{i,t}^k = A_s S_{i,t}^k + B_s W_{i,t}^k - C_s S_{i,t+1,\text{min}} \) varies in time dependent on the inflow \( W_{i,t}^k \). When \( W_{i,t}^k \) increases the straight line \( r_{i,t}^k \) correspondingly shifts to the left, inducing changes in the values \( S_{i,t}^{\prime} \) and \( S_{i,t}^{\prime\prime} \), dividing zones III and IV. It can therefore be concluded that Equation 3.13b defines, a priori, only the inactive zone, while the other zones are functions of the current period total net inflow.
value. As a result, particular care must be taken to achieve a relatively accurate forecasted daily inflows $W_{t}^k$, in real-life operational management of the system.

### 3.3.3. Attenuation of storage peaks

In the problem of flood protection (also called attenuation of storage peaks), the goal is to find for a set $S_{i,0}^\beta$ of initial storages and operating rules $r_i^\beta$, a reference storage $\beta_i S_{i,t}^*$ which can guarantee minimization of flood hazard for future years with inflows similar to or not worse than those of $W_{(k-1)t}^{k-1}$. The solution to the problem is therefore, the determination of a set of daily initial storages and operating rules, $(S_{i,0}^\beta, r_i^\beta)$, which can guarantee, for each year $k$ out of the reference set, that the storage $s_{i,t}^k$ at the end of any day $t$, will not exceed the reference storage $\beta_i S_{i,t}^*$. Naturally, solutions to the problem exist only if the value of $\beta$ is sufficiently high. Furthermore, if such a solution exists, then the so-called "maximum release policy", $r_{i,max}$ (independent of $\beta$) given by:

$$r_{i,t} = r_{i,max}(s_{i,t}^k) = N(s_{i,t}^k)$$

(3.14),

would also be a solution for the same initial storages $S_{i,0}^\beta$. It can therefore, be concluded that, any set $(S_{i,0}^\beta, r_{i,max})$ represents a solution provided the highest storage contained in $S_{i,0}^\beta$ does not exceed a determined maximum initial storage $S_{i,0,max}^\beta$. The following mathematical programming problem called "problem 0" can be solved to obtain $S_{i,0,max}^\beta$.

#### 3.3.3.1. Problem 0 (Determination of $S_{i,0,max}^\beta$)

Similar to the preceding Sub-section (3.3.2.1), the problem consists of determining $S_{i,0,max}^\beta$ as follows:

$$S_{i,0,max}^\beta = \max_{S_{i,0}} s_{i,0}^k \quad k = 1, \ldots, m; \quad i = 1, \ldots, n$$

(3.15a)

Subject to:

$$s_{i,0}^k = s_{i,0}^k \quad k = 1, \ldots, m; \quad i = 1, \ldots, n$$

(3.15b)

$$s_{i,t}^k = A_s s_{i,t}^k + B_r r_{i,max}^k (s_{i,t}^k) + C_w W_{i,t}$$

Once the value of $s_{0,\max}^\beta$ has been determined, the value of $s_{\tau,\max}^\beta$ at which spilling is triggered to prevent future flood damage, can be determined by solving the following problem (as in Sub-section 3.3.2.2):

### 3.3.3.2. Problem $\tau$ (Determination of $s_{\tau,\max}^\beta$, $\tau = 1, 2, ..., T-1$)

Define

$$s_{\tau,\max}^\beta = \max(s_{i,t}) \quad k = 1, ..., m; \quad i = 1, ..., n$$

(3.16a)

Subject to:

$$s_{i,0} = s_{i,0} \quad k = 1, ..., m; \quad i = 1, ..., n$$

(3.16b)

$$s_{i,t+1} = A_s s_{i,t}^k + B_r r_{i,\max}^k (s_{i,t}^k) + C_r W_{i,t}^k \quad k = 1, 2, ..., m; \quad t = \tau, ..., T-1; \quad i = 1, ..., n$$

(3.16c)

$$r_{i,\max}^k (s_{i,t}^k) \leq R_{i,\max} \quad k = 1, ..., m; \quad t = 0, ..., T-1; \quad i = 1, ..., n$$

(3.16d)

$$s_{i,t}^k \leq \beta_{i,t} S_{i,t}^- \quad k = 1, ..., m; \quad t = 0, ..., T-1; \quad i = 1, ..., n$$

(3.16e)

$$s_{i,T}^k \leq s_{\tau,0,\max}^\beta \quad k = 1, ..., m; \quad i = 1, 2, ..., m$$

(3.16f)

Equations (3.15d) and (3.16d) specify the upper limit of the release while Equations (3.15e) and (3.16e) represent constraints on the relaxed reference storage or the trigger level at which flood release should start. They specify that at the end of any operating period (day), the storage of the reservoir should be less than the revised reference storage $\beta_{i,t} S_{i,t}^-$. Those constraints were imposed to prevent spill of water if a reservoir is not above its full capacity (revised reference storage level), since it does not make any sense to spill water at such a time. Indeed, without those constraints spills could occur at any time even if a reservoir's storage is below the relaxed reference storage. Common sense therefore
suggests the prevention of such spills because their occurrence in one day can mean less water than normal released the second day without any benefit incurred. Here also, the end-of-the-planning-horizon storage is effectively constrained in Equations 3.15f and 3.16f. This is to guarantee an effective performance of the system under any hydrological event, especially flood events. Equation 3.15f avoids lending water to the beginning-of-planning-horizon (initial) storage during the interval of operation. Equation 3.16f ensures that the worst of the historically observed daily inflow sequences would just avoid causing flood damage in any day and constrains the end-of-the-planning-horizon storage to a value lower than or equal to the maximum initial storage.

The problems stated above can be solved by simulating the system’s behavior for different prescribed or guided values of the initial storage. As in Sub-section 3.3.2, all the solutions \((S_{i,t}^\beta_i, r_{i,t}^\beta_i)\) can be derived from the reference solution defined through solving the above problems. If all the future solutions are such that \(r_{i,t}^\beta_i \leq r_{i,max}(S_{i,t})\) then the performance of the system would not deviate from the reference solution. This can only be true provided that the reservoir is sufficiently empty and/or the inflow is sufficiently low. In other words, if, in a year \(k\) and day \(t\)

\[
A \cdot S_{i,t}^k + B \cdot W_{i,t}^k \leq C \cdot S_{i,t+1,max}^\beta_i + D \cdot N(S_{i,t}^k)
\]

(where \(A, B, C,\) and \(D\) represent matrix coefficients encoding the system layout and the inter-action among its constituent elements), then any water release \(r_{i,t}^k\) between

\[
\max\{0, A \cdot S_{i,t}^k + B \cdot W_{i,t}^k - C \cdot S_{i,t+1,max}^\beta_i\} \text{ and } N(S_{i,t}^k), \text{ with } N(S_{i,t}^k) \leq R_{i,max}, \text{ as depicted}
\]

in the shaded area of Figure 3.5, will give rise to a storage \(S_{i,t+1}^k\) smaller than or equal to \(S_{i,t+1,max}^\beta_i\). By definition \(S_{i,t+1,max}^\beta_i\) is the maximum value of the storage at time \(t+1\) that can guarantee the satisfaction of the constraints from time \(t+1\) up to the end of the planning horizon. In summary, any pairs \((S_{i,t}^\beta_i, r_{i,t}^\beta_i)\) defined as below (Equations (3.17a) and (3.17b)), are solutions to the problem of flood protection:

\[
S_{i,t}^\beta_i = \{S_{i,t}^0: 0 \leq S_{i,t}^0 \leq S_{i,max}^\beta_i\}
\]

\[
\min\{N(S_{i,t}^k), R_{i,max}, \max\{A \cdot S_{i,t}^k + B \cdot W_{i,t}^k - C \cdot S_{i,t+1,max}^\beta_i, 0\}\}
\]

\[
\leq r_{i,t}^\beta_i(t, S_{i,t}^k, W_{i,t}^k) \leq \min\{N(S_{i,t}^k), R_{i,max}\}
\]

Figure 3.5 gives a full interpretation of Equations 3.17a and b. In that Figure, the storage axis is divided into three possible release zone: namely I, II, III. In zone I any water release decision is possible. The system manager might even want to store all the incoming
Figure 3.5: The set of releases that can guarantee flood protection. (Reference: Orlovski et al., 1984)
inflows for future use. This can be done by closing all the gates of the control structure without any worry of worsening the future operational performance of the system or causing any flood damage (Orlovski et al., 1984). In zone II, although different options might still be possible, the manager may yet have to be more cautious and aware of potential flood occurrence when the storage and/or inflows increase. Lastly, in the third zone, which can adequately be called the "spilling zone", the manager is forced to release the maximum allowable volume of water through effective monitoring of the control structure gates. In real-life operation however, the manager may have to release more water than the maximum allowable if a flood more severe than that considered in the reference set occurs. The spilling zone is similar in concept to that described in the concept of reservoir zoning developed by the US Army Corps of Engineers (1982).

3.4. Feasible solutions to the combined problem of water demand satisfaction and flood control.

By 1) taking the intersection of the sets defined by Equations (3.13a) and (3.17a); and by Equations (3.13b) and (3.17b); and by 2) suitably re-arranging the various terms, it can be demonstrated that any pair \((S_{i,0}^{\alpha \beta}, R_{i,0}^{\alpha \beta})\) such that:

\[
S_{i,0}^{\alpha \beta} = \{S_{i,0}^{a}, S_{i,0 \text{min}}^{a} \leq S_{i,0}^{a} \leq S_{i,0 \text{max}}^{a}\}
\]

\[
\min(N(S_{i,t}^{k}, R_{i,\text{max}}), \max\{A \cdot S_{i,t}^{k} + B \cdot W_{l,t}^{k} - C \cdot S_{i,t+1 \text{max}}^{\alpha \beta} \cdot R_{i,t}^{\alpha \beta}\})
\leq r^{\alpha \beta} (t, S_{i,t}^{k}, W_{l,t}^{k})
\leq \min(N(S_{i,t}^{k}, R_{i,\text{max}}), \max\{A \cdot S_{i,t}^{k} + B \cdot W_{l,t}^{k} - C \cdot S_{i,t+1 \text{min}}^{\alpha \beta} \cdot R_{i,t}^{\alpha \beta}\})
\]

is a feasible solution of the problem described in Sub-section 3.3.1.

Interpretation of Equation 3.18b, represent the constraints on the feasible operating rules, is given by Figure 3.6. In the Figure, the storage axis is divided into six zones. The first, (I), and the last, (VI), representing the "inactive" and "spilling" zones respectively, while the second zone, (II), is similar to the "buffer zone". At the beginning of zone II, i.e., point \(S_i^m\) (just below storage \(S_i^{\alpha \beta, \text{min}}\)), water rationing will be triggered and the manager would have no other options but to release the reduced water demand \(\alpha_i R_i\) to provide for essential water needs. Following the buffer zone is the "conservation zone", \((S_i^m < S_i < S_i^k)\). In this zone which is further subdivided into three sub-zones (III, IV, and V), the system manager is presented with a wide range of possible water releases. He/she can freely choose, out of this range of possible water releases, the desired volume of water
to release without concern about causing any damaging effect to the system and its performance. Moreover, the three sub-zones of the conservation zone can be used to illustrate the declining importance of water demand satisfaction versus flood protection. While moving from sub-zone III to V the possible volume of water to release increases as storage increases. The increase of storage may rapidly lead the water level to the top of the conservation zone (top of zone V) or into the spilling zone (zone VI: above storage $S_{i,t}^{\beta_{i,t},\text{max}}$). Therefore, in zone V, it is possible and certainly wise to release water at the maximum allowable rate, thus preventing the occurrence of future floods. Inversely in zone III, while the storage is decreasing toward the bottom of the zone, water releases tend toward the reduced reference release. Therefore, to save water to compensate for possible future periods of low inflows, it is possible and preferable to release a volume of water equal to the reduced water demand. In zone IV on the other hand, either of the aforementioned management policies is allowed. Consequently, an ideal operating condition would be to have, at all times, all the reservoir storages in zone IV.

The straight lines $A. S_{i,t}^{k} + B. W^{k}_{i,t} - C. S_{i+1,t}^{j}$ and $A. S_{i,t}^{k} + B. W^{k}_{i,t} - C. S_{i+1,t}^{\alpha_{i},\beta_{i}}$ will become one $A. S_{i,t}^{k} + B. W^{k}_{i,t}$ line if ever the pair $(\alpha, \beta)$ cannot be guaranteed at all. This obviously can happen if $S_{i,t}^{\alpha_{i},\text{min}} > S_{i,t}^{\beta_{i},\text{max}}$ on some day $t$. However, if

$$S_{i,t}^{\alpha_{i},\text{min}} \leq S_{i,t}^{\beta_{i},\text{max}} \quad t = 0, 1, \ldots, T-1 \quad (3.19)$$

then feasible solutions to the problem will always exist. It can therefore, be said that the shaded area in Figure 3.6 is more or less sensitive to the time of the year and to the values $\alpha$ and $\beta$ of the two indicators. Moreover, if $\alpha$ and $\beta$ are such that

$$S_{i,t}^{\alpha_{i},\text{min}} < S_{i,t}^{\beta_{i},\text{max}} \quad t = 0, 1, \ldots, T-1$$

then, it can be said that the corresponding solutions given by Equations 3.18 are dominated, since $\alpha$ can be increased and $\beta$ reduced simultaneously until Equation 3.19 is obtained with the equality sign holding in at least one constraint. This can be a useful test for finding efficient or semi-efficient operating rules. However, because of the high workload which might be involved in performing trial and error simulation to finding the solution $(\alpha, \beta)$ such that the inequality in Equation 3.19 is satisfied with an equality sign holding in at least one constraint, a more direct search method has been devised and described in the following section.
3.5. Efficient solutions to the combined problem of demand satisfaction and flood control.

In this sub-section, description of a simple and more direct search method for finding efficient and semi-efficient solutions to the two-objective problem outlined in Sub-section 3.3.1 is given. The method is divided into two steps. The first step consists of computing, for a given value $a_i^*$ smaller than or equal to $a_{i,\text{max}}$ (see Figure 3.3), the corresponding minimum value $\beta_i^*(a_i^*)$ of the second indicator $\beta_i^*$; while the second step consists of using Equation (3.18) to determine the feasible solutions $(S_{i,\text{min}}, \beta_i^*(\alpha_i^*), r_i^*(\alpha_i^*), \beta_i^*)$. The determined solutions, as illustrated by points X and Y in Figure 3.3, are either efficient or semi-efficient. A similar procedure starting from a given value $\beta_i^*$ of the flood indicator could also be followed to first determine the corresponding maximum value $\alpha_i^*(\beta_i^*)$ of the deficit indicator, and secondly to determine by Equation 3.18, the solutions $(S_{i,\text{max}}, \alpha_i^*(\beta_i^*), r_i^*(\alpha_i^*), \beta_i^*)$. As above, these solutions are either efficient or semi-efficient (as illustrated by points X and Z of Figure 3.3). Consequently, to define if a solution is efficient or semi-efficient, one would only need to use sequentially the two procedures described above. For instance, if the first procedure applied with $\alpha_i^* = \alpha_{i,X}$ gives, as illustrated by X on Figure 3.3, $\beta_i^*(\alpha_{i,X}) = \beta_{i,X}^*$. And if the second procedure applied with $\beta_i^* = \beta_{i,X}^*$ gives for $\alpha_{i,X}^*$ the same value from which the process started, namely $\alpha_{i,X}$, then it can be declared with certainty that the set $(\alpha_{i,X}^*, \beta_{i,X}^*)$ is an efficient solution. However, if one starts from $\alpha_{i,Y}$, point Y on Figure 3.3 will first be obtained, but then point B, with $\beta_{i,Y}^* = \beta_{i,\text{min}}^*$, will be generated out of the second procedure. This proves that $(\alpha_{i,X}^*, \beta_{i,\text{min}}^*)$ is not an efficient solution.
Figure 3.6: The set of releases that can guarantee satisfaction of demand and flood protection at the same time (not scaled). (Reference: Orlovski et al., 1984).
Since the second step of the method of finding efficient and semi-efficient solutions has already been discussed in Sub-section 3.4, only the first step will be described in the following. For this, an assumption is made that a value \( \alpha_i^* \) of the deficit indicator is given. The operating rules which can ensure satisfaction of the reduced water demand \( \alpha_i^*, R_{i,t}^- \), can therefore be defined through Equation 3.13b. In particular, among all the operating rules that can be defined by Equation 3.13b, the operating rule \( r_{i,\text{max}}^* \) corresponding to the right-hand side of Equation 3.13b, ie.,

\[
r_{i,\text{max}}^*(t,s_{i,t}^k, w_{i,t}^k) = \min \{ N(s_{i,t}^k), R_{i,\text{max}}, \max \{ A_i s_{i,t}^k + B_i w_{i,t}^k - C_i a_i^* R_{i,t}^- \} \}
\]

is the one which minimises the flood indicator \( \beta_i \). Consequently, the following simple mathematical programming problems can be formulated to determine \( \beta_i^*(\alpha_i^*) \):

\[
\beta_i^*(\alpha_i^*) = \min \beta_i \quad i = 1, \ldots, n
\]

Subject to:

\[
s_{i,0} = s_{i,0} \quad k = 1, 2, \ldots, m; \quad i = 1, \ldots, n
\]

\[
s_{i,0} \geq s_{i,0}^\alpha_i^* \quad i = 1, \ldots, n
\]

\[
s_{i,t+1}^k = A_i s_{i,t}^k + B_i w_{i,t}^k - C_i a_i^* r_{i,\text{max}}^*(t,s_{i,t}^k, w_{i,t}^k) \quad k = 1, 2, \ldots, m; \quad t = 0, \ldots, T-1; \quad i = 1, \ldots, n
\]

\[
r_{i,\text{max}}^*(t,s_{i,t}^k, w_{i,t}^k) \leq R_{i,\text{max}} \quad k = 1, 2, \ldots, m; \quad t = 0, \ldots, T-1; \quad i = 1, \ldots, n
\]

\[
s_{i,t}^k \leq \beta_i s_{i,t}^- \quad k = 1, 2, \ldots, m; \quad t = 0, \ldots, T-1; \quad i = 1, \ldots, n
\]

\[
s_{i,t}^k \leq s_{i,0} \quad k = 1, 2, \ldots, m; \quad i = 1, \ldots, n
\]

In the above formulated problems, constraint (3.21c) is needed to guarantee Equation 3.13a, whilst constraint (3.21g) guarantees the satisfaction of the terminal condition stated in Equation 3.9b. Solutions to the set of problems 3.21 can be obtained by simulating the multi-reservoir system behaviour with initial storages \( s_{i,0}^\alpha_i^* \) and operating rule \( r_{i,\text{max}}^* \) for all inflow sequences \( \{ w_{i,t}^k \} \) of the reference set. For any fixed value \( s_{i,0}^\alpha_i^* \), the solution of the set of problems 3.21, if it exists, is given by (see 3.21f):

\[
\beta_i^*(\alpha_i^*|s_{i,0}^\alpha_i^*) = \max_{1 \leq k \leq m} \max_{0 \leq t \leq T-1} \left( \frac{s_{i,t}^k}{s_{i,t}^-} \right)
\]
where $s_{i,t}^k$ are the values obtained by simulation and $\beta_i^*(\alpha_i^*) \leq \beta_i^*(\alpha_i^*|s_{i,0})$. Therefore, if all constraints (3.21g) are satisfied with the strict inequality sign, then all the $s_{i,t}^k$ (and therefore $\beta_i^*(\alpha_i^*|s_{i,0})$) can be lowered by lowering the initial storage $s_{i,0}$. If the above situation arises, the behavior of the reservoir system must repeatedly simulated with a smaller value of the initial storage until at least one of the $m$ constraints (3.21g) is satisfied with the equality sign. The resulting value of $\beta_i^*(\alpha_i^*|s_{i,0})$ is the solution $\beta_i^*(\alpha_i^*)$.

### 3.6. Conclusion

This section gives a description of the mathematical model to be used in the study. The algorithms of the model are similar to those developed by Orlovsky et al. (1983) in their min-max approach to reservoir storage control. Considering the relationship between the approach developed here and that put forward earlier by Orlovsky et al. (1983), the earlier approach, although developed for multi-purpose reservoir storage control, only focused on a single-reservoir system. Thus, it merely guarantees the effective performance of a single-reservoir without looking into the complexity of two-reservoir system inter-reservoir relationships. The extra complexity of the present approach has been introduced to formalise a realistic two-reservoir system management approach.

The implementation of the min-max approach developed for this study is characterised by two simulation stages. The first stage is to be performed at the onset of the control process and consists in determining the “effective” degrees of satisfaction of the goals. This will be done by first solving, for a given sequence of flows, the mathematical-programming problems termed “Problems 0”. The sequence of flows, also referred to as the reference set of inflows, consists of $k$ one-year-long flow values given by the system manager, or determined from historical data. These values are particularly well-suited for testing the reliability of any operation rule and the future performance of the system.

The second stage involves solving a sequence of mathematical-programming problems termed “Problems $\tau$”. Solutions to “Problems $\tau$” will be used as constraints in the real-time mode of operation of the system. These constraints are dependent upon the information obtained from solving Problems 0. The two above-defined problems can be solved by using a simulation technique. The results will be used, as a range of solution constraints, to determine through an optimisation or simulation or a combination of a simulation and optimisation programming model, the real-time efficient day-to-day operational policy of the system. A fuzzy logic controller concept is suggested for the task.
Other important properties of the min-max approach are:

1. Firstly, that the operating rules can be interpreted in terms of storage allocation zones or more properly, in terms of probable release zones. In the most general cases, four zones can be identified - dead (or inactive), buffer, conservation, and spilling zones. The conservation zone can be further sub-divided into three zones (Figure 3.6).

2. Secondly, that the boundaries between the zones defined above are not fixed a priori as is the case with the present mode of operation of the WRS. They vary as a function of the volume of daily initial storage water plus the forecast daily inflows. This property recognises a precise role for real-time inflow predictors, thus making effective use of any similarity between future, current, recent past and historical hydrological events. This suggests a totally new way of daily operating the WRS.

3. Thirdly, and indeed most importantly, that whenever the storage is in the conservation zone, the manager can select any value of the release within the prescribed set of the possible releases. This introduces an operational flexibility that will certainly be welcomed by the WRS’ managers, who are not always interested in just satisfying optimum power generation, but also considering the effective satisfaction of other (secondary) objectives.
CHAPTER 4

DESCRIPTION OF STORAGE CONTROL VARIABLES

4.1. Introduction

Smoothing out the variability of water flow through control and regulation to make it available when and where it is needed is one of the primary functions of reservoir system management. A typical example is the regulation of lakes where the runoff of the upstream catchment is stored and used to satisfy water supplies, hydro-power generation, irrigation, recreational uses, maintaining environmental and ecological qualities, and other activities to downstream users while avoiding too large floods or water shortage at the site. In this chapter, the characteristics of storage control problems will be described.

In Section 4.2, the definition of the planning horizon is given while in Section 4.3, the state and the constraints on control are formulated. Section 4.4 describes how decisions on operating the system are made as a function of information available to the operator and how the inflows are evaluated.

4.2. The planning horizon and water year

In this study the planning horizon is characterised by a finite but very large number of “time intervals” indexed by \( t = 0, 1, 2, ..., \tau, \tau + 1, ..., T \). The time-interval \( t \) represents a day and the horizon a year with \( T \) days. Therefore \( T \) is equal to 365 (or 366). The starting month of the planning horizon determines the distribution of annual flows and some of their statistical characteristics. Different starting months have been used in other studies. Some examples are the calendar year; the water year with various starting months depending on climatic or geographic zone, and country; and the financial year. From a water resources management point of view, the water year is often the best choice in describing the planning horizon.

In their early works McMahon and Mein (1986), Kritsky and Menkel (in Svanidze, 1980) pointed out that the generic elements of stream flow should influence the selection of a water year. They suggested that the separation point of successive water years must be where the connection between years appears to be weakest. As a consequence, they advised against starting the year during a period of likely high inflow. Their arguments were that,
for some years, more than one flood event could occur partly or completely during the high period, while in other years no flood events might occur. Adopting Kritsky and Menkel's (in Svanidze, 1980) suggestion, Linsley and Franzini (in McMahon and Mein, 1986) recommended that annual values of stream flow should represent a period beginning and ending during a time of low flow. Following the suggestion of Kritsky and Menkel's (in Svanidze, 1980) and the recommendation of Linsley and Franzini (in McMahon and Mein, 1986) a planning horizon extending from July each year to June the following year was adopted here. This is because, low inflows in the WRS occur in winter with its minimum usually at the end of June and the beginning of July (see Figures 4.1 and 4.2).

Figures 4.1, 4.2 and 4.3 of the inflow patterns in the WRS, illustrate that indeed there is a good correlation with the decision made in terms of the chosen planning horizon.

Figure 4.1. Lake Te Anau monthly inflow (Reference: Waiau River: Report to the Waiau River Working Party by Riddell, Freestone and Nutting, 1993). The inflow data shown were recorded from 1927 to 1991. The power station became operational from 1970.
Figure 4.2. Lake Manapouri monthly inflow (Reference: Waiau River: Report to the Waiau River Working Party by Freestone and Riddell, 1993). The inflow data shown were recorded from 1933 to 1991. The power station became operational from 1970.

Figure 4.3. Mararoa river monthly flow (Reference: Waiau River: Report to the Waiau River Working Party by Freestone and Riddell, 1993). The flow data shown were recorded from 1977, when the Lake Manapouri control structure became operational, to 1991 inclusive.

4.3. State equation and control constraints

Implicit in effective reservoir-system management is an understanding of the water balance of catchments and the reservoirs in them. The water balance or continuity of a multiple reservoir system for a given planning horizon can be formulated as follows:
\[ S_{i,t+1} = F(S_{i,t}, r_{i,t}, W_{i,t}, t) = A_s S_{i,t} + B_w W_{i,t} + D_r r_{i,t} \quad t = 0, 1, ..., r, r + 1, ..., T \] (4.1)

where

\[ F(\cdot) \] is a transformation operator,
\[ S_{i,t} \] is the state variable representing reservoir storage at the beginning of the \( t \)th day \((t=0, 1, ..., r, r + 1, ..., T)\) in reservoir \( i \) \((i=1, 2, ..., )\),
\[ r_{i,t} \] is the control variable representing water releases from reservoir \( i \) during day \( t \),
\[ W_{i,t} \] is the input variable representing inflows into reservoir \( i \) during day \( t \),
\[ A, B, \text{ and } D \] are matrix coefficients encoding the system layout and the interaction among its constituent elements.

The problem under consideration is focused on the determination of operation rules of the form:

\[ r_{i,t} = r(t, S_{i,t}, W_{i,t}), \quad 0 \leq t \leq T \] (4.2)

with \( r_{i,t} \) subject to the control constraint

\[ 0 \leq r_{i,t} \leq N(S_{i,t}), \quad 0 \leq t \leq T \] (4.3)

where

- \( r(\cdot) \) represents the release function,
- \( N(\cdot) \) is the stage-discharge function and represents a given function of water release versus reservoir storage.

In this study, an assumption is made that the function \( N(\cdot) \) is differentiable such that

\[ N(0) = 0, \quad 0 \leq \frac{d}{ds}(N(S_i)) < 1, \quad S_i \geq 0 \] (4.4)

Properties (4.4) imply that \( N(S_i) \leq S_i \) for all \( S_i \geq 0 \); where \( S_i \) is the storage (amount of water) in the reservoir \( i \). Consequently, with the non-negativeness of the supplies (the inflows of water) and the initial storage, it can be guaranteed that \( S_{i,t} \geq 0 \) for any \( t > 0 \).

4.4. Information structure and control laws

4.4.1. Information structure

In developing the conceptual foundation of the reservoir control approach proposed in this study, a multi-stage decision-making process was designed. The process was designed to mimic the operators’ decision and monitoring process. It was perceived that
for any day, \( t \), the operator chooses a value of the corresponding control variable \( r_{i,t} \) on the basis of the information available. In other words, during any day \( t \), the operators make decisions according to the following information: 1) the state of the system i.e., the existing reservoir storage and channel flows, 2) the prediction of the net inflows during day \( t \), and 3) the pre-defined operation policy. With this information they endeavour to derive the best control laws for day \( t \).

It is also assumed that during any day \( t \), the operators model their knowledge with regard to possible future values of the inflows from day \( t+1 \) up to infinity, in the form of a set of infinite sequences of inflows. This set of inflows can be denoted by \( \mathcal{W}_{t+1}^{\infty} \) and its elements by \( \mathcal{W}_{t+1}^{\infty} = (w_{t+1}, w_{t+2}, \ldots) \). Similar notation \( \mathcal{W}_{t+1}^{T} \) \((T \geq t)\) can be used to describe a set of finite sequences of inflows \( \mathcal{W}_{t+1}^{T} = (w_{t+1}, w_{t+2}, \ldots, w_{T}) \) within a planning horizon of length \( T \).

In order to give a description of the sets \( \mathcal{W}_{t+1}^{\infty} \), a "reference set" \( \mathcal{W}_{0}^{T-1} \), used to generate the elements of the set \( \mathcal{W}_{t+1}^{\infty} \), is introduced. \( \mathcal{W}_{0}^{T-1} \) consists of sequences of supplies, each of length \( T \). As specified earlier (see Section 4.2) \( T \) represents the length of each planning horizon or seasonal fluctuation of inflows. Orlovski \textit{et al.}, (1984) in their study stated that detailed analyses made in the fields of management science have proved that decision makers seldom consider long-term expected values of physical and/or economical indicators as representative measures of system performance. They continue by demonstrating that, indeed, very often managers only focus their attention and effort on avoiding dramatic failures when the system is under stress. This suggests that in real-world systems, most decision-makers are risk-averse even if this entails a worse average performance of systems under their management. The management of the WRS is no exception. As a matter of fact, the aim of this study is to promote the pronounced interest in avoiding severe failures of the system during extreme hydrological events like those of the years 1988 and 1992. Consequently, the set \( \mathcal{W}_{0}^{T-1} \) should be defined using:

1. past observation of inflows for a number of years that include extreme hydrologic events (most severe drought and flood) together with
2. sequences of inflows generated using statistical techniques and/or
3. possibly some hypothetical sequences of inflows that the decision-maker considers to be particularly well suited for testing the reliability of any operating rule.
Therefore, the input sequence elements of $W^{*T-1}_0$ must be selected with particular care as the solutions depend on them. For convenience the planning horizons, years, will be numbered with the integer variable $k=1,2,\ldots$. The structure of the sets $W_{t}^{*}$ can be described as follows. For any day $t$ of the form $t = (k-1)T$ (beginning of a year), the associated set $W_{(k-1)T}^{*}$ consists of an infinite series of inflow events from the reference set $W_{0}^{*T-1}$. In essence, this means that only the a priori information contained in the reference set will be useful for the decision maker at the end of any year when modelling the inflows of the following year. This signifies

$$W_{(k-1)T}^{kT-1} = W_{0}^{*T-1}, \quad W_{(k-1)T}^{*} = W_{0}^{*}, \quad k \geq 1; \quad T = 365 \text{ or } 366 \quad (4.5).$$

Another assumption was that, at any day, $t$, the observations made in terms of water inflow into the reservoir system by the decision makers from the beginning of the year up to a day $t$, reduce the risk of over- or under-estimation of future inflows for the remaining part of the same year. Therefore, for any year $k \geq 1$ and day $t \in \{(k-1)T, kT - 1\}$ the set $W_{i=1}^{kT-1}$ may contain only some of the sub-sequences belonging to the set $W_{0}^{*T-1}$. In conclusion, for any $k \geq 1$ and $t \in \{(k-1)T, kT - 1\}$ the set $W_{i=1}^{*}$ is the ensemble of all sequences resulting from linking together the series of a sequence of the set $W_{i=1}^{*T-1}$ with a sequence of set $W_{0}^{*}$.

Assuming that the decision maker has perfect foresight of the set $W_{0}^{*T-1}$ at the beginning of the control process in year $k$, then the information available to him at any day of that year can be denoted by the tuple $(t, s_i, w_i, W_{t+1}^{kT-1})$.

### 4.4.2. Operating rule

The control law or operating rule in this study is perceived as a rule for calculating a value of a controllable volume of water to be released from the reservoir-system on any day $t \in \{(k-1)T, kT - 1\}$, $k \geq 1$, and for any tuple of information described above. Therefore, an operating rule can be described by the function $r(t, s_i, w_i, W_{t+1}^{kT-1})$, where $r$ is used as its short notation and $r_t$ as the short notation for the value of this function corresponding to a fixed value of $t$ and the other variables.
4.5. Summary

Through this section, it was shown that some reference inflow data sets $W^{*T-1}_0$ composed of either: 1) the combination of the historical data set, some statistically defined inflow data, and some inflow set suggested by the decision makers against which they intend to protect the system, or 2) a combination of two of the above, is needed in the development of the model. The inflow sequences in the data sets $W^{*T-1}_0$ are assumed to represent minimum and maximum input estimates or other extreme levels against which any sound operational policy is to be developed. Therefore, any operational management policy developed using these input data sets will certainly be able to cope with future net inflow not worse than those of $W^{*T-1}_0$. It is expected that operational management in any year using this model will present some advantages compared to those obtained by means of stochastic methods.
CHAPTER 5

APPLICATION OF THE STORAGE CONTROL COMPUTER MODEL

5.1. Introduction

In this chapter the developed min-max storage control model is applied to the WRS. The techniques used to select the reference storage and releases, and the critical hydrologic years and data, are also described. The stochastic nature of hydrologic events implies that a deterministic formulation of the reference water flows and lake levels would yield no explicit statement of reliability with which the system operator will meet the effective performance of the system in the future. Therefore, an implicit reliability chance constraint approach was adopted in the determination of Te Anau reference releases and Lakes Te Anau and Manapouri reference storages. The Manapouri reference releases were suggested by ECNZ based on the corporate management policy. The effective water rationing and flood flow allocation storage pool triggering parameters in the event of extreme hydrological events were also defined. The values thus defined are used to initiate, as a function of the above-mentioned and determined and/or suggested reference values, the development of the effective operating rules of the WRS. A sensitivity analysis was also conducted to define the economic value of operating the reservoirs in the system at different maximum and minimum storage levels.

5.2. Determination of the reference releases and storage of the system

As described in Chapter 3, the min-max storage control approach involves the combination of two solution steps in one integrative mode of operation. Achievement of this task requires the selection or determination of the preferred releases (reference releases), and reference storages.

5.2.1. Determination of reference releases

5.2.1.1. Lake Manapouri proposed reference releases

It is important that in any comprehensive approach to WRS management, environmental needs are considered. The question is however, “how?” One may ask if it is absolutely necessary that environmental requirements be fulfilled in their entirety regardless of any other conditions and at any price? Or if a trade-off between them and
other conditions is possible? The answers are numerous and differ from system to system and from manager to manager. One certain thing however, is the necessity to incorporate them in any comprehensive or multi-objective planning or operational management process of any reservoir-system. Therefore, it is suggested that they be used as constraints imposed on operation rules. In that context, the Electricity Corporation of New Zealand, ECNZ, which operates the WRS, proposed an integrative approach considering wildlife and fish habitat, flow regimes, flood control, water supply, hydro-electric power generation and water-based recreation activities. This is to find a way to conduct truly comprehensive basic operational management on a broad basis that balances the environmental objectives against their economic interests in producing electrical energy for sale. Consequently, after extensive community consultation and careful assessment of scientific studies, ECNZ proposes to instigate for future management, the following flow regimes (Figure 5.1), for the power station and the Lower Waiau River below Manapouri lake control structure:

![Flow Regimes Graph](image)

Figure 5.1: Reference releases of Lake Manapouri. The date represents Month (1 to 12), Week (first to fifth), and Day (Monday being day 1 and Sunday day 7).

1. A flow not less than 460 m$^3$/s through the power station at any given time.
2. A continuous minimum flow of 12 cumecs from 1 May to 30 September, 14 cumecs in April and October, and 16 cumecs at all other times. The objective is to: 1) enhance the fisheries habitat, 2) reduce nuisance algal growths, 3) improve water quality, and 4) restore some aspects of the river landscape (ECNZ, 1996).
3. Two “flushing” flows, of not less than 35 cumecs for 24 hours, released during the winter months of June and August.
4. One “river mouth opening” flow per year of not less than 150 cumecs for 24 hours during autumn (March to May) and further such flow during spring (September to November) each year. These flows will be released only if necessary to ensure the mouth of the river is sufficiently open to enable passage of migratory fish during autumn and spring (Southland Regional Council, SRC, 1996). This provision is subject to compliance with gazetted guidelines for release of such flow.

5. Seven recreational flows of not less than 35 cumecs, for 24 hours, released on the fourth Sunday of each month between October and April inclusive or such alternative dates as are agreed with the SRC. Two of these flows may be increased to not less than 45 cumecs for specific events (such as jet boating) subject to ECNZ’s ability to comply with the Lake guidelines.

5.2.1.2. Determination of Lake Te Anau reference releases

5.2.1.2.1. Introduction

Lake Te Anau control structure is operated so as to provide sufficient water for the efficient operation of Lake Manapouri. Consequently the determination of the preferred releases from Te Anau are not only a function of the inflows into Te Anau from its own catchment, Te Anau storage and the preferred water releases from Te Anau during periods of trout spawning, but also of the inflows into Manapouri from its own catchment (ie., precipitation, local tributary streams), the volume of water diverted into Manapouri from the Mararoa river, the storage volumes in Manapouri, and most importantly, Lake Manapouri water supply satisfaction. Therefore the accuracy with which the effective daily water releases from Te Anau, that optimise Lake Manapouri daily operational capacity, is obtained is primarily a function of the future Manapouri reference releases.

5.2.1.2.2. Selection of Te Anau reference releases

As specified in Sub-section 3.3.1, the goal of water demand satisfaction of the WRS as a whole is to find an operating rule that satisfies the maximisation of the ratio, $\alpha$, between Manapouri daily releases and its reference releases during periods of extremely dry hydrological events (drought). Optimum performance occurs when the ratio $\alpha$, which ranges from zero to one, is equal to one. Therefore, assuming a scenario dry hydrological
event and considering the constraints on Manapouri storage, where the end-of-the-day storage cannot be less than a defined minimum allowable storage (see Sub-section 2.4.1.1), an optimum operation rule can be defined if and only if:

\[ s_{3,t+1} \geq s_{3,\text{min}} \]  \hspace{1cm} (5.1)

where \( s_{3,\text{min}} \) is the Manapouri absolute (allowable) minimum storage volume.

Adopting the symbols and continuity defined in Section 2.4, the water balance of Lake Manapouri can be written as:

\[ s_{3,t+1} = s_{3,t} + w_{3,t} + d_{2,t} + r_{1,t} - r_{3,t} \]  \hspace{1cm} (5.2)

and yields a Manapouri total daily release of:

\[ r_{3,t} = s_{3,t} + w_{3,t} + d_{2,t} + r_{1,t} - s_{3,t+1} . \]

Substituting \( s_{3,t+1} \) into Equation 5.1 yields:

\[ s_{3,t} + w_{3,t} + d_{2,t} + r_{1,t} - r_{3,t} \geq s_{3,\text{min}} \]

or

\[ s_{3,t} + w_{3,t} + d_{2,t} + r_{1,t} - \alpha R_{3,t} \geq s_{3,\text{min}} \]  \hspace{1cm} (5.3)

where \( \alpha R_{3,t} = r_{3,t} \) and \( R_{3,t} \) is the Manapouri desired reference release. Assuming \( \alpha = 1.0 \) (i.e., optimum operation), Equation 5.3 becomes

\[ s_{3,t} + w_{3,t} + d_{2,t} + r_{1,t} - R_{3,t} \geq s_{3,\text{min}} \]

and by re-arrangement, yields

\[ R_{3,t} \leq w_{3,t} + d_{2,t} + r_{1,t} + s_{3,t} - s_{3,\text{min}} \]  \hspace{1cm} (5.4)

where \( R_{3,t} \) and \( s_{3,\text{min}} \) are deterministic values and \( r_{1,t}, d_{2,t}, w_{3,t} \) are independent random variables. In the case of a dry hydrological event scenario in Manapouri and an optimum operation, \( s_{3,t+1} \) will be equal to \( s_{3,\text{min}} \), and therefore, Equation 5.4 will yield:

\[ R_{3,t} \leq w_{3,t} + d_{2,t} + r_{1,t} \]  \hspace{1cm} (5.5)

This indicates that water demand requirements in extremely dry hydrological events and/or when the reservoir storages are very low, would be met if and only if the inequality in Equation 5.5 is true. In practice however, future reservoir inflows are not known with certainty. There is therefore, no absolute guarantee that in the future the sum of controlled and un-controlled inflows \( (w_{3,t} + d_{2,t} + r_{1,t}) \) will satisfy the inequality in Equation 5.5. As a generally held concept, the most reliable indicator of future inflows is the recorded historical inflow data series. The longer the record in general the more severe the observed extremes of the hydrological events and the lower the probability of violating the constraints in practice (ReVelle, Joeres and Kirby, 1969). In New Zealand, weather
patterns are unpredictable even in the short term (a few days), and there is no reliable method of predicting them in the longer term on which to base operation decisions’ (Electricity Shortage Review Committee, 1992). Deriving from all these reasons, one may estimate that, for n years of historically recorded inflows, there is a probability \( p \) that during each day \( t \), daily release from a reservoir will lie outside the range of the n years of recorded inflows (ReVelle, Joeres and Kirby, 1969). Therefore, in the absence of any information to the contrary, one might expect that in the future, the probability of not fulfilling the Manapouri daily water demand or reference release requirements (see Equation 5.5) would also be equal to \( p \). These observations suggest that a deterministic formulation of the Manapouri future inflows would certainly result in two shortcomings. First, the deterministic formulation will surely yield no explicit statement of the reliability with which the system will meet the preferred performance in the future. Second, the system reliability would have been fixed fortuitously by the specific postulated input sequence (ReVelle, Joeres and Kirby, 1969).

Equation 5.5 is now restructured in a stochastic formulation. Hydrological events are stochastic in nature, and therefore, inflows into Manapouri from: its own catchment, Mararoa streamflow, and Te Anau controlled releases (regulated since 1970), are only known, on a particular day, with some probability. That is, in day \( t \) of the year a preferred Manapouri inflow is treated as a random variable \( R_{3,t}^- \) having the cumulative probability distribution

\[
F_{W3,t + d_{3,t} + w_{3,t}} (R_{3,t}^-) = P[W_{3,t} + d_{3,t} + w_{3,t} \geq R_{3,t}^-] \quad (5.6)
\]

The constraints on the inflow in Equation 5.6 are expressed as limitations on the allowable risk of violating the system’s performance requirements. The objective of Equation 5.6 is therefore, to assess from the historical data, and with a special emphasis on the Te Anau controlled release, the reliability with which Manapouri total daily mean inflow will exceed the given reference release. Equation 5.6 can then be transformed into:

\[
P[W_{3,t} + d_{3,t} + r_{1,t} \geq R_{3,t}^-] = P[r_{1,t} \geq R_{3,t}^- - (d_{3,t} + w_{3,t})]
\]

\[
= F_{R_{3,t}^-} (R_{3,t}^- - (d_{3,t} + w_{3,t})) \quad (5.7)
\]

Equation 5.7 represents the probability distribution of controlled releases from Te Anau.

If in addition to the deterministic value \( R_{3,t}^- \), arbitrary values of \( d_{3,t} \) and \( w_{3,t} \) are proposed, then an estimate of this probability (Equation 5.7) can be obtained from the empirical frequency function of Te Anau historically observed daily release data. The closeness of this estimate to unity will indicate sufficient availability of water from Te Anau in any day to satisfy preferred supply. And as a consequence, the proposed values of
$d_{2,t}$ and $w_{3,t}$ will be acceptable from the water demand satisfaction point of view. If, however, this probability is less than some specified value $\psi$, then the proposed values of $d_{2,t}$ and $w_{3,t}$ will not provide sufficient dependable water supply into Manapouri. The choice of $\psi$ is that of the river-system manager/operator.

It is not necessary to use trial and error to determine the set of $(d_{2,t}, w_{3,t})$ pairs yielding sufficiently reliable water demand satisfaction. This is because the set already contains all $(d_{2,t}, w_{3,t})$ pairs for which the inequality

$$F_{r_{1,t}}(R_{3,t} - (d_{2,t} + w_{3,t})) \geq \psi$$

is true. This inequality may be called a chance constraint on $d_{2,t}$ and $w_{3,t}$. For mathematical programming purposes, it is preferable to rewrite the chance constraint in the form of its certainty equivalent:

$$(R_{3,t} - (d_{2,t} + w_{3,t})) \geq \alpha_{1,t}(\psi)$$

where $\alpha_{1,t}(\psi)$ is the 100$\psi$ percentile of the controlled Te Anau outflow $r_{1,t}$ solution of $(R_{3,t} - (d_{2,t} + w_{3,t}))$ for the inequality $F_{r_{1,t}}(R_{3,t} - (d_{2,t} + w_{3,t})) \geq \psi$.

Consequently the solution values of $d_{2,t}$ and $w_{3,t}$ must satisfy the constraint:

$$R_{3,t} - \alpha_{1,t}(\psi) \geq d_{2,t} + w_{3,t}.$$  \hspace{1cm} (5.9)

This probability representation is adopted because of the diverse advantages it can provide. It can for instance enable the formulation of an effective solution to the problem of “the impossibility of absolutely ensuring a specific performance of a reservoir fed by random inputs” (ReVelle, Joeres and Kirby, 1969). But the level of reliability at which each requirement is satisfied might be under the direct control of the system operator. The probability representation attaches a statement of reliability to the mathematical representation of water demand satisfaction requirements (ReVelle, Joeres and Kirby, 1969). Another related advantage of the probability representation is the clarity with which it presents the operational significance of a decision rule. It emphasises that a preferred or reference release rule is merely an aide to an operator’s judgment in deciding how much to release during a period $t$. And that if the rule is followed, the release commitment will be compatible with the reservoir performance requirements with a specific degree of reliability (ReVelle, Joeres and Kirby, 1969).
5.2.1.2.2.1. Waiau river system lakes and river outflow and inflow distributions

Conventional cumulative distribution function plots of: daily releases from Te Anau; daily inflow into Manapouri (excluding Te Anau outflow); Mararoa river at Cliffs gauging station streamflow; and the total inflow into the Waiau river-system as a whole were obtained (Figures 5.2, 5.3, 5.4, 5.5).

Flows in the system have been subjected to control since the Manapouri Power Station became operational in September 1969. The system’s lakes’ storage capacities have been increased and control structures built to control upstream flows to cater for hydro-electricity generation. This has resulted in an alteration of the natural pattern of the flows. The increased storage capacity of the lakes has induced a marked shift in the daily means of water flows. This is as a result of hydro-electric reservoirs artificially retaining water during periods of abundance in order to supply peak power to a national or sub-regional network during periods of high energy need. Thus the periods 1932-1969 and 1970-1996 can be assumed to have each, different but stationary characteristics of flows.

Figures 5.3 and 5.6 show that since 1970 inflows greater than 250 cumecs have occurred for a higher percentage of time in Te Anau than before 1970, and that in Manapouri, the inflows have also increased. These correspond with the predominance of high annual inflows in the 1970s and 1980s. In addition to the obviously higher inflows in the system, the most significant features are:

- The lower proportion of high and low inflows to Lake Manapouri. Inflow greater than or equal to 600 cumecs was common in Lake Te Anau compared to Lake Manapouri. This is due to the fact that Te Anau has a larger river-fed catchment area compared to Manapouri. Hence, low inflows into Manapouri can be augmented by the flow from Te Anau in periods of dry spells. This makes decisions of how much water to release from Te Anau very important to the overall performance of the system.

- The inflow and outflow distributions of the entire recorded historical data (from 1932 to 1996 inclusive) and of the recorded data pre (years 1932 to 1969) and post (years 1970 to 1996) the construction of the power station are similar. As discussed earlier the study is concerned in determining effective operating policies that will complement or improve the existing operating rules. It is therefore considered appropriate to use only the post power station (after 1970) flow distribution to define solutions for future operation.
Figure 5.2: Lake Te Anau outflow distribution.

Figure 5.3: Lake Manapouri local inflow distribution.

Figure 5.4: Mararoa river at Cliffs streamflow distribution.
Figure 5.5: Waiau river system total inflow distribution.

Figure 5.6: Lake Te Anau inflow distributions
Figures 5.7 and 5.8 show that outflows from Lakes Te Anau and Manapouri pre- and post-control have similar seasonal patterns. This shows volume conservation and limited controllable storage as carry-over month to month.
5.2.1.2.2. Selection of the preferred Te Anau controlled outflows

To solve Inequality 5.9 in sub-section 5.2.1.2.2, the constraint on $d_{2,t}$ specified in Sub-section 2.4.1.4 was applied. Figure 5.4 shows that in any day $t$, it is 77 percent likely that $d_{2,t}$ will not exceed $40 \text{ m}^3/\text{s}$. With this value of $d_{2,t}$ ($d_{2,t} \leq 40 \text{ m}^3/\text{s}$) assumed fixed a priori for each day $t$ of the year, and a given percentile of the probability distribution of the daily mean outflow from Te Anau; a value of $w_{3,t}$ can be determined and its probable daily distribution $\varphi$ assessed. From Figures 5.2 and 5.3, Table 5.1 is therefore derived. Table 5.1 includes rows “$\psi$”, “$w_{3,t}$” and “$\varphi$”.

**Table 5.1:** Percentiles of the probability distributions of daily outflow from Te Anau and their corresponding percentiles of the probability distribution of Manapouri local inflows.

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$w_{3,t}$</th>
<th>$\varphi$</th>
<th>$R_{3,t}^{-} - \sigma_{1,t}(\psi)$ ($R_{3,t}^{-} = 474 \text{ m}^3/\text{s}$)</th>
<th>$R_{3,t}^{-} - \sigma_{1,t}(\psi)$ ($R_{3,t}^{-} = 495 \text{ m}^3/\text{s}$)</th>
<th>$R_{3,t}^{-} - \sigma_{1,t}(\psi)$ ($R_{3,t}^{-} = 505 \text{ m}^3/\text{s}$)</th>
<th>$R_{3,t}^{-} - \sigma_{1,t}(\psi)$ ($R_{3,t}^{-} = 610 \text{ m}^3/\text{s}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi = 0.05$</td>
<td>507</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.10$</td>
<td>431</td>
<td>44</td>
<td>64</td>
<td>74</td>
<td>179</td>
<td></td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
<td>4</td>
<td>24</td>
<td>34</td>
<td>139</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-</td>
<td>0.98</td>
<td>0.95</td>
<td>0.918</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.15$</td>
<td>388</td>
<td>86</td>
<td>107</td>
<td>117</td>
<td>222</td>
<td></td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
<td>46</td>
<td>67</td>
<td>77</td>
<td>182</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-</td>
<td>0.80</td>
<td>0.686</td>
<td>0.60</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.20$</td>
<td>361</td>
<td>113</td>
<td>134</td>
<td>144</td>
<td>249</td>
<td></td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
<td>73</td>
<td>94</td>
<td>104</td>
<td>209</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-</td>
<td>0.64</td>
<td>0.50</td>
<td>0.45</td>
<td>0.175</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.25$</td>
<td>340</td>
<td>134</td>
<td>155</td>
<td>165</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
<td>94</td>
<td>115</td>
<td>125</td>
<td>230</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>-</td>
<td>0.50</td>
<td>0.40</td>
<td>0.35</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.30$</td>
<td>320</td>
<td>154</td>
<td>175</td>
<td>185</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
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<td>264</td>
<td>285</td>
<td>295</td>
<td>400</td>
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</tr>
</tbody>
</table>
In rows "ψ", the second column contains the values $\varphi_{1,t}(ψ)$ representing the reference values that, in any given day, Te Anau mean daily outflow can be greater than or equal to. The probability of this happening is "ψ" fraction of time. The other columns of rows "ψ" contain the values $R_{3,t} - \varphi_{1,t}(ψ) = d_{2,t} + w_{3,t}$ associated with $\varphi_{1,t}(ψ)$ for different values of the reference releases $R_{3,t}^-$. $R_{3,t}^- = 474 \text{ m}^3/\text{s}$ in the third column of the top-most row represents the mean of the reference releases 472 m$^3$/s, 474 m$^3$/s, and 476 m$^3$/s (see Bullet 2 of Sub-section 5.2.1.1). The value $R_{3,t}^- = 474 \text{ m}^3/\text{s}$ is used as representative of the Lake Manapouri reference release values specified in Bullet 2 of Sub-section 5.2.1.1, to determine a Lake Te Anau appropriate reference release.

The rows "w$_{3,t}$" contain the reference values that the inflows into Manapouri, from its own catchment, exceed or equal in any day $t$ for $\varphi$ of time. This is assuming $d_{2,t}$ is lower than 40 m$^3$/s.

The values in the rows "φ" represent the percentage of time that the daily Manapouri inflows, from its own catchment, are greater than or equal to the values $w_{3,t}$ in the rows immediately above them. The dashes '-' represent negative or zero values.

Water releases from Te Anau are controllable and inflows into Manapouri from its own catchment are random. Therefore, selection of Te Anau water release policies that will contribute to enhancing the effectiveness of the WRS’s water demand requirements should be such that $\varphi$ is close to unity and yet $\psi$ not too close to zero. As a consequence, an $r_{1,t} \geq 388 \text{ m}^3/\text{s}$ water flow with $\psi = 0.15$ was chosen as the Te Anau controlled release policy that can effectively guarantee water releases from Lake Manapouri to satisfy the

<table>
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<th>0.10</th>
<th>0.095</th>
<th>0.045</th>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$w_{3,t}$</td>
<td>-</td>
<td>434</td>
<td>455</td>
<td>465</td>
</tr>
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<td>$\varphi$</td>
<td>-</td>
<td>0.04</td>
<td>0.03</td>
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</table>
reference releases 474 m$^3$/s, 495 m$^3$/s, and 505 m$^3$/s. The values 495 m$^3$/s, and 505 m$^3$/s represent the Lake Manapouri reference releases specified in Bullets 3 and 5 in Sub-section 5.2.1.1. This is conditional on $w_{3,t}$ being greater than or equal to 46 m$^3$/s, 67 m$^3$/s, and 77 m$^3$/s respectively in any day $t$. These constraints can be achieved with a coefficient of reliability $\phi$ equals 80%, 68.6%, and 60% of the time respectively. For Manapouri reference release $R_{3,t} = 610$ m$^3$/s (see Bullet 4 in Sub-section 5.2.1.1.), a value $r_{1,t} = 431$ m$^3$/s was chosen as the representative effective controlled Te Anau outflow. This outflow can be achieved or exceeded $\psi = 10\%$ of the time, if the Manapouri uncontrolled inflows $w_{3,t} = 139$ m$^3$/s. The $w_{3,t} \geq 139$ m$^3$/s is true $\phi = 31\%$ of time on any given day.

As demonstrated above, in periods of drought, a Te Anau effective water release rule can be determined if and only if the following constraint is satisfied:

$$s_{1,t+1} \geq s_{1,\text{min}} \quad (t=0,1,\ldots,T) \quad (5.10)$$

where $s_{1,\text{min}}$ represents Te Anau minimum allowable storage volume. Deriving $s_{1,t+1}$ from the water balance of Te Anau and substituting it into Equation 5.10, yields

$$s_{1,t+1} = s_{1,t} + w_{1,t} - \alpha R_{1,t} \geq s_{1,\text{min}} \quad (5.11)$$

Assuming Te Anau optimal releases are always achieved (ie., $\alpha = 1.0$ and $s_{1,t} \geq s_{1,\text{min}}$), even during the cases of a worst scenario dry hydrological events, Equation 5.11 will yield:

$$w_{1,t} - R_{1,t} \geq 0$$

or

$$w_{1,t} \geq R_{1,t} \quad (5.12)$$

Similarly, due to the stochastic nature of hydrological events which implies that inflow in Te Anau during a day $t$ of any given year is an independent and random variable, the cumulative probability distribution function will yield:

$$F_{w_{1,t}}(R_{1,t}) = P[w_{1,t} \geq R_{1,t}] \quad (5.13)$$

where the constraint on the inflow is expressed as the limitation on the allowable risk of not being able to discharge the reference release $R_{1,t}$ during a period of drought. Equation 5.13 is therefore the formulated mode of assessing the reliability with which a preferred controlled release from Te Anau can be achieved. Consequently, if an assumption is made that the values of $r_{1,t}$ selected above (ie., 388 m$^3$/s and 431 m$^3$/s (Figure 5.9)) represent the deterministic values of $R_{1,t}$, the probabilities $F_{w_{1,t}}(R_{1,t})$ that inflow into Te Anau will exceed those $r_{1,t}$ values in any day $t$, are 20% and 17.4% respectively.
Figure 5.9: Te Anau reference releases. The date represents Month (1 to 12), Week (first to fifth), and Day (Monday being day 1 and Sunday day 7).

5.2.1.2.3. Conclusion

It can be concluded that an effective operation rule for water demand satisfaction in the system during a period of extremely dry hydrologic events can be obtained that satisfies the Manapouri reference releases specified in Sub-section 5.2.1.1, and provides end-of-the-day storages greater than or equal to the given minimum allowable storage with levels of reliability:

- $\psi = 20\%$ and $17.4\%$ for $w_{1,t} = 388 \text{ m}^3/\text{s}$ and $431 \text{ m}^3/\text{s}$ respectively, and
- $\eta = 80\%$, $68.6\%$, $60\%$ and $31\%$ for $w_{3,t} = 46 \text{ m}^3/\text{s}$, $67 \text{ m}^3/\text{s}$, $77 \text{ m}^3/\text{s}$ and $139 \text{ m}^3/\text{s}$ respectively with the condition that $d_{2,t} < 40 \text{ m}^3/\text{s}$.

The probability that the streamflow in Mararoa river will not exceed $40 \text{ m}^3/\text{s}$, in any given day $t$ out of the recorded data, is $77\%$.

5.2.2. Determination of reference storages $S_i^-$

Determination of a reference storage of a reservoir $i$ in any year $k$ consists in finding a flood storage volume $V_i$, such that at the end of a day $t$ there is a freeboard $C_i - s_{i,t+1}^k$ such that the inequality

$$C_i - s_{i,t+1}^k \geq V_i$$

(5.14)
where \( C_i \) is the capacity of reservoir \( i \), is satisfied. This equates to saying that the decision at the beginning of a day \( t \), on how much water to release during the day, should not lead to insufficient flood storage volume at the end of the day \( t \), given the extreme of the hydrological record. Consequently, in terms of decision-making, to effectively operate a reservoir system, choices must be made that lead to selecting at the beginning of each day those operating rules that result in the end of the day storages satisfying the inequality:

\[
S_{i,\text{min}} \leq S_{i,k} \leq C_i - V_i
\]

(5.15)
given the extremes of the hydrologic record. The longer the record the higher the likelihood of observing a worse extreme hydrologic event that would lead to selecting an appropriate value of \( V_i \) and hence not violating the constraint. Therefore, with over 60 years of recorded data, it can be assumed that the selected \( V_i \) will represent the appropriate value.

### 5.2.2.1. Determination of flood storage volume \( V_i \)

The solution to the problem of peak flood flow attenuation is to provide a flood storage volume such that higher than average inflows can be safely contained. The effective flood storage volume in this study is assumed to be the volume that can contain, by default, the highest ever recorded inflow. This assumption is made as a substitute for an experienced or preferred high inflow that the system manager wants to prevent system operating failure against.

The highest inflow recorded in Te Anau and Manapouri from 1926 to 1995 and from 1932 to 1995 respectively are:

- 5287 m\(^3\)/s, corresponding to a 60 years return period inflow of one day intensity for Te Anau, and
- 3572 m\(^3\)/s, corresponding to a 200 years return period inflow of one day intensity for Manapouri (Riddell, Freestone and Eaton, 1993).

Therefore, the flood storage volumes \( V_i \) for Te Anau and Manapouri can be computed as

\[
V_1 = 5287 \times 86400 = 4.5679 \times 10^8 \, m^3, \quad \text{and} \quad V_3 = 3572 \times 86400 = 3.0862 \times 10^8 \, m^3
\]

respectively.

### 5.2.2.2. Reference storages

Assuming the capacity of each reservoir equals its own maximum allowable storage, \( S_{i,\text{max}} \), the reference storages for Te Anau and Manapouri can be computed as:

\[
S_i = S_{i,\text{max}} - V_i
\]

(5.16)
Equation 5.16 gives for Te Anau and Manapouri \( S^{-}_T = 7.1104 \times 10^{10} \) m\(^3\) and \( S^{-}_M = 1.21275 \times 10^{10} \) m\(^3\) respectively. Conversion of the reference storages into water level (see Tean.dat and Manap.dat in Appendix D) yields the corresponding reference water levels 202.0 m and 176.2 m (above mean sea level, Deep Cove datum) for Te Anau and Manapouri respectively. These values of the reference storages are assumed constant throughout any given year.

Out of the over 60 years of recorded hydrologic events, the probability that storages in the lakes at the end of any given day \( t \) will not exceed the reference storage values determined above are around 44% and 3% for Te Anau and Manapouri respectively (see Figures 5.10 and 5.11). It can therefore be concluded that the probability for the inequality in Equation 5.15 being violated (i.e., flood occurrence), for the determined reference storage above, is lower in Te Anau and higher in Manapouri in any day \( t \). This is due to the difference in storage capacity between Lakes Te Anau and Manapouri and the source of inflow into Manapouri. Lake Te Anau, in addition to having a larger storage capacity compared to Lake Manapouri, releases a large part of its water into Lake Manapouri, which also receives water from its own catchment.

### 5.2.2. 3. Determination of the reference set \( W^{-}_{0} \)

Of the inflows in the WRS, Te Anau and Manapouri catchment inflows represent, a priori, the ones that are indispensable for simulating the system behaviour. Water from Mararoa River is not less important since its water is allowed to be diverted into Manapouri when the river's flow is lower than 40 cumecs (before the 1996 resource consent was granted to ECNZ (SRC, 1996)). Due to the lower rainfall often experienced in the Mararoa catchment, water diverted from the Mararoa River only contributes less than 10% of the Manapouri inflows (Riddell, Freestone and Nutting, 1993). Moreover, when the Mararoa River water is in flood (greater than 40 cumecs before the 1996 resource consent requirements) or reaches 30 NTU (after the 1996 resource consent requirements), it does not contribute to Manapouri inflow. Consequently, in determining the critical inflow sequences or critical hydrologic years of the reference inflow set \( W^{-}_{0} \), the Mararoa River flows were not taken into consideration since they were assumed more or less insignificant for the time being. They were however used in determining the operating rules.
Figure 5.10: Lake Te Anau level distribution.

Figure 5.11: Lake Manapouri level distribution.
Due to the poor quality of the pre-1932 data, the recorded Manapouri catchment inflows are usually taken to start in 1932. As a consequence, the year 1932 is selected as the beginning of the recorded inflow data for the whole system.

Inspection of the annual mean inflows into the lakes (Figures 5.12 and 5.14) indicates for both lakes simultaneously, drier than average hydrologic years in 1937/1938, 1950/1951, 1952/1953, and 1973/1974 and wetter than average hydrologic years in 1957/1958, 1982/1983, 1983/1984, and 1988/1989. Particularly noteworthy is the lowest annual mean value of 1950/1951, and the highest annual mean value of 1957/1958. Significant in the assessment of the severity of a drought or a flood flow during a given period is the duration of such an event within that period. Therefore annual, 6-monthly, monthly and weekly mean inflows into both lakes have been plotted (Figures 5.12 to 5.15) for inspection.

Inspection of the 6-monthly mean inflow plots (Figures 5.13 and 5.14) indicates that the inflow values were:

1. lower than average during:
   - the first half of years 1947, 1951, 1971 and 1974 for both lakes, and
   - the second half of years 1974 and 1976 in both lakes.

2. higher than average during:
   - the first half of the year 1958,
   - the second half of the years 1970, and 1988 in both lakes, and 1936 in Te Anau.

Also noticeable are the peak flood flows in the first half of the year 1983 and the second half of years 1946, 1957, 1980 and 1982 in both lakes. The first half of 1948 in Te Anau and second half of 1969 in Manapouri also show higher than average inflow. The results of this inspection only give an overall picture of the years with critical hydrologic events. More detailed information is required to determine with some precision the critical years needed for simulation of the system behaviour. Consequently, the monthly and weekly mean inflow graphs (Figures 5.12 and 5.15) were inspected. The inspection shows that:

1. The longest floods in terms of an event averaged over one month occurred during the hydrologic years 1957/1958, 1982/1983, 1988/1989 in both lakes. The longest flood flows in terms of weekly mean inflows occurred in both lakes in 1988/1989. Those particular ones represent the highest flood flows in Manapouri over the whole record. The inspection also showed the occurrence of the largest
short duration floods in terms of weekly mean flows in both lakes in 1946, 1967, 1978, and 1983.

2. Monthly mean inflows reach their lowest in 1952/1953, 1970 and 1971 in both lakes, in addition to 1933, 1947, 1965 and 1977 in Manapouri. Further detailed inspection through the weekly mean inflow graphs (Figures 5.13 and 5.15) showed that:

- The 1952/1953 drought was the most severe over a long duration in both lakes, with its lowest inflow having a return period of 100 years (Riddell, Freestone and Nutting, 1993).
- Sustained low inflows over relatively long periods can also be observed during 1974 and 1976. Although those inflows are not the lowest, they represent critical hydrologic events in terms of drought over a longer period.
- Both lakes experienced drought in the first half of 1971.

The inflow sequences of the reference inflow set $W^{*T-1}$ should be those with critically high and low flows against which the operational failure of the system is to be prevented. Consequently:

- 1957/1958 was selected to represent an extremely wet year,
- 1982/1983 to represent a wet year,
- 1988/1989 was selected to represent high inflows preceding average inflows,
- 1952/1953 was selected to represent an extremely dry year,
- an average to high inflows preceding lower than average inflows was represented by 1975/1976, and finally
- 1974/1975 was selected to represent a lower than average preceding an average inflow year.
Figure 5.12: Lake Te Anau monthly and weekly mean inflows. 260 m$^3$/s and 220 m$^3$/s represent the median values of the annual and 6-monthly mean flows of Te Anau respectively. Relatively wet and dry periods are plotted respectively above and below those values.
Lake Manapouri at annual mean flow (cumecs)

Figure 5.13: Lake Te Anau annual and 6-monthly mean inflows. 260 m$^3$/s and 220 m$^3$/s represent the median values of monthly and weekly mean flows of Te Anau respectively. Relatively wet and dry periods are plotted respectively above and below those values.
Figure 5.14: Lake Manapouri annual and 6-monthly mean inflows. 132 m³/s and 128.6 m³/s represent the median values of the annual and 6-monthly mean flows of Manapouri respectively. Relatively wet and dry periods are plotted respectively above and below those values.
Figure 5.15: Lake Manapouri monthly and weekly mean inflows. 120 m$^3$/s and 105.7 m$^3$/s represent the median values of monthly and weekly mean flows of Manapouri respectively. Relatively wet and dry periods are plotted respectively above and below those values.
5.3. The min-max computer model and its application to the WRS

5.3.1. Determination of the minimum and maximum initial storages

5.3.1.1. $\alpha_i^k = \min (\frac{r_{i,t}^k}{R_{i,t}})$ values: search procedure and solutions.

Te Anau reference releases were defined as a function of the Manapouri reference releases (see Sub-section 5.2.1.2.). Manapouri reference releases represent the ECNZ preferred water release to satisfy hydro-electricity generation as well as environmental and non-environmental water needs of the system. Therefore, an assumption can be made that both lakes' reference releases are known values. Furthermore, it can also implicitly be assumed that a Te Anau reference release during any day $t$ will satisfy to the degree of probability specified in section 5.2.1.2.2.2, its corresponding Manapouri water demand or reference release. Consequently, the search for the minimum and maximum initial storages can be done separately for each one of the two reservoirs. It is therefore suggested that, since inflows to Manapouri are mainly composed of the Te Anau controlled releases, solutions are first obtained for Te Anau. The optimum solutions in terms of Te Anau controlled water releases can then be used as controlled inflows into Manapouri in defining Manapouri solutions.

As described in Chapter 3, the problem of determining $\alpha_i^k = \min (\frac{r_{i,t}^k}{R_{i,t}})$ is divided into two sub-problems. The first sub-problem consists of determining the so-called minimum initial storage, $s_{i,0,min}^k$. The technique of determining $s_{i,0,min}^k$ consists of searching for the minima of various individually defined, through computer simulation, beginning-of-planning-horizon initial storage values. The individual initial storage values are defined such that they are bounded by given minimum and maximum allowable storage values (the physical constraints of the reservoir), and are less than or equal to the associated derived last-day-of-the-planning-horizon end-of-the-day storage, ie, end-of-day $t=T$ storage (see Equations 3.5). Importantly, the derived end-of-day $t=T$ storage must also be an active storage, i.e., it must be bracketed by the given allowable minimum and maximum storages. Therefore, in the process of determining $s_{i,0,min}^k$, an initial storage $s_{i,0}$ is first selected at the beginning-of-the-planning-horizon (ie, day $t=1$ of the planning horizon). Then, the developed Fortran computer simulation programs, Tanau_2h.For and Mnpr2_.For for Te Anau and Manapouri respectively, incorporating Equations 3.5c to 3.5e (given on diskette.
in Appendix D), computes each and every one of the end-of-the-day \(t=1,2,\ldots,T\) storage values. The daily storage values must satisfy the physical constraints of the reservoir. If the physical constraints of the reservoir are not satisfied for an end-of-the-day storage or Equation 3.5e is not satisfied at the end of the computation, then the computation is repeated with a lower or higher value of the selected initial storage until the physical constraints are, or Equation 3.5e is satisfied. In other words, the process is cyclic and continues until a solution is obtained. The value or step length \(\Delta x\) that the initial storage is increased by or decreased by is an arbitrary chosen volume of water. This value can be made small or large according to the accuracy and refinement desired. The solution "beginning-of-the-planning-horizon initial storage" such that the inequality in Equation 3.5e and the physical constraints of the reservoir are simultaneously satisfied, is then put in storage and the search process repeated for another pre-defined critical year \(k\), until a solution is obtained for each and all the critical years in the reference set. The search process is repeated until a series of the same solution values of the "beginning-of-the-planning-horizon initial storage" (see Equation 3.5b) is found for all the selected \(k\) critical years. The lowest of those values is selected (see Equation 3.5a) to represent the optimal solution \(S_{t,0,\min}^k\). An effective solution is such that, the inequality "greater than or equal to" in 3.5e can be satisfied with an equality in at least one critical year. The procedure adopted avoids movement in the search process in an infeasible direction. Due to the non-convexity of the constraint set, the solutions obtained by this technique are not guaranteed to be globally optimal. Therefore a number of different starting points were used to make sure that a good solution was found. The defined \(S_{t,0,\min}^k\) and its associated end-of-the-planning-horizon storages for each of the selected \(k\) critical years are recorded in columns (2) and (3) of Tables 5.2 and 5.3.

The computer models developed, and described above, to determine \(S_{t,0,\min}^k\) for Te Anau and Manapouri, are composed of a main program and two sub-routines each. The main program starts by evaluating a selected initial storage to define if it is constrained between some pre-defined physical boundaries. If it is not, a volume of water \(\Delta x\) is added to or subtracted from it accordingly until the physical constraints are satisfied (Figure 5-16). Secondly, the main program calls upon the sub-routines INMSTOR and INISTOR (for Manapouri and Te Anau respectively) to determine, as described in the first paragraph of this sub-section, the solution value \(S_{t,0,\min}^k\) (Figure 5-16). In the search for an end-of-the-day \(t=T\) storage, subroutines INMSTOR and INISTOR call upon subroutines MANPREL
and TEANREL (for Manapouri and Te Anau respectively) to compute, as described in the first paragraph of this sub-section, each day \( t=1, 2, ..., T-1 \) end-of-the-day storage by taking into account: 1) the defined beginning-of-the-planning-horizon initial storage, 2) the given daily total net inflows into the reservoirs, 3) the reservoirs' and rivers' physical constraints, and 4) the environmental and non-environmental requirements. MANPREL and TEANREL determine for each day \( t = 1, 2, ..., T-1 \) of the planning horizon the feasible largest daily controlled outflows that are less than or equal to the corresponding reference releases. The computations in MANPREL and TEANREL proceed such that the end-of-the-day storages satisfy the physical and statutory constraints (see Figure 5-16).

The solution to the second sub-problem consists in determining: 1) the daily largest controlled water releases that are less than or equal to their corresponding daily reference releases, and 2) the associated end-of-the-day storages. In the procedure adopted and described in Figure 5.17, a beginning-of-a-day \( t=\tau, ..., T \) (beginning with day one of the planning horizon) storage value is first selected. Then, s (3.6c) and (3.6d) incorporated in DH_Tanau.for and DH_Mnpr.for (see Appendix D) for Te Anau and Manapouri respectively are used to derive the solution controlled water release and the associated end-of-the-day \( t=\tau, \tau+1, ..., T \) storage. If at the end of the search, the end-of-the-day \( t=T \) (computed from \( t=\tau \)) storage is greater than or equal to the minimum storage, \( S_{i,0,\min}^k \), then the selected beginning-of-day \( t=\tau \) storage and its associated controlled release are solutions. If not, a new value of initial storage at day \( t=\tau \) is selected, as a function of the previous day \( t=\tau-1 \) storage (solution initial storage plus net inflow) and the range of possible solution releases (Equation 3.6d), and the computation procedure is repeated. The procedure is similar to that of a Markov chain of order one in that the outcome of each trial (at day \( t=\tau \)) depends on the outcome of the directly preceding one (that of day \( t=\tau-1 \)), but is independent of the outcomes of all former trials (those of days \( t = \tau-2, \tau-3, ..., 1 \)). This procedure is repeated until solutions are found for day \( t=\tau \). Then, the storage solution for the end-of-day that day \( t=\tau \) is used as the beginning of the day \( t=\tau+1 \) storage and the search procedure continues until day \( t=\tau=T \). During the search, a number of different starting values of initial storages at the beginning-of-the-planning-horizon are used to make sure that the beginning storage is the same for all \( k \) critical years (Tables 5.2 and 5.3), and to make sure that at least one end-of-the-planning-horizon initial storage (i.e., starting from any day \( t \)) is obtained equal to or approximately equal to \( S_{i,0,\min}^k \).
Figure 5.16: Flow chart of the Fortran simulation models Tanau_2h.For and Mnpr2_h.For to determine $S_{i,0,min}$ for Te Anau and Manapouri respectively.
Figure 5.17: Flow chart of the fortran simulation models DH_Tanau.for and DH_Mnpr.for to determine $p_{i,t,\text{min}}$ for Te Anau and Manapouri respectively.
The whole search procedure is performed for each one of the selected \( k \) critical years separately. Then, the solution water releases, associated with each of the solution beginning-of-the-days \( t = 1, 2, \ldots, T \) storage, are searched for daily minima. The search process is performed starting from day \( t=1 \) to day \( t=T \) over all the \( k \) selected critical years. In other words, the solution controlled water release of day \( t=1 \) of year \( k=1 \) is compared to that of day \( t=1 \) of year \( k=2 \) and that of day \( t=1 \) of year \( k = 3, \ldots, m \), to select the lowest water release value of all the days \( t=1 \) of the \( k \) critical years. Similarly, solution controlled water releases of days \( t = 2, \ldots, T \) of year \( k=1 \) are compared to those of days \( t = 2, \ldots, T \) for years \( k = 2, \ldots, m \) respectively, to select the lowest water release values of those days \( t = 2, \ldots, T \). The ratios \( \alpha_k = \min(\frac{r_{i,t}}{R_{i,t}}) \) between each of those values \( r_{i,t} = r_{i,t,\min} \) and their appropriate \( R_{i,t}^{-} \), representing the meaningful indicators of damages suffered by the system users out the reference set of inflows, are illustrated in Figures 5.18 and 5.19.

5.3.1.2. \( \beta^k_i = \max(\frac{S_{i,t}^k}{S_{i,t}^+}) \) values: Search Procedure and Solution.

Similar to the above problem, the problem of determining \( \beta^k_i = \max(\frac{S_{i,t}^k}{S_{i,t}^+}) \) is divided into two sub-problems, solutions of which are provided by two sub-computer-simulation-models Tean_1h.For and DH_Tean1.for for Te Anau and Manap1_h.for and DH_Manap.for for Manapouri (see Appendix D, on diskette). The first sub-model consists of determining the so-called maximum initial storage value, \( S_{i,0,\text{max}} \). The search technique is essentially identical to that of determining \( S_{i,0,\text{min}} \). The main differences are that: 1) the beginning-of-the-planning-horizon (day \( t=1 \)) initial storage must be less than or equal to that of the end-of-the-planning-horizon (day \( t=T \)); 2) the controlled water releases must be a function only of the open-gate stage-discharge (see Equations (3.7d)); and 3) the largest daily storages, for which operating rules exist and that are less than or equal to a pre-defined maximum allowable storage and their associated releases, instead of the largest daily releases, are defined. Similarly, in addition to the requirement that the releases and storages be positive, they must satisfy the physical constraints of the reservoirs and rivers (Equations 3.7e and 3.7f). The physical constraints of the rivers are defined by analysing: the historical available inflow; the controlled and un-controlled outflow data; the
relationship between flood hazards, inflows and water storages; the physical limitations on lakes and rivers in the system; and the maximum ever released volume of water through the reservoirs' control structures. The results of the analysis suggested that the allowable highest releases are $R_{i,max} = 1386 \text{ m}^3/\text{s}$ and $1930 \text{ m}^3/\text{s}$ for Te Anau and Manapouri respectively. The physical constraints on storage as described in the “Waiau river system operational guideline” (see Section 2.4.1.1.(b)) were applied. The constraints on releases as defined in section 2.4.1.2 were included in the computer model. Similar to the $\alpha_i^k$ problem, the search process is repeated until the common largest solution initial storage value for all the $k$ critical years, which satisfy the inequality in Equation 3.7g, is found. The solution represents the so-called maximum initial storage $S_{i,0,max}$ and its corresponding end-of-the-planning-horizon storage for each one of the $k$ critical years are recorded in columns (6) and (7) of Tables 5.2 and 5.3. The value $S_{i,0,max}$ is used in sub-model two to search, in a similar way to the $\alpha_i^k$ problem, all the daily solution maximum storage values and their associated $\beta_i^k = \max(S_{i,T}^{\leq})$. Similarly, during the search process, a number of different starting values of the initial storages are used to make sure that a least one end-of-the-planning-horizon initial storage (i.e., $S_{i,T}$ for day $t=T$ computed starting from the day $t=\tau$) is obtained equal to $S_{i,0,max}$. An example of the solutions is given in columns (8) and (9) of Tables 5.2 and 5.3. The ratio values $\beta_i^k$ thus computed are represented in Figures 5.18 and 5.19.

Te Anau and Manapouri results are shown in Figures 5.18 and 5.19 respectively, in the space (demand satisfaction $\alpha$, flood indicator $\beta$) of the indicators. All the points of the curves between $\alpha=0.0$ to $\alpha=0.1$ in Figure 5.18, and between $\alpha=0.0$ to $\alpha=0.42$ in Figure 5.19 represent semi-efficient solutions. All other points on the curve represent efficient solutions. The points contained in the space between the curve and the $\beta$ axis represent the dominated solutions. The points ($\alpha=0.004$ and $\beta=1.015347$) and ($\alpha=0.0$ and $\beta=1.0131407$) represent the performance achieved by the WRS managers in real life for Te Anau and Manapouri respectively. These points are in the dominated solutions region and are too large to be represented on the graph. The low value of the $\alpha$ indicators of the performance achieved by the WRS managers in real life correspond to the 1976 dry spell in Te Anau and to the intentional Manapouri power station shut down in addition to the Manapouri control structure closure in Manapouri. The 1988 flood is represented by the high value of $\beta$ in
both lakes. The figures show that the results of the approach are particularly attractive because the potential improvements are not at all negligible.

Figure 5.18: Efficient, semi-efficient and dominated solutions for Lake Te Anau.

Figure 5.19: Efficient, semi-efficient and dominated solutions for Lake Manapouri for the selected Te Anau (α, β).
Table S.2: Te Anau beginning- and end-of-planning-horizon solution initial storages for the selected k critical years.

<table>
<thead>
<tr>
<th>Critical Years</th>
<th>DEMAND SATISFACTION (10^9 m^3)</th>
<th>FLOOD PROTECTION (10^9 m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( S_{i,0}^k )</td>
<td>( S_{i,T}^k )</td>
</tr>
<tr>
<td></td>
<td>( S_{i,1}^k )</td>
<td>( S_{i,T}^k )</td>
</tr>
<tr>
<td>52/53</td>
<td>7.070286</td>
<td>7.083437</td>
</tr>
<tr>
<td>57/58</td>
<td>7.070286</td>
<td>7.470199</td>
</tr>
<tr>
<td>74/75</td>
<td>7.070286</td>
<td>7.165468</td>
</tr>
<tr>
<td>75/76</td>
<td>7.070286</td>
<td>7.107713</td>
</tr>
<tr>
<td>82/83</td>
<td>7.070286</td>
<td>7.180192</td>
</tr>
<tr>
<td>88/89</td>
<td>7.070286</td>
<td>7.100916</td>
</tr>
</tbody>
</table>

Note: 1) Columns \( S_{i,0}^k \) contain the beginning-of-planning-horizon initial storage; columns \( S_{i,1}^k \) contain day \( t=1 \) initial storage and columns \( S_{i,T}^k \) contain the end-of-planning-horizon (i.e., \( t=T \)) initial storage.

5.3.2. Determination of the trigger values \( S_{i,t,min}^a \) and \( S_{i,t,max}^b \)

To determine the values of \( S_{i,t,min}^a \) and \( S_{i,t,max}^b \) a series of simulations were performed using Fortran computer simulation programs incorporating Equations 3.10 to 3.12 and 3.14 to 3.16 respectively (see Programs Tanau_2.for, D_Tanau2.for, Teanl.for and D_Tean1.for for Te Anau and Programs Mnpr2.for, D_Mnpr2.for, Manapl.for and D_Manap.for for Manapouri on diskette in Appendix D). The search procedure is similar to that described in Section 5.3. The simulations were performed under the following assumptions:

1. During a period of drought or impending drought, water supply must not exceed \( \alpha_i R_{i,t}^* \).
2. In any day \( t \), the reservoirs’ maximum storage must not exceed the modified reference storage \( \beta_i S_{i,t}^* \).

As was explained in Sub-section 5.3.1.1, the determination of an effective or preferred combination of Te Anau and Manapouri’s efficient solution \((\alpha, \beta)\) is a function of the Te Anau solution. Therefore, the effective values of the daily minimum storage, \( S_{i,t,min}^{\alpha_i} \), level
at which to trigger flood alleviation must first be defined for Te Anau. This solution with its associated \((\alpha, \beta)\) solution will be used to define Manapouri's \((\alpha, \beta)\) sets and the solutions \((S_{\alpha,\beta}^{\alpha'}, S_{\alpha,\beta}^{\beta'})\). The different Te Anau \((S_{\alpha,\beta}^{\alpha'}, S_{\alpha,\beta}^{\beta'})\) are graphed in Figures 5.20 A); B); and C).

Figure 5.20 shows that only graph B, of the three graphs shown, is a solution. In graph A), \(S_{\alpha,\beta}^{\alpha'} < S_{\alpha,\beta}^{\beta'}\) toward the end of the planning-horizon. This indicates that the pair \((\alpha, \beta)\) cannot be guaranteed. In graph C), \(S_{\alpha,\beta}^{\alpha'} > S_{\alpha,\beta}^{\beta'}\) throughout the whole planning-horizon. This is an indication that the computed solutions are dominated and can be improved. In graph B) the proposed pair \((\alpha, \beta)\) gives a fairly good result with a large difference between \(S_{\alpha,\beta}^{\alpha'}\) and \(S_{\alpha,\beta}^{\beta'}\) throughout the planning-horizon with the exception of one day at the end of the planning-horizon where \(S_{\alpha,\beta}^{\alpha'} = S_{\alpha,\beta}^{\beta'}\). This indicates that the solution is not dominated. This demonstrates that the pair \((\alpha, \beta) = (0.67; 1.00825)\) is an efficient Te Anau solution. It is therefore used to define Manapouri's set of \((\alpha, \beta)\) pairs (see Figure 5.19) and the \((S_{\alpha,\beta}^{\beta'}, S_{\alpha,\beta}^{\alpha'})\) values (see Figure 5.21). In Figure 5.21, graph B) represents the set of Manapouri efficient solutions. The pair \((\alpha, \beta) = (0.74; 1.012666)\) represents therefore, an efficient pair of Manapouri water demand satisfaction and flood alleviation indicators. The combination of Te Anau's set \((\alpha, \beta) = (0.67, 1.00825)\) (see Table 5.3) with Manapouri's \((\alpha, \beta) = (0.74, 1.012666)\) was therefore suggested for adoption as the effective combination in determining the WRS daily maximum and minimum storage levels at which water rationing and flood alleviation must be triggered in periods of extreme hydrological inputs and in defining the system's operating rules.

Figures 5.20 B) and 5.21 B) show that the difference between \(S_{\alpha,\beta}^{\beta'}, S_{\alpha,\beta}^{\alpha'}\) is maximal throughout the planning-horizon except for the June and mid-July to mid-September periods in Te Anau, and the October and May to June periods in Manapouri. This is an indication that the conservation zone remains quite large throughout the year, and therefore shows the existence of a whole range of effective operating rules. At the
Table 5.3: Manapouri beginning- and end-of- the selected k critical planning-horizon’s solution initial storages function of selected Te Anau’s ($\alpha^*$, $\beta^*$).

<table>
<thead>
<tr>
<th>DEMAND</th>
<th>SATISFACTION</th>
<th>FLOOD</th>
<th>PROTECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

Te Anau proposed solution $\alpha^* = 0.67$ and $\beta^* = 1.00825$

$$S_{i,0,\min}^k = 1.211126*10^{10} \text{ m}^3$$

$$S_{i,0,\max}^k = 1.229874*10^{10} \text{ m}^3$$

<table>
<thead>
<tr>
<th>Critical years</th>
<th>$S_{i,0}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,T}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,1}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,T}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,0}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,T}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,1}^k$ $(10^{10} \text{ m}^3)$</th>
<th>$S_{i,T}^k$ $(10^{10} \text{ m}^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>52/53</td>
<td>1.211126</td>
<td>1.214275</td>
<td>1.211126</td>
<td>1.214275</td>
<td>1.229874</td>
<td>1.226911</td>
<td>1.229874</td>
<td>1.215228</td>
</tr>
<tr>
<td>57/58</td>
<td>1.211126</td>
<td>1.280731</td>
<td>1.211126</td>
<td>1.280930</td>
<td>1.229874</td>
<td>1.228804</td>
<td>1.229874</td>
<td>1.226579</td>
</tr>
<tr>
<td>74/75</td>
<td>1.211126</td>
<td>1.253106</td>
<td>1.211126</td>
<td>1.253105</td>
<td>1.229874</td>
<td>1.227394</td>
<td>1.229874</td>
<td>1.214609</td>
</tr>
<tr>
<td>75/76</td>
<td>1.211126</td>
<td>1.222234</td>
<td>1.211126</td>
<td>1.221363</td>
<td>1.229874</td>
<td>1.228168</td>
<td>1.229874</td>
<td>1.215711</td>
</tr>
<tr>
<td>82/83</td>
<td>1.211126</td>
<td>1.269119</td>
<td>1.211126</td>
<td>1.267961</td>
<td>1.229874</td>
<td>1.228481</td>
<td>1.229874</td>
<td>1.232607</td>
</tr>
<tr>
<td>88/89</td>
<td>1.211126</td>
<td>1.241763</td>
<td>1.211126</td>
<td>1.240402</td>
<td>1.229874</td>
<td>1.227825</td>
<td>1.229874</td>
<td>1.216120</td>
</tr>
</tbody>
</table>

Approach of winter (June to August) the difference between $S_{i,T,\max}^\beta$ and $S_{i,T,\min}^\alpha$ narrows down in both lakes, and in June, $S_{i,T,\min}^\alpha$ becomes, not only greater than the reference storage, but almost, if not for one to two days, equal to $S_{i,T,\max}^\beta$. This forces the system operator to release only small amounts of water from Manapouri and Te Anau, starting from early May and June respectively, until the lakes are sufficiently full. Then the difference between $S_{i,T,\max}^\beta$ and $S_{i,T,\min}^\alpha$ increased in July, but shrinks again in Te Anau starting from mid-July to mid-September. The mid-July to mid-September period is a winter period characterised by high water demand for electricity generation and other water-based activities. It is also characteristic of the approach of high inflows. The minimal difference between $S_{i,T,\max}^\beta$ and $S_{i,T,\min}^\alpha$ over that period is characterised by $S_{i,T,\max}^\beta$ dropping low. The dropping of $S_{i,T,\max}^\beta$, means that the system operator is forced to release, from Te Anau, the maximum possible amount of water to guarantee water demand satisfaction in Manapouri and to ensure the allocation of a sufficient flood storage pool just before the potential floods. Indeed, this is what the WRS operators do every year over the winter period. The pronounced maximum of $S_{i,T,\min}^\alpha$ in October in Manapouri corresponds to a period of decreasing inflow to the lake.
A) \((\alpha, \beta) = (0.60; 1.00725)\)

B) \((\alpha, \beta) = (0.67; 1.00825)\)

C) \((\alpha, \beta) = (0.70; 1.00892)\)

**Figure 5.20:** Te Anau values of \(s_{\text{t, max}}^{\beta} \) and \(s_{\text{t, min}}^{\alpha} \) out of the reference set \(W_0^{\alpha, \beta} \) for different values of \((\alpha, \beta)\).
Figure 5.21: Manapouri values of $S_{i,l,\text{max}}$ and $S_{i,l,\text{min}}$ out of the reference set $W_{0}^{\text{ref}}$ for different value of $(\alpha, \beta)$ and Te Anau's efficient solution pair $(\alpha, \beta) = (0.67; 1.00825)$.
The results of the simulation show that there is plenty of room for improving the operational performance of the WRS and for limiting, if not eliminating, system failure. It can therefore be anticipated that, if the min-max control approach solutions are used in the daily operation of the WRS, then flexibility of operation and multiplicity of options for enhancing the daily operational performance of the system will prevail.

The approach demonstrates its strength by using past hydrological events and the similarity between the past, present and future inflows to determine its solutions. The results obtained, in terms of the daily values of $S_{\alpha, \text{min}}^\alpha$ and $S_{\beta, \text{max}}^\beta$, represent the effective lower and upper storage levels where water rationing and flood flow attenuation by spilling should be triggered. The daily values of $S_{\alpha, \text{min}}^\alpha$ and $S_{\beta, \text{max}}^\beta$ can be used in a computer model incorporating a simulation technique or an optimisation technique or a combination of both techniques to determine the day to day real-time water release values. However, optimisation techniques are often not too appealing to reservoir operators. Therefore, a fuzzy logic programming concept has been suggested as an alternative and is discussed in Chapter 6. This is because fuzzy logic concepts are closer to the way system operators think and therefore should more readily be accepted by them.

5.4. Sensitivity analysis

As currently managed, the levels of Lakes Te Anau and Manapouri are operated within gazetted operating guidelines (see 2.4.1.1). These guidelines are based on the three most extreme high and low lake levels measured within the 32 year recorded period, for natural conditions, pre-control. The Guardians of the Lakes and ECNZ recommended operating guidelines for the lake level after detailed assessment of environmental consequences of the different extreme lake levels for the lakes. Therefore, the developed control approach was only used to assess the behaviour of the system under the operating guidelines’ lake levels. Hence it is suggested to use the whole post Power Station recorded historical data to test the model for different (not included in the guidelines) extreme lake levels in order to determine the economic benefits, if any, to the system.

The vegetation zonation pattern was found to be generally similar for both lakes, with such factors as elevation, substrate, and exposure to prevailing westerly winds. This suggests a dominant control by natural lake level variation in both lakes (ECNZ, 1996). Furthermore it was assessed formerly in this study that Manapouri behaviour is predominantly dependent on Te Anau operation. It is therefore suggested to conduct the sensitivity analyses on Te Anau only and deduce a solution for Manapouri.
In the following sensitivity analyses, three high (203.8m, 204.3m, 204.8 m) and three low (200.5m, 200.86m, 201.1m) lake levels are used. The values 204.3m and 200.86m are, as mentioned before, the allowable maximum and minimum levels of Lake Te Anau as specified by the operating guidelines (see Sub-section 2.4.1.1). The analyses are conducted by varying the storage of the lake such that for a chosen high lake level three different low levels are consecutively associated. This approach is suggested because, for water demand satisfaction for economic values, if the high lake level is fixed in space and time, lowering the absolute minimum level can only increase the storage.

The results of the analyses are illustrated as box-and-whisker plots in Figures 5.22 to 5.24. In Figure 5.22, high81, med81 and low81 stand for the flood indicator samples for the high water level fixed at 204.8 m when the low lake level varies between 201.1m, 200.86m and 200.5 m (high med, low) respectively. High82, med82 and low82 stand for the water demand satisfaction deficit indicator samples for the high water level fixed at 204.8 m and the low lake level varying between 201.1m, 200.86m and 200.5 m respectively. Similar description is made for the samples high431, med431, low431; high432, med432, low432; and high381, med381, low381; high382, med382, low382 where 43 and 38 stand for the maximum lake level fixed at 204.3 and 203.8 respectively. High, med, and low represents the absolute minimum lake level at 201.1, 200.86 and 200.5 m respectively. 1 and 2 represent the flood indicator and the water demand deficit indicator respectively. In the box-and-whisker diagrams, the asterisks represent water supply deficit and flood hazard. The position of the asterisks determines the severity of the water supply deficit and of the flood hazard. The asterisks do not represent the $\alpha_i$ selected earlier. They only represent the density of the deficit or hazard. Therefore, the more an asterisk of a sample is at the right hand-side as compared to other samples' asterisks the less water deficit or flood hazard will occur. The vertical line (bar) in the boxes (second quartile) represents the median. The figures show that regardless of whether the allowable maximum or minimum storage level is increased or decreased there is no influence on the flood indicator. The asterisks in the flood indicator graphs are on the same vertical line. This can be explained by the fact that the min-max approach is performed in such a way as to avoid any possible flood hazard. The plots show that there might be some economic gain by operating the lakes low. This is illustrated by the location of the asterisks representing low82, low432 in the "water demand satisfaction indicator" graphs of Figures 5.22 and 5.23. These low82, low432 whiskers are not on the same vertical line as the high82, high 432, med82 and med432 whiskers. They are located slightly left and approximately equal 0.36 whereas the high82, high 432, med82 and med432 values are approximately 0.375.
Figure 5.22: Flood and water demand satisfaction indicators for the maximum allowable storage fixed at 204.8 m and the absolute minimum storage varying between 200.5, 200.86 and 201.1 m.

Figure 5.23: Flood and water demand satisfaction indicators for the maximum allowable storage fixed at 204.3 m and the absolute minimum storage varying between 200.5, 200.86 and 201.1 m.
Figure 5.24: Flood for the maximum allowable storage fixed at 203.8 m and the absolute minimum storage varying between 200.5, 200.86 and 201.1 m.

Figure 5.25: The medxx2 samples.

However the benefits will not be large as the difference between the whiskers is not large. There is nevertheless one interesting figure worth noticing, the stability of all the medxx2 samples (see Figure 5.25). This suggests that when the absolute minimum level imposed by the operating guidelines is used, the water demand satisfaction is fulfilled with the same degree no matter how high the maximum allowable lake level is set. In view of the analyses, it can be affirmed that the economic benefits to the WRS by operating the lakes at different extreme levels is minimal as compared to the need for protecting the fragile lake shore ecosystems. This sustains the claim that the lakes should continue to be operated in the range proposed by the Guardians of the lakes and ECNZ.

5.5. Summary

In this chapter, the search technique used to define the parameters that are essential to the determination of the trigger values for water rationing and storage conservation at the approach of drought and flood respectively, was explained in detail. The efficient solutions
(α, β) were obtained and used in the determination of the effective daily trigger values $S_{i,j,\text{max}}^{\beta'}$ and $S_{i,j,\text{min}}^{\alpha'}$. The values were computed for daily operation. It is anticipated that the policies derived in Chapter six using the daily trigger values and the fuzzy logic control concept will provide a substantial basis for effective operation of the system. The sensitivity analyses proved that the operating guidelines are essential for sustainable operation of the system.
6.1. Introduction

A real-world reservoir-system operation model can be very complex. For example, it has to incorporate all the input imprecision and uncertainties, while the output should fulfill all the system requirements such as meeting various demands and yet guaranteeing the non-violation of physical (Shrestha, Duckstein and Stakhiv, 1996) and environmental constraints of the system. The common approach generally used to solve the uncertainty problem in reservoir-system operation is probability theory. However this approach tends to be too abstract and in some cases complex (Russell and Campbell, 1996) and therefore, is often very unattractive to those responsible for making decisions about reservoir systems. Moreover, the probability approaches only deal with the imprecision and uncertainties that lie in the value of a variable, say inflow, and do not handle the extent to which this variable may belong to a given imprecise (fuzzy) set, say between medium and high inflow. Therefore, it is believed that the appropriate tool to handle such challenges can be found, not in ordinary (crisp) rules such as linear or dynamic programming (optimisation) or in probability rules, but in the fuzzy logic controller. As a consequence, in this chapter the water rationing and peak flood flow alleviation trigger values $s_{\alpha_{\min}}^\prime$ and $s_{\beta_{\max}}^\prime$ obtained in Chapter 5 are used in a rule-base for a fuzzy logic controller to develop a real-world operation model for the WRS. This has led to the determination of the “best compromise” future daily operational management policies (daily water releases and end-of-the-day reservoir storage values) that would guarantee a sustainable management of the WRS while enhancing its electricity generation. The fuzzy logic controller is described and the results of its application to the WRS is given and compared to what was achieved by the WRS’s operators in real-life. The fuzzy logic controller program was written in Fortran and provided on diskette in Appendix D. The daily databases used are also provided on diskette in Appendix D.

6.2. Real-time daily operational model

As suggested in the introduction, a fuzzy logic controller incorporating the solutions of the min-max approach obtained in Chapter 5, was adopted as the appropriate alternative to overcome the challenges of developing a daily operating model for the WRS. The min-max reservoir control approach can be related to an “implicitly stochastic approach” since it assumed, in the definition of its solutions, foreknowledge of inflows by
stating that they (inflows) belong to a (carefully selected) set of reference inflows whose upper and lower boundaries represent the upper and lower limits of future inflows. The min-max approach was used to determine the preferred water demand satisfaction and flood flow alleviation indicators together with their associated volume of water to release and volume of storage conservation to allocate for peak flows. Those values with the formulae for the determination of the upper and lower limits of the feasible water releases (Equation 3.18) in combination with the daily information (the beginning-of-the-day reservoir storage level, the anticipated daily net inflow, the time of the year) available or likely to be available, are programmed as a rule-base in the fuzzy logic controller for the purpose of operating the real-world WRS.

6.2.1. Fuzzy logic programming: concept and applications

The concepts of fuzzy logic and even the name were introduced by Zadeh (1965) who pioneered the development of fuzzy logic. Since its introduction, fuzzy logic has continued to develop and in recent years it has been significantly used for automatic control in commercial as well as non-commercial situations. Mamdani and Assilian (1975) first applied fuzzy set theory to the control of a laboratory steam engine. This experiment triggered a number of other applications such as: cameras, where fuzzy logic is used for automatic focusing; washing machines which automatically adjust their washing cycles in response to the size of the load and how dirty the clothes are; the warm water process controller (Kickert and Van Nauta Lemke, 1976); activated sludge wastewater treatment (Tong, Beck and Latten, 1980) amongst others. The control concept has also been applied in a diverse set of domains such as arc welding (Murakami et al, 1989); automobile speed control (Murakami and Maeda, 1985); cement kiln control (Umbers, King, 1980); reservoir-system operation (Russell and Campbell, 1996; and Shrestha, Duckstein and Stakhiv 1996); water purification process control (Yagishita, Itoh and Sugeno, 1985) among others. This list is not an exhaustive one. More applications have been cited in the literature, for example Berenji (1992). Among the applications, Hitachi’s automatic train controller is one of the most celebrated recent applications (Berenji, 1992).

The concepts and operational algorithms of fuzzy logic programming are given in many textbooks, for example Kosko (1992, 1993), Klir and Folger (1988), Klir and Bo Yuan (1995), McNeil and Thro (1994) and Zadeh and Kacprzyk (1992). The key ideas are that fuzzy logic allows for something to be partly this and partly that, with a degree of belonging described numerically by a membership number between 0.0 and 1.0, rather than having to be either all this or all that. Therefore, in contrast to ordinary (crisp) and
probability rules, fuzzy logic programming rules deal with the vagueness, imprecision and uncertainties in systems' operational rules and variables by allowing partial and simultaneous fulfilment of the rules and constraints. Moreover, the fuzzy logic programming introduces flexibility in operation and the construction of its rules can incorporate the experience of the decision-makers or system operators, the "system experts". It can therefore be anticipated that a fuzzy logic controller incorporating the min-max approach solutions will improve the effectiveness of WRS over what can be achieved using crisp rules such as a linear programming technique (optimisation technique) or probability rules.

To aid understanding of the fuzzy logic approach, a review of the definitions of fuzzy sets, fuzzy numbers and some of the fuzzy operations is given below.

1) **Fuzzy sets:** How do humans reason? Suppose a reservoir-system operator is asked to release, in addition to the required volume of water for power generation, a flow of water equal to 50 m$^3$/s for recreational uses on Saturday 14 April, or Sunday 15, April. Also assume that the hydrologic events forecasting service in the area anticipates a good chance of high inflow into the system on Saturday but only a slight chance for high inflow on Sunday. The reservoir-system operators will certainly decide to release the additional flow of water on Saturday. But how do they reach this answer? They reach it with rules. Rules associate ideas, and relate a thing, an event, a process, or a condition to another thing, event, process or condition. In natural and computer languages rules have the form of if-then statements (Kosko, 1993) whereas, fuzzy logic rules consist of a condition (IF-part) and a conclusion (THEN-part). If the inflow into a reservoir is not high enough, the water storage level in the reservoir will not rise sufficiently to satisfy the release of the required additional flow of water downstream. If the reservoir storage level does not rise sufficiently, the operators cannot release the required additional flow of water downstream. Inflow will not be high on Sunday. So the operators cannot release the required additional flow of water on Sunday. The inflow will be high on Saturday. If the operators cannot release the required additional flow of water on Sunday and if the inflow will be high on Saturday, they can release the required additional flow of water on Saturday. So they release the required additional flow of water on Saturday.

What is meant by the rule "if the inflow is high, the reservoir water storage level will rise"? A lot is meant by it. It can mean if the inflow is a little high, the reservoir storage level rises a little. Or if the inflow is extremely high, the reservoir storage level rises a lot. Assuming **inflow** is a fuzzy set, inflow can be low, medium, slightly high or extremely high. Then the linguistic **low, high, slightly high, and extremely high** stand for
fuzzy subsets of inflow. Inflows are low or high or a value in between low and high. It is a matter of degree. This degree of belonging is called the grade or degree of membership of the fuzzy subset to the fuzzy set. For example, in Figure 3.6, any storage in zone III has a degree of membership between one (1.0) and zero (0.0). Therefore, if S (for example zone III of Figure 3.6) is a fuzzy set, “A” (a set of storage values s) is called a fuzzy subset of “S” if A is a set of ordered pairs: A = \{\{s, \mu_A(s)\}; s \in S, \mu_A(s) \in [0, 1]\}; where \mu_A(s) is the grade or degree of membership of s in A. The function \mu_A(s) is called the membership function of A. The closer \mu_A(s) is to 1.0 the more s is considered to belong to A - the closer it is to 0 the less it is considered as belonging to A. Therefore, a crucial point in applying fuzzy methods is the assessment of membership functions. Usually the membership functions are made linear as this makes subsequent calculations easier (Russell and Campbell, 1996). In some other applications, they are treated as utility curves for individual objectives with an overall objective of maximising a weighted sum of the membership values (Gates et al, 1991), or are assessed by using a value function transformation.

Special cases of fuzzy sets are fuzzy numbers, which are generalisations of the usual concept of numbers (Shrestha, Duckstein and Stakhiv, 1996). A fuzzy subset A of a set of real numbers is called fuzzy number if there exists at least one s such that \mu_A(s) = 1.

Any real number can be regarded as a fuzzy number with a single point support and called a “crisp number” in fuzzy mathematics.

Different types of fuzzy membership functions, as described in the paragraph above, have been used in fuzzy logic control. However, four types are more common: the linear types including: monotonic (such as the branch extending from 1.0 on the \mu(s) axis to S1 on the horizontal axis of Figure 6.2), triangular, and trapezoidal shaped membership functions and bell-shaped (utility curves type) membership functions. Of the four cited, a combination of the first three is used in this study to describe a fuzzy storage set membership. The interval over which the membership function of a fuzzy number A is non-zero is called the support of A.

2) Fuzzy set operation: Assuming that A and B are two fuzzy sets with membership functions \mu_A and \mu_B, then the following operation can be defined. The complement of a fuzzy set A is a fuzzy set \tilde{A} with a membership function

\[ \mu_{\tilde{A}}(s) = 1 - \mu_A(s). \]

The union of A and B is a fuzzy set with the following membership function

\[ \mu_{A \cup B}(s) = \max\{\mu_A(s), \mu_B(s)\} \]

and called the Zadeh “OR” rule.

The intersection of A and B is a fuzzy set
\( \mu_{A \sqcap B}(s) = \min\{\mu_A(s), \mu_B(s)\} \). This operation is called the Zadeh "AND" rule.

The most successful application area of fuzzy systems has undoubtedly been the area of fuzzy control (Klir and Bo Yuan, 1995). A general fuzzy logic controller consists of four modules: a fuzzy rule-base, a fuzzy inference engine, and fuzzification/defuzzification modules. The interconnections among these modules and the controlled process are shown in Figure 6.1 below and the fuzzy controller is described in the next section.

6.2.2. Fuzzy logic controller: its architecture and application to the WRS

The fuzzy logic controller operates by repeating a cycle of the following four steps. First, measurements are taken of all variables that represent relevant conditions of the controlled process. Next, these measurements are converted into appropriate fuzzy sets to express measurement uncertainties. This step represents the fuzzification. The fuzzified measurements are then used by the inference engine to evaluate the control rules stored in the fuzzy rule-base. The result of this evaluation is a fuzzy set (or fuzzy sets) defined on the universe of possible actions. This fuzzy set (or fuzzy sets) is then converted, in the final step of the cycle, into a single (crisp) value that, in some sense, is the best representation of the fuzzy set (or fuzzy sets). This conversion is called the defuzzification. The defuzzified values represent actions taken by the fuzzy controller in individual control cycles.

Figure 6.1: Proposed WRS operating procedure: Fuzzy logic controller architecture.
In designing a fuzzy controller, one must first identify the main control parameters and second, determine the term set that is at the right level of quantification for describing the values of each linguistic variable. In this study, water storage is used as the main control parameter for water releases and a term set including linguistic values such as {low (LO); Medium Low (ML); Medium (ME); Medium High (MH); and High (HI)} (see Figure 6.2) is used to described the ranges or sets (different zones of Figure 3.6, and Equation 3.18). The storage and the releases belong to a given time. The boundary values of the control parameters are illustrated in Figure 6.2. Based on these values a rule-base is developed (and described below) using the control variables and the consequence values that may result.

In Figure 6.2 the following symbols represent:

\[ S_0 = S_{\min} \] (absolute minimum storage),

\[ S_1 = \text{Storage at the open-gate stage discharge } N(\alpha_i R^-_{i,i}) \text{, where } N() \text{ is the open-gate stage discharge function}, \]

\[ S_2 = \alpha_i R^-_{i,i} + S_{i,i+1,\min} \]

\[ S_3 = \alpha_i R^-_{i,i} + S_{i,i+1,\max} \]

\[ S_4 = N(S_{i,i}^k) + S_{i,i+1,\min} \]

\[ S_5 = \min\{\text{storage @ } R_{i,\max}; (N(S_{i,i}^k) + S_{i,i+1,\max})\} \]

\[ R_0 = 0.0, \]

\[ R_1 = \min\{N(S_{i,i}^k); \max\{S_{i,i}^k + W_{i,i}^k - S_{i,i+1,\max} \alpha_i R^-_{i,i})\}, \]

\[ R_2 = \min\{N(S_{i,i}^k); \max\{S_{i,i}^k + W_{i,i}^k - S_{i,i+1,\min} \alpha_i R^-_{i,i})\}, \]

\[ R_3 = \min\{N(S_{i,i}^k); (S_{i,i}^k + W_{i,i}^k - S_{i,i+1,\min} \alpha_i R^-_{i,i})\}, \]

\[ R_4 = \min\{N(S_{i,i}^k), R_{i,\max}\}. \]

The values representing \( S_0 \) to \( S_5 \) and \( R_0 \) to \( R_4 \) are derived from Figure 3.6 and Equation 3.18 to reflect the min-max reservoir control rules adopted.

The four modules of the fuzzy controller illustrated in Figure 6.1 are described in following sub-sections. The system dynamics of the proposed objectives of the system are measured in terms of water storage.
Figures 6.2: (a) and (b) represent reservoir storage and release membership functions respectively.

6.2.2.1. Fuzzification

Fuzzification means using the membership functions of linguistic labels to compute each term's degree of membership at a specific operation point of the control process. In this study for example, the mean daily storage values (sum of the initial storage and the forecasted net mean daily inflow) in the reservoirs during each day $t$ are matched against the membership functions of the linguistic labels illustrated in Figure 6.2 (a) to define their degree of membership. The mean daily storage values represent the input variables. The membership values are computed as in the following examples.

**Example 1:** Assume a daily mean storage $S_t$ is lying between $S_3$ and $S_4$, then $S_t$ belongs to the set \{Medium, ME\} as well as to \{Medium High, MH\}. Its membership $\mu_{S_t}$ can be computed as:
and illustrated as in Figure 6.3 below for the triangular membership functions ME and MH:

\[
\mu_{S_t} = \begin{cases} 
\frac{S_t - S_1}{S_4 - S_1} & \text{for } S_t \in ME \\
\frac{S_t - S_3}{S_4 - S_3} & \text{for } S_t \in MH
\end{cases}
\]  \hspace{1cm} (6.1)

**Example 2:** In this example involving the trapezoidal fuzzy membership functions assume that \( S_t \) is somewhere between \( S_1 \) and \( S_3 \), shown in Figure 6.2 a. In this case \( S_t \) belongs to the trapezoidal membership function ML as well as the triangular membership function ME. The membership value of \( S_t \) in the set ML can be computed as in Equation 6.2 below:

\[
\mu_{S_t} = \begin{cases} 
1 & S_1 \leq S_t \leq S_2 \\
\frac{S_3 - S_t}{S_3 - S_2} & S_2 < S_t < S_3
\end{cases}
\]  \hspace{1cm} (6.2)

**Example 3:** Assume that in January 1997 the mean storage \( S_t \) (including the inactive and controllable storages) in Te Anau is equal to \( 7.1282 \times 10^{10} \text{ m}^3 \), and belongs part in ME and part in MH. Assume also that:

- ME varies between \( 7.125 \times 10^{10} \text{ m}^3 \) and \( 7.129 \times 10^{10} \text{ m}^3 \) with the middle point equal to \( 7.127 \times 10^{10} \text{ m}^3 \), and
- MH varies between \( 7.127 \times 10^{10} \text{ m}^3 \) and \( 7.133 \times 10^{10} \text{ m}^3 \) with the middle point equal to \( 7.1286 \times 10^{10} \text{ m}^3 \).

Therefore, the degree of belonging (or membership) \( \mu_{S_t} \) of \( S_t \) to the fuzzy sets ME and MH is 0.4 and 0.75 respectively.
6.2.2.2. The Fuzzy control rule-base

A fuzzy rule-base system is defined as a set of rules consisting of sets of input variables \( A \) in the form of fuzzy sets with membership functions \( \mu_A \); and sets of consequences or outputs \( B \) also in the form of fuzzy sets. The actual assessment of the rule-base is a procedure where knowledge and/or available data are translated or encoded into rules describing, for example, how a storage, say, in the medium storage range with a degree of membership 0.6 will result in a, say, medium-low water release of a reservoir system. Therefore, a careful selection of fuzzy sets for different input and output variables is important to the smoothness of the control. There are various methods to assess and validate the control rule-base. The different methods to do this, as described in Bárdossy and Duckstein (1995) are as follow:

1. Expert's experience and knowledge.
2. Modelling the operator's control actions: the rules can be assessed by the expert directly, but available data should be used to update them.
3. Modelling a process: the rules are not known explicitly, but the variables required for the description of the system can be specified by the expert.
4. Self organization: only a set of observations is available, and a rule system has to be constructed to describe the interconnections between elements of the data set and to improve the controller's performance.

6.2.2.3. The decision-making logic

In modelling the WRS operation the decisions are made at the beginning of each day \( t \). The input variables are assumed equal to the reservoir storage. The reservoir storage is computed as the mean total net storage in the reservoir at the end of day \( t \) assuming no release of water is made. Therefore, the storage is computed equal to the initial storage in the reservoir at the beginning of day \( t \) plus the incoming flow (the net total flow into the reservoirs) of day \( t \). The output variables are the actual water released to meet the environmental and non-environmental demands. The rules are thus set in the general format below:

\[
\text{If the reservoirs' initial storage plus net incoming inflow in day } t \text{ is } S, \text{ then actual water release is } R_i. 
\]

Due to the partial matching attribute of fuzzy control rules and the fact that the preconditions of the rules do overlap, usually more than one fuzzy control rule can be triggered at one time. The methodology which is used in deciding what control action should be taken as the result of the firing of several rules is referred to as the conflict
resolution process (Berenji, 1992). The conflict resolution process or computation of fuzzy rules is also called fuzzy rule inference. It is a computation consisting of two main steps: aggregation and conclusion. The first step - aggregation- determines the degree to which the complete IF-part of the rules is fulfilled. Special fuzzy operators are used to aggregate the degree of membership of various preconditions. The second step uses the validity of the condition to determine the validity of the conclusion. An example of the process, especially the second step, is given in the following example. Assume that the following rules are given:

Rule 1: IF $S$ is $S_1$ then $Z$ is $R_1$
Rule 2: IF $S$ is $S_2$ then $Z$ is $R_2$,

where $S_1$ and $S_2$ are the fuzzy sets representing the input variables and $R_1$ and $R_2$ are the fuzzy sets representing the output variables. Now if $s_t$ is the daily net mean storage reading for fuzzy variable $S$ during day $t$, then its true value is represented by $\mu_{S_1}(s_t)$ and $\mu_{S_2}(s_t)$ for rule 1 and 2 respectively. The aggregation step is skipped here because only one input variable is used for each of both rules. The control outputs of rules 1 and 2 are calculated by applying the matching strength of their preconditions, $\mu_{S_1}(s_t)$ and $\mu_{S_2}(s_t)$ respectively, on those of their corresponding conclusion $\mu_{R_1}(s_t)$ and $\mu_{R_2}(s_t)$:

Rule 1: $\mu_{R_1'}(r) = \min(\mu_{R_1}(s_t), \mu_{S_1}(s_t))$
Rule 2: $\mu_{R_2'}(r) = \min(\mu_{R_2}(s_t), \mu_{S_2}(s_t))$.

where $r$ ranges over the values that the rule conclusions can take. This means that as a result of the given storage $s_t$, rules 1 and 2 recommend control actions with $\mu_{R_1'}(r)$ and $\mu_{R_2'}(r)$ as their respective membership functions. The conclusion step of the conflict-resolution process then produces:

$$\mu_R(r) = \max(\mu_{R_1'}(r), \mu_{R_2'}(r)),$$

where $\mu_R(r)$ is a pointwise membership function for the combined conclusion of Rule 1 and 2. The value of $\mu_R(r)$ is the output membership function and its associated result is fuzzy. The process used to derive the conclusion of rules 1 and 2 is the Zadeh "OR" rule. The result produced from the evaluation of fuzzy rules is, of course, fuzzy. In this study, it can be expressed as a medium and (or) medium high (large) volume of water to be released. Naturally, a reservoir control structure's gates cannot interpret such linguistic commands. Membership functions are therefore used to translate the fuzzy output into a crisp value. This translation is known as defuzzification.
6.2.2.3. Defuzzification

The objective of the defuzzification method is to derive a non-fuzzy (crisp) value that best contains the fuzzy value of the linguistic output variables. There are many defuzzification techniques reported in the literature (Berenji 1992, and Kosko 1992). However to select the method that is appropriate to a problem at hand, one needs to understand the linguistic meaning that underlies each defuzzification process. In practical applications, the only difference between defuzzification methods is whether they deliver the “best compromise” or the “most plausible” result, and whether they provide continuity or not. The definition of continuity in defuzzification methods means that an arbitrary small change of an input variable can never cause an abrupt change in any output variable. The purpose of this study will be therefore best served by using a defuzzification method that is continuous and delivers the “best compromise” solution. Two of such commonly used methods are the Centre of Maximum, CoM, and the Centre of Area, CoA.

The CoM computes crisp outputs as weighted means of the term membership maxima, weighted by the inference results as follows:

\[
R^* = \frac{\sum_{i=1}^{n} \mu_{Ri} r_i}{\sum_{i=1}^{n} \mu_{Ri}}
\]

(6.4)

where \( n \) is the number of rules with firing strength \( \mu_{Ri} (r) \) greater than 0.0 and \( r_i \) is the amount of control recommended by rule \( i \).

With the CoA method, assuming a control action with a pointwise membership function \( \mu_R (r) \) has been produced, the crisp output will be computed as the centre of gravity of the distribution (centroid) for the control action. The crisp value can then be computed as follows:

\[
R^* = \frac{\sum_{i=1}^{m} r_i \mu_R (r_i)}{\sum_{i=1}^{m} \mu_R (r_i)}
\]

(6.5)

where \( m \) is the number of quantification levels of the output \( R \), \( r_i \) is the amount of control output at the quantification level \( i \) and \( \mu_R (r_i) \) represents its membership value in \( R \). The CoA is the most frequently used defuzzification method in fuzzy logic control systems. In this study the CoA was appropriate to represent the control systems. A working example of the CoA is given below and illustrated in Figure 6.4.

Assume a reservoir storage \( s \) belongs in part to ME and in part to MH of Figure 6.2 a. Then the fuzzy release \( r \) belongs in part to ME and in part to MH of Figure 6.2 b. Assume that:
1. the release fuzzy set ME varies between 1.0 and 6.0, and the release fuzzy set MH ranges between 4.0 and 9.0 as illustrated on the “Defuzzification” graph of Figure 6.4, and

2. \( \mu_{s_{ME}} \) is equal to 2/3 and \( \mu_{s_{MH}} \) is equal to 1/4 then the defuzzified value for the concluding (or “best compromise”) release can be computed as:

\[
R^* = \frac{2 \times \frac{1}{25} + 3 \times \frac{2}{3} + 4 \times \frac{2}{3} + 5 \times \frac{1}{25} + 6 \times \frac{1}{4} + 7 \times \frac{1}{4} + 8 \times \frac{1}{4}}{\frac{1}{25} + \frac{2}{3} + \frac{2}{3} + \frac{1}{25} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = 5.736. 
\]

Using the CoM would have given:

\[
R_{CoM}^* = \frac{13 \times \frac{2}{3} + 18.5 \times \frac{1}{4}}{\frac{2}{3} + \frac{1}{4}} = 4.413. 
\]

The CoM method under-estimates the solution crisp value. In CoA, the greater the number of quantification levels of the output the more accurate the result. In this study the number of quantification levels \( m \) (Equation 6.5) of the output is defined such that the difference between the degree of membership \( \mu_R(r_{i2}) \) and \( \mu_R(r_{i1}) \) of \( r_{i2} \) and \( r_{i1} \) at the quantification level \( i_2 \) and \( i_1 \) is 0.2 (see fuzzy logic program in Appendix D). The value 0.2 was chosen after trial and error computations.

![Figure 6.4: Example of defuzzification using CoA method.](image)
6.3. Application of the fuzzy logic control model to the WRS

6.3.1. Tuning of the fuzzy logic controller

In modelling the WRS operation by the fuzzy logic controller, the input variables were determined to be a combination of: the beginning-of-the-day storage, the anticipated daily net inflow (assumed known), the reference release values, and the time of the year characterised by the daily minimum and maximum storage levels below and above which water rationing and storage conservation for flood alleviation should be triggered respectively. The output variables were defined according to Equation 3.18 (see Section 3.4) and the physical constraints of the rivers of the system. Equation 3.18 ensures an effective operation of the system. The rule system structure is then formulated as follows:

Rule 1: If inflow is LO then water release is LO
Rule 2: If inflow is ML then water release is ML
Rule 3: If inflow is ME then water release is ME
Rule 4: If inflow is MH then water release is MH
Rule 5: If inflow is HI then water release is HI

The calibration of the model was done on a daily basis using Equations 3.18, water release functions of the gate open stage-discharge and the sets of daily inflows from the selected reference planning horizons (see Section 5.2.2.3). The system’s water balance (Equation 2.2); the physical constraints on the reservoir, i.e., constraints on storage (see Section 2.4.1.1) and on diversion of water from Mararoa river into Manapouri (see Section 2.4.1.4); and the inter-reservoir zonal relationship (see Section 2.4.2) were also incorporated as rules. For simplification, it was assumed that the output on any particular day is represented by the derived flow of water.

The simulation was done so as to satisfy the specified constraints throughout the system and to keep, if possible, the two tandem reservoirs of the system in balance at all times. Therefore, the rules for individual reservoirs were formulated such that:

- when the release storage level of a reservoir is between the top of conservation zone and the top of spilling zone, i.e., zone VI of Figure 3.6, releases are made to attempt to draw the reservoir to the top of the conservation zone without exceeding the release value given by the right hand side of Equation 3.18b.
- releases are made equal to the reduced water demand when the reservoir storage is between the top of the inactive zone and the top of the buffer zone (zone II of Figure 3.6), and greater than or equal to the reduced water demand when the reservoir storage is greater than the top of the buffer zone.
• although no release is allowed in the inactive release zone (zone I of Figure 3.6), in the real-world system, it can be equal to a computed minimum volume of water. Therefore, when the storage level of a reservoir enters zone I of Figure 3.6 and is above the absolute minimum storage level (see Section 2.4.1.1), the releases are made equal to the minimum of 1) the current day inflow, 2) the volume of water equal to the open-gate stage-discharge releases and 3) the maximum permitted drawdown rate of water for the storage in the low range (see Appendix B).

• releases are made equal to a value greater than or equal to the flow value given by the left hand side of Equation 3.18b and less than or equal to the flow value given by the right hand side of Equation 3.18b until the top of spilling zone (zone VI of Figure 3.6) is exceeded. Then the excess water is spilled if sufficient outlet capacity is available. If insufficient capacity exists, a release is made to fit the available outlet capacity.

The operational rules of Te Anau were formulated such that:

• Releases are made to avoid as much as possible contributions to flooding Manapouri.

• Releases are made where possible to maintain Manapouri reference releases.

• Attempts are made to bring Te Anau, in the measure of possibility, to the same storage index level (see Section 2.4.2) as Manapouri, based on storage index level at the end of the previous time period. This is done such that the constraints of the releases are not violated. The objective is to meet downstream flow requirements and keep the reservoirs in balance. This balancing may occur immediately or may take several time periods depending upon the storage, inflows and water requirements of the system.

To achieve the release requirement imposed on Te Anau, the assumptions below are made. Assuming $s_{0,T}$ and $s_{0,M}$ are the initial storage (index level at the end of the previous time period) of Te Anau and Manapouri respectively, then:

if $s_{0,T}$ is LO and $s_{0,M}$ is ML, or ME, or MH, or HI or,

if $s_{0,T}$ is LO, or ML and $s_{0,M}$ is ME, or MH, or HI or,

if $s_{0,T}$ is LO, or ML, or ME and $s_{0,M}$ is MH, or HI or,

if $s_{0,T}$ is LO, or ML, or ME, or MH and $s_{0,M}$ is HI then the Zadeh “AND” inference rule is used to determine, out of rules 1, 2, 3, 4, and 5 above and the actual inputs, the concluding fuzzy membership of the outputs. The purpose is to release water from Te Anau
such as to fulfil the third requirement of the operational rules of Te Anau specified above. On the other hand:

- if \( s_{o,T} \) is LO, or ML, or ME, or MH, or HI and \( s_{o,M} \) is LO or,
- if \( s_{o,T} \) is ME, or MH, or HI and \( s_{o,M} \) is LO, or ML or,
- if \( s_{o,T} \) is MH, or HI and \( s_{o,M} \) is LO, or ML, or ME or,
- if \( s_{o,T} \) is HI and \( s_{o,M} \) is LO, or ML, or ME, or MH, then the Zadeh “OR” inference rule is used. This is to satisfy the first two requirements of the operational rules of Te Anau specified above.

### 6.3.2. Results

One of the influential motivations for this study was the 1992 drought with its associated electricity shortage experienced in New Zealand. It is therefore appropriate that analyses and simulations should demonstrate what effect, if any, the availability and application of the developed real-time fuzzy logic reservoir control model incorporating as rules the solutions of the min-max approach, might have had in mitigating the severity of the electricity shortage of 1992. The appropriateness of the model can be tested with the 1988 flood year experienced in The South Island of New Zealand. From the above described fuzzy logic control model, solutions were obtained for reservoir end-of-the-day water levels and outflows at Te Anau and Manapouri. The solutions are indicators of the potential flood mitigation and availability of hydro fuel for power generation and water for other secondary but important uses during the specified periods.

#### 6.3.2.1. The 1988 flood

A retrospective analysis of floods shows that the most extreme long-duration flood event in the catchment started in June 1988 with lake levels rising and peaking in September and October/November of that year. The October/November peak was the most significant by far with the November 4, 1988 flood being the largest since 1926 for both Te Anau and Manapouri (Freestone, 1992). It was therefore suggested that the model be tested for the period extending from September to November 1988. The results are presented in Figures 6.5 to 6.8. They indicate an improved operation of the system by demonstrating that the lake levels can be maintained well below the levels achieved by the operators. Previous studies (Freestone, 1992) suggested that in 1988, the “flood duration/interval” guidelines could not be satisfied because the flood, due to its extreme nature, could not be contained within the rules.
Figure 6.5: Te Anau water level during the extreme long-duration flood period of 1988 (October/November). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station to define the behavior of the WRS after the construction of a second tailrace at the power station.

Figure 6.6: Manapouri water level during the extreme long-duration flood period of 1988 (October/November). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station to assess the benefit of installing an additional tailrace tunnel and of a future increase water discharged through the power station.
Figure 6.7: Manapouri lake total control outflow during the extreme long-duration flood period of 1988 (October/November). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station to define the behavior of the WRS after the construction of a second tailrace at the power station.

Figure 6.8: Manapouri Power Station discharge during the extreme long-duration flood period of 1988 (October/November). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station to define the behavior of the WRS after the construction of a second tailrace at the power station.
This present proposed reservoir control approach demonstrates however, that it was possible to maintain the lake level well below the level achieved by the operators and to satisfy compliance with the flood duration/interval guidelines. Moreover, even though the simulated lake level was high in Te Anau it was still below that achieved by the operators over the flood period (September to November). The improved result is due to the ability of the fuzzy logic controller to incorporate the solutions of the min-max approach to provide the right volume of water at the right time. This demonstrates that, had the model existed prior to the flood in 1988, the operators would have been able to achieve a better operational performance. The choice of a large range of water release and the foreknowledge, each day, of the maximum storage at which flood flow mitigation should be triggered coupled with a reliable fuzzy logic controller would have been very useful to the operator. This would have contributed to an effective decision-making process in terms of the system operation and thus enhanced the performance. From Figures 6.5 and 6.8 it can be seen that, even a constant daily flow of 470 m$^3$/s can still be released through the power station while maintaining the simulated lake levels unchanged in both lakes, and drawing down the cumulative volume of water spilled. This indicates that, even if the capacity of the power station was increased, an improved performance of the system was still possible using the proposed control approach. The simulation was performed with a constant daily flow of 470 m$^3$/s (higher than the currently permissible 460 m$^3$/s) to define the behavior of the WRS after the construction of a second tailrace at the power station.

Figures 6.5 and 6.7 show that the simulated results follow almost the same pattern as the results achieved by the operator. Moreover, the simulated lake levels are lower than those achieved by the operator and are within the limits set by the system operational guidelines. This means that the model can adequately be used to operate the system with a lower risk of system failure during periods of flood similar to that of 1988.

During the whole period of October-November flooding the WRS operators operated the system with the control structure gates fully open in order to discharge the flood water and to prevent system failure (Freestone, 1989). This reflects the willingness of the operators to drain the reservoir and possibly inundate the downstream rivers and lake to avoid lakeshore flooding. The fully gate-open policy is reflected in Figures 6.5 and 6.6 by the nearly identical shape of the actual water level in both Lakes Te Anau and Manapouri. The identical shape of the lake level during that period reflects the close dependency of the operation of Lake Manapouri on that of Te Anau. Other studies (Freestone, 1992, Freestone, Carter and Rogers, 1991) also confirm the similarity of water level shape for both lakes during the October to November 1988 flood. The fuzzy logic
controller is deterministic in the sense that it guarantees that "the same input condition(s) will always result in the same output condition(s). Therefore, if properly tuned, the fuzzy logic controller coupled with the min-max approach solutions can become a powerful tool for operating the WRS. The output conditions can be assessed a day in advance thus helping the operator make the appropriate decisions.

6.3.2.2. The 1992 drought

The inflows into the WRS during the period extending from November 1991 to June 1992 were particularly low. The cumulative effect of this low flow event on hydro-storage was substantial, particularly in the South Island where the drought was most extreme (Electricity Shortage Review Committee, 1992). The Committee found that the extremely dry months of November and December 1991 resulted in storage being below expected in December. The Committee also noted that while January and February inflows were close to average, the inflows dropped below expected at the end of February and deteriorated from then onward at an accelerating rate. It was therefore adequate to test the proposed control model for the period November 1991 to June 1992 in order to assess its effectiveness in dealing with drought. The model was tested assuming that the total available water for power generation was used and the willingness of ECNZ to release a constant volume of water down the Lower Waiau River depending on the time of the year (Section 5.2.1.) was taken into account. The results of the simulation are shown in figures 6.9 to 6.13. The results indicate that if the proposed controller had been available and used at the time of the drought, the system's performance would have been better. The simulated cumulative Manapouri total controlled outflow was higher than that achieved by the operators for the same period (Figure 6.11). The simulated cumulative outflow is equal to $8.0023 \times 10^9$ m$^3$ compared to $7.8027 \times 10^9$ m$^3$ achieved by the system operators (see Figure 6.12). This corresponds to a 2.56 percent increase in volume of water for diverse uses including power generation. It is worthwhile noting that this was achieved with a constant 460 m$^3$/s maximum load through the power plant. The minimum constant low flow requirements down the Lower Waiau river is also satisfied most, if not all, of the time.
Figure 6.9: Te Anau water level during the extreme long-duration drought period of 1991/1992 (November 1991 to June 1992). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station. Maxpow = maximum water discharged (in m$^3$/s) at Manapouri power station.

Figure 6.10: Manapouri water level during the extreme long-duration drought period of 1991/1992 (November 1991 to June 1992). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station. Maxpow = maximum water discharged (in m$^3$/s) at Manapouri power station.
Figure 6.11: Manapouri Power Station discharge during the extreme long-duration drought period of 1991/1992 (November 1991 to June 1992). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station. Maxpow = maximum water discharged (in m$^3$/s) at Manapouri power station.

Figure 6.12: Manapouri total control outflow during the extreme, long-duration drought period of 1991/1992 (November 1991 to June 1992). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station. Maxpow = maximum water discharged (in m$^3$/s) at Manapouri power station.
Figure 6.13: Mararoa weir outflow during the extreme, long-duration drought period of 1991/1992 (November 1991 to June 1992). Two different flow rates, 460 m$^3$/s and 470 m$^3$/s were discharged through the power station. Maxpow = maximum water discharged (in m$^3$/s) at Manapouri power station.

(see Figure 6.13). Figure 6.13 indicates that, while the gates at the Mararoa weir remained closed most of the time throughout the operation of the system during the 1992 drought, there was actually a possibility to release water for both power generation, and secondary but necessary uses. This was achieved with both Te Anau and Manapouri lake levels (Figures 6.9 and 6.10) remaining within the storage limits imposed by the lakes operating guidelines. In both lakes, the simulated levels are even higher for most of January 1992 to May 1992. The pattern of the lake levels is also similar. This indicates that the proposed control system operates identically to the way the operators would have performed, but with the advantage of providing efficient solutions. The high lake levels, especially in Manapouri, mean adequate water head for power production and enough storage water for emergency uses and thus sustainable operation. Figure 6.13 shows that with the exception of the spills in November 1991 and those in February, March, April and May 1992 the gates at the Manapouri Control Structure were kept closed. The figure also shows that with the use of the fuzzy logic controller the minimum required flows of 16 m$^3$/s throughout November to May, 12 m$^3$/s from May to September and 14 m$^3$/s in April and October each year and other required flows (see Sub-section 5.2.1.1 and Figure 5.1) are fulfilled. The fuzzy logic controller demonstrates its robustness and effectiveness in achieving an
optimum performance throughout the low inflow period. Figures 6.9 to 6.13 inclusive indicate that it is possible to achieve an even better performance in the operation of the power station. This was demonstrated by setting the optimal flow of water to be released through the power station at 470 m³/s. Good results were obtained with the lake level remaining at similar levels to those corresponding to 460 m³/s. The results show that, had the proposed control approach existed, an effective operation of the system during the 1992 drought could have been achieved. The results demonstrate that there is plenty of room for a better and more effective performance of the system during drought periods.

6.4. Discussion and Conclusions

In order to clearly point out the advantages and potential of the proposed reservoir control approach over the current mode of operation it was assumed adequate to use a simple qualitative comparison of the simulated versus the observed operational results instead of an analytical comparison.

In the development of a reservoir control model, the construction of a fully stochastic model is often considered a better choice. However, this is often extremely difficult to achieve because of the: 1) excessive computational requirements and 2) high probability of non-availability of the required hydrological data. The excessive computational requirements has led to numerous investigations adopting purely deterministic models. The min-max model developed in this study, although deterministic in concept, is an exception to other purely deterministic models. This is because it approximates the solutions for the future system operational requirements by using all the available historically observed inflow sequences, or some extreme inflow sequences the system managers want to protect the system against, to define the most likely future solutions. The solutions are a set of daily upper and lower storage values at which peak flood flow mitigation and water rationing can effectively be triggered respectively, and a set of daily ranges of desirable releases. The approach’s solutions are equivalent to saying that the obtained flood mitigation and water rationing trigger storage values can guarantee the effective operation of the system under a type of future inflows not worse than those of the reference inflow set. The solutions obtained are therefore incorporated in the fuzzy logic controller where the storage conditions are mapped into favorable outflow conditions to achieve an optimum operation of the system. The solutions are determined separately for each and every day of the planning-horizon. And because of the deterministic properties of the fuzzy logic controller, any future storage conditions similar to those derived from the
reference inflows will result in similar outflow conditions defined (using the reference inflows) by the fuzzy logic controller. Therefore, the proposed controller represents a new and effective way of operating the WRS as the outputs to some known hydrologic conditions are known with certainty. Due to the time invariant and continuity properties of fuzzy logic control systems the transfer function describing the mapping of inflow into outflow will not change over time, and a minor change in inflows will not result in an abrupt change in the outflows. This shows the robustness of the controller as it ensures effective daily operation of the system from year to year as long as the future inflow sequences are not worse than those of the reference set. However, even if the future inflows are worse than those historically observed or those proposed by the system operators, the solution flood flow mitigation and water rationing trigger values can be updated, in relation to the new conditions, by re-running a simulation with the min-max controller. Therefore, as far as the flood mitigation and water rationing trigger values are adequately defined, the fuzzy logic controller is guaranteed to provide sound and effective operating rules for the system under any future hydrological events.

The proposed fuzzy logic controller, incorporating the min-max solutions as inputs and rules, was used to simulate the operation of the WRS during the 1988 flood and the 1992 drought periods. The results of the simulation indicate that the controller is suitable for operation under extreme hydrological conditions. This is proven by the fact that satisfactory results were obtained for both 1988 and 1992 extreme hydrological scenarios. During the 1988 flood period, the simulated reservoir levels were well below those achieved by the system operator. While during the 1992 drought, the simulated lake levels in both lakes were higher than those achieved by the system operators for most of the critical period. The higher lake levels during drought equate to more efficient power generation. The enhanced performance for the 1992 drought is also demonstrated by the release of more water downstream to the Lower Waiau as compared to what the system operators achieved.

The similarity in the pattern of simulated and operated reservoir levels indicates that the proposed controller behaves in a similar way to the operators. The only difference is that the operation is performed more effectively.

Notwithstanding the retrospective nature of the results of the simulation, it can be said that, if at the beginning of the 1988 flood or of the 1992 drought, this controller had been available, then the system operator would have achieved a more effective operational performance. However, the improved results do not mean that the system is being poorly operated by the present mode of operation. This is because:
1. The simulations were carried out as if the Manapouri power station was unaffected by its integration into a larger generation system (the national grid).

2. The complexity of identifying the idle times in the power plant which are caused by breakdowns and maintenance. Some of these operational halts occur randomly during any time of the year. They are difficult to consider in any simulation model because it is not certain if they will occur, when they will occur, how long they will last and what kind of repercussions they will have on the whole system.

3. The management of the system which must take into account many intangible effects and decisions that cannot be adequately considered with a mathematical model. This is mainly true when making flood regulation decisions, i.e., in zone V of Figure 3.6 where a conservative attitude prevails with regard to flood management.

The simulated performance does show, however, that there is room for refining the current operating process and that the proposed combination of min-max solution and fuzzy logic controller presents itself as an effective and robust tool to achieve this task.

The fuzzy logic control process adopted is applied by looking one day ahead as opposed to the current practice where the operating rules are applied by looking one week ahead. This introduces a more detailed and refined control process. The other advantages of the proposed mode of operation over the currently utilised approach are that:

1. Due to the properties of the min-max control approach, the problem of reliance on some probabilistic characterisations of the future inflows can be solved by considering that future inflows belong to some given sets whose boundaries represent the worst minimum and maximum inflows historically observed or some other extreme values against which the operators want to develop a sound operational policy.

2. Due to the time-invariant and continuity properties of the fuzzy logic controller, the transfer function describing the non-linear mapping of inflows into outflows does not change over time. Moreover minor deviation of future inflow values from those of the historically observed inflows will not cause abrupt change in the output results.

The proposed controller also demonstrates with its solutions that there exist:

1. New possibilities for improving energy production under drought while maintaining the reservoir levels within their acceptable limits.
2. New possibilities of operating the system effectively during a flood and providing adequate storage for flood flow, thus decreasing the risk of flood damage and non-compliance with the guidelines.

3. Water released through the Lower Waiau river for environmental and non-environmental uses can be augmented under any hydrological events.

4. The storage levels at which flood mitigation and water rationing is triggered in any day can be adequately pre-defined.

5. Daily flood flow mitigation and daily water rationing triggers rather than time invariant trigger values produce better operating performance.

Thus, the proposed control approach provides a sustainable mode of operating the WRS under a given reference set by defining for each day of the planning-horizon a set of admissible and effective control rules such that the system states remain within their acceptable limits while the operational performance is enhanced.
CHAPTER SEVEN

SUMMARY AND CONCLUSIONS

7.1. Introduction

In the study the development of an effective reservoir controller to complement or supplement the WRS operator’s decision-making process was undertaken. This is to contribute to enhancing the WRS operational performance with particular focus on minimising risk of failure during periods of extreme hydrological events. This springs partly from the 1992 water shortage events where the whole of New Zealand’s hydro systems experienced what was the worst hydrological events in 65 years of observations. The experience prompted the Electricity Corporation of New Zealand to search for better ways of operating its complex hydropower generation systems. Particular emphasis is placed, in this study, on the management of the WRS, which is one of the largest systems in terms of energy production in New Zealand.

To respond to the need of this study a methodology that integrates two important criteria: long-term efficiency (enhancing the performance of the system) and risk aversion (improving the worst case performance and avoiding dramatic failures), is proposed. It consists of:

- Firstly, solving a deterministic risk aversion optimum control problem through the proposed min-max control approach. The solutions to the problem are a whole range of possible daily releases and storage zones that guarantee the worst of a given reference inflow set would surely avoid causing system failure.
- Secondly, developing a daily system-operating model incorporating the defined range of daily releases and storage zones, the system water balance, the physical constraints and other secondary requirements and constraints on the system as decision-making rule-base, to solve the long-term efficient operational performance of the system.

The proposed operating model, i.e., a fuzzy logic controller incorporating the solutions of the min-max approach as a rule-base, although simple in essence, is a powerful and effective tool for real-time daily operations. Its application will allow the release, during each day, of the “best compromise” water volume out of a whole range of possible and effective values. The proposed fuzzy logic controller has proven robust, yet easy to fine-tune due to its rule-based structure which mimics the human way of thinking. It can be fine-tuned in the light of the operating experience gained over the years by system
operators, and the solutions provided by the min-max approach. Thus it offers a way for the WRS operators to make sound and effective decisions.

7.2. Summary

The 1988 flood in the South Island and the 1992 nation-wide drought events were used to test the performance of the developed fuzzy logic controller incorporating the solutions of the proposed min-max reservoir control concept for the WRS. The model performed satisfactorily. It simulates lake’s levels better than those achieved by the system operators during both 1988 flood and 1992 drought periods, especially in Lake Manapouri. During the flood event, the simulated Manapouri lake levels were well below those achieved by the system operators while during the drought event they were above those achieved by the system operators most of the time. In Te Anau, although compliance with the flood duration/interval guidelines can not be achieved, the simulated 1988 flood event lake’s levels were well below those achieved by the system operators. The simulated low lake levels in Te Anau during that event suggest that, contrarily to the finding Riddell, Freestone and Nutting (1993) it was possible to draw the Te Anau lake levels further down compared to those achieved by the system operators. This is an indication that the approach has dealt more satisfactorily with the 1988 flood event. The performances of the controller in dealing with the 1992 drought event water releases through the power station and to the downstream Lower Waiau river for secondary uses is also satisfactory. During the 1992 drought event the simulated cumulative volume of water released through the power station and the Manapouri control structure was 2.56% greater than that achieved by the system operators. This can be assessed as an improvement over the current mode of operating the system. The most important improvement is the simulated high lake levels in both lakes. The simulated high lake levels in Manapouri mean availability of more water head for power generation, thus an increase in power production efficiency.

The simulated lake levels for both 1988 flood and 1992 drought follow, to some extent, a similar pattern to those achieved by the system operator. This suggests that the operating rules from which the simulated effective solutions are selected have the same structure as those of the current mode of operation. Therefore, the proposed solutions are anticipated to be easily understood by the system operator and therefore easily accepted and adopted. This feature is a very important requirement in practice. Many control approaches have failed to be adopted by system operators just because their solutions appeared too different from the ones the system operators are achieving.
In finding solutions to complex problems there is usually no single alternative plan of action which is better than all the other. The improved performances reported by the use of the proposed approach are not the only possibilities, but are what is considered in this study as the best alternatives. They may be viewed as an upper boundary to the possible gains that could be derived from the use of the approach.

The proposed approach simulated the performance of the WRS during the 1988 flood and the 1992 drought satisfactorily and complied relatively well with the operational guidelines as compared to the current mode of operation. Where the guidelines were not complied with, the simulated results show an improvement in relation to the performance achieved by the operators. These demonstrate that sustainable performances of the WRS can be achieved during normal or extreme hydrological events not worse than those historically observed if the proposed controller is used.

The proposed approach also introduced flexibility and benefits in terms of constraints imposed on storage. The current operational guidelines impose three storage zones for the WRS reservoirs (section 2.4.1.1.) namely: low, main and high operating zones. These zones are fixed in time and space. This illustrates the rigid way the system is currently operated. As is commonly known, flows vary with seasons and the filling and emptying of reservoirs are function of flows and of the time of the year. Fixing the spilling trigger point (i.e., top of high operating zone) to a single value and the water rationing trigger point (i.e., top of low operating zone) to a single value throughout the whole year is not an effective and flexible way of operating a reservoir-system. Too late or too soon flood mitigation or water rationing can be detrimental to the effective operation of a reservoir-system. Therefore, the proposed min-max reservoir control approach solutions that suggest varying flood mitigation and water rationing trigger points throughout the year are advantages over the current operating guidelines of the WRS.

The dependence of releases on inflows, introduces the importance of: 1) forecasting future inflows, and especially 2) selecting the reference inflow set necessary for the min-max control approach in determining the risk aversion solutions. Studies were carried out at Lincoln University to forecast future inflows in New Zealand hydro-systems (Peters, 1996). Due to the deterministic and time invariant properties of the fuzzy logic control system, the overall performance of the proposed fuzzy logic controller can be assessed as not very sensitive to the reliability of the inflow predictor. Nevertheless it is anticipated that the incorporation of the inflow predictor approach developed at Lincoln University, into the controller’s rule-base will contribute in making it an effective real-time operating tool for the WRS.
The limitations in this study can be grouped as data-related and operation-related. As was previously discussed, the quality of the Mararoa stream flows pre-commissioning of the Mararoa weir are not fully reliable due to missing data. Moreover, the inflows to the lakes are not measured directly but are back calculated from measurements of lake’s levels, and the outflows from the lakes. As is commonly known, error can be introduced in measuring low and extremely high lake levels. Because of the scope of this study, errors in the inflow calculation were not assessed and corrected. Therefore, the validity of the simulated results is considered with these limitations. Another limitation (operation-related) is the consideration of the WRS as a stand-alone system and the non-introduction of the many intangible operational decisions such as the unscheduled closing of the power station for maintenance or dealing with breakdowns. These operation halts as discussed previously occur randomly during any time of the year and therefore are difficult to consider in any simulation model. A study involving integration of the WRS within the large national grid generation system and management that takes into account all the intangible effects are beyond the scope of this study. However, the advantage of a range of effective releases to choose from, introduced by the solution of the proposed approach, suggests the possibility and flexibility of dealing adequately with these intangible decision-making processes.

7.3. Conclusions

Analysis of the results of the simulation indicates that the proposed fuzzy controller incorporating the modified min-max reservoir control approach solutions as inputs and rules can be used satisfactorily to:

1. Enhance the effectiveness of the operational performance of the WRS during normal and extreme hydrological circumstances.
2. Minimise the risk of not meeting the WRS management requirements in terms of power generation, environmental (including reservoir physical constraints) and non-environmental requirements, in any normal and especially extreme (flood and droughts) hydrological events not worse than those historically observed, or suggested by the system operator.
3. Comply with the statutory guidelines of the lakes under any hydrological circumstances.

The results of the simulation show that during the 1988 flood and 1992 drought periods the simulated water levels were to a greater extent within the limits set by the statutory guidelines as compared to the water levels achieved by the WRS operators. The simulated performances show efficient power generations while more water was
released for secondary uses during the 1992 drought and less floodwater was released during the 1988 flood to the Lower Waiau River.

4. Lend a helpful hand to the operators who until now operate the system only through their own experience and the statutory guidelines.

5. Lend a helpful hand to experienced operators in training novice operators

Moreover, the results of the simulation show that the current mode of operation adopting one single water rationing trigger point and one single flood mitigation trigger point throughout the whole year is a rigid way of operating the system. The min-max reservoir control approach was successfully used to introduce flexibility and sound operating techniques by determining trigger values that vary with the seasons of the year.

7.4. Recommendations

From a training perspective, the whole approach, because it can accommodate more than one operational style or water release policy, is well suited to helping senior operators refine their techniques and to providing training for novice operators.

Although, for the purpose of this study, the performance of the WRS has been assessed as if it was a stand-alone system, it is in fact an integral part of the New Zealand overall national grid energy generation system. Therefore, it would certainly be useful to adapt the proposed control approach to assess the performance of the WRS as part of the overall national-grid energy generation system.
REFERENCES


techniques.


APPENDIX A

The Min-Max Approach to Storage Control Problems

A.1. Introduction

The intention of this appendix is to summarise the approach which Orlovski et al. (1983) have applied to solve the management problem related to a multi-objective single reservoir system (called Lake Como) located in the northern part of Italy. The notation of their work has been adopted so as to conform as far as possible with that of this thesis.

The authors have considered a two-objective storage control problem that they solved using a min-max approach. The approach permitted the determination of a set of control laws that guarantees given degrees of satisfaction of the objectives in the case of the worst sequence of water supply to the system out of a pre-specified set of possible sequences. Using this approach they were able to determine at any time a range of feasible water releases. Their approach, which has been modified in the thesis to solve the problem of a multi-objective two-reservoir system, is described in its original form in this appendix.

A.2. Goal Constraints

In essence, the goals in controlling a storage system consist in satisfying demands for the releases of water from the reservoir, and in attenuating storage peaks. Formally these goals can be expressed in terms of inequalities which are to be observed when choosing an appropriate control of the system. In the approach developed by Orlovski et al. (1983), the inequalities have the following form

\[ r_t \geq \alpha r^*_t, \quad t \geq 0, \]  
\[ s_t \leq \beta s^*_t, \quad t \geq 0 \]  

where \( r^*_t, s^*_t \) are pre-specified reference values for the minimal release and the maximal storage at which there are no losses or damage at time \( t \). The coefficient \( \alpha (\alpha \leq 1) \) and \( \beta (\beta \geq 1) \) are introduced to make these constraints more flexible. Ideally, the aim will be to find a control law which guarantees the satisfaction of the goal constraints with \( \alpha = 1 \) and \( \beta = 1 \). However, if such a control law does not exist, then the constraints with equations (A.1) can be relaxed by introducing some values \( \alpha < 1 \) and/or \( \beta > 1 \). In these cases the problem of controlling the system is for a multi-objective kind and the determination of solutions consists in providing the greatest possible values of \( \alpha \) and the lowest values of \( \beta \). In other words, the solution to the problem consists in providing the lowest possible water deficits...
and the lowest possible flood damages. Both $r^*_t$, $S^*_t$ are assumed periodic in the sense that

$$r^*_t = r^*_{t+T}, \quad S^*_t = S^*_{t+T} \quad (A.2)$$

for any $t \geq 0$, with $T$ the time period introduced in section 3.

The problem under consideration, therefore consists in determining pairs $(s_0, r)$ of initial states and control rules which guarantee the satisfaction of constraints

$$0 \leq r_t \leq N(s_t), \quad t \geq 0 \quad (A.3)$$

and Equations (A.1) for given values of $\alpha$ and $\beta$; with $N(s_t)$ equals the stage-discharge function defined in section 3. To be more specific, the following shall be introduced:

**Definition 1** $[(\alpha, \beta)$-feasibility of $(S_\xi, r)]$. Given an instant of time $\xi$ ($\xi$ represents an hour and $\xi=0, 1, \ldots, t$) of day $t$ ($t=24$ hours) and the set of possible sequences of inflows $W^{\alpha}_{\xi}$, a pair $(S_\xi, r)$ is called $[(\alpha, \beta)$-feasibility if the storage, from instant $\xi$ to the end of the day $t$ denoted by, $\varphi(\xi, t, S_\xi, r, W^{t-1}_{\xi})$ and the corresponding values of $r_t$ of the control variable satisfy the constraints in Equation (A.3) and Equations (A.1) for all $t \geq \xi$ and all $W^{t-1}_{\xi} \in W^{\alpha}_{\xi}$.

Similarly a pair $(S_\xi, r)$ will be called $\alpha$-feasible $[\beta$-feasible] if the control constraints stated in Equation (A.3) and the goal constraints (A.1a) [(A.1b)] are satisfied for all $t \geq \xi$ and for all possible inflow sequences. Thus a pair $(S_\xi, r)$ is $[(\alpha, \beta)$-feasible if it is both $\alpha$- and $\beta$-feasible.

The rest of the appendix will be devoted to the determination of the sets of $\alpha$-, and $\beta$-feasible pairs which will be denoted by $F^\alpha_\xi$, and $F^\beta_\xi$ respectively.

**A.3. Water demand satisfaction**

In this section, a way of determining a set $F^\alpha_0$ of $\alpha$-feasible pairs $(s_0, r)$ which is defined in the form of the direct product of two sets as follow below will be suggested.

$$F^\alpha_0 = S^\alpha_0 \times R^\alpha,$$

where $S^\alpha_0$ is the set of initial states having the form

$$S^\alpha_\tau = \{s_t \mid \text{there exits } r \text{ such that } (s_t, r) \text{ is feasible} \}, \tau \geq 0,$$
and $R^a$ the set of control laws which will be defined later in the appendix. In other words, the set $F_0^a$ can be obtained by separately determining the sets $S_0^a$ and $R^a$.

In what will follows, the control law which specifies the values $\alpha r_i^*$ for the control variable at any current instant of time $t$ will be defined as $r_{\text{min}}$, and refer to as the minimal release policy. Furthermore it will be used to determine the set $S_r^a$.

**Lemma 1.** For any $t \geq 0$, if $S_r \in S_r^a$, then the couple $(S_r, r_{\text{min}})$ is $\alpha$-feasible.

**Proof.** If $S_r \in S_r^a$ then, by definition, there exists a control law $r$ such that $(S_r, r)$ is $\alpha$-feasible. Assume $r_i$ and $s_i$ represent the corresponding values of respective variables for some fixed sequence $W_r^s \in W_r^s$. Furthermore, assuming that $r_{\text{min}} = \alpha r_i^*$ and $S_i$ represent the releases and storages obtained for the same sequence of supplies by applying the minimal release policy, then, from the state equation

\[ S_{t+1} = S_t + W_t - r_t, \quad t \geq 0 \] (A.4),

equation $S_{t+1} - S_{t+1} = (S_r - S_t) + (r_t - r_{\text{min}})$ can be obtained. But $S_r - S_t = 0$ and $r_t - r_{\text{min}} \geq 0$, since $r$ is $\alpha$-feasible, so that $S_t \geq S_i$ for all $t \geq \tau$. Consequently, using the $\alpha$-feasibility property of $(S_r, r)$ and the assumption that $N()$ is non-decreasing, $r_{\text{min}}$ is verify for all $t \geq \tau$ and all sequences $W_r^s \in W_r^s$. From the satisfaction of the above inequality, remark can be made that $r_{\text{min}}$ satisfies the control constraints stated in Equation A.3 and therefore the couple $(S_r, r_{\text{min}})$ is $\alpha$-feasible.

Lemma 1 implies that the set $X_r^a$ contains those and only those storages $S_r$ that satisfy the following conditions:

\[ \alpha r_i^* \leq N(s_i), \quad t \geq \tau, \] (A.5a)

\[ S_{t+1} = S_t + W_t - \alpha r_i^*, \quad t \geq \tau, \] (A.5b)

for all inflow sequences $W_r^s \in W_r^s$.

Another property of the set $W_r^s$, which lemma 1 can easily verified is:

\[ S_r \in S_r^a \Rightarrow s \in S_r^a, s \geq S_r. \]
Therefore, with the introduction of the value
\[ s^a_r = \min_{s \in S'^a_r} s, \tag{A.6} \]
the set \( S^a_r \) can explicitly be described as
\[ S^a_r = \{ s / s \geq s^a_r \}. \tag{A.7} \]

Problem of the type (A.6) are not practically solvable, since they involve an infinite number of constraints. Nevertheless, using the periodicity properties stated in equations 2.5 and A.2, it can be conclude that
\[ S^r_{kT} = S^r_0, \quad k \geq 1 \quad (k = \text{planning horizons defined in section 2}), \tag{A.8} \]
and consequently \( S^r_{kT} = S^r_0 \) for all \( k \geq 1 \). This fact allows to prove the following

**Lemma 2.** For any \( k \geq 1 \) and any \( \tau \in [(k-1)T, kT-1], \) if \( s_\tau \in X'^a_r \) then
\[ S^r_{kT} = \varphi(\tau, kT, s^r_\tau, r^{min}_r, W^{kT-1}_r) \geq S^a_0 \tag{A.9} \]
for any \( W^{kT-1}_r \in W^{kT-1}_r \).

**Proof.** If a assumption contrary to Lemma 2 is made, then using Equation A.8 on can demonstrate that for some planning horizon \( k \geq 1, \tau \in [(k-1)T, kT-1], \) \( s_\tau \in S^a_r \) and \( W^{kT-1}_r \in W^{kT-1}_r \), the pair \( (\varphi(\tau, kT, s^r_\tau, r^{min}_r, W^{kT-1}_r), r^{min}) \) is not \( \alpha \)-feasible, and therefore using Equation 2.5, it can be concluded that \( s_\tau \notin S^a_r \). This contradiction completes the proof.

The above result illustrates that by adding the constraint (A.9) to equations of the type (A.5), the set \( S^a_r \) can be described by means of a finite number of constraints. More particularly, the following sequence of mathematical-programming problems for determining \( S^a_r, \tau \geq 0 \) can be formulated:

**Problem 0.**
\[
\begin{align*}
    s^a_0 &= \min s_0, \\
    \alpha r^*_t &\leq N(s_t), \quad t = 0, ..., T-1, \\
    s_{t+1} &= s_t + w_t - \alpha r^*_t, \quad t = 0, ..., T-1, \\
    s_T &\geq s_0, \quad w^{T-1}_0 \in W^{T-1}_0.
\end{align*}
\]
**Problem** $\tau$ ($\tau \in [(k-1)T, kT-1], k1 \geq 1$).

\[ S_{\tau}^\alpha = \min S_t, \]
\[ \alpha r_t^* \leq N(s_t), \quad t = \tau, ..., kT-1, \]
\[ s_{t+1} = s_t + w_t - \alpha r_t^*, \quad t = \tau, ..., kT-1, \]
\[ s_{kT} \geq S_0^\alpha, \quad W_r^{kT-1} \in W_r^{kT-1}. \]

The appearance of the solution of Problem 0 ($S_0^\alpha$) in the formulation of Problem $\tau$ helps conclude that Problem 0 should be solved first. More importantly, it is worthwhile pointing out that Problem $\tau$ ($\tau > 0$) must be solved in real-time, since the set $W_r^{kT-1}$ is known only at time $\tau - 1$. If real-time computations are not feasible, then the reference set $W_0^{kT-1}$ (see Sub-section 2.3) can be used instead of the set $W_r^{kT-1}$ and the corresponding problem may be solved prior to starting the control process. In such case advantage is not taken of the observations obtained during the course of the process, and the values of $S_t^\alpha$ computed that way are generally greater than those computed using real-time information in the form of the sets $W_r^{kT-1}$.

In real-life situations, the determination of $S_t^\alpha$ values is very simple. In fact, if the number of sequences of the set $W_r^{kT-1}$ is finite, then it only suffices to solve the corresponding Problem $\tau$ for each inflow sequence $W_r^{kT-1}$ and select the maximal of those values.

Having define and solve the problem of system state variables, the set $R^\alpha$ of control laws which was mentioned earlier can now be defined.

**Definition 2 (set $R^\alpha$)** The set $R^\alpha$ consists of all control laws $r_t = r(t, s_t, w_t, W_r^{kT-1})$ satisfying the inequalities
\[ \min\{N(s_t), \alpha r_t^*\} \leq r_t \leq \min\{N(s_t), \max\{s_t + w_t - S_{t+1}^\alpha, \alpha r_t^*\}\} \] (A.10)
for all $k \geq 1$, $t \in [(k-1)T, kT-1]$. 
Theorem 1. If a control law \( r \in R^a \), then the pair \((S_0, r)\) is \( \alpha \)-feasible for any \( S_0 \in S_0^a \).

Proof. It is worthwhile noticing first that the right-hand side inequality in Equation (A.10) implies that the control constraint stated in Equation (A.3) is satisfied, while the left-hand-side inequality is equivalent to the goal constraint stated in Equation (A.1a) provided

\[
\alpha r_0^* \leq N(s_i)
\]  

Therefore, it should be proved that any control law \( r \) satisfying (A.10) gives rise to release \( r_1 \) and storage \( S_t \) that satisfy (A.11) for any \( t \), provided \( S_0 \in S_0^a \).

At time \( t = 0 \) the satisfaction of (A.11) is guaranteed by the definition of \( S_0^a \) as in Problem 0. As for the future, there are two cases: 1) either \( S_0 + \mathcal{W}_0 - S_1^a \geq \alpha r_0^* \), or, on the contrary 2) \( S_0 + \mathcal{W}_0 - S_1^a < \alpha r_0^* \). In the first case, Equation (A.10) implies \( S_1 = S_0 + \mathcal{W}_0 - r_0 \geq S_1^a \), i.e., \( S_1 \in S_1^a \), and consequently the definition of \( S_1^a \) (see Problem \( \tau \) for \( \tau = 1 \)) guarantees the satisfaction of (A.11) for \( t = 1 \). In the second case, (A.10) implies \( r_0 = \alpha r_0^* \), and therefore the definition of \( S_0^a \) (see Problem 0) guarantees the satisfaction of (A.11) for \( t = 1 \).

If \( S_1 \in S_1^a \) at time \( t = 1 \), the above argument can be repeated and proof provided that (A.11) holds for \( t = 2 \). If on the other hand, \( S_1 \not\in S_1^a \), the following two cases can be considered: either 1) \( S_1 + \mathcal{W}_1 - S_2^a \geq \alpha r_1^a \) or 2) \( S_1 + \mathcal{W}_1 - S_2^a < \alpha r_1^a \). As was demonstrated above, in the case, (A.10) implies \( S_2 = S_1 + \mathcal{W}_1 - r_1 \geq S_2^a \), so that the definition of \( S_2^a \) guarantees the satisfaction of (A.11) at time \( t = 2 \). Similarly, in the second case, (A.10) implies \( r_1 = \alpha r_1^* \), and therefore, knowing that \( r_0 = \alpha r_0^* \), the satisfaction of (A.11) for \( t = 2 \) is guaranteed by the definition of \( S_0^a \). Therefore, by recursively using the same arguments the theorem is proved.

This theorem in conjunction with Definition 2 guarantees that the resulting product set \( F_0^a = S_0^a \times R^a \) introduced earlier in this section contains only \( \alpha \)-feasible couples.
(S₀, r). In fact, it can easily be verify that for any α₁ ≥ α² any α₁-feasible couple (S₀, r) is also α²-feasible. In other words, for any α₁ ≥ α² the following is verify
\[ F₀^{α₁} ≠ φ \Rightarrow F₀^{α²} ≠ φ. \]  (A.12)

From Theorem 1 conclusion can be made that the multistage decision making control process that provide for the satisfaction of the goal constraints stated in Equation (A.1a) consists in the following:

- the decision-maker at any instant of time t uses the set \( W^{r_{t+1}}_t \) to calculate the values \( s'^t \) through finding the solution to Problem τ for \( τ = t + 1 \) and then
- he chooses any value of \( r_t \) (release during time interval t) that satisfies inequalities stated in Equation (A.10). Naturally, the initial state of the system at time \( t = 0 \) must belong to the set \( S^α_0 \) obtained through solving Problem 0.

### A.4. Attenuation of storage peaks

The problem of attenuation of storage peak is similar to that of demand satisfaction described in the section above. Similar to the preceding section where it was proved that demand satisfaction can be facilitated by higher initial storages \( (S^α_0 \) is bounded from below) and realised by applying the minimal release policy, it is natural discerning here that the storage peaks increase with the initial storage and that their attenuation is realised by applying the maximal release policy where \( r = N(s_t) \) at any time t. Indeed, in a regulated reservoir system, the lowest possible flood is obtained by keeping the control structure gates permanently wide open.

In this section the set \( F^α_0 \) of β-feasible couples \((s₀, r)\) similar in form to that of \( F^α_0 \) will be determined. \( F^α_0 \) can be written as: \( F^α_0 = S^α_0 \times R^β \). All the proofs are very similar to those already demonstrated in section A.3 since the results obtained there have their dual that can formally be obtained by replacing the set \( S^α_0 \) with the set \( S^α = \{s_τ | \text{there exists } r \text{ such that } (s_τ, r) \text{ is } β\text{-feasible}\} \) and \( r^\text{min} \) with the maximal release policy \( r^\text{max} \) given by
\[ r^\text{max}_t = r(t, s_t, W_t, W^{r_{t+1}}_t) = N(s_t). \]

The dual of Lemma 1 is the following
Lemma 3. For any $\tau \geq 0$, if $s_\tau \in S_\tau^\theta$ then the pair $(s_\tau, r_{\max})$ is $\beta$-feasible.

Proof. The proof is a direct consequence of the property $dN(s)/ds<1$, which implies $N(s_\tau) - N(s_\tau') \leq s_\tau - s_\tau'$ for all $s_\tau \geq s_\tau' \geq 0$. In fact, if $s_\tau \in S_\tau^\theta$, then, by definition, there exists a control law $r$ that gives rise to releases $r_\tau$ and storage $s_\tau$ which satisfy the constraints stated in Equations (A.3) and (A.1b) for $t \geq \tau$. On the other hand, the storage $s_\tau$ and the release $r_{\max} = N(s_\tau)$ obtained at time $t$ by applying the maximal release policy are such that $s_{\tau+1} - s_{\tau+1} = s_{\tau+1} - s_{\tau+1} N(s_{\tau}) - r_\tau$. However, $r_\tau \leq N(s_\tau)$, since $(s_\tau, r)$ is $\beta$-feasible; consequently $s_{\tau+1} \leq s_{\tau+1} + N(s_{\tau}) - N(s_{\tau})$. Therefore, $s_\tau \geq s_\tau$ implies $s_{\tau+1} \geq s_{\tau+1}$. Since $s_\tau = s_\tau$, inequalities $s_\tau \leq s_\tau \leq \beta s_\tau$ are satisfy for all $t \geq \tau$, and this implies the Lemma.

Moreover, an explicit description of the set $S_\tau^\theta$ can be obtained by simply determining the value $S_\tau^\theta = \max_{s \in S_\tau^\theta} s$, since $S_\tau^\theta = \{s / 0 \leq s \leq s_\tau^\theta\}$.

The following property which is the dual of Lemma 2 also holds:

Lemma 4. For any $k \geq 1$ and any $\tau \in [(k-1)T, kT-1]$, if $s_\tau \in S_\tau^\theta$ then $s_{kT} = \phi(\tau, kT, s_\tau, r_{\max}^{kT}, w_{\tau}^{kT-1}) \leq s_0^\theta$ for any $w_{\tau}^{kT-1} \in W_{\tau}^{kT-1}$. The implication of this property is that the values $s_\tau^\theta$ can be obtained by solving the following mathematical-programming problems:

Problem 0.

\[
\begin{align*}
S_0^\theta &= \max S_0, \\
S_t &\leq \beta S_t^1, & t &= 0, \ldots, T-1, \\
S_{t+1} &= S_t + w_t - N(s_t), & t &= 0, \ldots, T-1, \\
S_\tau &\leq S_0, & w_{\tau-1}^T &\in W_{\tau-1}^{T-1}.
\end{align*}
\]

Problem $\tau$ ($\tau \in [(k-1)T, kT-1], k \geq 1$).
Having solved the preceding mathematical-programming problems, thus defining the set $S^\beta_0$, the set $R^\beta$ similar to that of $R^\alpha$ can be defined.

**Definition 3 (Set $R^\beta$).** The set $R^\beta$ consists of all control laws

$$r_i = r(t, s_i, w_i, W_r^{kT-1})$$

that satisfy the inequalities

$$\min\{N(s_i), \max\{s_i + w_i - s_i^\beta, 0\}\} \leq r_i \leq N(s_i)$$

(A.13)

for all $k \geq 1$, $t \in [(k-1)T, kT-1]$, where the properties of these control laws are given by the following:

**Theorem 2.** If a control law $r \in R^\beta$, then the pair $(s_0, r)$ is $\beta$-feasible for any $s_0 \in S^\beta_0$.

**Proof.** The proof to this theorem is similar to that of Theorem 1, and confer to Equation A.12, for any $\beta^* \leq \beta^\beta$

$$F_0^\beta^* \neq \phi \Rightarrow F_0^\beta^\beta \neq \phi.$$  

(A.14)

**A.5. Conclusion**

In this appendix the min-max storage control approach was described, and different ways of determining control laws and initial system state variables which guarantee given degrees of satisfaction of the objective constraints imposed on the system in the worst possible sequence of inflows out a pre-specified set of possible sequences were given.

The approach consists of two operational optimisation stages. The first stage consists in determining the optimum degrees of satisfaction of the water demand objectives by solving a certain mathematical-programming problem over the whole planning horizon (water year). This stage is performed prior to starting the control process and is done by using only the a priori information in terms of future inflows. The second stage similarly consists of determining solutions to a sequence of mathematical-programming problems
which are defined in real-time, since they depend upon the information obtained in the course of the control process.

The approach described in this appendix demonstrates a basic concept which for the reasons outlined in Section 1.3 have been chosen as the basis for the approach developed in this study. The control approach in this appendix was adapted to a multi-objective single reservoir system and therefore will require substantial and conceptual modifications in order to deal with the operational management problems of the multi-objective multi-reservoir system (WRS) proposed in this study.
APPENDIX B

Waiau River System Statutory Guidelines
(Freestone, and Eaton, 1993)

B.1. Statutory guidelines

Operation of the WRS is governed by a set of constraints on the reservoirs water storage and water releases named the “statutory guidelines”. The so-called Guardians of Lakes Manapouri and Te Anau ensure the compliance with the guidelines during the managerial operation of the system. The guidelines recognised three operation range or storage zones for each lake in the system, namely: main, high and low ranges. In the main operation range the operation of the system in terms of water release is largely unconstrained. In the high and low ranges the system is to be operated such as to limit the period spent within these ranges.

Another requirement of the guidelines is to direct turbid flows from Mararoa river down the Waiau River and so reduce discoloration of the lake. Sediment transport in the Mararoa occurs predominantly in floods and hence it is important that most of the Mararoa flood flows are discharged through the control structure.

B.2. High operation range

The high operation range is the zone where the water level is above 178.6 m and 202.7 m above mean sea level Deep Cove datum for both Lakes Manapouri and Te Anau respectively. The tables below describes the maximum number of days that can be spent in each zone determined by an interval of elevation, and the minimum number of days that is allowed for the recurrence of being in a particular zone.

For each lake in the high operating range:
1. If the ratio between “interval” and “previous duration” for any particular event of shorter
duration than specified in the tables equals or exceeds the ratio in the table, the
requirements of the guidelines are complied with.

2. If the interval duration ratio so calculated is less than the ratio in the table, then for the
purpose of compliance, the duration is considered to include the subsequent interval.

3. Periods of duration including subsequent intervals if appropriate are accumulated until
the required ratio is achieved.

4. Accumulated periods of duration, as defined in paragraph 3, should not exceed the
permissible maximum.

**Table B.1**: Lake Manapouri: maximum duration and Minimum interval between floods in
different operation ranges in the high operation range.

<table>
<thead>
<tr>
<th>Elevation (m amsl)</th>
<th>Maximum duration (continuous days)</th>
<th>Minimum interval between flood in this level (continuous days)</th>
<th>Interval/duration ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 180.5</td>
<td>1</td>
<td>100</td>
<td>100.0</td>
</tr>
<tr>
<td>Above 180.4</td>
<td>3</td>
<td>100</td>
<td>33.0</td>
</tr>
<tr>
<td>Above 180.1</td>
<td>9</td>
<td>100</td>
<td>11.1</td>
</tr>
<tr>
<td>Above 179.8</td>
<td>22</td>
<td>80</td>
<td>3.6</td>
</tr>
<tr>
<td>Above 179.5</td>
<td>35</td>
<td>40</td>
<td>1.1</td>
</tr>
<tr>
<td>Above 179.2</td>
<td>44</td>
<td>40</td>
<td>0.9</td>
</tr>
<tr>
<td>Above 178.9</td>
<td>99</td>
<td>20</td>
<td>0.2</td>
</tr>
<tr>
<td>Above 179.6</td>
<td>119</td>
<td>20</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Table B.2: Lake Te Anau: maximum duration and Minimum interval between floods in different operation ranges in the high operation range.

<table>
<thead>
<tr>
<th>Elevation (m amsl)</th>
<th>Maximum duration (continuous days)</th>
<th>Maximum interval between flood in this level (continuous days)</th>
<th>Interval/duration ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 204.3</td>
<td>1</td>
<td>100</td>
<td>100.0</td>
</tr>
<tr>
<td>Above 204.2</td>
<td>3</td>
<td>100</td>
<td>33.0</td>
</tr>
<tr>
<td>Above 203.9</td>
<td>10</td>
<td>60</td>
<td>6.0</td>
</tr>
<tr>
<td>Above 203.6</td>
<td>22</td>
<td>30</td>
<td>1.4</td>
</tr>
<tr>
<td>Above 203.3</td>
<td>39</td>
<td>30</td>
<td>0.8</td>
</tr>
<tr>
<td>Above 203.0</td>
<td>65</td>
<td>30</td>
<td>0.5</td>
</tr>
<tr>
<td>Above 202.7</td>
<td>125</td>
<td>20</td>
<td>0.2</td>
</tr>
</tbody>
</table>

B.3. Low operation range

The low operation range is the zone where the water level ranges from 176.8 m to 175.86 m and from 201.5 m to 200.86 m above mean sea level Deep Cove datum for both Lakes Manapouri and Te Anau respectively. The tables below describe the maximum duration (continuous number of days) that can be spent in each zone determined by an interval of elevation.

For both lakes in the lower operating range:

1. ECNZ will use its best endeavours to avoid low lake level during the equinoctial periods (March, April, October and November).

2. The annual total days below any particular elevation should not exceed twice the maximum duration specified for any one event below that elevation.
3. The rate of drawdown should not exceed natural rates of drawdown namely 0.05 m per day for Lake Manapouri and 0.03 m per day for Lake Te Anau. Both average over 4 days.

Table B.3: Manapouri operation guidelines in the low operation zone.

<table>
<thead>
<tr>
<th>Elevation (m amsl)</th>
<th>Maximum duration for any events (continuous days)</th>
<th>Maximum interval between drought in this level (continuous days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 176.8</td>
<td>107</td>
<td>214</td>
</tr>
<tr>
<td>Below 176.5</td>
<td>66</td>
<td>132</td>
</tr>
<tr>
<td>Below 176.2</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>At 175.9</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Table B.4: Te Anau operation guidelines in the low operation zone.

<table>
<thead>
<tr>
<th>Elevation (m amsl)</th>
<th>Maximum duration for any events (continuous days)</th>
<th>Maximum interval between drought in this level (continuous days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 201.5</td>
<td>88</td>
<td>176</td>
</tr>
<tr>
<td>Below 201.3</td>
<td>46</td>
<td>92</td>
</tr>
<tr>
<td>Below 201.1</td>
<td>21</td>
<td>42</td>
</tr>
</tbody>
</table>
APPENDIX C

Te Anau and Manapouri lakes control provisional gate structure rating.
Table C.1: Manapouri lake control provisional gate structure rating (m$^3$/s)

<table>
<thead>
<tr>
<th>Indices</th>
<th>1, A</th>
<th>2, B</th>
<th>3, C</th>
<th>4, D</th>
<th>5, E</th>
<th>6, F</th>
<th>7, G</th>
<th>8, H</th>
<th>9, I</th>
<th>10, J</th>
<th>11, K</th>
<th>12, L</th>
<th>13, M</th>
<th>14, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>HWL</td>
<td>175.87</td>
<td>176.0</td>
<td>176.2</td>
<td>176.4</td>
<td>176.8</td>
<td>177.2</td>
<td>177.6</td>
<td>178.0</td>
<td>178.6</td>
<td>179.2</td>
<td>179.8</td>
<td>180.2</td>
<td>180.6</td>
<td>181.0</td>
</tr>
<tr>
<td>Flow</td>
<td>0</td>
<td>4</td>
<td>17</td>
<td>36</td>
<td>89</td>
<td>157</td>
<td>239</td>
<td>333</td>
<td>494</td>
<td>677</td>
<td>881</td>
<td>1028</td>
<td>1183</td>
<td>1346</td>
</tr>
</tbody>
</table>

Note: HWL = Head water level in metre above mean sea level, deep Cove datum; flow in m$^3$/s.

Table C.2: Manapouri lake control provisional tail water level rating - January 1992.

<table>
<thead>
<tr>
<th>Indices</th>
<th>1, A</th>
<th>2, B</th>
<th>3, C</th>
<th>4, D</th>
<th>5, E</th>
<th>6, F</th>
<th>7, G</th>
<th>8, H</th>
<th>9, I</th>
<th>10, J</th>
<th>11, K</th>
<th>12, L</th>
<th>13, M</th>
<th>14, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWL</td>
<td>174.1</td>
<td>174.4</td>
<td>174.7</td>
<td>175.0</td>
<td>175.3</td>
<td>175.6</td>
<td>176.0</td>
<td>176.5</td>
<td>177.0</td>
<td>177.5</td>
<td>178.0</td>
<td>178.5</td>
<td>179.0</td>
<td>180.0</td>
</tr>
<tr>
<td>Flow</td>
<td>0</td>
<td>5</td>
<td>11</td>
<td>26</td>
<td>74</td>
<td>168</td>
<td>310</td>
<td>502</td>
<td>710</td>
<td>935</td>
<td>1175</td>
<td>1610</td>
<td>2140</td>
<td>3450</td>
</tr>
</tbody>
</table>

Note: TWL is the Tail water level; flow in m$^3$/s.

Table C.3: Provisional rating Mararoa at Cliffs - applies from 11th January 1992.

<table>
<thead>
<tr>
<th>Indices</th>
<th>1, A</th>
<th>2, B</th>
<th>3, C</th>
<th>4, D</th>
<th>5, E</th>
<th>6, F</th>
<th>7, G</th>
<th>8, H</th>
<th>9, I</th>
<th>10, J</th>
<th>11, K</th>
<th>12, L</th>
<th>13, M</th>
<th>14, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>level(m)</td>
<td>180.0</td>
<td>180.1</td>
<td>180.2</td>
<td>180.4</td>
<td>180.6</td>
<td>180.8</td>
<td>181.0</td>
<td>181.4</td>
<td>181.8</td>
<td>182.2</td>
<td>183.0</td>
<td>184.0</td>
<td>185.0</td>
<td>186.0</td>
</tr>
<tr>
<td>Flow</td>
<td>6.3</td>
<td>7.7</td>
<td>9.3</td>
<td>12.7</td>
<td>18.2</td>
<td>28.4</td>
<td>43.2</td>
<td>85</td>
<td>139</td>
<td>200</td>
<td>346</td>
<td>574</td>
<td>851</td>
<td>1180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indices</th>
<th>1,A</th>
<th>2,B</th>
<th>3,C</th>
<th>4,D</th>
<th>5,E</th>
<th>6,F</th>
<th>7,G</th>
<th>8,H</th>
<th>9,I</th>
</tr>
</thead>
<tbody>
<tr>
<td>HWL</td>
<td>179.22</td>
<td>179.3</td>
<td>179.4</td>
<td>179.5</td>
<td>179.6</td>
<td>179.8</td>
<td>180.0</td>
<td>180.4</td>
<td>181.0</td>
</tr>
<tr>
<td>Flow</td>
<td>0</td>
<td>23</td>
<td>78</td>
<td>151</td>
<td>238</td>
<td>450</td>
<td>701</td>
<td>1305</td>
<td>2420</td>
</tr>
</tbody>
</table>

Note: HWL = Head water level in metre above mean sea level, deep Cove datum; flow in m$^3$/s.

Table C.5: Te Anau lake control gate rating table (m$^3$/s).

<table>
<thead>
<tr>
<th>Indices</th>
<th>1,A</th>
<th>2,B</th>
<th>3,C</th>
<th>4,D</th>
<th>5,E</th>
<th>6,F</th>
<th>7,G</th>
<th>8,H</th>
<th>9,I</th>
<th>10,J</th>
<th>11,K</th>
<th>12,L</th>
<th>13,M</th>
<th>14,N</th>
<th>15,O</th>
</tr>
</thead>
<tbody>
<tr>
<td>level(m)</td>
<td>196.0</td>
<td>200.5</td>
<td>201.0</td>
<td>201.5</td>
<td>201.8</td>
<td>202.0</td>
<td>202.2</td>
<td>202.5</td>
<td>203.0</td>
<td>203.5</td>
<td>204.0</td>
<td>204.5</td>
<td>205.0</td>
<td>205.5</td>
<td>206.0</td>
</tr>
<tr>
<td>Flow</td>
<td>0</td>
<td>106</td>
<td>162</td>
<td>244</td>
<td>285</td>
<td>333</td>
<td>374</td>
<td>421</td>
<td>539</td>
<td>651</td>
<td>776</td>
<td>909</td>
<td>1063</td>
<td>1263</td>
<td>1386</td>
</tr>
</tbody>
</table>
NOTE:

- The rating of all the tables may be linearly interpolated.
- In Te Anau and Manapouri lakes control gate rating tables it is worthwhile noticing that the ratings are for gate openings with gates clear of water.
APPENDIX D

ROUTINES, SUBROUTINES AND DATA FILES

All routines and subroutines are Fortran coded.

D.1. MIN-MAX RESERVOIR CONTROL MODEL

D.1.1. Routines

The filenames with the extension .exe are representative of Fortran files with .for extension.

Teanlh. exe: This routine computes Te Anau feasible initial maximum storage for a critical year out of the historically observed data, and inputs daily releases from Te Anau into Manapouri.

Tanau2h. exe: This routine computes Te Anau feasible initial minimum storage for a critical year out of the historically observed data, and inputs daily releases from Te Anau into Manapouri.

Manapl_h. exe: This routine computes Manapouri feasible initial maximum storage for a critical year out of the historically observed data.

Mnpr2_h. exe: This routine computes Manapouri feasible initial minimum storage for a critical year out of the historically observed data.

DH_Tean1. exe: This routine computes Te Anau feasible daily maximum storage for a critical year out of the historically observed data, and inputs daily releases from Te Anau into Manapouri.

DH_Tanau. exe: This routine computes Te Anau feasible daily minimum storage for a critical year out of the historically observed data, and inputs daily releases from Te Anau into Manapouri.

DH_Manap. exe: This routine computes Manapouri feasible daily maximum storage for a critical year out of the historically observed data.

DH_Mnpr. exe: This routine computes Manapouri feasible daily minimum storage for a critical year out of the historically observed data.

Tean1. exe: This routine computes Te Anau efficient initial maximum storage for a critical year out of the historically observed data, and inputs daily releases from Te Anau into Manapouri.
Tanau_2. exe: This routine computes Te Anau efficient initial minimum storage for a critical year out of the historically observed data, and inputs daily releases from Te Anau into Manapouri.

Manap1. exe: This routine computes Manapouri efficient initial maximum storage for a critical year out of the historically observed data.

Mnpr2. exe: This routine computes Manapouri efficient initial minimum storage for a critical year out of the historically observed data.

D_Tean1. exe: This routine computes Te Anau efficient daily maximum storage for a critical year out of the historically observed data and inputs daily releases into Manapouri.

D_Tanau2. exe: This routine computes Te Anau efficient daily minimum storage for a critical year out of the historically observed data and inputs daily releases into Manapouri.

D_Mnap. exe: This routine computes Manapouri efficient daily maximum storage for a critical year out of the historically observed data.

D_Mnpr2.exe: This routine computes Manapouri efficient daily minimum storage for a critical year out of the historically observed data.

D.1.2. Subroutines

Inistor: Determines the end-of-the-planning horizon storage starting from a given day t.

Treleas: This subroutine determines the minimum of gate opening release and reference release for both Te Anau and Manapouri.

Tean_rele: This subroutine determines the minimum of gate opening release and reference release from Te Anau.

Fld: Determines the storage corresponding to the reference water release.

Ref_rel: Inputs the current Manapouri reference release.

R_releas: Inputs the current Te Anau reference release.

Mreleas: Inputs the current Manapouri gate fully opening release

Flood: Determines the relaxed reference release.

Referflw: Computes the Manapouri storage corresponding to the reference release.

Mandrel: Inputs the minimum compensation flow down the Lower Waiau river.
D.1.3. Data files

Waiau.txt: This file contains the daily inflows for Te Anau, Manapouri from its own catchment, and daily Mararoa streamflows. The first three columns contain in succeeding order the year-month-day, month-day and month-week-day (with 1 representing Monday, 2 Tuesday and so on). Column four contains Manapouri daily inflow, column 5 contains Te Anau daily inflow and the last column contains Mararoa daily streamflows. Manapouri, Te Anau inflows and Mararoa streamflows are represented by Eflow1, Eflow2 and Eflow3 in the routines.

Line 1: Characters (title of the columns)
Line 2-T: Integer in columns 1-3, and real in columns 4-6. T = the number of day data is available.

Tean.dat and Manap.dat: Contain the lake levels and their corresponding storage and release for Te Anau and Manapouri respectively.

Tean_1q.dat, Tean_2q.dat, DT_rel.dat, DT_rel2.dat: Contain the Te Anau feasible daily water release into Manapouri.

Line 1: Characters (title of the column)
Line 2-T: Real - Daily water releases with T = number of data.

D.2. FUZZY LOGIC CONTROLLER

D.2.1. Routines

Wai_syst. exe: This routine simulates the behaviour of the Waiau River-System to determine its optimum operation policies. The routine makes use of the available information to compute the input and output fuzzy set boundaries, the degree an input and output belong to the fuzzy sets and determines the crisp (defuzzified) output solution.

D.2.2. Subroutines

Treleas: Inputs the Te Anau gate opening release
Tstorage: Determines the relationship between a given Te Anau release and the reservoir water level.
R_release: Inputs Te Anau reference release.
M_storage: Determines the relationship between a given Manapouri release and the reservoir water level.
Elevation: Defines the relationship between a given Manapouri water storage and water level.

Fuznorml, Fuzlow: Fuzzy inferences.

Defuz: Defuzzification.

D.2.3. Data files

Tean_mm.dat and Mnpr_mm.dat: Contain Te Anau and Manapouri daily maximum and minimum solution storages at which flood mitigation and water rationing are triggered respectively.

**Line 1:** Characters (title of the columns)

**Line 2-T:** Integer in column 1 (date in month-day); Real in column 2-3 - Daily maximum and minimum storage; \( T = \) number of data.

Waiau11.txt, Waiau1.txt: These files contain the daily inflows for Te Anau, Manapouri from its own catchment, and daily Mararoa streamflows for the years 1988 and 1991/92 respectively. The first three columns contain in succeeding order the year-month-day, month-day and month-week-day (with 1 representing Monday, 2 Tuesday and so on). Column four contains Manapouri daily inflow, column 5 contains Te Anau daily inflow and the last column contains Mararoa daily streamflows. Manapouri, Te Anau inflows and Mararoa streamflows are represented by Eflow1, Eflow2 and Eflow3 in the routines.

**Line 1:** Characters (title of the columns)

**Line 2-T:** Integer in columns 1-3, and real in columns 4-6. \( T = \) the number of day data is available.
APPENDIX E

GENERATION OF MISSING MARAROA RIVER STREAM FLOW DATA

E.1. Introduction

Records of hydrologic processes such as stream flows are usually short and often have missing observations - the Mararoa River flow data set is no different. Therefore, one of the first steps in any operational management decision requiring the stream flow data is to fill in the missing values -if any- and/or to extend shorts records where required. The filling up of missing values or the extension of short records can be easily achieve with the existence of nearby sites with the same or longer records. In the following, an attempt was made to determine the Mararoa river missing stream flow values and to extend the record where needed, using the adjacent catchments rivers and lakes.

E.2. Mararoa stream flow data

The stream flow data set for the Mararoa River was obtained through Works Consultancy Services. Although the data set may not all be of a good quality, it remains the only available set. Consequently, any operational management decision requiring the river stream flow data would have to be dependent on it. From 1963 to 1967 a set of the Mararoa stream flow values was collected at the Mt York gauging station in order to investigate the feasibility of enhancing the Manapouri power station generation capacity by diverting Mararoa water into Lake Manapouri. The recording was stopped then re-started from 1974 at above the Lake Manapouri control structure weir gauging station, which was later replaced by the gauging station at Cliffs. These gauging sites are the most reliable recording sites on the river and data are still collected at Cliffs. The recorded data have however, been affected due to a number of reasons, thus generating, for periods of a day and/or up to weeks, some missing values. To synthetically fill in the gap created by the missing values the technique of linear regression was suggested. The technique involved establishing a correlation relationship between the Mararoa river stream flows and those of other gauging stations in the same river-system catchment and/or the adjacent catchments. The gauging station on Oreti River at McKellar’s flat, which was later replaced by the Three Kings gauging station was used. Those two gauging stations are located only several kilometers east of the Cliffs station and recording are maintained by the Southland Regional Council. Because the McKellar's flat recorded data are only archived from 1977
to 1986 and those of the Three Kings are recorded from 1986, it was suggested that the filling in of the Mararoa missing data be done in two stages. The first stage involving filling in the gaps in the data collected before 1986 using the McKellar's flat recorded data and the second stage involving filling in those gaps after 1986 using the Three Kings station's recorded data.

For the period prior to 1963, no known stream flow data were archived for the river. Therefore, an attempt was made to determine synthetically a stream flow data set for the pre 1963 critical years selected in Chapter five. There are no known available recorded stream flow data on the Oreti River pre 1963. It was suggested to use either of the Manapouri or the Te Anau inflow data separately or both combined, for the period of 1963 to 1967, to extend the Mararoa river stream flow back in time. The period of 1963 to 1967 was selected because it represents a period prior to the construction of the Manapouri lake control structure with available and reliable Mararoa river stream flow data.

E.3. Filling in the gaps in stream flow data

The critical years selected in Chapter five representing the post 1963 period were the planning horizons 1974/1975; 1975/1976; 1982/1983 and 1988/1989. Because the planning horizons are assumed to start on the first day of July of each year (see section 4.2), the 1974/1975 year data is considered short in that record in that year started in August 8, 1974. Therefore the technique developed in the following section will be used to extend the record from August to July. The only planning horizon selected that has missing values is 1988/1989. Data were missing from November 17, 1988 to December 13, 1988 and from March 22, 1989 to March 28, 1989. A simple linear regression technique based on the best (largest) R² was then suggested to fill in the gaps. The Oreti River, at the Three Kings station, recorded data for the period of July 1, 1988 to November 16, 1988 (period immediately prior to the missing data) were used in the process. The linear regression gave an R² = 81.05% and a slope equal to 0.390893 (assuming zero (0) intercept). Thus:

\[ \text{Mararoa river stream flows} = 0.390893 \times (\text{Oreti river stream flows}) \]
Figure E.1: Correlation graph of Mararoa versus Oreti stream flows for the period of July 1, 1988 to November 16, 1988.

Figure E.2: Mararoa observed and estimated stream flows using the determined linear regression equation for the planning horizon of July 1, 1988 to June 30, 1989.

Figure E.1 shows the correlation relationship between Mararoa and Oreti data for the period immediately prior to the missing stream flow values. Figure E.2 indicates the accuracy with which the developed linear regression relationship can estimate the missing
Mararoa stream flows. Although the regression slightly under-estimates the Mararoa stream flows at times, it shows that the observed and correlated flows follow the same pattern most of the time during 1988/1989. The closeness of the two lines in Figure E.2 proved that the developed regression equation can be used to determine the missing stream flow values for that planning horizon.

E.4. Extending the stream flow data

Visual observation of the pattern followed by the Lakes Te Anau and Manapouri inflows in rapport to the Mararoa river stream flows for the period between 1963 and 1967 (Figure E.3), and the physical location of the lakes in rapport to the river gauging station indicate that the Manapouri inflows are more suitable compared to Te Anau for estimating the Mararoa river stream flows. This is confirmed by the results in table E.1 below.

The results in Table E.1 were obtained by: firstly adopting the technique of the “ladder of powers” transformation (Hirsch et al., 1992) to transform the Manapouri and Te Anau inflows and, secondly subjecting the original and transformed inflow values to a linear regression using the “best subset” technique. The “Best subset search” tool on Minitab statistic software package was used to define the best equation using the best R². The year 1964/1965 inflow data were selected because they give better representation of the relationship between the flow pattern over the other years of the 1963/1967 period.

The runs were divided in 1-; 2-; 3-; 4-variable runs. Examination of the value of the square of the single/multiple correlation coefficient, R², indicates that the Manapouri un-transformed inflow variables give the highest correlation coefficient while the Te Anau inflow variables show no correlation. Detail examination shows that, further gain in R² is minor after two or more variables have been introduced when Manapouri un-transformed inflow variables are already used. Thus the addition of further variables when Manapouri un-transformed inflow variables are already in the regression equation will remove very little of the unexplained variation in the response.
Figure E.3 A) and B): Observed Manapouri, Te Anau and Mararoa river inflows for 1964-1965.

Table E.1: The “best K” subsets regression output from Minitab.

<table>
<thead>
<tr>
<th>Vars</th>
<th>R-sq (%)</th>
<th>Adj. R-sq (%)</th>
<th>C-p</th>
<th>S</th>
<th>Mnp</th>
<th>Mnp^2</th>
<th>TeAn</th>
<th>TeAn^2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>38.6</td>
<td>38.4</td>
<td>68.7</td>
<td>18.460</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>25.6</td>
<td>25.3</td>
<td>160.3</td>
<td>20.332</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>47.5</td>
<td>47.2</td>
<td>8.6</td>
<td>17.098</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>45.1</td>
<td>44.8</td>
<td>25.3</td>
<td>17.098</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48.5</td>
<td>48.0</td>
<td>3.7</td>
<td>16.960</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48.0</td>
<td>47.6</td>
<td>6.6</td>
<td>17.027</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>48.6</td>
<td>48.0</td>
<td>5.0</td>
<td>16.967</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Note: Mnp and TeAn represent Manapouri and Te Anau inflow variables respectively. ^2 represents the ladder power of 2 of the inflow variables. X represents the subset selected.
This is shown by the slight increase in $R^2$ from the set of 2- to 4-variables runs. The Manapouri inflows were therefore suggested to estimate the Mararoa river stream flows prior to the year 1963 and for the month of July 1974. In order to avoid estimating negative stream flow values the linear regression equation was defined forcing the solution through the origin. The equation derived can be written as:

Mararoa stream flow = 0.097897 \times \text{Manapouri inflow}

Figures E.4 a) and b) show that although the estimated Mararoa stream flow using the developed linear regression might have been slightly overestimated at time, they however follow more or less the same observed stream flow pattern. This proved that although the regression approach may be a bit crude, it does however, estimate the Mararoa flow with a relative accuracy.

Figure E.4a) and b): two examples of observed and estimated Mararoa river stream flows using Manapouri inflows.
E.5 Conclusion

As the archived Mararoa river stream flows are in time sequence, the post 1977 missing values can be reliably estimated using the Oreti river stream flows and a linear regression for the period immediately prior to and/or after the missing data. Any missing value for the period prior to 1977 can be estimated using Manapouri inflow data. It should be noted that although the two methods described are relatively crude modes of estimating flow data, they give reasonable indication of what mean daily flows can be expected in Mararoa river for the missing and short record. Moreover, the existence of the longer record on Manapouri helps to make estimates relatively realistic.