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DYNAMIC PLANNING
AND
PIG FATTENING

A thesis
submitted in fulfilment
of the requirements for the Degree
of
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by
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ABSTRACT

Farmers make decisions in a planning environment in which planning information is frequently imperfect and outcomes uncertain. Efficient planning systems must therefore be dynamic in nature. However, available systems which are capable of being directly applied to individual farms on a whole farm basis do not reflect the true nature of the planning environment. This study explores the nature of these problems through developing a dynamic planning system for pig fattening.

The study considers the nature of the pig fattening problem and specifies the important factors that planning models must allow for to represent the realistic planning environment. The features of currently available models are reviewed leading to a statement of the improvements required. A realistic model of pig fattening is developed and possible solution algorithms are considered. As the models require detailed pig growth information a simulation model designed to predict response from alternative feeding patterns was developed. The output from this model together with the output from a model which calculates growth distributions and posterior probabilities on potential growth provide some of the input data for a stochastic multi-period linear programming model of the problem. The remainder, feed cost information, is obtained from least cost models formulated to represent the features of the dynamic planning problem.

With imperfect information, planning involves continual re-planning as new observations are made. Consequently only the solution to the first period of the model is likely to be implemented. To ensure that the first period decision set is optimal it is shown that a minimum number of periods must be included in the planning model. The minimum number is referred to as the planning horizon. Methods of determining a planning horizon are reviewed and it is concluded that available methods do not provide a general method for all planning situations. A method is developed for the pig fattening problem.

Finally, a number of planning experiments were carried out to demonstrate the value of the planning models and systems developed. The results indicate the potential value of applying sophisticated planning methods to individual farms and provide a means to examine the condition under which detailed individual farm dynamic planning can be worth while.
PREFACE

Many studies commence with the objective of deriving theories and methods which will make a significant contribution to knowledge. This study was no exception. The realities of extending the frontiers of contemporary knowledge means that most research projects provide only a small contribution. Taken together, the results of many projects may, however, quietly lead to an important development. It is hoped that this study provides a component of this process.

The work was carried out over a period of several years on a part-time basis with all the ensuing frustrations. Due to a change in employment as well as other factors, a total of five supervisors were involved in the course of the study. To all I am grateful for the encouragement and help provided. Professor W.O. McCarthy provided the initial encouragement and Professor J.B. Dent was presented with the difficult task of trying to make sense out of the draft of a study in which he had not been involved. In the interim, Professor J.D. Stewart, who first introduced me to an understanding of management and provided a deep respect for the need to be pragmatic, Dr L. Howard of the University of Queensland and Professor W. Musgrave were all involved.

I am also grateful to Mr S. Filan for providing me with an insight into the Burroughs Corporation's Tempo manual, and Dr M. Blackie for programming and editing assistance. The experience of Mrs Marion Mischler in typing the thesis was a considerable help, not to mention her perserverance.

The study is dedicated to farmers and my family.
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"We are just emerging from the period in which planning techniques, both at the farm and national level, were rather simple and naive ..." (Heady, 1971, p. 8). While simple techniques such as budgeting have played a major contribution to farm planning, and always will, our improved understanding of the nature of decision problems and the development of techniques to utilise this understanding means it is important to explore ways of introducing sophisticated planning at the farm level. Farmers have traditionally devoted much of their time to technical questions (Brownlee & Gainer, 1949), as have many professional advisers, and this emphasis will undoubtedly continue while the owner-operator farm forms the basis of an agricultural industry. Therefore methods which enable both farmers and advisers to utilize economic decision making developments may make a major contribution to an efficient industry.

Over the last twenty years many planning developments have concentrated on enterprise selection and individual enterprise efficiency under static, and largely certain, conditions. The planning environment, however, is commonly non-certain so that the problem is not only enterprise selection but also ensuring the selected products are efficiently produced through detailed dynamic planning and action using continuously observed outcomes and changing forecasts. Where the effective enterprise choice is not extensive, possibly as a result of obvious economic dominance, detailed dynamic planning becomes the major question. This means sophisticated planning techniques which can be applied to the individual farm can potentially be of considerable use.
The use of techniques such as linear programming and systems simulation in the enterprise selection and efficiency problems have largely been used on representative farms (Carter, 1963). Attempts at individual farm application have met with mixed success (Baker, 1971). This may be due in part to a failure to allow for the dynamic nature of the problem and also to the fact that the enterprise selection and efficiency problem is only one of many decision questions. It is not uncommon to find from survey information that managerial ability is more important than enterprise selection where obviously dominated enterprises are discarded (Stewart, 1964).

With recent developments in computer hardware and the formulation of software systems that remove much of the tedious and expensive hand work required in preparing, coding and punching data (Baker, 1971), the cost associated with implementing individual farm planning systems is declining. On the other hand system development costs are of major significance. This implies planning systems should only be developed where it is anticipated they can be used for many farms over many years. Due to the world wide similarities, pig fattening is a typical case.

The objective of this study is to consider the nature and structure of dynamic planning systems through the development of a model for pig fattening. As any economic analysis must rely on the underlying technical relationships, the study includes an examination of the general nature of these relationships though stress is not placed on an exhaustive treatment of all technical questions. As feed costs are a major component (Ryan, 1972) of total costs methods of improving the efficiency of feed use are considered in some detail. The other major area is the determination of a planning horizon as this is essential to the development of a dynamic planning system. The developments are not taken through to the farm applicational stage though an indication is given of their potential benefits. The economic usefulness of the systems developed must depend
on the specific conditions found in any one area.

The presentation is structured by initially considering the relevant decision variables (chapter II) and then using this background to review planning models (chapter III). In chapter IV a model which conceptually includes the major decision problems is presented and in chapter V alternative solving methods are discussed. The details of a linear programming model used to represent the decision problem are covered in chapter VII, while the technical relationships, important in devising efficient feeding systems leading to the coefficients for the linear programming model are discussed in chapter VI. The concept of a planning horizon and the development of a method to determine a horizon are treated in chapters VIII and IX. Finally, the results of experiments with the models are presented and an overall conclusion to the study given in chapters X and XI.

In the development of the conceptual planning models the objective has been to develop detailed formulations. This approach is considered necessary to enable deficiencies in current technical and economic knowledge, as well as computational algorithms, to be assessed. It also allows an assessment, sometimes on a subjective basis, of apparently justifiable simplifications. For practical applications it will be seen that some of the detail considered may need to be simplified.
CHAPTER II

THE DECISION VARIABLES IN PIG FATTENING

1. INTRODUCTION

To assess the features that must be included in the planning model it is necessary to list and consider the decision variables involved. Similarly, as many pig enterprises are involved with breeding and other related activities it is important to consider the relationships between the fattening and other possible components. In the following discussion it is assumed a fixed endowment of buildings, feed mixing and storage facilities exist and that the labour supply is fixed.

The chapter is structured by initially listing the components of a general pig enterprise and then discussing the decision variables within each component. The emphasis is placed on the fattening unit. Finally, the relationships between the components are specified.

2. THE COMPONENTS OF A PIG FATTENING ENTERPRISE

For management purposes it is useful to define components on the basis of general decision areas rather than on physical criteria. Table 1 contains a diagrammatic representation of the components. An individual pig enterprise will contain some but not necessarily all the components.

The diagram stresses that fattening activities are dependent on the major decision areas of:

(i) weaner supplies
(ii) feed supplies
(iii) marketing
(iv) financial factors
(v) labour and machinery organisation.
TABLE 1

Major Decision Units of a General Pig Enterprise

Feed Growing  →  Feed Purchasing  ↔  Stock Replacement  →  Stock Purchasing

Feed Storage and mixing  ↔  Weaner Provision

Pig Fattening

Marketing (pigs & feed)  ↔  Effluent disposal

Financial
1. Cash
2. Tax
3. Ownership

Management

Labour and Machinery
A feed growing unit is included as some enterprises will have sufficient land to produce a varying proportion of the feed required. The weaner production unit is assumed to include the sow activity. The stock replacement unit is therefore solely concerned with the rearing of replacement stock and associated selection decisions. Stock purchasing may involve both weaner purchase and breeding stock purchasing.

As the fattening unit is the primary area of concern in this study the discussion on the decision variables will concentrate on this area. Some of these variables will also have implications in other areas, an example being feed provision decisions.

3. THE DECISION VARIABLES

3.1 The Acquisition of Stock for Fattening and Sow Replacement

Where stock are being purchased the major question is the method of acquiring the necessary stock. A range of options exist including open market purchase through to co-operative contract arrangements, possibly involving vertical integration. Decisions made will determine whether the fattening and weaner production units have available a constant or fluctuating supply. Where the enterprise relies on the open market both:supplies and price will be related stochastic variables.

An important factor in an acquisition policy will be the genetic potential of stock. Co-operative organisations will enable a degree of control while in an open market system the genotypic density function will have a comparatively wide range. Similarly the response differences between male and female stock must be considered.

\[1\] This may involve purchasing stock from specific breeders based on historical information.
Relevant questions where weaners are produced are the numbers to be provided over time and the stage of development at which they should be transferred to the fattening unit. This implies more than age, as feeding policies will influence the state of the weaners at time of transfer. Using only a breeding policy means the fattening unit must accept a supply which is difficult to change rapidly. To ensure that the operations of the fattening unit are not constrained requires the production of weaner numbers that can satisfy any demand. (Excesses can be sold.) The cost of such a policy will depend on the marginal value product of weaners at different points in time compared with their market value. Facility capacities may prevent the consideration of such a policy. (A purchasing policy can clearly provide greater flexibility.) Where considerable excesses are not produced the supply of weaners to the fattening unit will be stochastic.

An important question in a breeding policy is whether to run a specific pathogen free (S.P.F.) system. Andrilenas (1964) in a survey of United States pig producers, found that S.P.F. systems reduced feed costs by 9.5%. This is achieved at a direct cost of the necessary precautions and the indirect costs of being forced to work within the constraint of a given supply of weaners through time. Such systems may also place restrictions on genetic change through constraints on the acquisition of breeding stock. Partial S.P.F. systems may be possible in a buying policy through the development of co-operating groups which largely isolate themselves from other producers.

There are, of course, many other decision variables within the weaner production unit. Examples are the time of weaning, feeding decisions, the level of preventative medication, intensity of supervision at farrowing, whether to castrate males and mating programme decisions particularly with respect to the use of hybrid vigour. The values given
to these variables largely effect the major variables of quantity, quality and stage of development of weaners produced through time.

3.2 The Provision of Feed Supplies

Included within this unit are the feed growing, feed purchasing and storage and mixing units. The function of these units is to satisfy the demand for feed from the stock replacement, weaner production and fattening units. This demand involves both quantity, quality and timing considerations and is dependent on the interactions between these two groups in that demand is a function of cost. Further, there may also be simple physical limitations on the quality and quantity supplied due to the facilities available and the availability of feed types.

An enterprise with sufficient land may have the choice of producing a proportion of its own feed supplies. Decisions in this area should depend on the opportunity cost of competing crop and stock activities as well as the availability of equipment and managerial expertise. Where feed is produced there may be a choice between grain and pulse crops and will depend on the cost of substitutes available on the open market. Another factor to be considered is that the quantity (and quality) of home produced supplies will be stochastic though this is less important where additional supplies can be readily purchased. Many enterprises must rely entirely on purchased feed. Such units commonly allocate 60-80%\(^2\) of their total budget to feed. The alternative acquisition methods available include purchasing as required from feed firms, entering into contracts with feed firms, purchasing source ingredients on the free market, making contracts with firms dealing in source ingredients or with the grain and pulse crop producers themselves, and finally, organising co-operative structures with

\(^2\) See, for example, Ryan (1972) and Meat and Livestock Commission (1974).
groups of producers. The latter may include both feed users and producers. Decisions made in this area will affect costs and the availability of different feeds through time. The facilities available for feed storage and mixing will influence an optimal feed supplying system. Storage enables the purchase of feed, both source ingredients and mixed material, at times of lower cost. Important decision variables are therefore the timing and quantity of feed purchases in order to satisfy a demand at least cost. Constraints on these operations are the level and type of storage available and the access to cash though on-farm feed storage can be augmented though renting. This could be in the form of paying a storage increment to grain and pulse crop producers for on-farm storage.

On-farm feed mixing can reduce feed costs appreciably and also influence quality (Hanley, 1971) though limits are imposed by the available plant and access to contract equipment. Decisions must be made regarding the number of different feed types to be prepared and the frequency with which they are to be mixed as frequency may be important due to chemical and physical changes with storage. The plant available affects quality through grinding and measuring efficiency.

Two other important decision variables fall in the control area. One is feed testing and subsequent action to control moisture levels and deterioration due to microbial action. The other is the testing of source ingredients for nutrient content in order to maintain a pig intake of given nutrient levels.

3.3 The Marketing Unit

Marketing decisions largely involve the disposal of fat and breeding stock though enterprises with a crop producing capacity may be involved with crop disposal and those with storage facilities in feed trading.
The primary decision variables revolve round whether to sell on the open auction market, sell on a weight/grade/price basis, use contracts, or some combination. Contract operations can involve decisions whether to form co-operative groups to obtain greater market leverage. Questions of commitment to a vertically integrated system are also relevant and, like other contract systems, may involve questions of quantity, quality and timing.

Other problems in this area include decisions regarding the conformation and weight of pigs when being considered for sale. Success in estimation will be non-deterministic. Also questions of slaughtering and transport methods may be relevant where these affect the quality of the product.

Inter-actions between the marketing and production units clearly exist. Decisions within the marketing complex influence the prices received for a defined product and contractual agreements may impose constraints on the fattening unit through requirements to supply specified quantities of given types through time. In the extreme case the acceptance of a contract may entirely dictate actions within the fattening and other units in order to meet the contract. In this case the problem is one of providing the given pigs at least cost.

3.4 Financial and Related Units

Included in this unit are the provision of operating funds through time, the development of income tax policies and the question of ownership organisation. These units are outlined together, due to their influence on the short run cash situation. Long term debt arrangements are also important in this respect.

Cash decisions largely involve ensuring an adequate supply of cash at any point in time. Questions of sources of cash and the
efficient use of surpluses are important where large feed inventories are kept and an irregular stock selling policy is followed. Use of income tax options enables income tax to be minimised but such policies must be related to the effect they have on liquidity as well as their direct costs. Similarly, given largely owner-operator type enterprises, ownership organisation can influence income tax commitments through profit sharing.

Decisions within this area therefore influence the cost of cash and the availability through time. The cost may be determined by either the opportunity or borrowing costs. An optimal tax policy may mean different timing of buying and selling decisions compared with the case where income tax is ignored. Further, where the owners have a non-linear monetary utility function the direct incorporation of income tax effects may influence fattening unit operations as they will reduce the marginal monetary return of decisions.

3.5 The Labour and Machinery Unit

It is assumed that the enterprise has a given complement of labour and machinery. Labour demand per se will depend on the physical facilities available. Labour decisions involve questions of incentive schemes and the degree of responsibility allocated to labour, as well as decisions regarding the level of intensity to devote to specific tasks. In large enterprises the allocation of labour between jobs is important given differing quality between labour units. Similarly, machinery decisions revolve round such factors as the level of repairs and maintenance and optimal replacement times. Questions of whether to use machinery to its full capacity to substitute for labour are also relevant.

Decision activity in this area results in a specific level of technical efficiency being achieved and therefore a defined response level for a given cost.
3.6 The Fattening Unit

3.6.1 Introduction

In a multi-unit enterprise the decisions made within this unit are directly related with actions in the other units as the other units provide stock and feed through time. The important characteristics of these profiles are the quantities, the quality in terms of genetic quality and feed quality, the stage of development of the animals and the number of different feed mixes available. Similarly, supplies of labour, cash and machinery are made available. Given these supplies the unit must also operate within the constraints of the physical fattening facilities available. Likewise, actions within the fattening unit place demands on the other units in terms of stock, feed, labour, machinery and cash requirements so that all units must be integrated.

For discussion purposes the decision variables will be considered under a number of sub-headings ranging from the physical environment to questions of decision frequency.

3.6.2 The Environment and Growth

The physical facilities put limits on the extent of the control possible over the environment. Temperature, humidity and air circulation all affect growth response so that the effect of maintaining these parameters at given levels must be assessed against the cost involved. If temperature is maintained within the zone of thermal neutrality energy for heat dissipation or maintenance is not required. For the purpose of this study it is assumed house temperatures are maintained within this zone. Feeding space, feeder type, the watering system, pen layout and so on all affect intake (Rao, 1968) but are all constant in the short run.

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3 It appears the zone of thermal neutrality is approximately 16-21°C for fattening pigs though there are interactions with other environmental parameters such as humidity. See Sorenson (1962).
except that there will be an interaction between stocking rate and the
effect of these factors. Pen size similarly affects response through
its effect on group size and the resultant effect on the social structure
of the group.

Stocking rate is a potentially important decision as while
stocking rate interacts with pen size it also directly affects health
and group social structures and therefore growth responses. Stocking
rate should not be considered in isolation as while a physically sub-
optimal stocking rate reduces physical efficiency, it may make efficient
use of limited space. Space has an opportunity cost so where this is
high consideration of heavy stocking rates could be important. Bryant
and Ewbank (1969, 1972, 1974), among others, have carried out a number of
experiments designed to explore the effect of stocking rate on social and
economic factors. Unfortunately the evidence available is insufficient
to produce a continuous stocking rate-response function. The most
directly useful of the Bryant and Ewbank trials used stocking rates of
0.56, 0.77 and 1.19 m² per pig (starting with 20 kg. liveweight mixed
sex pigs) and indicated that only at the 0.56 m²/pig rate were liveweight
gain and voluntary food intake adversely affected. In this study it is
therefore assumed that fixed space requirements are necessary at different
liveweights.

A related factor affecting growth is the compatibility of individual
pigs in a pen. Aggressive pigs may disrupt the social structure so that
pen reformation may be useful. Such moves will lead to new social
structures being formulated and resultant growth disruption until a new
equilibrium is attained. Evidence in this area is largely subjective

4 Some work has also been done on the effect of other environmental
factors on social structure. See, for example, Ewbank (1973).
so that decisions must similarly be subjective. The nature of this
decision area also makes an objective analytical approach difficult except
on a case to case basis.

Maintaining the environment at as near an abiotic level as
possible is another potentially important decision area. The use of a
specific pathogen free system relates to this problem as well. Controlling
disease may involve clearing the fattening house of stock so that there is
an interaction with buying and selling policies.

The net effect of the available facilities and decisions made is
to give a particular environment which in turn defines a stochastic
production function on the basis of which feeding decisions must be made.

3.6.3 The Variability of Stock Output

An important factor for managerial simplicity is whether to use
a constant input of weaners and a relatively constant output of fat stock.
Whether such a policy has economic advantages will partially depend on the
market situation particularly with respect to contract opportunities. The
primary decision problem in such a policy revolves around feeding questions
to give a particular type and quantity of growth. To provide more general
conclusions it is assumed that varying the input and output flow is an
option. Under stable prices and costs, however, an optimal system will
tend to a relatively constant policy.

3.6.4 Pen Formation for Response Uniformity and Space

Utilisation

For efficiency of feed use it is desirable to form groups of
pigs that will respond in a relatively uniform manner. There are probably
three important factors in determining the nature of response. Firstly,

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5 For an extensive coverage of growth and development see Lodge, G.
the physical state of the pigs in terms of their liveweight and conformation particularly with respect to fat levels, secondly the genotype of the pigs\textsuperscript{6}, and, thirdly, the sex of the pigs though the differences between castrates and gilts of similar genotype and state is not great. Some workers\textsuperscript{7} also consider age to be a relevant parameter. The benefits of forming uniform pens must be assessed against the chance of achieving uniformity and the work involved in the measuring necessary as well as the possible social structure disruption. The return from forming similar groups will depend on the extent of the genetic variability within the incoming population. If the distribution of genotypes is unimodal and has a small variance, the payoff will not be large. Similar comments apply to the physical state of the pigs.

Pen reformation can take place at any time though with increasing liveweight the stress resulting from the formation of new social structures increases. Pen formation at the weaner stage will usually require mixing pigs from different groups so that they should be made as uniform as possible. This formation requires information on the physical state and genotype of the pigs. Observations on liveweight and fat cover through ultrasonic techniques provide a guide to the physical states of the pigs, though observational errors can be significant, particularly with respect to fat levels. To estimate the genotype of individual pigs, observed responses in the weaner production unit or during the initial period in the fattening unit can be used, though such estimates will be uncertain. A feeding regime which is specifically designed to provide observations quickly and more accurately leading to genotype estimations may be useful even though such rations may be sub-optimal in an immediate growth

\textsuperscript{6} The nature of response will be discussed at some length in later chapters.

\textsuperscript{7} For early work in this area see McMeekan (1940).
The number of pens available imposes a constraint on pen formation decisions. Given an intake of weaners the number of empty pens available limits the number of like groups that can be formed so it becomes important to place priorities on the observable characteristics. Thus, for example, it is important to assess whether forming homogenous sex groups will provide greater feed use efficiency then forming what are considered to be homogenous genotype groups from a potential response point of view. The extreme case would involve forming sub-groups within sex groups. A major difficulty in considering these problems is the lack of detailed response information based on individual sexes. While there is sufficient evidence to indicate that sex is important (Agricultural Research Council, 1967), many of the nutrient balance and response studies are based on either mixed sex groups or are limited to one sex. Accordingly it is assumed sex is included as a component of genotype though this is clearly an area requiring further consideration as more technical information becomes available.

Even where the number of pens severely restricts group formation, it will be useful to have estimates of the characteristics of pigs in a pen so that appropriate feeding can be carried out. The simplest observations in this respect is the average liveweight of the group.

The other important factor in pen formation is the effect it has on space utilisation. Forming a group of pigs at the weaner stage and leaving them in the same pen can initially give a sub-optimal stocking rate and an overcrowding at the later stages. An important factor in deciding on a policy will be the opportunity cost of the space used.

A number of options for the efficient use of space are available depending on the physical facilities available. For equally sized pens the options range from accepting initial under-utilisation so that the
stocking rate is designed for the pigs when they are at near sale weights, to the other extreme of stocking to give efficient initial use of space. A possible variation is to sell or remove pigs from a pen through time to maintain a near optimal stocking rate. Separated pigs can form new groups but at the cost of the stress resulting from mixing unfamiliar pigs. Where different sized pens are available, either through having a range of fixed sizes or moveable partitions, optimal space levels can be more closely adhered to. As the majority of sheds currently being constructed have moveable partitions or a range of pen sizes the models developed assume this form of structure.

The net effect of decisions on pen formation leads to defining a particular response function over time from which feeding decisions can be made. This results from the effect of stocking rate on response and on the pen uniformity achieved.

3.6.5 Feeding, Growth and Disposal

The decision area of major importance is feeding and disposal. In feeding decisions the decision unit is the pen as there are usually no facilities for individual treatment though in some cases a group of several pens may form a unit. For disposal or sale decisions, the unit can be the individual pig.

The major variables in feeding are the quantity and quality of feed offered to a pen. Quality refers to the utilisable nutrient content though this may not be independent of quantity. The feeding regime adopted is intended to produce a given type of growth or response so that the basic decision variable is the required state or condition of the pigs at different points in time. Where a number of alternative feeding patterns can be used to achieve the state the problem is to find the least cost one.
Given the nutrient requirements the cheapest mix of source ingredients to supply these must be determined. This least-cost mix problem raises many specification problems such as the appropriate amino-acid balances and the rates of substitution between nutrients. These questions will be considered in some detail in a later chapter. Related to these problems is the question of the optimal physical nature of the feed presented. For example, experience\(^8\) has indicated the use of wet mixes has reduced disorders through lowering dust levels.

In considering the quantity of feed to be used per unit of time, intake limits must be taken into account. Feeding *ad libitatum* can reduce the labour requirement since feeding need not be on a daily basis and accurate weighing is not essential. The use of restricted feeding involves a decision on the frequency of feeding though it appears there is little advantage in feeding more than twice per day.\(^9\) Restricted feeding can lead to irritability so that this factor combined with the costs of measuring and feeding may mean the use of less concentrated feeds is worthwhile. This is effectively *ad lib.* feeding but with intake limitations controlling the quantity of feed consumed. In assessing this approach the effect of using "filler" feeds on the utilisation efficiency of nutrients must be considered.

Another question inherent in the quantity-quality decisions is the number of times the feed type should be changed throughout the life of a pen. Where the nutrient balance requirement changes with the stage of development of pigs, the question of changing the ratio of nutrients becomes important as well as changing the quantity. If the balance requirement continuously changes a continuously changing feed mix would

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9 See, for example, Agricultural Research Council (1967).
be appropriate. Using many different mixes however creates problems as control of the system is difficult, the facilities for mixing and feeding may form a constraint and sudden dietary changes can cause digestive problems. Where a range of feed types are used, control of the system is simplified if batch intakes are large compared with the total numbers that can be housed.

Health and medication questions are problems related to feeding and growth. The use of growth stimulants and drugs in feed mixes can lead to greater efficiency in feed use though response will depend on the conditions in any particular case. Diagnosis and treatment of disorders and diseases as they occur are also problems related to this general area.

The final decision in the feeding and growth system is that of disposal or sale, whether individually or in groups. An important relationship is the correlation between observable characteristics and the grade placed upon the pig where pigs are being paid for on the basis of grade and a price differential exists. Many grading systems rely on dressed weight, fat cover and a number of visual characteristics such as conformation and colour. A reasonable estimation of dressed weight can be obtained from the liveweight and estimates of the fat cover from ultrasonic testing, though many producers would use eye and feel appraisals. Whatever method is used, and this is another information problem, a decision must be made regarding the estimated return from the pigs compared with the possible additional net return by holding the animal or animals for a longer period of time. Current feed costs and market movements must be allowed for in these calculations. Similarly the demand for space by incoming and pigs on-hand must be taken into account and thus the value of the space. The frequency of pig sales must also be considered within practical selling facility limitations.
The more frequently sales are made the more likely pigs can be sold at what is regarded the optimal condition.

While it is not the purpose of this chapter to discuss the detailed nature of response, it is important to introduce generally some of the characteristics of response so that the feeding decision variables can be put into perspective. The feeding profile through time affects the liveweight of the animal but it may also affect other factors such as conformation, the dressing percentage and the fat content of the carcass. Where these factors are important in grading any such relationships must be assessed to enable feeding decisions to be made. For example, where the type of fat on an animal affects the grade, because pigs are monogastric, the type of feeding in the later stages of growth affects the type of fat laid down. Similarly, if fat levels can be influenced by feeding it is important to consider the return from depositing fat compared with the cost. The energy requirement of fat deposition is greater than that for protein deposition, so when the cost of providing a high energy ration is considerable the influence of fat levels through feeding becomes a critical question.

Feeding decisions will clearly influence the rate of gain, and the conversion efficiency, these being two statistics commonly quoted in feeding trials. A high rate of gain means the total feed used for maintenance over the lifetime of the pig will be less compared with a slow rate of gain ending at the same liveweight. Similarly, the total use of fattening shed space (space days) used by the pig will be less. To assess the importance of rate of gain the cost of concentrated feeds necessary for high rates must be compared with the opportunity cost of space used by a slower rate of gain. Similarly, given equal rates of gain between pigs, a high conversion ratio can be achieved through the use of concentrated feeds but this will not necessarily be optimal as a
poorer conversion ratio may mean a feed of a significantly lower per-unit cost can be used so that total cost per unit gain is less. These relationships are obviously related to the rate of gain problem. It is therefore possible that a slow rate of gain and a low conversion ratio will be optimal, particularly where the opportunity cost of space is low.

Another relationship of potential importance is compensatory gain. It appears pigs which are restricted in growth prior to a weight of approximately 23 kgs will subsequently make up the weight lost\(^\text{10}\). However, Lucas (1967) concludes that deprivation at higher weights does not have the same effect. Thus, compensatory gain is not a decision problem for fattening situations other than to influence the potential growth relationships of incoming weaners. To assess this factor it is important to know the age of incoming weaners in relation to their live-weight.

Optimal values of the decision variables in the feeding area must clearly depend on the current state of pigs and the current cost and price levels as well as expectations of these parameters. These future expectations are important as current decisions influence future pig conditions and therefore potential sale revenues.

3.6.6 Decision Frequency and Control

Decision making within the fattening unit should be a dynamic operation as the conditions under which planning takes place are continually changing. Growth response is stochastic as the intake of utilisable nutrients, the environment and the disease components are all random variables. Similarly, the knowledge state regarding the response function of the various pens is non-certain. This means planned-for pig states have a varying chance of being achieved. Likewise, estimates of price

\(^{10}\) See, for example, Lucas et al. (1959).
and cost distributions and other endogenous variables will often require updating through time. This real-world situation has two important planning implications. Firstly, there is a need for recording what is happening within the system so that decisions can be made whether to re-adjust the decisions. The problem here is one of deciding what information should be observed, how frequently and the methods of observation. Such information enables updating, for example, the price and cost estimates, and the likely response functions. Secondly, such information may indicate that planned feeding and disposal programmes should be altered. An optimal recording-decision adjustment process must be determined by considering both these areas simultaneously. An important question will be the frequency with which decisions should be updated. The frequency with which new observations are made clearly places an upper limit on feeding programme changes. An optimal frequency must largely depend on the variance of the distributions, the costs of recording and the returns from maintaining a feeding and disposal programme best suited to the current and expected future state of the system where the 'state of the system' includes both endogenous and exogenous variables.

3.7 A Management Problem

The discussion has been largely concerned with outlining the decision variables which directly affect the physical outcomes from the system. A further problem is the choice between alternative methods that can be used in finding the optimal levels of the physical decision variables. The problem is one of determining an optimal planning, execution and control system for the enterprise. (Another problem, not considered in this study, is day to day decision making. Examples are ensuring inputs are obtained on time and the daily allocation of responsibilities to employed labour.)
A major factor in selecting a decision method is determining the objective function. This involves isolating the factors which contribute to utility and the relationship between them. This problem is currently one of the major areas under study by many workers, particularly with respect to quantification methods (Lin, et al. 1974). The objective function used must be specified on a normative basis as the analysis is concerned with determining actions that should be taken. This does not necessarily mean, as is sometimes assumed, that this will be a simple monetary objective function. In a few cases a normative function may be the same as a positive function. Where this occurs, however, there will be no gains to management research as the decision makers are entirely rational.

Another management factor is the provision of information for planning and control. The critical decisions are the methods to use in obtaining information and in estimating relationships. Conclusions must depend on the potential benefits of improved information so that management is involved in value of information type problems. Somewhat similarly, decisions must be made on the extent of non-certainty to recognise. In some cases, assuming that a stochastic variable is deterministic will be a justifiable simplification. The extent of information necessary for control purposes must also be decided. This involves decisions regarding the variables to observe and the frequency of observation. The selection of a planning method or model to use is related to the information problem both in a strategic planning sense and in a control and re-planning sense. Information required depends on the planning methods used and vice versa and both depend on the costs and returns of alternative planning and information systems. At one extreme management may rely entirely on a

\[\text{\footnotesize\textsuperscript{11}}\text{These range from asking an advisory officer to maintaining on farm systems using, for example, various forecasting techniques. See, for example, Rodgers (1974).}\]
subjective, intuitive type approach while at the other the unit could operate a sophisticated model specifically designed for the particular unit. Dynamic programming has been used in this way (Meyer and Newett, 1970). Such a system is unlikely to be justifiable except for large integrated systems. In the middle lie budgetary control type systems which are being commercially used at present. Such systems usually rely on a comparative budgeting system, possibly computerised, to make policy decisions and these decisions are then used in a model to predict future expected states. Actual states are then compared with predicted, commonly on a monthly basis, to provide a simple control system. Divergences are then used to trigger a re-planning which might be subjective or involve the re-activation of the budgeting system.\textsuperscript{12} Such systems remove much of the calculational tedium, but fail to provide an entirely objective planning device. An important question in this area concerns the importance of the various decision variables and therefore which ones warrant detailed analysis compared with subjective estimation.

In this study many of the factors raised in this section will not be considered. It will be assumed that updated price and cost information is available as well as control information. The question of an optimal planning method is, however, a major part of this study. A planning method which is as close to reality as possible is to be developed. The planning method will cover the whole planning system of planning and control. A further stage must be to appraise the method compared with less correct but less costly techniques. The method developed in this study is unlikely to have direct use on other than large units at this stage though the development of data generation systems may reduce the cost of implementation to a point where such systems can have general use (Computer Systems, 1970).

\textsuperscript{12} For an example of such a system see Blackie (1974).
4. THE RELATIONSHIP BETWEEN THE FATTENING AND OTHER UNITS

As this study is concerned with the fattening unit, it is important to summarize the links used through the interfaces with other units.

The feed unit, through its buying, selling, producing, storing and mixing operations provides a flow of feed of different quality through time. The important link is the cost of such feed. As the feed unit can sell mixed feed or basic ingredients, whether immediately after purchase or following a period of storage, the real cost to the fattening unit of feed supplied is the market opportunity cost of the feed. Fattening unit decisions should therefore rely upon projections of the market value of feed (usually a stochastic variable) rather than the cost of providing the feed. This means that part of the profits of the total system may be attributable to the operations within the feed unit.

The other link between feed and fattening concerns physical limitations. The facilities available will place a limit on the quantity and the number of different mixes that can be supplied. Similarly, where cash is an effective constraint feed provision, particularly feed storage, must be integrated with fattening operations.

In the provision of weaners the same principles apply. Weaners can be sold so that the market provides a weaner cost for use in making fattening unit decisions. Because product specification in stock is subjective, such estimates must be in the form of a probability density function. (In the case of feed, market price estimation is relatively straightforward due to ease of specification.) Where all weaners are purchased the valuation problem does not arise, though there is still a problem of market forecasting.

Whether weaners are home produced or purchased there may be physical limits on the supply both in a quantity and timing sense. In
this study a purchasing system without limits is assumed. Again, a further limitation on operations will occur where cash is constraining. The other factor affecting fattening decisions is the quality of the stock being provided. In a perfect, economically rational market the prices should reflect quality. In reality this does not occur so an essential information flow between weaner provision and fattening is quality information.

While the interfaces between the labour, machinery, cash, and taxation units are clearly important, in many cases the restrictions and relationships involved are ignored in this study. This is assumed in order to make the understanding of the principles involved more straightforward. Effectively, it is assumed that these factors do not impose operative constraints and that taxation can be handled independently of the fattening problem.

Other assumptions are that the management unit provides sufficient planning information. Similarly, it is assumed that a decision has been made to use an integrated planning and control system with frequent updating. The study, then, is concerned with the development of such a system.

5. PERSPECTIVE

Discussion within this chapter has outlined the general nature of the problems facing management of pig meat producing enterprises. While some simplifying assumptions have been made, it is clear that the problem to be considered is complex. This description of the problem enables an examination of currently available models and provides a background for developing an improved model in subsequent chapters.
CHAPTER III

A REVIEW OF PLANNING MODELS

1. INTRODUCTION

Models currently available do not fully represent the real situation both in terms of their form and in the decision variables included. To provide the background for the development of an improved model existing models are considered in this chapter. The review is not restricted to pig fattening models as other forms of stock production contain similarities, particularly with respect to the general form of the planning problem.

The review is structured by initially considering the use of the neo-classical production model (Heady, 1952) in the pig fattening case and then discussing more recent advances.

2. THE NEO-CLASSICAL MODEL

The work of Heady et al. (1961) is a typical example of the use of the model and forms a basis for the discussion. The study involved using feeding experiments to determine a production function in which the dependent variable was total liveweight gain over the fattening period while the independent variables were total input of corn and soya bean meal. These energy and protein sources were fed *ad lib.* in various proportions.

The derived function was used to form isoquants representing the combinations of corn and soya bean meal resulting in a particular total liveweight gain. The price ratios and isoquants gave the least cost feed combination. As it appeared meat prices followed a seasonal pattern the feeding experiments were also used to derive a function defining the time taken to consume given intakes of corn and soya bean meal. The 'profit
maximising' liveweight was determined from a profit function based on the total liveweight gain function and the relevant prices and costs. The price assumed was based on the expected time of slaughter estimated from the time-feed consumption relationship.

For applicational purposes optimal feed proportions and slaughter weights were derived for a range of price ratios. In this use the model is clearly simple to operate but must, however, be questioned on a number of points, as it has been in recent years. Besides conceptual problems and the inclusion of only two feeds, the use of a general function in giving farm recommendations ignores the differences which exist between farms and managers.

3. SHORTCOMINGS OF THE NEO-CLASSICAL MODEL

3.1 Introduction

The neo-classical model described is only a part of the total short run model for a farm producing several products. Naylor and Vernon (1969) have appraised the general model as far as its applicability to positive analysis is concerned. The problem in this case, however, is one of assessing the model from a normative point of view. This is achieved by firstly considering the general form of the model and whether it mirrors the nature of the real world environment. Secondly, by examining whether all the decision variables are included and thirdly by questioning the form of the physical production relationship it assumes.

3.2 The General Form of the Model

3.2.1 The Nature of the Objective Function

The model uses an objective function containing only pre-tax cash. Recent work\(^2\) has shown that many other outputs from the system

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1 For a brief general discussion on the usefulness of marginal analysis in applied animal production see Barnard (1970).

2 See, for example, Gasson (1974), and Simon (1957).
contribute to utility and should therefore be included. However, as an attempt to include all these factors in the system developed would confound the central issues being explored it is assumed that the objective is the maximisation of expected cash return. It is recognised that later developments must include these factors.

3.2.2 The Perfect Knowledge Assumption

The model assumes perfect knowledge as the factors affecting the optimal decision are assumed to be known. These include the shape of the response function, the state of the pigs when introduced to the system, the nutrient content of the feeds as well as the prices and costs. This assumption must not be confused with the certainty assumption as perfect knowledge can imply that density functions of random variables are known. As is commonly recognised the assumption is unrealistic. As knowledge about most parameters is imperfect there is the decision problem of how far to go in searching for information. Equally as important, as time progresses responses and other information are observed so that estimates of response functions and other factors can be updated. This means decisions may need to be changed as the information is updated. The possibility of re-planning therefore becomes an integral part of the planning system and must be allowed for when hypothesising a model of the system. Updating is separate from searching, in that data is being observed (e.g. weight gain of a particular pen of pigs) and can be made use of without extensive search expenditure.

3.2.3 The Certainty Assumption

The model assumes that all variables and relationships are single valued and the perfect knowledge assumption implies that the values and forms are known. In most cases prices and costs are not single valued and, similarly, the production relationship is stochastic being dependent on random variables such as disease incidence, genotype and group interactions.
Other random variables in the system are the nutrient content of feed and the physical characteristics of weaners entering the fattening system.

In this study no distinction is made between risk and uncertainty. It is reasonable to assume that random variables can at least be assigned a subjective probability distribution. In reality there are only a limited number of random variables whose distributions can be objectively quantified on the basis of relative frequency or logic. This occurs as time series or cross-sectional data is seldom directly comparable due to the continuous change in genetic material and technology and the basic differences between production units. Risk and uncertainty are therefore combined and referred to as 'non-certainty'.

The existence of non-certainty has three important implications. Firstly, where income variability effects utility it should be included as a component of the objective function. Secondly, data collection becomes more complex and confidence limits need to be placed on estimates of the distributions of random variables. In the extreme there will be planning situations where the confidence limits are so wide that the best planning system could be to make a random choice of decision variable levels after a preliminary analysis has excluded any alternatives that would be dominated under all conditions (Jensen, 1968; Powell & Hardaker, 1969). Thirdly, non-certainty has re-planning implications in that as the outcomes of events become known it may well be worth changing the value of decision variables.

3.2.4 Time as a Factor in a Model

The neo-classical model largely ignores time as being an important element of the production process. In effect production is assumed to

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3 For a discussion on subjective probability and the logic for its use see Dillon (1971).
be instantaneous as no allowance is made for such factors as the period of time taken to reach slaughter weight (except for price purposes) and the consequent effects on overall payoff. In animal production models time must be included as a factor for the following reasons.

(a) The response from feeding a given quantity and type of feed will depend on the time at which it is fed in relation to the beginning of the fattening period. Strictly, this response depends on the state of the animal at the time of feed input rather than absolute time. This factor is not so important, however, where ad lib. feeding must be used as this implies a dating of feed inputs through knowing the liveweight and the intake relationship.

(b) The length of the time lapse between weaner introduction into the system and slaughter affects the payoff from the system. There are two basic reasons for this. Firstly, with seasonal differences in prices and costs occurring the value of production depends on the time of sale. Heady et al. partially allowed for this by allowing the gross return to vary with time of sale. Secondly, while on hand each pig is consuming the services provided by the fixed resources. Thus, for example, available fattening shed space is being used thus preventing further weaners from being introduced into the system.

Ensuring the economically efficient use of the flow of services provided by the fixed resources raises the
The question of the objective that should be used. The neo-classical model attempts to maximise the return per animal without regard to how many batches can be fattened during any fixed time interval. Given the possibility of varying the fattening time the objective of maximising return per unit of time must be used if the system which maximises return over the total period of concern is to be determined. This problem is related to determining the optimal replacement time of an asset and has been the subject of many arguments in the literature, particularly in respect to the time to harvest forestry. Appendix I contains a discussion of this problem.

(c) Prices and costs not only vary seasonally but also follow trends through time. Similarly technical factors such as the genetic potential of stock can be expected to change. Thus the system cannot be regarded as one which is continuously repeated. Rather a model of the system must allow for such expected movements so that an optimal slaughter weight in different periods may vary. This means the optimal replacement time will vary for other reasons than just the direct effect of replacing sooner or later under the assumption of constant conditions.

Given conditions are changing the neo-classical model implication that a static-equilibrium exists is clearly deficient. A system can be said to be
optimal only if it is tagged with the set of conditions under which it is optimal and these will always be changing. In most cases the chance of ever implementing an optimal system is small as decisions are made before the conditions eventuate.

(d) Given that an optimal system is continually changing, the system being implemented may need to be continuously changed. However, there are physical limits such as cash constraints and management conservatism which frequently prevent an instantaneous change from one system to another. (The neo-classical model implies instantaneous change is possible.) A model of the system must determine an optimal sequence of events from the current system to an optimal system. Time taken to change is an integral part of the model and must be included, and such inclusion may give a sequence of events and an optimal system different from that where it is assumed change is instantaneous. In many cases there is a possibility that the particular sequence of changing conditions which occurs means that an optimal system is never attained where this is defined as the resource structure and system which is optimal for the current prices and costs.

(e) The instantaneous production implication of the neo-classical model ignores the opportunity for re-planning which occurs in reality (though the instantaneous assumption does not preclude the recognition of many variables being stochastic, as for example, the density function of meat yield can be allowed for in estimating
the gross return parameters). Once time is introduced into the model there is the recognition that decisions are implemented in a sequence and can be made in a sequence so that they can be changed. The reasons why change may be necessary have already been outlined. Thus, a model of the system should include the possibility of re-planning. The re-planning interval length is likely to vary with the state of the system and the change which does occur though a decision to implement original plans can be regarded as an active re-planning decision.

3.2.5 Recognition of the Entire Production Entity

The neo-classical model in its simplest form takes as its unit an average pig. The farmer cannot, however, base decisions on a single pig as he is dealing with a total system and there are interactions between decisions made for a given pig group and the outcome from the total system. The decision unit must be the entire system except in those cases where a sub-unit can be shown to be independent.

In particular, a decision model must recognise the use of fixed resources by the production units and the constraints imposed on the system by other factors such as marketing contracts. Thus, for example, the stocking rate used in pens and the resultant use of fattening house space becomes a factor whereas the neo-classical model ignores this problem. Also, by using the whole system as the decision unit any size relationship can be incorporated into a model.

3.2.6 The Current Planning State

It has been stressed that the realistic planning environment of imperfect knowledge, non-certainty and the recognition of the time lapse between input and output gives rise to the need to include a re-planning
possibility in the decision model. Furthermore, as time progresses
decision points which were in the future become the current point in
time. Accordingly, decisions made and implemented at the current point
in time must clearly be dependent on the actual current state of the entire
system rather than on any prediction of the state made in the past. The
decision model must allow for the system to potentially be in any one of
a range of possible future states, but once the future state becomes the
current state and re-planning occurs this must be carried out from the
base of the actual current state in existence as any response will be a
function of, for example, the current liveweight of animals in a given pen.
The neo-classical model ignores this aspect. It takes the average pig
and determines the set of decisions to be implemented throughout the life
of a pig. Describing the current state involves more than giving the
numbers and condition of each type of pig on hand as, for example,
expectations regarding future time subscripted prices and cost expectations
are relevant variables in determining an optimal policy. An optimal
decision set must therefore be state subscripted and will vary as the
current state varies through time. A complicating factor in this system
is that in most cases the variables (state variables) giving the current
state of the system will not be known with certainty. An example is the
non-certainty attached to the conformation of a pen of pigs.

3.2.7 The Nature of the Production Unit

The neo-classical model is based on the response function of
an 'average pig'. The decision unit in many cases is a pen of pigs so
that the model assumes all pens are aggregated and have the same
production function. This creates aggregation problems as, for example,
taking all pigs which are slaughtered in a period it is possible that
the gross return calculated from the average weight and price will not
equal the actual gross return. Most pigs are sold at a price based on
a grading system and price changes above and below some mid-point are
A decision model should recognise that pigs are not all identical. This could mean a range of possible production functions need to be included in the model given pigs can be classified with at least some minimum chance of success and therefore can be grouped to enable different treatments. Similarly pigs do not have to be all sold at the same weight and condition. Pigs responding at a different rate to some standard may need to sold at a different stage.

The response function should recognise that feeding can influence the type of output over and above simple liveweight. Further, given the recognition of the re-planning possibility, output from a system in any period has the two major components of the output of saleable meat and the output of pigs in various states which become the inputs for the next planning period. These states can be described using such state variables as liveweight, age and estimated genotype.

3.3 The Decision Variables Included

The discussion on the deficiencies of the form of the neo-classical model has indicated that many decision variables are ignored or treated incorrectly by the model. In order to emphasise the implications of using such a model some examples are briefly given.

A major factor is that in each decision period (which could well be a week), an updated value for most decision variables is required. These include the numbers and classes of animals to be disposed of and the numbers and classes of weaners to be brought into the system as well as the quantity and quality of feed to be given to each group of animals. Further, in each period, growth responses are observed so decisions must be made on whether to revise estimates of response functions and therefore anticipated treatments.
3.4 The Form of the Production Relationship

The form and details of any decision model must depend on the nature of the technical relationships involved in the production process. Incorrect specification not only leads to erroneous decision variable levels but also to the exclusion of many that in reality exist.

The simple feed - liveweight relationship assumed in the neo-classical model leads to errors as, for example, assuming gross return is totally dependent on liveweight leads to ignoring the problem of output quality.

Other assumptions which can be questioned are:

(a) that response is a function of initial liveweight rather than of other factors such as the body composition and predicted genotype,

(b) that response is independent of stocking rate, environmental control, disease control and other management practices,

(c) that response is deterministic,

(d) that feeding cannot influence quality.

Rather than provide detailed evidence of these factors at this point, the development of a response relationship will be left until technical sections of a trial model are discussed in chapter VI.

4. DEVELOPMENTS IN MEAT PRODUCTION DECISION MODELS

4.1 Introduction

Since the development of the neo-classical model there have been a number of improvements made both to specifically pig orientated models as well as to other types of meat production models. In order to facilitate the development of an improved model representatives of these
improvements are considered. The models discussed will be presented in as near to a chronological order as possible but at the same time maintaining a topic orientation.

4.2 The Form of the Production Relationship

Dean (1960) recognised that response in a given period is dependent on treatments in previous periods so that decisions in any one period cannot be made independently. He developed milk production functions in which the feed intake in a number of preceding periods formed the independent variables. These functions, together with feed consumption functions, were used to derive a profit equation for estimating the optimal feed inputs in each period. While milk production is not directly analogous to meat production it can be expected to have the same form. Growth in any one period depends on what has occurred in previous periods if for no other reason than the maintenance requirement will have changed. When deciding on a policy for a pig in any period such functions are necessary so that given the current state of the pig the future response for this current state can be estimated. This implies that current state can be used in place of previous treatments.

In line with the concept that treatment in any period should depend on the state of the cow and other environmental factors, Heady et al. (1964) carried out a number of experiments designed to relate weekly milk production in a given period to a number of variables including the inputs of various feeds, the age of the cow, stage of lactation, and the live-weight. The form of the equations estimated were logically sound and the $R^2$'s and $t$ values were relatively good though there were clearly problems of multi-collinearity. The equations were used to produce the usual isoclines and thus the profit maximising milk production levels and least cost feed input combinations.
The major contribution of the work of both Dean and Heady et al. was to show that response in any period is a complex of many factors and that it is possible to observe and make use of some of them in determining a feeding system in a given period.

Dean used only two periods and Heady et al. determined the optimal decision in each period independently. In meat production it is clearly important to treat all periods as being dependent in that, for example, the liveweight in any period is a function of the treatment in previous periods. Duloy and Battese (1967) have recognised and stressed this aspect in developing a model of meat production. They also raised the question of quantity as well as the type of feed to be used in each period whereas Dean and Heady et al. assumed ad lib. feeding. Duloy and Battese considered intake limits and, assuming these are a function of liveweight, defined quantity fed as being some fraction of the maximum intake possible for a given liveweight. It was pointed out that the real quantity decision variable is the level of this fraction rather than the actual quantity consumed as this is a random variable. This recognition is important in obtaining least squares estimates.

Assuming the type of ration is fixed their model had the following form:

(i) Intake was defined as:

\[ R_t = k Y_{t-1} \]

where

\[ Y_t = \text{liveweight at the end of period } t, \]
\[ k = \text{a fraction so that the maximum value of } k \text{ gives the capacity intake,} \]
\[ R_t = \text{intake during period } t, \]

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4 Also see Battese et al. (1968) and Townsley (1970).
Change in liveweight was given by (assuming a linear production function):

\[ \Delta y_t = x_0 + x_1 y_{t-1} + x_2 r_t + u_t \]

This assumed liveweight change depends on starting weight as well as feed consumed (liveweight affects the maintenance requirement).

This led to the following relationship for total liveweight:

\[ y_t = x_0 + y_{t-1} + x_1 y_{t-1} + k x_2 y_{t-1} + u_t \]

or

\[ y_t = x_0 + A y_{t-1} + u_t \]

where

\[ A = (1 + x_1 + k x_2) \]

Starting from the initial liveweight this gives:

\[ y_t = A^t y_0 + (1 + A + A^2 + \ldots + A^{t-1}) x_0 + u_t + A u_{t-1} + A^2 u_{t-2} + \ldots + A^{t-1} u_1 \]

(This relationship is very difficult to work with.)

Accordingly the profit equation had the form of, assuming a unity discount factor and constant prices and technology:

\[ \Pi = (p_y y_t - S - V_T)/T \]

where

\[ \Pi = \text{profit per unit of time}, \]
\[ p_y = \text{price per unit of output}, \]
\[ S = \text{set up costs per animal}, \]
\[ V_T = \text{total feed costs}, \]
\[ T = \text{the number of periods for which the process is run}. \]

The decision variables are the total time \( T \) and \( k \) (the proportion of liveweight in each period to be fed). As it stands the model is incomplete as \( V_T \) is dependent on \( k \). Thus:
\[ V_T = \sum_{t=1}^{T} V_t \]

where

\[ V_t = \text{feed costs in period } t. \]

and

\[ V_t = p_k y_{t-1} \]

where

\[ p_k = \text{per unit feed costs.} \]

This gives

\[ V_T = p_k k(y_o (1 + A^1 + A^2 + \ldots + A^{T-1}) + x_o ((T-1)A^0 + (T-2)A^1 + \ldots + A^{T-2}) + u_1 (A^{T-2} + A^{T-3} + \ldots + A^0) + \ldots + u_{T-1} A^0) \]

This model is complex even though it assumes a linear production function and does not consider the type of ration. Battese et al. (1968) were forced to work with a quasi reduced form in defining the relationship for pig fattening experiments in which type of ration was also considered\(^5\).

Meat production functions are clearly of this recursive nature. Attempts must be made to develop models that enable manipulation and that can be solved relatively easily. Once non-linearities are recognised the Duloy and Battese model becomes impractical. Since these developments a number of dynamic programming models have been formulated in an attempt to allow for the principles introduced above and are discussed in a following section.

The Duloy and Battese model also recognises the need to maximise returns per unit of time as the time to slaughter is one of the decision

\(^5\) They experimented with combinations of separated milk and grain. The total quantity of each ration was also varied in relation to the liveweight of the animals. The effect of the rations on quality and therefore price were also explored.
variables. They do not, however, relate the time to slaughter with the use of resources. The next section considers models that emphasise the importance of time and use of fixed resources.

4.3 Time and the Use of Resources

Stewart and Thornton (1960) attempted to devise an operationally useful model which considered the fixed resource requirements of animals by developing a linear programming model of a pig enterprise which had activities representing the different weights at which pigs could be slaughtered. Available rearing and fattening house facilities were defined in terms of pig days. Each activity was estimated to require a given number of pig-days in each house and, similarly, labour supply and the requirements per unit of activity were defined in man-hours. Resource availabilities used were the total supplies in a year so that the solution did not indicate the time sequencing of production nor did it consider an optimal feeding policy for each slaughter weight. The general form of the model was:

(i) \[
\text{Max } Z = \sum_{j} x_j c_j \\
\text{where}
\]
\[
x_j = \text{level if } j^{th} \text{ activity}
\]
\[
c_j = \text{gross margin of } j^{th} \text{ activity, these being pigs slaughtered at different weights so that the gross margin includes feed costs.}
\]

(ii) Subject to a number of constraints of which the following is a typical example:

Fattening accommodation (standard pig days)

\[
54750 \geq 17.8x_1 + 33.0x_2 + 55x_3 + \ldots
\]

Trant and Winder (1961), in considering broiler production, go further than Stewart and Thornton by allowing explicitly for the time at which batches of broilers should be sold and replaced with a new batch
as well as considering the use of fattening house space. Their objective function is one of maximising return per unit of time in a similar way to Duloy and Battese (1967), but they also allow for the fattening house requirement by the birds. Their model consisted of:

\[
\text{maximise } \Pi = n^{-1} (v - c - f)
\]

where \( \Pi \) = profit per unit of time,
\( n \) = production period length,
\( v \) = gross revenue per lot of broilers,
\( c \) = variable costs per lot of broilers,
\( f \) = fixed costs of a batch of broilers.

This gives the decision rule:

\[
\frac{dv}{dx} = \frac{dc}{dx} + \frac{dn}{dx} \left( \frac{v - c - f}{n} \right)
\]

It was assumed \( x \), the level of input (feed), was a function of time.

To allow for fattening house space requirements the gross return function was estimated by calculating the number of birds that could be fattened assuming they were taken to a given weight before slaughter and combining this information with the price per bird. Batch size was determined on the basis of the per bird space requirement at slaughter weight so that the house would be sparsely populated when the chicks were first introduced. As to be expected, it was found that when chick and feed costs are low and meat prices high, the profit maximising system consisted of producing light birds. This meant high numbers of birds per lot and a large number of lots per year.

Hoepner and Freund (1964) also explicitly allowed for replacement time as well as resource use in a model for broiler production. Their resource use allowance, however, was endogenously allowed for in the model. They assumed body weight and feed consumed was a function of
Their model had the following form:

(i) \[ W = \frac{M}{1 + AR^t} \]

(ii) \[ F = d_o + d_1 t + d_2 t^2 \]

where

\( W \) = liveweight at time \( t \),
\( M \) = liveweight at maturity,
\( A \) = number of periods for the bird to grow from the initial weight \( W_0 \) to \( M \),
\( R \) = \( e^{-yt} \) where \( y \) represents the proportionate rate of growth in time \( t \) relative to \( t-1 \),
\( F \) = total feed consumed.

(iii) \[ B = \frac{52}{1+2} \]

where

\( B \) = number of batches per year,
\( Z \) = constant time interval between batches necessary for cleaning (in weeks).

(iv) \[ Z = \left( C_0 + C_1 t + C_2 t^2 \right)^{-1} \]

where

\( Z \) = number of birds per square foot.

(v) This gives a function for the number of birds per square foot per year:
\[ BZ = \left( \frac{52}{t+2} \right) \left( C_0 + C_1 t + C_2 t^2 \right)^{-1} \]

(vi) The annual gross return per square foot is given by:
\[ \text{Return} = WP \cdot BZ \]

where

\( P_w \) = price per unit.
(vii) The profit equation follows from all these relationships and has the form:

\[ \Pi = \frac{WP_{BZ} - FP_{BZ} - P_{BZ} - F}{W} \]

where

- \( P_f \) = per unit feed cost,
- \( P_c \) = chick cost,
- \( F \) = fixed costs per batch,
- \( \Pi_t \) = profit per square foot per year with replacement after \( t \) periods.

This model assumes *ad lib.* feeding with a constant ration and for these and other reasons is unsatisfactory. Recognising the fallacy of ignoring the ration composition problem Hoepner and Freund developed a further model in which the protein - energy ratio of the feed was allowed to vary and therefore to be one of the decision variables. Thus the weight and feed consumption functions were extended to:

\[ W = f_w(p, c, t) \]
\[ F = f_f(p, c, t) \]

where

- \( p \) = protein content of the feed,
- \( c \) = energy content of the feed giving the same \( \Pi_t \) relationship as before except \( W \) and \( F \) were defined as above.

As with Trant and Winder's model, the space requirement is based on the requirement of birds at their slaughter weight\(^6\). Recognising that this leads to understocking at the early stages of each batch, Hadar (1965), developed a model which recognises that available fattening space can be more efficiently utilised. In pig production, unlike broiler production, it is not common to slaughter the entire stock at one point. Hadar's

---

\(^6\) None of the models discussed consider that the space input affects the production function. They assume a fixed requirement exists according to liveweight.
model allows birds to be sold off at a sequence of ages so that heavy stocking occurs initially and, as the birds grow, some are sold off to provide space. Similarly, he allows batches to be staggered so that full utilisation can be achieved. The model can then determine the optimal combination of these alternative approaches. Hadar also included in the model an allowance for price changes with time. The detailed structure of Hadar's model is given below:

(i) let \( x_t^i = \) the number of birds of age \( i \) on hand at the end of week \( t \) (the model assumes a fixed feeding system so that age is synonymous with weight),

\[
\cdot \cdot \cdot x_t^i - x_{t+1}^{i+1} = \text{the number of birds sold at the end of week } t \text{ as it was assumed only birds of age 0 could be purchased.}
\]

(ii) The net revenue from the sale of birds is given by (ignoring chick costs):

\[
x_t^i (x_t^i - x_{t+1}^{i+1}) = (p_t^i - q_t^i) (x_t^i - x_{t+1}^{i+1})
\]

where

\[
p_t^i = \text{price of } i \text{ week old birds in week } t \text{ (obtained by using a weight time function)},
\]

\[
q_t^i = \text{total feed cost for a bird held for } i \text{ weeks and sold in week } t \text{ (obtained by using a feed consumption function)},
\]

\[
r_t^i = \text{net revenue per bird sold in week } t \text{ of age } i.
\]

(iii) The objective function to be maximised (slightly modified):

\[
\sum_{i=1}^{S-1} \sum_{t=1}^{T-1} (x_t^i - x_{t+1}^{i+1}) + \sum_{i=1}^{S-1} (x_t^i - x_{t+1}^{i+1}) + \sum_{t=1}^{T} x_t^S x_{t}^S - \sum_{t=1}^{T} p_t^i x_t^i
\]

where

\[
p_t^i = \text{price per chick in period } t,
\]

\[
T = \text{total number of weeks considered},
\]

\[
S = \text{maximum age birds are allowed to reach}.
\]
It can be seen from the second term that the system is assumed to repeat itself every $T$ weeks (a year). Maximising total return from $T$ periods with a repeating system is the same as maximising the return per time period except for the problem of varying within year seasonal prices.

(iv) The objective function defined ignores space requirements so the following constraints were added:

\[ \sum_{i=1}^{S} s^i x^i_t < b, \quad t = 1, 2, \ldots, T \]

where

- $b =$ available space,
- $s^i =$ a function giving the space requirement for an $i$ week old bird.

\[ x^i_t - x^{i+1}_t \geq 0 \quad i = 1, 2, \ldots, S-1 \]

\[ x^i_t - x_t^{i+1} \geq 0 \quad t = 1, 2, \ldots, T-1 \]

It will be noted that Hadar has not considered feeding questions at all.

While the models presented above cover most of the developments except for models based on dynamic programming discussed below, mention of Dillon's (1968) work in bringing together production models should be made. He defined a number of cases where time is important and developed the general form models of these cases should take. The cases defined are, where

- $Y =$ output,
- $X_i =$ level of $i$th input,
- $t =$ time to harvest or slaughter.

(i) $Y = f(X_i, t)$ (e.g. hay yield)

(ii) $X_i = f_i(t)$ (e.g. feed input assuming ad lib. feeding)
so that
\[ y = f(t) \]
or
\[ x_1 = f(t, x_2, x_3, \ldots) \]

(iii) \[ Y = f(X_i^t) \] (e.g. wool production where the input of a given quantity of feed has a different response depending on the time of year.)

(iv) \[ Y = f(X_i, F) \]

where
\[ F = f(X_i \text{ in the previous year}) \]

Thus, \( F \) defines the state of the system at the start of the year. (e.g. crop yield depends on fertiliser input as well as the soil fertility which depends on the previous year's fertiliser input.)

Dillon's model for meat production is mainly based on type (ii) above and is largely the same as Hoepner and Freund's (1964). He does, however, mention the use of linear programming in determining least cost feed mixes so that a function of the form:
\[ p = p_o + pp P + pc C \]
where
\[ p = \text{feed price/unit}, \]
\[ P = \text{protein content of feed}, \]
\[ C = \text{energy content of feed}, \]
\[ pp = \text{price per unit of protein}, \]
\[ pc = \text{price per unit of energy}, \]
can be used within the model. These comments introduce the question of determining the details of least cost mixes.
4.4 Developments in Finding Least Cost Feed Mixes

The neo-classical approach to finding a least cost feed mix involves developing iso-quants for total liveweight gain, in the case of Heady et al. (1961), for two feed inputs. While a later chapter will be devoted to the development of an improved model it is relevant to consider some of the advances that have been made.

The early linear programming models concentrated on the simple least-cost problem without considering the profit function of the total entity. An example is given by Alexander & Hutton (1957) in which the typical approach of finding the least cost mix of ingredients which satisfies certain minimum requirements of protein and energy content subject to a bulk constraint is used. The minimum nutrient requirement in these models is based on standards designed to give maximum liveweight gain under ad lib. feeding. Later developments (Taylor, 1965) followed the same principle but used the results of animal experiments to define the minimum requirements of minerals, vitamins and amino acids to achieve maximum liveweight gain.

Dent (1964) combined the least-cost linear programming model with the neo-classical model thus enabling the consideration of a large number of ingredients as candidates for a least-cost mix compared with two used by Heady et al. (1961). Dent made use of feeding trials using bacon pigs to estimate a liveweight production function based on the levels of protein and energy inputs. The selected equation was used to estimate iso-quants for varying average rates of gain per day (and therefore introducing the time element). The advance on the earlier models was the use of iso-quants and linear programming. A range of points on an iso-quant were taken and for the combinations of energy and protein specified by each point a least cost linear programming model was used to determine the optimal mix. Each solution was then compared ex post
to determine the least cost point on the iso-quant. While Dent did not stress the use of the least cost points for different average rates of growth, these can be used in the neo-classical way to determine a profit maximising growth rate.

The major assumption in Dent's approach is that substitution occurs between protein and energy to produce an identical output. This assumption must be questioned on nutritional grounds and is discussed in chapter VI. The existence of an iso-quant of Dent's form may be illusory and may have occurred as the production functions used had extremely low \( R^2 \) values suggesting an erroneous production relationship assumption. Another problem was that the output recorded was simple liveweight rather than type of liveweight. As a movement is made around an iso-quant the total weight of output may stay the same but it is possible that the type of output changes. As an energy source is substituted for protein the level of fat in the output may increase while the lean content decreases.

Dent's model does have a number of other problems. Examples are that the model is inherently static, fixed resource requirements are ignored and it is based on average growth rates over large weight ranges. Dent et al. (1970, (a), (b) & (c)) did, however, extend the ideas expressed in the model in a series of papers. Experiments were specifically carried out to study the quality aspects of production with particular emphasis on the effect of varying the input of the essential amino acid lysine. Further experiments were conducted to test the use of linear programming in feed mix compounding.\(^7\)

A further improvement on Dent's model, largely of a computational nature, was made by Townsley (1968). Townsley formulated the following model:

\(^7\) See Dent & Casey (1967) for a general discussion of the use of linear programming in animal nutrition.
(i) minimise \( Z = c'p \)

(ii) subject to \( f(p) = \bar{y} \)

\[ \begin{align*}
   Ap & < b \\
   p & > 0
\end{align*} \]

where

- \( c' \) = a vector of per unit ingredient costs,
- \( p \) = a vector of ingredient levels,
- \( \bar{y} \) = the average rate of liveweight gain which is parametrised to give the 'expansion path',
- \( A \) = a matrix of nutrient contents per unit of the ingredients,
- \( b \) = a vector of average daily nutrient requirements.

The two components of \( b \) representing the energy and protein requirements are varied in repeated solutions to give points on an iso-quant for \( \bar{y} \).

It was then noted that this problem could be stated as:

Maximise \( y = f(p) \)

\[ \begin{align*}
   c'p & = k \\
   Ap & < b \\
   p & > 0
\end{align*} \]

where \( k \), a cost restriction, is parametrised.

This approach maximised the daily rate of gain for a given cost restriction so that when the cost restriction was parametrised the expansion path was mapped out. The advantage of the approach is that quadratic programming can be used provided the production function can reasonably be approximated with a quadratic function. Each solution provides the optimal point on each iso-quant. Dent's approach required many solutions to find this point. Further, Townsley emphasised the profit maximising objective and noted that a solution to this problem could be
obtained using quadratic programming. The model used was:

\[(i) \quad \text{maximise} \quad \Pi = \frac{P_Y - P_W}{Y - W} f(p) - c'p\]
\[(ii) \quad \text{subject to} \quad A p \leq b \]
\[p \geq 0\]

where

(i) it is assumed pigs are purchased at \(W\) lbs and sold at \(Y\) lbs liveweight and that the objective is to maximise average profit per day,

(ii) \(P_W\) and \(P_Y\) are the prices of pigs of \(W\) and \(Y\) lbs respectively,

(iii) \(f(p)\) is a function giving the average liveweight gain.

Townsley's model is equivalent to the profit maximising neo-classical model but with the added advantage of being capable of allowing for the complexity of the multi-nutrient least cost problem with substitution. As with Dent's model it has a number of problems such as a failure to recognise the constraints imposed by the fixed resources.

Dent does not make it clear whether the experiments on which the functions were based used ad lib. feeding. Whatever the case, the problem of considering the nutrient concentration of the feed and appetite limits with respect to a least cost mix were not specifically accounted for. A model has been developed to consider these problems and is discussed in the chapter on least-cost mixes. Since their development a model which also considers this problem has been published. Kennedy (1972), in considering beef fattening, used a least-cost model of the following general form to determine the feed mix to use in achieving a given growth rate:
(i) Minimise Ration Cost  =  c x

(ii) Subject to  

\[ E \preceq e x \]
\[ m \preceq M x \]
\[ p \preceq p X \]
\[ A \geq d \]
\[ o = -d + l x \]
\[ o = -Nd + e x \]

where

\( c \) = a vector of per unit ingredient costs,
\( x \) = a vector of ingredient levels,
\( E \) = the average metabolisable energy requirement per day for a given growth rate and liveweight (liveweight must be specified to allow for the maintenance requirement),
\( e \) = a vector of per unit of ingredient energy contents,
\( m \) = a vector of average per day mineral requirements,
\( M \) = a matrix of per unit of ingredient mineral contents,
\( p \) = the average per day protein requirement,
\( p \) = a vector of per unit ingredient protein contents,
\( A \) = the maximum intake restriction level,
\( d \) = the dry matter content of the mix,
\( l \) = the sum vector,
\( N \) = the energy concentration of the mix.

In ruminants the efficiency of energy utilisation depends on the energy concentration and similarly the appetite restriction depends on the energy concentration as well as the liveweight. To determine the feed mix for a given liveweight and rate of gain the above model is solved for a range of metabolisable energy requirements and intake restriction levels and the minimum minimorum selected. Frequently the case with
model development is that other developments are ignored. In this case
the possibility of substitution was ignored.

4.5 Feeding and the Length of the Decision Periods

The developments discussed in which feed type and quantity have
been considered have largely worked on average requirements over long
periods of time. To overcome these problems using continuous relation-
ships leads to extreme complexity which renders practical solving impossible.
Duloy and Battese's (1967) work demonstrated this point. A number of
workers have developed discrete models based on continuous production
relationships. Hadar's (1965) model, though being concerned with the
fixed resource requirement rather than the details of feeding, is an
example. A model specifically designed to consider feeding decisions
over small decision periods was developed by Meyer and Newett (1970).
The model was based on a dynamic programming algorithm for a beef feed-
lotting problem in which the decision variables were the weight of an
animal to buy and sell, the total holding time, the number and length of
feeding periods and the ration to feed within each period. In looking
at these factors Meyer and Newett stress that animal response in any
period is a function of the current state of the animal. Their model
has two state variables, liveweight and cumulative time from animal
purchase and two decision variables, ration type and period length. The
model was also repeated for a range of starting and final liveweights
effectively increasing the decision variables by two.

As candidates for ration type a least-cost linear programming
model was used to find mixes which were optimal for a range of net energy
contents\(^8\). This can be called a 'nested approach' and is similar to
Dent's (1964) approach of solving a sub-problem (least cost for given

---
\(^8\) See Lofgreen & Garrett (1967) for an explanation of net energy concepts.
energy and protein content) and using the answers as candidates for inclusion in the optimum to the overall problem. Given a particular liveweight, Meyer and Newett determined the liveweight increase resulting from using each of the rations on an ad lib. basis for a varying length of period. This led to a new liveweight and cumulative time value for the state variables. To determine the new liveweight, information on the energy requirement for maintenance and production was used. Within periods, sub-periods of two days were used to update the liveweight for maintenance requirement calculations.

Meyer and Newett's model had the following general form.

(i) Maximum profit per beast = $Z$

$$Z = \max_{t_i, k, o, f} \left( \sum_{j=1}^{n} C_j \cdot T - BP(W_o)W_o \cdot \sum_{j=1}^{n} C_j \cdot T - BP(W_o)W_o \right)$$

(ii) Subject to $T < T_e$

where

$t_i$ = length of period $i$, \hspace{1cm} (i = 1, 2, ..., n),

$x_k$ = quantity of feed $k$ consumed \hspace{1cm} (k = 1, 2, ..., n),

$w_o$ = initial purchase weight,

$w_f$ = sale weight,

$T$ = total time to sale,

$SP$ = sale price per unit of weight \hspace{1cm} (dependent on $T$ & $w_f$)

$C_j$ = cost of feed consumed in period $j$ \hspace{1cm} (j = 1, 2, ..., n),

$FC$ = fixed costs per unit of time,

$BP$ = buying price per unit of weight,

$T_e$ = maximum time to sale.

The objective, $Z$, was determined using dynamic programming through the following recurrence relationships:
(i) \[ L_n(W_{n-1}, T_{n-1}) = \max_{t_n, X_k} \left( L_{n+1}(T_{n-1} + t_n) - C_k X_k \right) \]

where

\[ L_{n+1}(T) = SP(T, W_f) W_f - FC.T, \]

\[ T_i = \text{cumulative time to period } i, \]

\[ C_k = \text{cost per unit of feed}. \]

(ii) \[ L_i(W_{i-1}, T_{i-1}) = \max_{t_i, X_k} \left( L_{i+1}(W', T_{i+1} + t_i) - C_k X_k \right) \]

where

\[ W' = \text{liveweight achieved by feeding ration } X_k \text{ for a time of } t_i. \]

In determining the L values many side conditions had to be satisfied which meant that the model was considerably more involved than suggested above. The dynamic programming model had to be repeated for a range of starting and finishing weights as well as a range of the number of feeding periods. As such it is extremely complex. It took 102 minutes to solve on an IBM 365/50 (64k Core) for a problem having only three possible starting weights and a single finishing weight. This complexity is partially due to allowing the feeding periods to be variable and the need to determine ending weights from following through the effect of feeding a particular ration. If it was assumed each period was of fixed duration the model could endogenously decide whether to use the same feed for several periods effectively achieving the same end. Similarly, rather than simulate the effect of various rations the calculation of feed requirement to give a fixed set of liveweights would remove the problem of allowing for variable ending liveweights.

Meyer and Newett's model does not include a number of other advances previously made. Examples are the maximisation return per animal rather than per time period, using the total entity and not allowing for sub ad lib.
feeding.

5. FURTHER DEVELOPMENTS REQUIRED

There is clearly a need to develop a model which combines all the advances as well as including the remaining deficiencies as far as possible. None of the models discussed remove the perfect knowledge assumption and they all tend to assume the existence of a static equilibrium solution as they do not include the possibility of updating plans in each period. Meyer and Newett's dynamic programming model approaches replanning as dynamic programming solutions provide optimal actions for all starting states in all time periods. However, such optimal actions only apply provided none of the original relationships assumed change. It should also be noted that most of the models were based on a single average animal whereas a total planning system must include the total production entity thus allowing for the current total state of the system at any planning point. Also, particularly in pig fattening, the decision unit must largely be based on a pen of pigs and recognition must be given to the possible grouping of pigs on the basis of expected response. This would recognise the existence of a range of response functions.

Doubts have been raised about the general form of the production relationships commonly used particularly with respect to the nutrient substitution relationships. Further developments must examine these relationships as the technical relationships must dictate the form of any decision model. Similarly attempts must be made to include all the decision variables which a recognition of the real planning situation introduces.

6. RECENT CONTRIBUTIONS

Following the development of the ideas included in this study a number of other papers on decision models for meat production have appeared.
Kennedy (1972) has developed a dynamic programming model similar to Meyer and Newett's but with an allowance for several batches. Rather than maximise profit over the variable life of one beast Kennedy takes a total planning period of two years. This is divided into decision periods of twenty-eight days. At each decision point a decision is made whether to sell the animal and therefore whether to purchase another or to invest the proceeds until the end of the two year period. Where an animal is held a decision must be made regarding which growth rate to feed for. Least cost rations for each growth rate were constructed using the model discussed in section 4.4. While Kennedy assumes that if an animal is on hand at the end of the two year period it must be sold, his model does attempt to maximise return per unit of time. (However, the model is still based on a single animal, on perfect knowledge, and ignores fixed resource requirements.) The model also time subscripts all prices and costs so that a changing market situation is allowed for. Kennedy notes that while dynamic programming provides a solution at any point in time no matter what the starting state, the model can be re-run at the start of each period. The difficulties in using dynamic programming are discussed and he stresses the major problem of 'dimensionality'\(^9\) with respect to the number of state variables.

Fawcett (1973) has questioned the existence of substitution between nutrients to give identical growth using a slightly different reasoning to that suggested in section 4.4. It is based on the idea that growth requires a defined nutrient intake so that excesses of a particular nutrient are wasted. Fawcett considers that substitution is "more imaginary than real" and that it has appeared to occur because production relationships have been derived from experimental results using groups of pigs. These factors are

\(^9\) See Bellman and Dreyfus (1962).
considered in more detail in chapter VI.

In an attempt to partially allow for feedback and re-planning, Blackie (1974) developed a deterministic simulation model of a pig breeding and fattening unit to enable a comparison to be made between alternative policies. Having selected a policy, the model is used to predict outcomes on a monthly basis so that actual outcomes can be assessed. If discrepancies exist, or conditions change, this is noted so that the manager can decide whether any action is required. This could involve re-using the model as a comparative tool. Another simulation model, designed to compare alternative levels of fixed investment in housing as well as breeding and fattening decisions, has been developed by Dent (1971). This model did not have the control features of Blackie's model. Such models have emerged in response to the failure of the analytical models such as the neo-classical model and later developments to provide a realistic representation of the complex dynamic planning problem. These simulation models are detailed budgets which, being computerised, can be used to rapidly compare alternative systems. If sufficiently realistic analytical models cannot be developed it may be necessary to continue to explore the development of such models particularly with respect to reducing the cost of implementing them.
CHAPTER IV

A CONCEPTUAL MANAGEMENT MODEL

1. INTRODUCTION

To consider the possible components of an operationally useful model it is desirable to develop a model which includes all the variables that may conceivably be important. This approach shows where available data is deficient and provides an objective base for assessing possible simplifications. Within this framework the general form of a model which can be used to provide period by period decisions is developed. The model will be based on an activity analysis approach as the complexities of the real world make the manipulation of realistic continuous models difficult.

The discussion firstly provides a base for later developments by giving a general statement of the planning problem and some further comments on the re-planning function. Secondly, the model itself is developed by initially considering the individual animal case followed by the single pen case, as this is the basic decision unit, and finally the multi-pen situation is introduced. These developments will not consider operational solving algorithms. This problem forms the topic covered in the following chapter.

2. THE GENERAL PROBLEM

The general decision problem faced by pig producers (and by the producers of many other products) can be represented by the following statements.

(i) At any decision point, time \( t \), find values of

\[ x_i^t \]

the decision variables \( i = 1, 2, \ldots, m; \ t = 1, 2, \ldots, T \)

which maximises the objective, \( Z \):

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1 See Telser & Graves (1968) for a discussion of the errors introduced by a discrete approach.
\[ Z = \sum_{t=1}^{T} f_1(U_t) \]

where

\[ U_t = \text{expected utility from the decision period starting at } t. \]

(ii) Expected utility is some function of a range of variables:

\[ U_t = f_2(Y_t^j, P_t^j, C_t^i, X_t^i, S_t) \]

where

\[ Y_t^j = \text{physical output of product } j \quad (j = 1, 2, \ldots, n) \text{over period } t. \]

\[ P_t^j \text{ and } C_t^i = \text{the product and input prices respectively} \]

\[ \text{associated with } Y_t^j \text{ and } X_t^i. \]

\[ S_t = \text{the state of the total entity at time } t. \]

(iii) Production is dependent on a range of factors embodied in a third function:

\[ Y_t^j = f_3(S_t, E_t, X_t^i) \]

where

\[ E_t = \text{the physical production environment in the period commencing at } t. \]

(iv) The state of the entity depends on previous events and decisions:

\[ S_t = f_4(S_{t-1}, E_{t-1}, X_{t-1}^i) \]

(v) The choice of \( X_t^i \) is dependent on \( S_t \) (particularly the physical facilities available).

While this decision framework\(^2\) does not formally state the re-planning requirement, it is implied as the current state and expected conditions

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\(^2\) Similar statements have been made by many workers, particularly for growth models. See Dorfman et al. (1958), Eisgruber & White (1970), Harle (1968), Simon (1955) Boehlje & White (1969), Renborg (1970). Despite the recognition of the problem few attempts at developing operational systems have been made. An example is the work leading to recursive programming. See Day (1963) and Day & Kennedy (1970).
will be constantly changing. Even where a model which includes an
allowance for the system to be in a range of possible states in future
periods is used (for example, a dynamic programming formulation) re-planning
is required as non-anticipated states can eventuate. Similarly, as discrete
models can not include the infinite range of states that are possible, the
planning system will be more accurate if planning occurs using the actual
state observed in the current period. This planning system of planning-
exteuction-re-planning can be defined as 'continuous' planning and execution
where continuous means planning occurs at all points at which it is
physically possible to alter or make new decisions.\(^3\)

In attempting to include the dynamic and non-certain decision
environment into planning models many workers have proposed the use of a
strategy approach. Hildreth (1957) defines a strategy as a function
designating an action to be followed given a particular event. However,
given imperfect knowledge, the concept of a strategy can be incorrect as
it implies that decisions originally made are implemented according to
the observed events. This system is only correct where the expected
conditions originally used in formulating the strategies still hold or
where anticipated prices and costs are assumed to be components of the
starting state (though, as an optimal system may exhibit stability over
a range of conditions (Modigliani & Cohen (1961), Renborg (1962), Nuthall
(1971), re-planning may be unnecessary). However, as updated knowledge
in future periods cannot be conceived with certainty, it is still necessary
to include the strategy concept in the model formulated to allow future
plans to be dependent on the state occurring.

Accepting the conceptual need for continuous planning does not
necessarily mean a formal system is warranted. Many successful farmers
owe in part their success to an inherent ability to subjectively operate

\(^3\) Dependent on such factors as marketing arrangements and the physical
facilities available.
a continuous planning system. There will also be productive processes in which decision flexibility is limited and potential condition changes small. For example, Byrne (1970) found that, compared with implementing an initially determined plan, the use of re-planning provided only marginal benefits. Byrne, however, used yearly re-planning periods and decisions were made with a model which did not recognise non-certainty (nor, therefore, inherently included the re-planning possibility).

Similarly the nature of the individual's objective function will affect the need for continuous planning. Simon (1957) implies this point when he discusses the satisficing type objective and the search procedures which might be used in achieving a satisfactory outcome. Using the same production process as the previous period may provide a satisfactory outcome so that formal re-planning is unnecessary. Strictly this can be regarded as continuous planning in which the decision has been to make no change.

As well as the need for a detailed recording system (Boulding, 1952), continuous planning also requires continual market observations to indicate whether price forecasting systems used need to be reviewed. Effectively part of the continuous planning system is reviewing recording and forecasting systems.

With the above formulation of the general structure of the planning model required, the following sections contain a development of the components of the system for the pig fattening problem.

3. A MODEL FOR AN INDIVIDUAL ANIMAL

3.1 Introduction

In order to develop a model based on the decision unit of a pen it is useful to initially consider the individual animal. This is

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4 Logan & Bullock (1970) and Patrick & Bisgruber (1968) discuss the importance of these reviews.
achieved by considering animal response, feeding questions, genetic variability and the nature of the outcome distribution. Quantification of the components is considered in later chapters.

3.2 **Individual Animal Response**

There are two basic schools of thought on what are regarded to be the important state variables affecting response. The first considers that the critical state variable is the age of the animal and the other that the current physical state of the animal is the determinant (Lodge & Lamming, 1968). This study is based on the second approach for reasons which are discussed in chapter VI.

Accordingly the genotype of a particular animal and its starting state define a treatment response relationship. This relationship describes changes in body weight and conformation, the important characteristics of which are usually lean, fat, and dressing percentage. The genotype of an animal will determine the maximum response possible in terms of growth or change of the various body components. Feeding and environmental effects will then determine the actual change which occurs from within the feasible set. (Changes can involve weight loss of the various components.) The general response relationships for a period may be stated as:

\[ y = f(s, t, e) \]

\[ y \in Y \]

\[ Y = f(s) \]

where

\[ y = \text{a vector of body components representing the end of period state of the animal,} \]

\[ s = \text{a vector of body components representing the state of the animal at the beginning of the period,} \]
\( t = \) a vector of treatment levels for the various possible treatment types (e.g. feed, disease control),

\( e = \) a vector of environmental component levels,

\( Y = \) the set of possible end states.

Where the period length is greater than a day the response relationship must allow for the daily change in the maintenance requirement so that if a constant quantity of feed is provided the feed available for growth, if any, is changing.

The new state of the animal provides a vector of body components which is the input for the following response period. Alternatively, if the decision is to sell the animal, the vector gives rise to a quality rating which partially determines the price received (partially as grading systems are usually somewhat subjective). The vector component levels will determine dressing out percentages.

The new state of the animal is non-certain as it is dependent on non-certain and not fully controllable factors. These include:-

(a) the utilizable nutrient content of feed (net energy, biological protein value, etc; the probability density function of nutrient content of different feeds tends to normality (Crampton & Harris, 1969)).

(b) the requirement by the animal for utilizable nutrients for some given response. This will vary with fattening house "climatic" factors (temperature, humidity), disease incidence (stress), noise and other disturbance factors including social structure.

(c) the feed intake level. This will vary with factors causing stress such as social structure disturbances.
(d) errors in applying the type of treatment planned.

(e) the genotype of the animal. This determines such factors as the potential efficiency of feed use for maintenance and for production of both lean and fat, disease resistance, appetite level and aggressiveness.

Response variability can, of course, be controlled to a certain extent through shed environmental control systems, feed testing and disease precautions. Such methods may shift the expected value of the response as well as reducing the variance. Furthermore, the nature of the variability will be affected by the time of year through seasonal climatic changes, which also affect disease vectors, and the state of the animal. (An example is that of fat cover affecting the incidence of temperature variations on feed requirements.)

As each of the above factors follow a distribution they combine to give a response distribution for an individual animal under a given treatment. For decision making this distribution must be adjusted to allow for observational errors as the state vector(s) is non-certain (the animal cannot be dissected). Similarly, the knowledge of animal genotype is non-certain. (It can be regarded as a component of the state vector.)

4.3 Treatment Decisions

Given the response relationship for an individual animal the problem in any period is to determine an optimal treatment, including methods of observing s and of affecting e. For a particular observed state there is a set of expected states to which it is technically feasible to 'move' the animal at the end of the period. To reach each state a least cost treatment can be determined. The problem is one of deciding which of the alternative end states the animal should be in through considering the costs and expected payoff resulting from either the direct
sale or from having a pig closer to a sale state. These decisions clearly involve considering several future periods and must allow for the fact that the potential state movements in future periods are non-certain as the state of pig at the end of the first and later periods is non-certain. The possible states of an animal at any future point in time define a set of possible expected, ending states for that period, one for each of the possible states. To choose between the expected end states in any period requires information on the expected cost of the state movement and their probabilities in each of the periods.

It is necessary to determine the nutrient requirements of each state movement to give the cost estimates. The changing maintenance requirement and the change in body weight and confirmation define a requirement for the various nutrients (net energy, net protein, and thus amino acids, vitamins and minerals). These nutrients can be divided into essential and non-essential groups. The essential nutrients must be provided in sufficient quantity so that the net utilization by the animal is sufficient to provide for the anticipated maintenance and body change.

In order to provide the animal with these requirements there are a range of source ingredients with varying nutrient contents and cost. Thus, there exists the classical least cost problem of selecting a mix of source ingredients which provide the state movement nutrient requirements at least cost subject to the intake restriction. As the costs of the source ingredients, particularly for future periods, is generally non-certain, the cost of providing any state movement is non-certain. Similarly the net nutrients provided by the mix are non-certain.

Where a sequence of periods are considered it may be necessary to place further restrictions on feed mix construction as sudden feed mix changes, due to requirement and cost changes, can lead to growth depressing
effects (Davidson, 1966). In these cases the type of mix in any one period must be based on the mix used in the previous period. This feed mix problem can be solved independently of the overall problem as the results provide cost information for each possible state movement. Using the approach outlined requires that each state movement cost for each period be pre-determined.

There are alternative approaches to specifying state movements or alternative actions. For example, rather than specifying a change and determining the necessary feed requirements a range of feeds could be specified and the resultant animal change simulated. Such an approach, however, would tend to involve a greater number of calculations as ensuring an adequate range of end states are represented would require considerable experimentation.

4.4 Genetic Variability

Every animal is genetically different except for piglets formed from a fertilised egg which has split. However, many traits are economically insignificant or immaterial and can therefore be ignored (except for any correlations with important factors). The major factors of economic significance are the maximum potential response, the nature of this response in terms of fat and lean growth and the dressing out percentage, the efficiency of feed use both for fat and lean production as well as maintenance, and the nature of appetite limits. There is an infinite range of possible values of these traits though for planning purposes it is possible to form groups or classes which can be regarded as exhibiting certain response and efficiency characteristics. The importance of allowing for these different classes in planning will depend on the variability of the stock being fattened and whether there is any way of determining differences in a predictive sense. (In milk production there are significant economic gains resulting from feeding based on production observations where concentrate
feeding is used (Broster, 1974). Feed costs in pig fattening are equally as significant.) Bichard (1968) noted that while there is only a small variation in genetic ability regarding metabolic efficiency, there is considerable variation in maintenance requirements, mature size (potential response and rate of growth) and these tend to be positively correlated. Similarly, he notes there is considerable variability in the way productive energy is partitioned between body tissues (lean and fat).

Assuming a number of genetic classes can be defined there is at present no way of deciding with certainty the genetic class of a weaner being introduced into the system. However, given the population of pigs from which weaners are being drawn it is possible to estimate a probability distribution of the chance of an individual weaner being one of the possible classes. The ideal way of doing this would be to take samples and treat them in a perfectly controlled environment with a perfectly controlled treatment thus removing all variability other than genetic. Even with this approach the sampling errors mean that the derived distribution is not certain. The problem is that removing all variability would be extremely costly even if it could technically be achieved (knowledge is not available on all factors affecting response). A practical approach in estimating the distribution is one of using records from progeny and boar testing stations which attempt to maintain a constant environment and treatment as well as from constant treatment trials carried out within the enterprise. A problem is that it is not certain that the treatments and environment are constant, particularly between batches. Thus;

(a) it cannot be decided with certainty whether a pig responding in a defined way is of a given genetic class, and

(b) pigs responding in the same way from different groups
cannot be regarded with certainty as having the same genotype.

However, given the general distribution form which the response of a given genetic class follows due to treatment errors and environmental variability, it is possible using analytical techniques to approximate the parameters of the distributions and the probability of each class. Such a method is discussed in a following chapter.

A further piece of information sometimes available is the response of parents or lines. This can be used by dividing pigs into groups and using the above approach to produce a conditional probability distribution.

The genetic value of the population providing weaners is dynamic so that estimation of genetic class distributions can never be other than partially subjective. A real question is whether such estimates will be sufficiently accurate, or close enough to the risk takers true beliefs, to give a positive utility pay off. This is accepted at this point.

The importance of recognising a range of possible genetic classes comes from the different treatment requirements necessary to obtain a defined state movement. Furthermore, the range of possible state movements in any period for a given starting state will depend on the genotype. Explicitly stressing the importance of genotype, this component of the state vector, $s$, can be isolated so that:

$$ y = f_1(s, g, t, e) $$

where

$$ g = \text{a vector of components expressing the genetic ability of the animal for each economic trait.} $$

There are a number of reasons why it is efficient to use a feed mix which is specifically designed for the genotype of the animal (this
could be in terms of both quantity and type. For example, genotypes exhibiting a greater potential liveweight response may require more concentrated mix to overcome appetite restrictions, and genotypes with a large potential to lay down lean may require a protein-energy balance specifically designed. If fed the same feed, a pig with a low potential to lay down lean may de-aminate the protein and deposit fat. There will also be cases where animals of different genotypes should be sold earlier or later due to their different potential responses. (Depending on the significance of all these factors it may be economic to invest in facilities giving a large number of pens.)

As genotype cannot be predicted a feed designed for a particular genotype will provide inefficient state movements if in fact the pig is some other genotype. In assessing the type of feed to use the chance of a pig being various genotypes must be allowed for. A hedging mix somewhere between optimal mixes for a range of individual genotypes may be optimal depending on the probabilities and consequences. To design a feed mix to give a defined expected state clearly requires a knowledge of the probability distribution of genetic classes as the outcome distribution will depend on the genetic class probabilities.

3.5 Outcome Distributions and the Genetic Class Distribution

It can be assumed that observational and treatment errors are part of the environmental variability so that the outcome distribution depends on the environmental and genetic class probabilities. For a given observed animal state and treatment:

\[ P(y_o^d) = \sum_q \left( p(y_o^d/g_q^d) \right) \left( p(g_q^d) \right) \]

where

- \( P \) designates probability,
- \( y_o^d \) = the \( d^{th} \) possible observed outcome,
- \( g_q^d \) = the \( q^{th} \) genotype class.

An outcome distribution can take on an infinite range of forms as the genotype class probabilities will depend on the population from which the pigs are selected and on the selection methods.

Assuming the genotype class probabilities are known, the outcome distribution then depends on the environmental probabilities. The difficulty is that the genotype of an animal cannot be determined with certainty so that trials cannot be carried out to determine \( p(y^d_o)/g^q \). The nearest approach is to use as near to a perfectly controlled environment and treatment as possible on samples of pigs at low liveweights to put them into what appear to be genetically similar groups (dissecting some to get correlations between observations and actual states) and then subsequently exposing them to commercial treatments to obtain a frequency distribution.

Logic suggests, however, that the outcome distribution for a given genotype will approach normality. In plants, where genetically identical individuals can be obtained through vegetative means, the outcome distribution approaches a normal distribution though clearly the distribution is non a-symptotic. Further, though detailed evidence is not available, it is likely that observational and treatment errors approach normality. The same applies to the environmental components.

Using a normal distribution with the extremes adjusted requires a knowledge of the standard deviation and the mean for a given treatment and state. Estimates of these must rely on using information from testing stations and farm records for what appear to be constant treatments. It is clear, however, that these must be partially subjective. Estimates of mean responses can be obtained from experimental evidence,

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and from simulation approaches based on experimental data (as will be used in this study).

With time, observations are made regarding the response of the animal so that estimates of genotype probability can be updated. As it is impossible to observe the environment that has occurred in the first and later periods (for example, it is difficult to record the effect of sub-clinical levels of various disease vectors), the observed response cannot lead to a certain knowledge of genotype. Statistical decision theory can, however, be used to revise the probability distribution through using the historic response information.

At the end of the first and later periods, after the response has been observed, Bayes' formula can be used to update the probabilities. Thus:

\[
p (g^q/ y^d) = \frac{p (g^q) \cdot p (y^d/ g^q)}{\sum_q p (g^q) \cdot p (y^d/ g^q)}
\]

where

- \( p (g^q) \) = a priori probability of \( g^q \) occurring,
- \( p (y^d/ g^q) \) = a priori probability of outcome \( y^d \) given \( g^q \),
- \( p (g^q/ y^d) \) = probability of \( g^q \) given \( y^d \) has occurred.

A problem in obtaining revisions is determining the \( y^d \) that has actually occurred. The new liveweight can be measured though observational errors are still possible. In a group of pigs, samples can be used but this immediately introduces errors. The other major factor in giving the state, fat level, cannot be determined accurately even with the use of electronic apparatus so that further observational errors potentially exist. Thus, there is the additional random variable of observational errors involved. If \( y^d_o \) is observed, \( y^d_a \), the actual outcome, may be some other value, Thus

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6 See, for example, Eidman et al. (1967).
\[ p \left( g^q/y^d_o \right) = \sum_d p \left( y^d_a/y^d_o \right) p \left( g^q/y^d_a \right) \]

As a pig develops and observations are made some \( p \left( g^q \right) \) may become zero as the response precludes their possible occurrence. Whether this will occur will depend on the type of treatments used and the initial \( p \left( g^q \right) \) (as it effects the extent of the genetic class distributions overlap.) Further, pigs of the same physical state may have different genotypic distributions. This will depend on their past treatments and responses.

At the end of any period a new genotype distribution can be calculated and used to determine a new outcome or response distribution for use in the next decision period. In planning, the future periods are considered but not observed so that in each period the possible responses must be allowed for. Each resultant state gives a different revision of genotypic probabilities and thus a different outcome distribution.

4. THE DECISION UNIT - A PEN

4.1 Introduction

The discussion enables a model based on a pen unit to be developed. A model including a pen response and information system allowing for disposal decisions based on a pen is required. The question of forming pens of pigs of similar predicted genotype must also be considered.

4.2 Pen Response and Outcome Distributions

To consider a pen response situation, possible pen compositions must first be considered. Incoming weaners of a given state have a distribution representing their possible genotypes so that when a pen is made up, possibly of varying states, there exist a number of possible genotype combinations. Further, due to state observational errors, the
state composition of the pen is non-certain. Assuming all pigs in the pen are the same state the possible genotypic combinations can be represented by a vector \( c^t \), \( t = 1, 2, \ldots \) with components representing the number of animals of each genotype. Where the pen includes a range of states the possible pen structure must be described by having one vector for each state. Thus the matrix \( C^T \) (\( T \) has a greater range than \( t \) to allow for varying numbers in a given state resulting from possible observational errors) is made up of vectors \( c^t \), the number of vectors depending on the number of possible weaner states.

With given probabilities on genetic classes each possible \( C^T \) has a derived probability. As a simple example, consider a pen of three pigs of the same state and assume they can be one of two possible genotypes with probabilities \( p(g^1) = x \) and \( p(g^2) = 1 - x \). The possible pen compositions and probabilities are:

(i) Compositions: 
\[
\begin{align*}
\begin{bmatrix}
3  \\
0
\end{bmatrix},
\begin{bmatrix}
2  \\
1
\end{bmatrix},
\begin{bmatrix}
1  \\
2
\end{bmatrix},
\begin{bmatrix}
0
\end{bmatrix}
\end{align*}
\]

(ii) Probabilities: 
\[
\begin{align*}
p(c^1) &= x^3; \quad p(c^2) = 3 \cdot x^2(1-x); \\
p(c^3) &= 3 \cdot x(1-x)^2; \quad p(c^4) = (1-x)^3
\end{align*}
\]

If either \( c^2 \) or \( c^3 \) occurs there is no doubt about the genotypes of the individuals in the pen for future response estimation. If \( c^1 \) or \( c^2 \) occurs, provided the outcome distributions of the genotypes has some common outcomes, there is still doubt as the environment is not observed. In reality, due to the large number of genotypes, there is only a small chance of \( c^1 \) or \( c^2 \) type outcomes occurring. Further, due to observational errors the chance of being able to conclude with certainty regarding genotype is extremely small.

Assuming that the environment experienced by all pigs in a pen is identical (micro-environments do not exist though strictly this
is a possibility), for a given $C^T$ there is an outcome distribution for a given treatment, the possible outcome sets are determined by the actual environment. The outcome set can be described using a matrix of the form:

$$\bar{y}^d = \text{matrix of vectors} \begin{bmatrix} y_1^d \\ \vdots \\ y_n^d \end{bmatrix}$$

where

$$y^d = \text{a vector of body components for animals of state } d.$$  

$$n = \text{the number of animals on hand at the end of the period of state } y^d. \text{ This must be explicitly included in the outcome to allow for deaths or other problems necessitating disposal.}$$

Each possible $\bar{Y}^d$ has a probability of occurrence based on the environmental possibilities:

$$p(\bar{Y}^d) = p(C^T) p(\bar{Y}^d/C^T)$$

But, $\bar{Y}^d$ could occur from possible alternative pen structures (particularly after allowing for deaths), so:

$$p(\bar{Y}^d) = \sum_{T} p(C^T) p(\bar{Y}^d/C^T)$$

To obtain pen outcomes the individual animal response relationship for a given genotype should strictly be adjusted to allow for the social ordering in the pen. Furthermore, to allow for the possible pen genotype/state composition, and therefore the varying place in the social order of a particular state/genotype combination, there should strictly be a number of response relationships to allow for differing stress levels resulting from the social ordering. Data of this nature does not exist.

At the end of each period observations of pen structure and response can be used to update the $p(C^T)$ information in much the same way as for the individual animal case. Thus:
updated \( p^{(T/Y^d)} \) = \( \frac{p^{(C^T)} p^{(Y^d/C^T)}}{\sum_p p^{(C^T)} p^{(Y^d/C^T)}} \)

(Note that \( p^{(Y^d/C^T)} \) will be zero for many \( Y^d \).)

The new \( p^{(C^T)} \) are used to determine the new \( p^{(Y^d)} \) for future decision periods. Again, when planning at any particular point there will be a number of possible, anticipated, new \( p^{(C^T)} \) for each future period.

These are used in the dynamic model to determine the future possible effects of any current decision. Further, as for the individual animal case, two pens that are identical in numbers and apparent states may not have the same set of \( p^{(C^T)} \) as their past treatment could be different.

To include observational errors (both state and numbers) in the system, the updated \( p^{(T/Y^d)} \) must be adjusted as for the individual animal case. Thus:

\[ p^{(C^T/Y^d)} = \sum_{Y^d} p^{(Y^d/Y^d)} p^{(C^T/Y^d)} \]

These observational errors should be considered as in a practical situation the work involved in carefully assessing each pen means subjective assessment must commonly be used.

Treatment decisions in a pen context involves deciding on the \( Y^d \) to be achieved. For a given pen state (numbers in each state) there is a feasible set of possible \( Y^d \) and for each a least cost feed mix to provide the pen state movement. Such a mix may well be some kind of compromise as each state group in a pen can seldom be optimally treated. This creates a problem in designing a feed mix. For a given pen state movement each pig type (genotype, state combination) requires a given set of net nutrients and has certain appetite limits.\(^7\) Part of these requirements will be common to each pig so that additional requirements

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\(^7\) Though, for a given state, appetite variability of fattening pigs is small - see Holub (1969).
may mean some pigs get an excess, depending on appetities, precluding some individually possible state movements. The linear programming least cost problem therefore has the following general form:

(i) minimize \( Z = cx \)

(ii) subject to 
(a) \( b_d \leq Ax^d, d = 1,2,.... \)
(b) \( x^1 = x^2 = ..... \)
(c) \( x \geq 0 \)

where 
- \( c \) = a vector of per unit source ingredient expected costs,
- \( x^d \) = a vector of ingredient levels,
- \( b_d \) = a vector of nutrient and other requirements for the \( d^{th} \) type of animal to give a defined response.
- \( A \) = a matrix of per unit of ingredient nutrient supplies.

The requirement \( x^1 = x^2 = ..... \) gives a single mix for the pen (it is clear that this restricts possible pen state movements).

This general model of the pen situation adds considerable complexity to the decision problem in terms of the calculations required. In reality where weaners are being obtained from the same farm or from a limited number of breeders the amount of genetic variability is likely to be limited. This means only a small number of genetic classes need to be defined. The number must depend on the response differences and the economic consequences. Furthermore, the number of pens available in many cases allows pens of pigs of the same apparent state to be made up so that the matrix \( C^T \) will have few vectors.
### 4.3 Pen Re-Formation

Given initial responses it may be desirable for efficiency of later response to re-group pigs on the basis of state and predicted genotype distribution. The relative priority and weighting of these two factors depends on the influence each factor has on future response efficiency. Any such re-grouping simplifies feed design and enlarges the set of possible state movements. On the other hand re-grouping creates social ordering disturbances so that the gains must be related to the temporary growth check which will result. It appears the degree of check increases with increasing liveweight so that gains are more likely to accrue if re-formation is carried out as soon as reasonable information is available.

Where a single pen of weaners are divided into, say, two new pens on the basis of their state, the possible new pen compositions \((C^T)\) have probabilities obtained from the updated probabilities. Given the observed pen state \(Y^d\), there can be a number of possible \(C^T\). If divided in two, the matrix \(C^T\) is approximately halved in terms of the number of columns (states) to give the new pen structures. These have the same probabilities as the pre-halved pen. This provides new outcome matrices for future treatments (with a reduced number of possible states).

Where pigs of the same state are selected from several pens, the \(c^t\) distribution is obtained from the probabilities of the \(c^t\) possibilities for each individual pen. Thus, for example, taking 2 pigs from each of 2 pens where there are 2 genotypes the calculations are:

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8 A feeding programme designed to allow a full genetic expression in the initial periods may provide net benefits as the rate of learning is important. See, Ying (1967).

(a) Original pens:

<table>
<thead>
<tr>
<th>Pen 1 (Nos.)</th>
<th>Pen 2 (Nos.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{c_1}{c_1} \quad \frac{c_2}{c_2} \quad \frac{c_3}{c_3} )</td>
<td>( \frac{c_1}{c_1} \quad \frac{c_2}{c_2} \quad \frac{c_3}{c_3} )</td>
</tr>
<tr>
<td>genotype 1</td>
<td>2 1 0</td>
</tr>
<tr>
<td>genotype 2</td>
<td>0 1 2</td>
</tr>
<tr>
<td>probability</td>
<td>a b c</td>
</tr>
</tbody>
</table>

(b) Possible new pen compositions:

<table>
<thead>
<tr>
<th>( \hat{c}_1 )</th>
<th>( \hat{c}_2 )</th>
<th>( \hat{c}_3 )</th>
<th>( \hat{c}_4 )</th>
<th>( \hat{c}_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>genotype 1</td>
<td>4 3 2 1 0</td>
<td>genotype 2</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
</tbody>
</table>

(c) Probabilities:

\[
\begin{align*}
p(\hat{c}_1^1) &= (a \times d) ; \\
p(\hat{c}_2^2) &= (b \times d) + (a \times e) ; \\
p(\hat{c}_3^3) &= (c \times d) + (b \times c) + (a \times f) ; \\
p(\hat{c}_4^4) &= (b \times f) + (c \times e) ; \\
p(\hat{c}_5^5) &= (c \times f) 
\end{align*}
\]

Where there are several pens with pigs of the same state they could be formed into two groups. It would be efficient to group them so that their genotype distribution has as small a range as possible. Where a given combination of states are taken from several pens and formed into at least two pens so that their genotypic distributions have a narrow range, the new probabilities are formed in the same way as the above example except for the added complication of the different states.

An optimal re-formation is clearly a complex problem due to the large number of possible re-formation combinations and the assessment of the advantages of each type. In an activity-type analysis this involves defining an activity for each possible method. Which system is chosen then depends on the possible future outcomes and their probabilities. In
reality the pen numbers available and costs associated with servicing many pens will restrict the extent of re-formation.

4.4 **Marketing Pigs**

Information regarding pen structure is important in assessing the value of a pen when considering the possibility of disposal. The two possible disposal methods are either to sell the whole pen or a fraction at the end of any period.

For selling the entire pen the factors involved in determining the expected value and the method are:-

(a) given the observed state of the pen (matrix \( y^d_o \)) the actual state can take on a number of values with associated probabilities - \( y^d_a \) with \( p_d \).

(b) the component values of the vector \( y^d_a \) determine the grade of the particular pig. (Note: several \( y^d_a \) may give rise to the same grade.)

(c) a pig of state \( y^d \) has a chance of being graded into a number of grade-weight categories \( (z^w, w = 1,2,\ldots) \) with associated probabilities (such probabilities can be determined from dissection, chemical, colour and weighing tests on previously commercially graded pigs). That is,

\[
p(z^w/y^d) = p^w.
\]

(d) the probability of a pig of observed state \( y^d_o \) being graded and weight classified \( z^w \) is:-

\[
p(z^w) = \sum_{d} p(y^d_o/y^d) p(z^w/y^d) \quad \text{(many } p(z^w/y^d) \text{ will be zero)}.
\]

(e) from the current planning moment the price received for
a given $z^w$ is a random variable (except for contract situations). Thus, the expected value of a pig of observed $y^d_o$ is given by:

$$E(V) = \sum_w p(z^w) \left[ \sum_u p(R^u/z^w) (R^u/z^w) \right]$$

where

- $R^u/z^w$ is the $u$th possible pig value given $z^w$, $u = 1, 2, \ldots$
- $p(z^w)$ comes from (d) above.

(f) given a whole pen $(\overrightarrow{Y}^d)$, the expected value of the pen is the sum of $E(V)$ multiplied by the number of pigs of that state determined for each vector (except for the last component which is the number of pigs of that state). Thus, where $E(V^d)$ is the expected value for $y^d_o$,

$$E(V^d_o) = \sum_v E(V^d) x$$

where

- $x = \text{the last component of the vectors in } \overrightarrow{Y}^d_o$
- $v = \text{the number of vectors in } \overrightarrow{Y}^d_o$

Where part of a pen is sold off the calculations are identical except $\overrightarrow{Y}^d_o$ is divided into the relevant sub-matrix. The remaining sub-matrix, with the same updated composition probabilities, $p (C^T)$, is the new pen for future treatment analysis.

5. THE TOTAL PRODUCTION ENTITY

5.1 Introduction

The developments outlined have considered weaner purchase, pen formation, pen response and pen disposal. It remains to consider the total management model with respect to the total state of the system at any current planning moment and constraints placed on the system due to fattening shed space limits and the number of feed mixes that can be used.
Furthermore the multi-period structure of the system needs stating and the method of using the model outlined.

5.2 The Starting State of the Complete Unit and Pen State Movements Through Time

The current state of the entity consists of a number of pens of pigs ($P_i$, $i = 1, 2 \ldots$), each with an observed $Y_o^d$ and each with a set of $C^T$ with associated probabilities. Thus, where $t$ gives the time period and is equal to 1,

$$t[P_1/Y_o^d/P_1(C^1), P_2(C^2), \ldots]$$

$$i = 1, 2, \ldots$$

$$d = 1, 2, \ldots$$

describes the current pig state (for the entire entity state, information is also required on pen sizes, prices, costs and so on). Some $P_i$ may have the $d$ on $Y_o^d$ equal to that representing an empty pen.

This description may have already allowed for any purchase, sale, re-formation decisions or it can be assumed these are considered in the current period to give an adjusted state for consideration of treatment decisions (which must be considered together).

Assuming the first, this current state has occurred as a result of:

(a) $t^{-1}[P_1/Y_o^d/P_1(C^1), \ldots]$

(b) Treatment decisions.

(c) Environmental event.

(d) Purchase, sale and re-formation decisions.

Consider (d). Pen $P_i$ has either been emptied, partially emptied, or emptied and re-filled with weaners so that a current state has occurred which is different from a simple treatment/outcome change. Whatever the case this gives a new $Y_o^d$ and an associated set of $p(C^T)$. Regarding
re-formation, those $P_i$ which were filled in $t-1$ with weaners are candidates for re-formation. Similarly, whatever action is taken, and this can be influenced by what pens were emptied, this gives rise to a new $Y_o^d$ and an associated set of $p(C^T)$ for this set of pens. Pens emptied can also be filled with weaners.

Given the adjusted $t[P_i/Y_o^d/P_1(C^1) \ldots \ldots]$, the effect of any treatment decision set can be evaluated (one treatment per pen). Taking one of the pens, the total entity being a combination of the situation for each pen, the treatment used gives rise to an outcome distribution for the period, each event having an updated $p(C^T)$ set. Thus, the following represents the events:

$$
\begin{align*}
Y_o^d & \xrightarrow{\text{with set } p(C^T)} Y_o^1 \\
Y_o^1 & \xrightarrow{\text{}} Y_o^2 \\
& \quad \quad \quad \quad \quad \vdots
\end{align*}
$$

Taking the situation for each pen there is a set of possible

$$
t + 1[P_i/Y_o^d/P_1(C^1) \ldots \ldots],
$$

each with a probability (environmental).

Each of these possible entity states gives rise to a new set of feasible purchase, sale, re-formation, treatment decision. The decisions anticipated for purchase, sale, and re-formation give the adjusted

$$
t + 1[P_i/Y_o^d/P(C^T) \ldots ]
$$

which in turn defines the set of feasible treatments. Thus, the total time path of the system can be represented as:-
Let \( t^R_a \) represent the possible entity states, \( a = 1,2, \ldots \).

### Diagram

**Actions and environmental events**

1. \( t^R_a \)
2. \( 2^R_1 \) with \( p_1 \) → \( 3^R_1 \) with \( p_3 \)
3. \( 2^R_2 \) with \( p_2 \) → \( 3^R_2 \) with \( p_4 \)
   .
   .
   .

The problem is to determine an optimal action for each period for each of the possible states the system can be in at each period. The types of actions taken determine the set of future actions that are feasible (and thus the need to consider many periods).

### 5.3 Some Constraints on the Entity State Movements

Besides biological feasibility the set of feasible state movements is partially determined by the number of different feed mixes that can be used in any period and by any limits on weaner purchase and sale numbers.

Limits on the number of feed mixes requires the model to record which of the possible treatments use the same feed mix (type rather than quantity) and to select treatments (pen state movements) such that the number of different pen treatments is less than or equal to some defined number.
Having limits on the number of feed mixes that can be used leads to a possible transformation for solving purposes. Provided the maximum number of possible mixes is less than the number of pens there is likely to be no need to record which pen a particular pig is in where each pen has only one state of pigs. If pigs are identified with their state, so that groups of identical state/genotype distribution combinations are recognised and treated as a separate entity, individual pens within the group may not need to be recognised. The maximum number of possible mixes is probably sufficient to ensure such a group is treated identically rather than split up into treatment groups that do not coincide with the original pens. If a split which does not coincide with the original pens occurs, in cases with a variable pen size facility such a division is physically possible. Further, as the number of mixes is maintained at a maximum figure, this does not lead to greater feed mixing complexity.

5.4 Fattening Shed Space Constraints

The current state of the entity must clearly satisfy the constraint imposed on stock numbers and associated states by the available space. The set of future state movements is constrained by what is feasible with respect to the space available. Under the assumption that a pig of a given state has a fixed space requirement, each group of pigs in a pen has a fixed space requirement. With moveable pen partitions the problem is to ensure that at any time the total space requirements do not exceed that available rather than on an individual pen basis. Thus,

\[ H \geq \sum_{i} h^{s_i} n^{s_i} \]

where

- \( h^{s_i} \) = space requirement (square metres (m²)) of a pig of state \( s_i \).
- \( n^{s_i} \) = number of pigs of state \( s_i \).
- \( H \) = total space available (m²).
For practical reasons a continuous time space feasibility situation cannot be considered. It is assumed that if space is made available for the starting state of the entity, sufficient space is available for any of the feasible state movements within the period. The significance of this depends on the period length. In terms of the time path of the entity, anticipated decisions must be such that no matter what environment occurs the possible state movements must provide a space feasible state. This clearly places restrictions on the set of possible actions. If information was available on the effect of stocking rate on response the space limitations could be made less restrictive. Thus, decision variable levels in all time periods must be selected such that:

\[ \sum_{i} h_{i}^{s_{i}} \text{ resulting from } t_{i} R_{a} \leq H \]

for all \( t \) and all \( a \).

5.5 The Total Decision Model

Returning to the general statement made in section 2 of this chapter, the operation of all the relationships and the nature of the feasible sets and constraints can now be followed for the pig fattening case. Utility is taken as the expected cash return so that \( U_t = E(\text{net cash return}) \) from pen sales, less weaner and feed purchases. \( y_{j}^{t} \) is the output of pigs graded as grade/weight \( j \). \( x_{t}^{i} \) are the pen treatment, weaner purchase and pen re-formation decisions, there being one set for each possible state the system is anticipated to be in for each time period. Similarly, the \( S_{t} \) represent the pen states for each possible state at time \( t \) as well as the fattening shed space available. The set of possible decision variable levels will vary with the anticipated state at any time period. There will be several anticipated states as it is non-certain what the actual state will be. The feasible decision set is dependent on the constraints, the number of possible mixes, limits on
weaner purchases as well as $S_t$.

The method of using the model under continuous planning is:-

(a) solve given the current state, $S_t$.
(b) implement first period decision variable values.
(c) observe new state; update $p(C^T)$ and estimates of all distributions.
(d) Return to (a).

In some cases small changes in estimates of future conditions will mean that the decision variable values for period 2 (and later in some cases) for the particular end of period one state which occurs will remain optimal.

In the general planning problem there are specific cases where opportunities for major changes in the decisions are limited. Further, in a practical situation the costs of continuous planning may mean a policy of only formally re-planning at strategic points will be optimal. Such cases arise where the production period is long and opportunities for buying and selling only occur occasionally. For example, in cropping operations the number of time points where significant changes in the system are possible are limited. Such a practical approach implies subjective continuous planning between the strategic points.
CHAPTER V

SOLUTION METHODS

1. INTRODUCTION

The discussion of the management model has not considered practical methods of obtaining solutions. This chapter will consider the possible methods and briefly outline the form of the model to represent the conceptualised problem. A later chapter will discuss simplifications possible. To consider possible methods it is first necessary to consider details of what components of the non-certainty must be specifically allowed for.

2. SIMPLIFICATIONS IN THE NON-CERTAINTY TO BE RECOGNISED

The simplest approach to non-certainty is to replace all random variables with their expected values. Whether this approach is acceptable depends on the amount of bias that can arise in the decision variables. There are four components of the problem potentially leading to bias if handled incorrectly. The first is the objective function. If the second and higher moments of the random variables influence the level of utility the model used must allow these moments to influence the value of the decision variables. In this case, as the objective is the maximisation of expected return, it is only necessary to obtain an unbiased estimate of total expected return.

To derive the unbiased estimate, unbiased estimates of the expected values of the variables contributing to total return are required. This leads to two other components. If expected values are used to define the resource requirements of production possibilities, and to define the levels of resource supplies, the resultant activity levels may not be
unbiased expected values. Effectively, this is a problem of ensuring a feasible production system. A system which is feasible for the expected values may not be feasible for some of the other value combinations, thus leading to biased estimates. Further, when using activity expected values in conjunction with expected costs and returns, a biased estimate of the expected objective contribution may be obtained. Such errors are usually due to non-independence between output and the prices and costs, as later examples will show.

For later use, these two problem areas are termed the 'feasibility' problem and the 'expected value unbiased estimate' problem. The second is clearly dependent on the first in some cases.

In the 'feasibility' problem, the resource limiting production is available fattening house space. The total available supply in any decision period is certain so that problems of using the expected value do not arise. However, the production activities' (pens of pigs) requirement for space is a random variable. While in the current planning period the space requirement can be estimated with certainty, requirements in future planning periods are not certain as the pen response function is non-certain. If the expected space requirement is used to determine a feasible production system in any period, any anticipated production system may require more space than is available and therefore be infeasible. Conversely, excess space may be available. Using expected space requirements therefore leads to biased estimates of the anticipated output. To give unbiased estimates the range of possible outcomes must be considered together with their probabilities.

In the 'expected value unbiased estimate' problem the relevant

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1 For a more detailed discussion of these problems see Madansky (1960) and Tisdell (1973).
variables are the purchase price of weaners, the cost of providing a treatment, the grade given to pigs and the price received for a given grade of pig. An unbiased estimate of the expected cost of buying a group of weaners, of providing a treatment and of the sale price of a pen of pigs are required as these are all independent components of the objective function. As the number of weaners purchased and the distribution of purchase price are assumed to be independent variables, using the expected purchase price gives an unbiased estimate of the expected cost of a group of weaners. Similarly, using the expected cost of feed ingredients leads to an unbiased estimate of the expected cost of a unit of a feed mix (treatment). The two components of the sale value of a pen of pigs are the grades and prices so that using the expected state of a pen of pigs can, and usually will, lead to a biased estimate of its value. This occurs as grade and price are not independent variables. Each possible pen state must be considered to give each possible numbers/grade combination. To give the expected value of a particular pig of a given grade the expected price of that grade can, however, be used as

\[ E(C \times u) = C \cdot E(u) \]

where

\[ C = \text{ a constant (kgs. of meat of a given grade)} \]
\[ u = \text{ a random variable (price per kg. for the given grade)} \]

This means that in assessing the value of a pen of pigs, after applying some treatment, the range of possible outcomes must be considered. For each possible outcome the possible grade/number combinations must be assessed to allow for grading errors and each of these combined with the grade expected price data and weighted by the probabilities. It has already been noted that range of outcomes must also be considered for feasibility reasons.
The fourth area of possible bias results from the multi-period and stochastic nature of the problem. These decision problems can be viewed as a series of random trials. The outcome of the first leads to one of a range of trials in the second period and so on. By considering each possible outcome of the first trial, all possible second period trials are taken into account. If the expected value of the first trial is used as a surrogate for the distribution this precludes entry to many second period trials. This means the value of alternative actions in the first trial cannot be correctly assessed. An example is where favourable cash outcomes resulting from some action in a period enables investment in a lumpy asset requiring a large cash input in the following period. If the expected cash outcome was used the possibility of the investment could be precluded and therefore give a biased estimate of the value of the action. In that cash is assumed to be non-limiting and that the outcome distribution is taken into account, this form of bias will not occur in the pig problem.

3. CHOICE OF METHOD

The problem requires a method which recognises the multi-period nature of the problem and can allow for the stochastic nature of the pen response. All other variables can be treated as certain through using expected values. Allowing for stochastic response inherently allows for non-certainty regarding pen composition, both in terms of pig state and genotype. Given activity analysis must be used, the choice lies between dynamic programming, linear programming and the experimental approach commonly referred to as systems simulation. Dynamic programming is impractical due to the dimensionality problem derived from the number of variables necessary to describe the state of the system. These would be the fattening space available and the numbers of pigs
within each possible weight, conformation, and genotypic probability group (pen composition possibilities). The number of possible combinations in this latter group are far greater than the two or three that would be possible using dynamic programming.²

Systems simulation³ (S.S.) is a potential solving method as all the features of the problem can be allowed for within such a model. Being an experimental approach S.S. requires considerable experimentation due to the large number of decision variables resulting from the multi-period nature of the problem. Furthermore, with the continuous planning approach such experimentation must be carried out at each planning point. Some economies would occur as a result of initial experimentation providing indications of the set of decision variables that are likely to provide improved objective function values. However, a major objective of the study is to explore the problem of determining a planning horizon. To make comparisons it is desirable to estimate the optimal system for the given conditions. Systems simulation, being an experimental approach, cannot be guaranteed to provide the optimal system except through complete enumeration. Further, it will be shown later that it is necessary to have derived valuation estimates for use in the planning horizon problem. These are difficult to obtain using S.S. In view of these factors S.S. was not used. It was, however, used for determining some of the technical information required as an input to the model selected.

Given the need to use an activity analysis approach to the problem, poly-period linear programming is a technically feasible technique provided the non-certainty which must be actively accounted for can be incorporated

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² The dimensionality problem is discussed in most general discussions on the use of dynamic programming. See, for example, Throsby (1964) and Johnston (1965).

³ For general discussions on the use of systems simulation in management see, for example, Hardaker (1967) and Anderson (1974).
into the model. This is possible, at least to a minimum level. In a continuous planning problem the advantage of linear programming (L.P.) is that a single solution is potentially all that is required at each planning point. L.P. also provides valuation information for use in the planning horizon problem. Because L.P. satisfies the basic requirements of the problem it was used. This does not imply it is ideal, as will become evident. Conceptually dynamic programming would be more suitable and S.S. could represent certain sections of the problem more realistically. Overall, L.P. fits the requirements more appropriately than the alternatives. As noted by Dreyfus (1956), if a problem is essentially linear in its assumptions, after transformations if necessary, the efficiency of the simplex method or its derivatives means linear programming should be a preferred technique.

4. THE INTRODUCTION OF TIME AND NON-CERTAINTY INTO LINEAR PROGRAMMING MODELS

Linear programming was first used to solve allocation problems in which time was ignored. These models assumed that the transition from the current state (and system) could be carried out instantaneously at no cost. The unrealistic nature of this assumption for many cases led to models such as Loftgard and Heady's (1959), and Swanson's (1955). These models only partially recognised limitations to change and the links between periods. However, they introduced time by using a multi-period approach with all variables time subscripted. Capital formed the link between periods. Candler (1960) noted that problems with only one resource being determined by previous actions could be solved using a single period parametric approach.

Later developments in poly-period L.P. have shown how models can be constructed that account for the complete initial state of a system and allow for endogenous changes in all resources (the state of the system)
from period to period. Examples are the work of Stewart and Thornton (1962) in considering the endogenous determination of resource levels, Byrne & Healey (1969) in showing how the movement of stock numbers can be considered, Pearse (1963) in considering cash reconciliation problems in development and Cartwright (1968) for the inclusion of tax incentives.

A major problem in multi-period problems is deciding on the number of periods to be included in the model, both from the point of view of the physical size of the problem and of ensuring the optimal first period decisions are formed in a continuous planning situation. The optimal first period decision problem is considered later but it should be noted that size problems have been studied. Dantzig (1959), for example, discusses problems which can be divided into independent segments and Rae (1970) has noted that adding additional periods to a basic model of adequate length does not alter the first period decisions obtained. Thus, size can be minimised.

None of the developments referred to above allow for any non-certainty. Initial developments on the inclusion of non-certainty into L.P. models were for timeless models. Latterly developments have included both timeless and multi-period problems. The majority of the developments assume that only some of the variables are stochastic. In some cases it can be assumed that, while the majority of variables are stochastic, a large proportion of the resultant effects on objective variability can be accounted for by assuming the components of objective functions are random variables, after suitable adjustments, so that the other variables can be assumed to be certain. The type of model used, however, must depend on the problem both in terms of the form of the objective and the types of variables which are stochastic.

Linear programming models that recognise the non-certainty in some or all of the variables can be broadly classified according to whether
they allow for time considerations. Time is important for two reasons - firstly, whether a knowledge of the random variable outcomes will be known before decisions are made and, secondly, whether in future periods it can be anticipated that a range of outcomes are possible so that possible alterations to the plan can be considered. All decisions for some total period of time do not have to be made at the current planning moment.

Models which do not allow decisions to be updated can be multi-period models. An example is given by Johnson, Tefertiller and Moore (1967). Their model assumes outcomes are known before planning so that for each set of possible values the problem was solved and the results used to approximate the objective distribution. Such models are at best poor approximations as they not only ignore the real possibility of sequential adjustments but assume observation of future outcomes before planning. A range of models which do not assume a prior knowledge of random variable outcomes, but ignore sequential planning through replanning as outcomes become known, have been developed. These include the objective function variance minimization model (Markowitz, 1959) and its surrogate of minimisation of mean absolute deviation (Hazell, 1971), and the 'safety-first' (Sengupta, 1969 and Webster et al. 1975) and chance constrained (Charnes et al. 1959 and Symonds, 1967) and game theory type models (McInerney, 1969; Hazell, 1970 and Maruyama, 1972). The majority of these models assume that all non-certainty occurs within the objective function coefficients or that this is a reasonable approximation. Exceptions are the chance constrained problems in which one or more constraints are not allowed to be violated except with a minimal probability and the 'Truncated Maximin' model of Maruyama (1972). While the problem to be solved conceptually dictates the choice of model, comparisons have shown that the choice is not critical in many cases where
the objectives are similar (Hazell, 1970; Merrill, 1965; Boussard, 1969; Sengupta et al. 1963).

As reality in most cases allows plans to be changed as outcomes occur, models which reflect this possibility must, at least conceptually, be preferred. Allowing for re-planning implies random variable outcomes are not known prior to initial planning at any point. The first L.P. model to reflect a strategy approach was the two stage model (Madansky, 1962). This model is appropriate for problems in which decisions must be made in the first period before outcomes are observed and then a further set of decisions can be made after observing the outcomes. The non-certainty involved is primarily that of the input-output coefficients though non-certainty in the objective function and resource level coefficients can be incorporated through problem transformations. Cocks (1968) developed this model for the multi-period case and to allow for non-certainty in all the coefficients. Applications of the model are limited but two examples are those of Rae (1971), a vegetable crop production problem, and Yaron and Horowitz (1972), in a limited way, of a farm growth problem. The major problem of the multi-period discrete stochastic programming model is the physical size necessary to represent many periods and many states of nature. This is a problem of models in which probability distributions are discretely represented. Other than the variance and mean absolute deviation minimization models, non-certainty is discretely represented in the linear programming models discussed.

To select a model for the pig case the requirement of allowing for continuous planning as the stochastic outcomes are observed must be met. As feasibility must be guaranteed in any period (to ensure decisions in later periods are realistic) models using some kind of chance constrained, safety first or game theoretic approach are precluded.
Thus, a form of discrete stochastic programming must be used. If feasibility is not ensured in each period the objective function can potentially be biased as one period leads to another and infeasibilities compound.

5. A LINEAR PROGRAMMING MODEL OF THE PROBLEM

5.1 Introduction

In order to consider the transformations and simplifications that may be possible and necessary it is useful to define a linear programming (L.P.) model which mirrors the general model developed in the last chapter. It will be clear that modifications are necessary as the model contains a large number of variables required to be integer. The model is developed initially by considering a single period and then the multi-period case is introduced.4

5.2 The First Period

The current state of the system is described by defining the types of pens that can exist and recording the number of pens of each type on hand. Thus, for each \( c^T \) as defined in section 4.2 of chapter 4, a variable is defined (the observed pen structure must be used and set equal to the number of pens of each type. In defining pen types a discrete approach must be used as potentially an infinite number of types exist. Two pens which are observed as being physically identical may be in different groups if their genetic class distributions are different. The pen type variables, whose values will be integer levels, form part of the b vector. Let these components be \( b_i, i = 1,2,\ldots, a \).

Using activity analysis to describe possible actions the four

4 The conventional terminology of using \( c, b, A \) and \( x \) to represent the objective function coefficient vector, the requirements vector, the input-output matrix and the decision variable vector, respectively, is used.
types of activity required are pen treatment or feeding activities, pen sale and reformation activities, and weaner purchase activities. For each pen type there are a range of possible expected ending states for the period. For each state movement a defineable feed requirement exists. This gives a treatment activity or vector with components $a_{ij}$ where $j$ ($j = 1, 2, \ldots, d$) refers to the particular treatment. There is a set for each pen type. Let $a_j$ refer to the vector representing the $j^{th}$ treatment for the $i^{th}$ pen type. Let the level of a treatment activity be $x_j$. Together these form part of the vector $x$. As activity $a_j$ uses a pen of type $i$, coefficient $a_{ij}$ is unity. The only other non-zero components of $a_j$ relate to the pen type produced by the treatment and the shed space requirement. The $x_j$ can only take on integer values, the level of which is constrained by the number of pens of type $i$ ($i = 1, 2, \ldots, a$). For each $a_j$ there is a $c_j$ representing the expected cost of the particular treatment.

Pen sale, reformation, and weaner purchase are assumed to occur at the start of a period so that the starting state is that before such adjustments. The treatment activities define pen state movements after such adjustments. Sale of pigs can occur through either a complete or part pen being sold. Selling a part pen is a special form of pen reformation so its description is included below. Whole pen sale is described by a vector $a_j$, $j = d + 1, d + 2, \ldots, d + e$. The only non-zero component of this vector is a unit coefficient in position $a_{ij}$. It has an associated $c_j$ equal to the expected return from selling the pen. The number of pens of type $i$ sold is given by $x_j$ ($j = d + 1, d + 2, \ldots, d + e$). Again these must only take on integer values. Similarly, weaner purchase activities have vectors $a_j$, $j = d + e + 1, d + e + 2, \ldots, d + e + f$. The only non-zero component of such vectors is minus one in position $a_{ij}$. The associated $c_j$ is the expected cost of buying the
number and type of weaners necessary to form a pen of type \( i \). The level of purchase activities is given by \( x_j^i (j = d + e + 1, \ldots, d + e + f) \), and these values must again only take on integer values.

Pen reformation can only occur for pens of pigs less than a given liveweight. These activities, \( a_j^i \), \( j = d + e + f + 1, \ldots, d + e + f + g \), represent dividing a pen into components and putting each component into another pen. For each pen that can potentially be reformed there is a range of possible divisions. Thus the components of \( a_j^i \) consist of a unit coefficient in position \( a_{ij}^i \) and negative coefficients in a number of other positions representing the fraction of the different type of pen provided. The sum of such coefficients may not equal one as some of the new pen types may have a greater or lesser number of pigs in them. Again, \( x_j^i (j = d + e + f + 1, \ldots, d + e + f + g) \), the number of pens of type \( i \) reformed in a defined way, must only take on integer values. Under the assumption that the act of reformation is costless, the associated \( c_j^i \)'s are zero except where the reformation involves the sale of part of a pen in which case \( c_j^i \) is the expected return. The remaining pigs must stay in the same group. The sum of all contributions being made to a given pen type must be integer valued. To ensure this a dummy activity and constraint must be added for each pen type that can be formed from reformations. Such constraints have the following form:

\[
0 = \sum_{j=d+e+1}^{d+e+f+g} a_{ij} x_j^i - x_{D_i} D_i
\]

\( x_{D_i} \) integer,

for post-subscript \( i = a+1, a+2, \ldots, a+h \);

thus \( b_i = 0 \) for \( i = a+1, a+2, \ldots, a+h \),

where \( h \) = the number of pen types that can potentially be made up from reformations;

thus, pre-subscript \( i \) refers to the pen types that can be made up from reformations. This changes with post-subscript \( i \).
\[ i_{aij} \text{ fractional contribution to pen type } i \text{ by reformation } j. \]

The \( c^i_D \) are zero.

Besides the constraints imposed on the \( i^x_j \) by the current state of the system \( (b^i_1, i = 1, 2, \ldots a) \) and the constraints described above, pen treatment activities are constrained by the fattening shed space available and a limit on the number of feed mix types that can be used.

Let \( b^a_{a+h+1} \) represent the area of utilizable space available and therefore \( i^a_{a+h+1,j} (i = 1, 2, \ldots, a; j = 1, 2, \ldots d) \) be the pen space requirement by the \( i^{th} \) pen type and the \( j^{th} \) treatment. Thus

\[
 b^a_{a+h+1} \geq \sum_{j=1}^{d} i^a_{a+h+1,j} i^x_j \\
 j = 1, 2, \ldots d \\
 i = 1, 2, \ldots a
\]

This assumes that if a mid-period space requirement is used the slight initial under utilisation and later over utilisation is insignificant. It also assumes that response variability expressed within a period is insufficient to warrant allowing for a range of possible space requirements.

To ensure that no more than \( b^a_{a+h+2} \) mixes are used a series of equations are required to count the number of pens using each type of mix together with a number of dummy equations and dummy integer activities to limit the number of mix types to no more than \( b^a_{a+h+2} \). Thus:

\[
 0 = \sum_{j=1, 2, \ldots d}^{i^x_j, \text{post subscript } i = a+h+3, \ldots a+h+p.} \sum_{i=1, 2, \ldots a}^{i^x_j} -q^x_D k + x^i_D_k, \quad k = 1, 2, \ldots p \\
 0 \geq \sum_{k=1}^{p} x^i_D k \\
 b^a_{a+h+2} \geq \sum_{k=1}^{p} x^i_D k
\]
x_D_k must be integer, k = 1, 2, ..., p

where q = a constant greater than the number of pens that
can use the same mix;

Thus b_i = 0 for i = a+h+3, ..., a+h+p

and b_i = 0 for i = a+h+p+1, ..., a+h+p+k

and p (and k) is the number of different mix types that can exist.

The c_D_i (i = a+h+3, ..., a+h+p) and c_D_k (k = 1, 2, ..., p) are zero.

The only other constraints not formally defined are the constraints linking
the current state of the system to the levels of the treatment, sale,
purchase and reformation activities. Thus a series of the following
constraints are required:

\[ b_i \geq \sum_{j=1}^{d} a_{ij} x_j + \sum_{j=d+1}^{d+e} a_{ij} x_j - \sum_{j=d+e+1}^{y} a_{ij} x_j + \sum_{j=w}^{y} a_{ij} x_j \]

for each i = 1, 2, ..., a

where w = d+e+f+1

y = d+e+f+g

To interpret this relationship it is necessary to recall that
the components have the following definitions:

(a) j = 1, 2, ..., d represent pen treatment activities,
    j = d+1, d+2, ..., d+e represent pen sale activities,
    j = d+e+1, d+e+2, ..., d+e+f represent pen purchase
    activities,
    j = d+e+f+1, ..., d+e+f+g represent pen reformation
    activities;

(b) all coefficients are unity except those associated with
    the reformation activity. Some of these will be positive
    and other negative and non-integer;
5.3 The Multi-Period Model

The matrix form for the later periods is basically the same as that of the first period except that it is not known with certainty what the starting state will be. As a result of the observed first period starting state will be. As a result of the observed first period starting state and the decisions made there exists an outcome distribution. Dividing this into ranges provides a series of possible outcomes which lead to the starting states of the subsequent period. Each such starting state has a definable probability. A starting state consists of the number of pens of each type where the term 'type' encompasses both the physical state as well as probabilities on genetic classes. The state of nature occurring defines the genotype probability distribution and this leads to a new outcome distribution for the following period.

Each first period treatment activity vector must be augmented with components to express the possible outcomes for the possible second period starting state. Similarly for subsequent periods. These components take the form of negative ones in the equations representing pen types for each possible second period starting state. As there are a number of these, several sets of pen type constraints are required. The equations reconciling pen supply and use in the second period and subsequent periods will therefore have the following form:

$$0 \geq \sum_{j=1}^{d} a_{ij} x_j^t + \sum_{j=1}^{d} a_{ij} x_j^{t+1}, \quad i = 1, 2, \ldots, a$$

where the superscript refers to the period.

(c) all the $x_j$ must be integer;

(d) some constraints do not have reformation activities as they represent a pen type not assumed to be reformable.
If there are $S$ possible outcome states there will be $S$ such sets of equations for each previous period starting state. Thus:

\[ 0 > \sum_{j=1}^{d} a_{ij}^p x_{ij}^p + \sum_{j=1}^{d} a_{ij}^{p+1} x_{ij}^{p+1} \]

\[ i = 1, 2, \ldots, a \]

\[ p = 1, 2, \ldots, S \]

where the pre-superscript $p$ refers to the $p^{th}$ possible outcome.

There is a set of treatment activities for each possible first period outcome, and similarly for subsequent periods. This assumes that the first period outcome is observed before the second period decisions must be implemented. This is the actual case.

Consequently the multi-period case has the following general matrix format:-

\[
\begin{align*}
 b_1^{1} & < A_{1} x_1 \\
 b_1^{2} & < A_{2} x_2 \\
 b_2^{1} & < A_{1} x_1 \\
 b_2^{2} & < A_{2} x_2 \\
 & \quad \quad \quad \quad \quad \vdots \quad \quad \quad \quad \quad \vdots \\
 b_3^{1} & < A_{1} x_1 \\
 b_3^{2} & < A_{2} x_2 \\
 b_3^{2} & < A_{2} x_2 \\
 b_3^{2} & < A_{2} x_2 \\
 \end{align*}
\]

where the subscript refers to the period and the superscript to the previous period outcomes. If there are $S$ possible outcomes in any period, there will be $S^2$ matrix constraints in the second period, $S^3$ in the fourth period and so on.

Finally, the objective function has the following form:

\[
Z = c_1 x_1 + \sum_{p=1}^{s} p^P (c_2^p x_2^p) + \sum_{p=1}^{s^2} p^P (c_3^p x_3^p) + \ldots \]

where $p^P$ is the probability of the $p^{th}$ outcome where an outcome refers to a combination of period outcomes for all previous periods, the $c_t^p$ vectors are the same as that discussed for the first period case except
the treatment activities for the final period must be given the expected value of having the pen on hand. The method of doing this is discussed in a later chapter.

The model is used by re-planning each period with updated variable values. These updates will involve $b_1$ and all $c_t^P$. Thus only $x_1$ is implemented except in the cases where stability analysis indicates, depending on the first period outcome, that $x_2^P$ is optimal for the updated values. Similarly for subsequent periods.

Due to the large number of possible pen types ($b_i$, $i = 1, \ldots, a$) and the exponential nature of the multi-period matrix structure with respect to the number of possible outcome states recognised, the model is extremely large. This is the major difficulty of using stochastic linear programming. For practical reasons it is only possible to consider a limited number of outcome states and genetic classes. Data availability and the significance of genetic variability mean that it is doubtful whether including many genetic classes is warranted. Furthermore, it will be shown in a later chapter that the model can be simplified through not having to recognise individual pens. This is achieved without affecting the correctness of the model and is a major advantage as the number of pen types is extremely large.

The development of the model has indicated the type of information necessary for its application. The following chapters will consider how this information can be obtained. This includes the development of response relationships and the resultant feed requirement determinations both in physical and least cost terms. Models are also required to provide response variability and genetic class probability updating information. The major problem of deciding the number of periods to be included in the model has also yet to be considered.
CHAPTER VI

PHYSICAL RESPONSE AND FEEDING

1. INTRODUCTION

Detailed response function information is required to quantify the general management model that has been outlined. This chapter contains a description of the development of a physical response model as well as a least cost feed model. This enables the determination of the least cost method of alternative pig state movements through having information on the range of state movements that are physically possible and the associated nutrient requirements.

Traditional feed trials have generally failed to provide this total production surface information though there are some exceptions (Dent, 1964). The emphasis has tended to be on such criteria as the rate of growth, feed conversion efficiency and minimizing carcass fat content. Conclusions, for example, have often been in the form of the best feed type to maximise the rate of growth. Such information does not enable a comparison between economic factors such as the decrease in feed costs resulting from slower growth and the resultant net effect on total returns. Some of this trial work and the limited response surface work, however, can be used to formulate a response model. As the information available precludes the development of a fully tested model the model developed will provide a basis for further specific experimentation. The availability of an adequate response model would remove the need for many of the feeding trials that are repeated in many countries.

Unexpected price and cost changes are seldom great enough to make state movements involving total weight losses profitable. The
model will therefore only consider weight gain situations, though losses in fat content are allowed. The development is forced to assume that observational errors are inherently included in currently available experimental evidence due to the lack of data. Genotypic groups are not considered till Chapter VII.

The present discussion is divided into four sections. The development of a response theory and the resultant potential response relationship is discussed first. This includes the problem of an appropriate state variable listing to enable response prediction for any pig state and for predicting sale values. Secondly, a method for determining the nutrient requirements for each state movement is developed. This leads to the construction of a least-cost feed mix model which gives the mix necessary to provide the required nutrient input. This is covered in the third section. The final section contains a discussion on the prediction of response for a given feed mix type and quantity. This is required for cases where the actual genotype is different from that assumed in constructing the feed mix and because it is difficult to determine exactly the feed requirement for a given state movement.

2. POTENTIAL RESPONSE

2.1 The Factors Determining Potential Response

The factors affecting growth and development of pigs were first studied in detail by McMeekan (1940). Since then many workers have considered the factors determining potential response. The work of Elsley, McDonald & Fowler (1964) is a recent contribution. Of the two theories on growth and development the first suggests potential growth and development is largely dependent on the chronological age of the animal. The concept proposes that body components grow and
develop, given sufficient feed, at different ages. If feed intake is restricted when a particular component is potentially increasing in weight, it is hypothesised that its growth will be restricted and subsequent feeding will not reinstate the component to its potential mass. The second theory proposes that potential growth and development is largely dependent on the current weight of bone and muscle together with associated skin and viscera. Chronological age is largely irrelevant. This latter theory is proposed particularly by Fowler (1966 & 1967). By applying it to data used to develop the first theory, Elsley et al. (1964) found that if the fat content of carcasses is excluded from the development relationships, the second theory is substantiated. These workers conclude that fat is essentially a variable component of a carcass and will depend on the level and type of feeding. The bone-muscle ratio is regarded as being fixed for any given total weight of bone and muscle despite the kind of feeding system used. Similarly, muscle conformation appears to be solely dependent on the total bone and muscle weight with the exception of the head.

Evidence from growth studies in sheep and cattle also support the second theory. Examples are given by Yeates (1964) and Butterfield & Johnson (1967). Some conflicting evidence does exist, however. Two examples are given by Nielsen (1964) and Robinson (1964). Nielsen found that pigs with an older chronological age at 20 kg liveweight subsequently had a greater growth rate and lower feed conversion efficiency than pigs reaching 20 kg liveweight at an earlier age. Without data on carcass conformation the suggestion that age affects subsequent response cannot be substantiated. Similarly, Robinson (1964) found that compensatory growth occurred following initial feed restrictions, though no differences in conversion rates were observed. The problem of interpretation remains one of knowing the type of growth that has occurred. In contrast to these studies Widdowson (1967) maintains that feed conversion efficiency in
subsequent growth is not affected by the time taken to reach a particular liveweight.

On balance it appears that potential growth and development depends only on the current weight of bone, muscle and fat and that chronological age is not important except possibly in extreme cases. These cases are unlikely to occur in commercial operations.

2.2 State Description for Response Determination

To determine possible pig state movements in any period the factors determining potential growth and development must be recorded. The discussion indicates the factors are the weight of bone plus muscle and of fat. Body fat has been classified by Fowler (1966) into essential and variable. For a given weight of bone and muscle there will be a particular level of essential fat but variable fat will depend on past feeding. A limit to variable fat deposition exists so that to determine potential growth and development the current variable fat level must be recorded. Thus, the state of a pig can be described using the two state variables of 'bone plus muscle weight' and 'variable fat weight'. This assumes for a given bone plus muscle weight there will be a given skin and viscera weight. Alternatively, liveweight and variable fat weight could be used.

A state description is also required for assessing the sale value of a pig. The state variables required depend on the grading system being used. Currently the New Zealand system is based on the 'head on' hot carcass weight and the fat depth at position 'C' (at the eye muscle over the last rib) as measured by an intrascope. Pigs in a particular grade, as determined by these two measurements, can be downgraded by a subjective appraisal of a number of factors. Examples are fat colour and odour, muscle colour and skin blemishes. The lack of detailed objective data relating feeding and other actions to these factors means
they must be largely ignored except to place restrictions on feed mixes to prevent feeds being used which are known to give non-acceptable carcasses. As there is a direct relationship between carcass weight and bone plus muscle weight, bone plus muscle weight can be directly used in estimating the sale value. Similarly, there is a relationship between the state variable fat weight and fat depth at 'C' (McMeekan, 1941) so that variable fat weight can be used in estimating the grade.

2.3 The Response Relationship

The problem in obtaining response relationships is the lack of suitable trial data. Given that response depends on the bone plus muscle weight, information is required on bone plus muscle weights together with the weight of other body components. Obtaining this information requires expensive sequential killing and dissection trials. Currently the only available extensive data are those obtained by McMeekan (1940). However, there is extensive information on liveweight growth rates so that McMeekan's data were used to derive body component relationships and recent information to give the growth rates. This allows for improved genetic stock and better information on feeding than was available at the time McMeekan carried out his experiments. To get bone plus muscle growth, information from the liveweight growth and the body component relationships were used. Table 6.1 gives the derived relationships.

A lack of detailed information regarding the division of total fat into essential and variable exists. This is reflected in part by the variable fat relationship statistics. The division used was based on data from McMeekan's variable plane of nutrition growth studies. While there is considerable evidence from other trials indicating the variable nature of fat levels according to feeding this is not in a form enabling limits to be placed on essential and variable fat. Trial work
TABLE 6.1

**Body Component and Growth Relationships***

Where \( x = \text{bone + muscle weight (kgs)} \)

1. Skin (kgs) \[ = 0.1547 x^{0.8714} \ (r = 0.99, \text{S.E.} = .044) \]
2. Offal (kgs) \[ = 0.4728 x^{1.037} \ (r = 0.99, \text{S.E.} = .014) \]
3. Essential fat (kgs) \[ = 0.2449 x^{1.085} \ (r = 0.98, \text{S.E.} = .09) \]
4. Maximum variable fat (kgs) \[ = 0.0108 x^{2.0399} \ (r = 0.79, \text{S.E.} = .582) \]
5. Muscle (kgs) \[ = 0.6002 x^{1.0884} \ (r = 0.99, \text{S.E.} = .009) \]
6. Maximum growth of bone + muscle (kgs) over one week (G):

(a) Up to and including a starting bone + muscle weight of 26.3 kgs:

\[ G = 0.1391 + 0.1841 x - 0.0035 x^2 \]

\( (R^2 = .93, \text{SE}_1 = .0328, \text{SE}_2 = .0014) \)

(b) Greater than a starting bone + muscle weight of 26.3 kgs:

\[ G = 2.56 \text{ kgs.} \]

* Based on data from McMeekan (1940) and Agricultural Research Council (1967).

Specifically designed to provide this information is required.

While it would have been possible to produce a maximum growth relationship of the form,

\[ \text{Growth} = f \ (\text{initial weight; time}), \]

the complex nature of such a relationship gives rise to computational difficulties. As feeding decisions are frequently updated on a weekly basis a relationship giving maximum weekly growth was used in preference. This includes a grafted relationship approach as once the maximum rate of growth is attained the growth becomes constant within the weight limits used in this study.

---

1 See Laird (1965).
The maximum liveweight growth these relationships give (predicted) compared with progeny testing data, as quoted by the Agricultural Research Council (A.R.C.) (1967) is given in Table 6.2.

**TABLE 6.2**

Predicted and Actual Liveweight Growth

<table>
<thead>
<tr>
<th>Week No.</th>
<th>Predicted LW. (kgs) at start of week</th>
<th>Actual LW. (kgs) at start of week (progeny testing data)*</th>
<th>Actual - Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.7</td>
<td>22.7</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>24.9</td>
<td>25.6</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>29.12</td>
<td>29.8</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>33.83</td>
<td>34.4</td>
<td>0.57</td>
</tr>
<tr>
<td>5</td>
<td>39.04</td>
<td>39.2</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>44.75</td>
<td>44.6</td>
<td>- 0.15</td>
</tr>
<tr>
<td>7</td>
<td>50.88</td>
<td>49.8</td>
<td>- 1.08</td>
</tr>
<tr>
<td>8</td>
<td>57.41</td>
<td>55.5</td>
<td>- 1.91</td>
</tr>
<tr>
<td>9</td>
<td>64.22</td>
<td>61.1</td>
<td>- 3.12</td>
</tr>
</tbody>
</table>

* Average of Landrace and Large Whites for both gilts and castrates.

The predicted liveweights tend to give higher figures at the greater starting weights but this is to be expected as the feed used in the progeny testing is not designed to give maximum liveweight growth including fat.

3. **NUTRIENT REQUIREMENTS**

3.1 **Introduction**

The response relationships define maximum growth in any period in terms of bone, muscle and variable fat weights. Designed growth can be at any level within these limits. However, in selecting potential state movements, the fact that the nutrient requirements for all other
functions must be met before the variable fat requirement must be allowed for. Other body component changes are defined by the body component relationships. In order to provide the feed necessary to achieve the required growth, information is required on the maintenance requirements and growth requirements for the type of growth occurring.

3.2 Maintenance Requirements

The majority of work determining maintenance requirements has used liveweight as the independent variable. It might be expected, however, that nutrients required to maintain a given liveweight will depend on the ratio of body components as the dynamics of tissue maintenance could well vary with different tissues. In the absence of investigations designed to explore this factor, maintenance requirements are based on liveweight.

The previously outlined body component relationships do not allow for intestinal and stomach contents. To obtain a liveweight figure the total body weight must be adjusted. A number of figures have been quoted in the literature ranging from 123 (Clausen, 1953), to 103 (Houseman, et al. 1973) per cent of body weight. The variability will depend on many factors including the type of feed and time of last feeding. A compromise figure of 110 per cent was used.

Maintenance requires protein, energy, mineral and vitamin intakes. Little work has been done on the mineral, vitamin and amino acid balance requirements for solely maintenance so that it is assumed that growth requirements include the maintenance requirements.

Maintenance requirements are frequently quoted as a simple energy requirement but tissue dynamics, enzyme manufacture and so on require an intake of protein. Crampton and Harris (1969) give a figure of 0.9125 $W^{-0.75}$ gms of digestible protein ($W = LW$ (kgs)) as the intake
necessary to make up endogenous urinary losses. While this figure assumes the biological value of the protein is 100%, they noted that in cows the actual requirement was approximately three times this requirement. With pig feeding the biological value of the ingested protein will be high provided the amino acid balances are satisfactory so that for the lack of better information a figure of 1.22 $W^{-75}$ gms of digestible protein was used (75% biological value).

A range of figures for the energy maintenance requirement have been proposed. These depend in part on the addition to basal metabolism required to cover energy requirements for activity and on the efficiency of metabolisable energy use to produce the net energy assumed.

Many activity requirement estimates fall within the range 120 (Kielanowski, 1966) to 135 (Mitchel, 1962) per cent of basal metabolism. Estimates of the efficiency of metabolisable energy use for maintenance are frequently around 81% (A.R.C., 1967). The consensus, as discussed by the Agricultural Research Council (1967), appears to be that the maintenance energy requirement is close to $1.017 W^{-56}$ megajoules (M.J.) per day of metabolisable energy (M.E.). Some more recent work by Verstegen, et al. (1973) suggests a figure of $0.475 W^{-75}$ M.J.M.E. per day. This provides lower estimates for liveweights below 50 kgs. Taking a conservative approach, the former estimate has been used. Not all workers regard liveweight as being the only determinant of maintenance requirements. For example, Kotarbinska & Kielanowski (1969) derive an equation in which the level of intake is included suggesting the rate of growth may affect maintenance. Further work is required, however, before such a suggestion can be used.

While the maintenance requirements are based on a per day requirement, feeds are designed for use over a period. Where a fixed quantity of feed is offered per day the nature of any growth will change as the
residual after the dynamic maintenance requirement will change. The calculation of the feed input necessary assuming a constant input and type to give a defined type of growth over a period is therefore complex. Where, however, the requirements are based on a constant growth rate and an average maintenance requirement over a relatively short period, such as one week, the errors will not be great. This approach is used. To determine the average daily maintenance requirement therefore requires the estimation of the sum of the changing daily requirement, day by day. Furthermore, where the protein requirement is specifically allowed for, the energy content of this protein allowance (15.75 M.J.M.E./kg) must be deducted from the total energy requirement as given by $1.017 \ W^{56} \ M.J.M.E. \ per \ day$.

3.3 Production Requirements

The nutrient requirements for growth depend on the amount and type of growth. They consist of energy requirements for fat production as well as the work carried out in laying down muscle, the protein requirement for muscle deposition and the associated requirements for minerals and vitamins. The protein must contain the necessary balance of amino-acids. In the following estimates the energy, protein and mineral requirements for producing the skin, bone and viscera are included within the fat and muscle requirements.

While some workers consider the requirements for growth vary with the rate of gain and the liveweight, the consensus is that requirements are constant (Crampton & Harris, 1969; Carr, 1972). Requirements could vary with liveweight as the chemical composition of gain varies. For example, McMeekan (1940) gives figures of the lipid and protein content of fat and muscle varying from, respectively 76.7% to 85.9% and 19.0% to 21.6%. However, no data are available to enable allowances for these changes.
Energy requirements for growth are specified as metabolisable energy. This allows for the difference in efficiency of M.E. use for production compared with maintenance. The A.R.C. report a range of estimated efficiencies of M.E. use to give net energy covering 65% to 70%. An efficiency of 68% is assumed.

For fat production the estimates of the energy requirements are relatively consistent. The requirement assumed is 48.8 M.J.M.E./kg of deposition. This is based on Kielanowski's (1972) estimates of 48.78 M.J.M.E./kg of fat in one experiment and 53.5 M.J.M.E./kg of lipid deposition from another experiment. After allowing for fat containing 90% lipid this later estimate becomes 48.64 M.J.M.E./kg of fat.

Muscle production requirements are based on a protein requirement and an energy requirement for the work involved in laying down the muscle and associated body component changes. Most estimates of the protein (N x 6.25) content of muscle are close to 20% so this is used. For example, Crampton and Harris (1969) suggest a figure of 20% whereas the A.R.C. give a range of 19-23%. Using 24.24 M.J.M.E. as the energy content of 1 kg of protein leads to an estimate of the energy requirement for the work involved in depositing a kilogram of protein. Using pigs of 2.5 to 8.5 kgs, Kielanowski (1965) estimated the energy cost to be 7.2 M.J.M.E. Kotarbinska and Kielanowski (1969), however, using heavier pigs, obtained an estimate of 21.94 M.J.M.E. whereas Thorbek (1970) gives a figure of 35.76 M.J.M.E. Most workers regard sucking pigs to be unrepresentative of growing pigs and Kotarbinska's et al. estimate was determined using an assumed maintenance requirement greater than generally accepted so a compromise between the later two estimates of 29 M.J.M.E. was used.
Many workers note that the efficiency of digestible protein use is not 100% and this varies with liveweight. For example, Thorbek (1969) gives efficiencies ranging from 57.7% to 44.4%. Usually such estimates ignore the maintenance protein requirements. Adjusting Thorbek's figures for the maintenance requirements gives efficiencies ranging from 65% at 30.3 kg body weight to 51% at 85 kg body weight. It is never clear in such studies whether the efficiencies quoted are due in part to protein intake being greater than can be utilized so that deamination takes place. If this is the case, particularly with increasing weight, efficiency would be expected to decline. Accounting for these factors, an efficiency of 65% is assumed so a gain of 1 kg protein requires 1.54 kgs D.P. as well as the energy requirement.

Protein intake in excess of requirements is de-aminated and used as a source of energy. This can occur as either the ration contains insufficient energy to allow all the protein to be deposited or there is an excess of protein so that it is de-aminated and stored as fat. Thus, protein can substitute for energy but clearly the reverse cannot occur. Crampton and Harris (1969) note that the efficiency of use of the energy content of digestible protein is around 80%. Armstrong (1969) found a similar figure of 83%. However, such energy is not as effective as carbohydrate energy for fat production. Breirem and Homb (1972) give an efficiency figure if 93%. Using 83% and 93% respectively gives an effective efficiency of 77%. Thus, using 24.25 M.J.M.E. as the content of 1 kg of D.P. gives an effective M.E. of 18.67 M.J. for fat deposition.

Estimates of protein requirements assume that the amino acid balance is satisfactory. It is possible that the efficiency of protein use reported in experiments is in part a function of an imbalance of amino acids. Given a better understanding of the requirements, this
efficiency may be improvable. Most stated requirements are expressed as a percentage of the total diet. Logic would suggest that a percentage of the protein requirement would be more accurate so that total requirements would vary with the type of growth planned. For the purposes of this study the A.R.C. recommendations are used to ensure an adequate balance.

Finally, any growth requires an adequate intake of minerals and vitamins. Again it would be expected that requirements depend on the amount and type of growth whereas most workers give a simple daily or percentage of diet requirement. This probably reflects an inadequate total understanding, though the recommendations have been shown to prevent deficiency symptoms and in this sense are adequate. To simplify the feed compounding problem it is assumed vitamin and mineral requirements are met by adding standard quantities of prepared concentrate mixes designed for local conditions.

4. NUTRIENT INPUTS AT LEAST COST

4.1 Introduction

Section 4.4 in Chapter III covered the development of least cost models within the general context of planning models. These models emphasised the determination of least cost mixes giving an average rate of growth expansion path. The feeding problem within the total planning model proposed, however, is one of determining the least cost method of achieving potential state movements over a short period. This information is then used as an input to the profit maximising model. This section contains the development of such a model. It is based, at this stage, on the requirements of a single, average genotype, pig. The problem will be seen to be readily approximated by a linear activity analysis model so linear programming is used.
Basing the mix formulation on an average daily basis for a period (say one week), the objective is to find the ingredient levels, vector $x$, which minimizes the daily expected cost $cx(z)$, where $c$ is the vector of per unit of ingredient expected costs. This must be achieved subject to the mix satisfying a number of general requirements. The major requirement is that the mix must contain the required nutrients though there are a number of other requirements associated with this. The mix must satisfy the quantity intake limits imposed by the pig and must allow for the effects of the fibre content through its effect on metabolizable energy. Furthermore, substitution between certain nutrients is possible so that this must be allowed for through specifying its possible form and limitations on its type and extent. Finally, limitations on the extent of feed type changes possible through time to prevent possible growth disturbances that can occur from frequent, radical changes in diets can be necessary (Davidson, 1966). The model will be developed through specifying, in general form, the requirements and limitations listed.

4.2 The Requirements

4.2.1 The Nutrient Content

The nutrient requirements for energy and protein, both maintenance and production, give four equality constraints in the linear programming formulation of the problem. Equalities are necessary as any excesses in the mix would provide an expected growth different from the required state movement. The energy constraints must represent the energy requirement and supplies with the energy content of protein deducted. Similarly, the amino acid balance requirements and the mineral and vitamin requirements give rise to a series of minimum constraints. These constraints do not need to be equalities as excesses are wasted or stored without affecting the type of growth. While the efficiency of M.E.
use for maintenance and production varies, provided all requirements are expressed in M.E. terms, this difference does not require separate energy equalities for maintenance and production as some workers have used (Lofgreen et al. 1968). Algebraically, the nutrient requirements are:

\[
\begin{align*}
  b_1 &= \sum_j a_{1j} x_j \\
  b_2 &= \sum_j a_{2j} x_j \\
  b_k &= \sum_j a_{kj} x_j, \quad k = 1, 2, \ldots, l
\end{align*}
\]

where

\[
\begin{align*}
  b_1 &= \text{M.J.M.E. requirement per day for maintenance and production,} \\
  b_2 &= \text{D.P. (kgs) requirement per day for maintenance and production,} \\
  b_k &= \text{the daily minimum requirement of the } k^{th} \text{ amino acid, mineral or vitamin,} \\
  x_j &= \text{level of the } j^{th} \text{ ingredient (kgs), } j = 1, 2, \ldots, n, \\
  a_{1j} &= \text{the per unit supply of M.J.M.E. in the } j^{th} \text{ ingredient,} \\
  a_{2j} &= \text{the per unit supply of D.P. (kgs) in the } j^{th} \text{ ingredient.} \\
  a_{kj} &= \text{the per unit supply of an amino acid, mineral or vitamin in the } j^{th} \text{ ingredient.}
\end{align*}
\]

4.2.2 Limits on Intake

It is only in the last few years that feed mix models have recognised that the real problem is to achieve a nutrient intake at least cost without regard to the total weight of feed except for total intake restrictions (Mohr, 1972). Previously the models assumed the problem was to determine an optimal formulation per unit of bulk. Allowing the nutrient concentration of the mix to vary may provide a
cheaper nutrient intake provided any effects of concentration on availability are allowed for. For pigs the efficiency of M.E. use is not consistently affected by the nutrient concentration. Lodge, et al. (1972) for example, found in some cases that a lower energy concentration gave an increased efficiency, but not in others. The A.R.C., on the other hand, report on studies in which efficiency has increased and on others where a decline has occurred. There are probably many more factors involved than are currently recognised. For this study it is assumed efficiency is not affected by intake quantity. It is worth noting also that Cole et al. (1967a) found that the fibre content of the diet did not influence the rate of feed passage.

Other experiments by Cole et al. (1967b) have shown that pigs adjust their intake to ensure the utilisable level of energy intake is maintained at a constant level. A pig will attempt to consume sufficient feed to give maximum potential growth though there is recent evidence to suggest feeds of very high energy concentration will give 'super maximum' growth (Cole et al. 1972). There is, however, clearly a limit to intake for physical digestive rate and palatability reasons so that if the energy concentration of the feed is extremely low the energy intake will be insufficient to promote maximum potential growth. Provided palatability is not a problem, the effective limit on intake is either the maximum energy requirement or, where this is not satisfied, the physical-digestive limit. In that potential state movements are all based on genetically feasible growth levels, for the feed mixing problem the essential factor in determining maximum intake is therefore the physical-digestive limit.

Using intake data a number of workers have estimated intake relationships. Most of these relationships use liveweight as the
explanatory variable\(^2\), though the A.R.C. use both liveweight and age. The problem with these relationships is that they are based on intakes of feed which provide the pigs' energy requirements and do not express the upper physical limits on intake. The approach used here was to find the feeding trial reported in the literature which used the lowest energy concentration. This was the work of Cole et al. (1967) in which they obtained an average daily intake of 3.65 kgs over a liveweight range of 38-105 kgs using a feed containing 12.43 M.J.D.E. Detailed information for all weights was not, however, given so that information from Headley et al. (1961) was used to give the intake for different liveweights after an adjustment to give an average of 3.65 kgs. Using the data quoted gave the following relationship:

\[
\text{Maximum intake (kgs)} = 0.2728 \times x^{0.6165}
\]

where \(x = \text{liveweight (kgs)}\)

and \(r = 0.99 \& \text{S.E.} = 0.009\).

It is probable that this estimate is conservative as in the Cole et al. experiment energy requirements were still met. Given any period in which liveweight is increasing, the limiting day for a feed intake which is constant is the last day (as maximum intake/kg LW. declines with increasing LW.). To counteract partially the conservative intake relationship, the mid-point liveweight is used to give the maximum intake. Thus, the following constraint is added to the model:

\[
I \geq \sum_j x_j
\]

where

\[
I = .0278 \times x^{0.6165}
\]

and units of \(x_j\) are expressed in kgs.

\(^2\) For example, see Brody (1945) and Headley et al. (1961).
4.2.3 Nutrient Substitution

Carbohydrate cannot substitute for protein in muscle growth so that an isoquant for a given liveweight change, such as the isoquants derived by Dent (1964), is not a true isoquant as the type of growth in terms of muscle and fat to give a constant liveweight change can vary markedly. This is clear when it is considered what can happen as rations giving the same liveweight growth are varied from high energy-low protein to low energy-high protein mixes. The first ration will give high fat growth and possibly no muscle growth. As the protein content is increased lean growth increases but the energy previously going into fat production is now partly being used to deposit the protein so that the same liveweight growth is maintained but of a different type. As more protein is introduced into the ration the proportion being used for muscle production as against being used as a source of energy changes. Due to the efficiency differences the apparent isoquant is curvilinear. (Using pen averages would also tend to smooth out the apparent isoquant.)

On the other hand it is known that protein can be de-aminated by the animal and used as a source of energy so that protein can substitute for energy. Thus, a ration model must allow for a limited form of substitution in developing rations which will provide a given type of growth. This is considered in detail below.

In general it is known that substitution between other nutrient requirements can occur. This gives rise to essential and non-essential nutrients. The non-essential requirements can be anabolised by the animal and, in many cases, form a wide range of alternative organic materials. While some of these bio-chemical pathways are understood, there is a lack of substantiated quantitative data (Lewis, 1962). Furthermore, the economic significance of including detailed substitution

...
relationships for relatively low priced mineral, vitamin and amino-acid requirements is small so that these forms of substitution are ignored in the model. This does not mean substitution is precluded as the mineral, vitamin and amino-acid requirements used implicitly assume anabolisation occurs.

While protein as a source of energy tends to have a higher price than carbohydrate sources, there will be cases where it will be economic to allow de-amination. To include this form of substitution in a given growth type isoquant requires a careful consideration of de-amination efficiency and the conditions under which it will occur. De-amination will not occur if there is sufficient energy from other sources. In allowing the possibility of de-amination it must be ensured that the type of growth does not change. These conditions give three cases:

(a) in any case where the state movement incorporates the maximum potential muscle growth, substitution can occur. Protein provided in excess of that required for maintenance and muscle growth will be de-aminated and substitute for other energy sources.

(b) in any case where some variable fat growth is required and muscle growth is less than the potential maximum, substitution cannot be allowed to occur. If more protein than that required for maintenance and muscle growth is provided, the additional protein will give rise to extra muscle growth and a lower variable fat growth. Thus the type of growth would change.

(c) in any case where zero variable fat and less than the potential muscle growth makes up the state movement, substitution can occur if the starting state of the animal does not include
any variable fat. Protein in excess of the maintenance and muscle growth requirement will be de-aminated to take the place of other energy rather than provide additional muscle growth since there is no excess energy to the essential requirements to enable additional muscle growth. Thus, the type of growth is not altered. But, if the starting state of the animal includes some variable fat, de-amination cannot be envisaged as additional protein would be laid down as muscle through using the stored energy in the fat. Thus the growth would be different from the desired level.

For cases (a) and (c) an isoquant can be formed. Its general form will be a linear segmented 'curve' in which the number of segments will depend on the case. Taking the general case and sequentially increasing the protein intake as the intake of energy is reduced produces the following segments (sequential segments have a decreasing negative slope):

(a) as protein intake is increased above the maintenance and muscle growth requirement the de-aminated protein substitutes for maintenance energy at a given level of efficiency.

(b) further protein eventually substitutes for energy used in the work of deposition at a lower efficiency.

(c) finally, protein intake additional to that used in (a) and (b) substitutes for energy deposited as fat at a lower efficiency than that in (b).

While there is no direct experimental evidence showing that this hypothesis occurs, the evidence on growth relationships and efficiencies
of energy use support it. If additional protein substitutes for energy use in fat production before the other use the efficiency changes maintain the same decline. Metabolizable energy being used for fat production, when directed to maintenance, has a greater net energy yield so offsetting the lower efficiency of the energy from de-aminated protein being used in fat production.

Where the M.E. requirements are determined after allowing for the efficiency differences between maintenance and growth, the effective 'curve' reduces to two segments, one for maintenance and deposition energy costs and the other for fat deposition. (As noted in section 3.3, 1 kg of D.P. provides 20.13 M.J.M.E. over the first segment and 18.67 M.J.M.E. over the second.)

Algebraically, this substitutional system can be represented using the following equations:

\[(i) \quad b_1 = \sum_{j} a_{1j} x_j + 20.13x_t + 18.67x_s\]

\[(ii)\quad b_2 = \sum_{j} a_{2j} x_j + 20.13x_t\]

\[(iii)\quad b_3 = \sum_{j} a_{3j} x_j - x_t - x_s\]

where

- \(b_1\) = total energy requirement,
- \(b_2\) = protein requirement,
- \(b_3\) = energy requirement for maintenance and deposition work,

and

- \(a_{3j} = a_{1j}\) all \(j\).

and

- \(c_t = c_s = 0\)

---

3 See the earlier discussion on growth relationships, particularly variable fat, and nutrient requirements.
The second relationship ensures protein is not used for fat deposition, after de-amination, at the higher level of efficiency. This equation must be added to those previously defined and the total energy and protein requirement relationships modified to include the substitution variables \( x_t \) and \( x_s \).

### 4.2.4 Fibre Content

The A.R.C. report on many experiments designed to quantify the effect of increasing the fibre content of rations. In most cases the effective energy content declined compared with the apparent energy content. They quote the following equation for this effect:

\[
\text{Digestible energy as a \% of gross energy} = 91.33 - 2.19x
\]

where

\( x = \% \text{ of crude fibre in the diet.} \)

Using the equation to predict the fibre effect, the A.R.C. found it agreed with the results obtained by most workers.

Using this relationship and assuming \( \text{M.E.} = .916 \text{ D.E.} \) leads to the relationships:

\[
\text{M.E.} = (.8366 - .0201x) \text{ G.E.}
\]

Thus, \( \frac{d\text{M.E.}}{dx} = -.0201 \text{ G.E.} \)

Given \( \text{M.E.} = .8366 \text{ G.E.} \) leads to:

\[
\text{Decline in M.E. for each additional fibre \%} = .024 \text{ M.E.}
\]

In order to allow for these effects the M.E. content of each feed must be adjusted to allow for the fibre content. As this is a linear effect this can be done prior to solving the model.

The A.R.C. also note that evidence exists to suggest that the fibre content affects the protein availability. As this evidence is not extensive, this possible effect is ignored.
4.2.5 **Limits on Feed Change**

Sudden changes in diet may lead to metabolic disturbances and subsequent growth checks. As objective data on such effects are not available the economics of diet changes in response to sudden ingredient price changes cannot be considered. However, where considered necessary subjective limits can be placed on changes. To allow for this problem it is necessary to ensure that planned current diets are similar in ingredient content to those used in the past period. Similarly, planned diets for a number of future periods may need to be similar to that planned for use in the immediately proceeding period. In practice diet changes would be introduced in stages, the time involved being dependent on the type of change.

If diet change limitations are important, feeds for any period cannot be designed as independent problems. This requires that the model specified above be formulated as a multi-period problem. Given four weekly sub-periods over which a defined state movement is required in each sub-period leads to a nutrient requirement specification in each sub-period. Each specification set gives a component of the total model. The link between each component is the set of restrictions limiting changes in the quantities of ingredients included in each mix. These restrictions must allow some flexibility as to obtain the required type of growth may require changes in the energy-protein ratio.

Let 'a' and 'b' represent, respectively, the upper and lower fractional change subjectively assessed to be allowable for an ingredient in a subsequent period. For the current planning period the feed mix used must satisfy:

\[ j^{b_{1+1}} = bx_j^o \leq x_j^1 \quad \text{all } j \]

and

\[ j^{b_{1+2}} = ax_j^o \geq x_j^1 \quad \text{all } j \]
where

\[ x^0_j \] = the \( j^{th} \) ingredient level used in the last sub-period.

\[ x^1_j \] = the \( j^{th} \) ingredient level planned to be used in the current sub-period.

For the subsequent sub-periods the requirements take the following forms:

\[ b x^q_j \leq x^{q+1}_j \leq a x^q_j \quad \text{all } j \]

where

\[ x^q_j \] = \( j^{th} \) ingredient level in the \( q^{th} \) sub-period.

This leads to the operational requirements:

\[ 0 \leq x^{q+1}_j - b x^q_j \quad \text{all } j \]

\[ 0 \geq x^{q+1}_j - a x^q_j \quad \text{all } j \]

Where the state movements within each sub-period involve appreciable growth, the factor 'a' must allow sufficient quantity increase to provide the increased energy and protein requirements. Any simple quantity increase is acceptable from a growth disturbance concern. Thus 'a' should be based on the most restrictive of the energy or protein increase required between sub-periods together with the flexibility allowance. Similarly 'b'. In that growth may vary between the sub-periods, 'a' and 'b' may change for each sub-period. Placing these restrictions on feed mixes may mean state movements which were previously possible now become infeasible.

4.3 The Least Cost Model in Total

The components of each sub-period problem and the constraints on diet change between sub-periods gives rise to the following total model where the components are defined in matrix terms using:
(a) Components

(i) The within sub-period requirements of:

(1) \[ b_1 = \sum_j a_{1j} x_j + 20.13 x_t + 18.67 x_s \]

(2) \[ b_2 = \sum_j a_{2j} x_j - x_t - x_s \]

(3) \[ b_3 = \sum_j a_{3j} x_j + 20.13 x_t \]

(4) \[ b_k \leq \sum_j a_{kj} x_j \quad \text{All } k = 4, 5, 6, \ldots, d \]

(5) \[ l \geq \sum_j x_j \]

are summarised as:

(A) \[ b^q \geq \lambda x^q, \]

where \( q \) represents the sub-period.

Similarly, the daily cost of the \( q^{th} \) sub-period mix

is given by:

(B) \[ z^q = c^q x^q \]

(ii) The restrictions on the 1st sub-period mix based

on the mix actually used in the last week of:

(6) \[ j_{d+1} \leq x^1_j \quad \text{all } j \]

(7) \[ j_{d+2} \geq x^1_j \quad \text{all } j \]

are summarised by:

(C) \[ d \geq B^1 x^1 \]

Note that:

\[ x^q = (x^q_1, x^q_2, \ldots, x^q_n, x^q_s, x^q_t) \]

and

\[ u^q = (x^q_1, x^q_2, \ldots, x^q_n) \]
(iii) The restrictions on the mix in subsequent sub-periods based on the mix used in the preceding sub-periods of:

\[(8) \quad 0 < x_{j}^{q+1} - b_{j}^{q} \quad \text{all } j\]

\[(9) \quad 0 > x_{j}^{q+1} - a_{j}^{q} \quad \text{all } j\]

are summarised as:

\[(D) \quad 0 < B_{u}^{q+1} - R_{u}^{q}\]

(b) The Total Model

For four sub-periods the model is:

Minimize \[ Z = \sum_{q=1}^{4} c_{q} x_{q} \]

Subject to:

(i) \[ b_{1}^{1} > A_{x_{1}}^{1} \]

\[ d > B_{u}^{1} \]

\[ b_{2}^{2} > A_{x_{2}}^{2} \]

\[ 0 > - R_{u}^{1} \text{ } + B_{u}^{2} \]

\[ b_{3}^{3} > A_{x_{3}}^{3} \]

\[ 0 > - R_{u}^{2} \text{ } + B_{u}^{3} \]

\[ b_{4}^{4} > A_{x_{4}}^{4} \]

\[ 0 > - R_{u}^{3} \text{ } + B_{u}^{4} \]

and

(ii) \[ x_{q} > 0 \]

Milling and mixing costs are ignored. As the total weight in a mix can vary part of the problem could be to consider these costs as well as transport costs. These costs, however, are relatively insignificant compared to the ingredient costs when the possible weight variations are accounted for.
The model also assumes a number of other simplifications which are regarded as warranted to meet practical considerations. Strictly, a model with daily sub-periods and one which allows the daily growth to give the required end state to be endogenously determined would be more accurate. This would reflect the dynamic maintenance requirement more precisely and permit a non-constant growth rate within each week. Such a model should require that the mix type used on any day within a week should be identical. Only the quantity should change. Furthermore, the practical use of such mixes would require sophisticated feeding equipment. Model specification would be complex and the computational costs unwarranted.  

Variability of mix quality may also be important. The nutrient content of ingredients is stochastic though, in this study, feed variability is accounted for by assuming it is one of the components leading to a stochastic response function. However, it is possible to formulate mixes such that the nature of the variability is partially controlled. Two examples of such models are given by Rahman & Bender (1971), and Chen (1973). By ensuring, for example, that the mix provides the required nutrients with a minimum probability shifts the growth distribution to the right. (The only way to ensure an 'exact' nutrient content is to use an extensive testing system.) As the assumed objective in this study is expected profit, the profit distribution shape is irrelevant so that methods of altering the ingredient distribution shape are ignored.

5. PREDICTING RESPONSE

The model defined, when adjusted for a particular genotype, does not indicate the type of response that would occur if the pig was

---

4 Also recall that it was assumed growth takes place in an environment of thermal neutrality. For range systems this may be inappropriate. For a model which allows for thermal stress see Brokken (1971).
of a different genotype. As the genotypic class of a pig cannot be predicted with certainty it is necessary to develop a state movement prediction model. This enables an outcome distribution to be formulated, for use in the planning model, for a given mix being fed to a pig of non-certain genotype.

The form of this model largely follows from the principles, relationships and constants given in the sections on potential response and nutrient requirements. Some adjustments are necessary for a particular genotype.  

For a given daily feeding pattern over a period and the starting bone plus muscle and variable fat weights, it is possible to predict, on a daily basis, the movements of the state variables. The procedure involves determining the maintenance energy and protein requirements so that any surplus energy and protein in the feed intake available for growth can be estimated. Feed intake may be less than feed offered due to the intake restriction. Surplus nutrients give an estimate of the daily growth. A daily system is necessary for accurate maintenance requirement prediction. An outline of the model is given in Appendix II.  

It is difficult to obtain suitable data to validate the response model as most feeding trials do not provide sequential dissection information. To obtain a detailed conclusion would require trials to be run which are specifically designed for validation purposes. The model was tested, however, against the most suitable data available in the literature. This was work on the effect of energy and protein intakes carried out by Lodge, et al. (1972). A comparison is given in table 6.3.

5 Included in Chapter VII is a discussion of the genotypic differences assumed.

6 A listing of the program is available on request.
TABLE 6.3

Actual* Against Predicted Response

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. weight gain (gms/day)</td>
<td>654</td>
<td>678</td>
</tr>
<tr>
<td>Feed conversion (kgs feed/kg gain)</td>
<td>2.6</td>
<td>2.59</td>
</tr>
<tr>
<td>Lean % of carcass.</td>
<td>51.1</td>
<td>47.5</td>
</tr>
<tr>
<td>Fat % of carcass.</td>
<td>35.7</td>
<td>34.4</td>
</tr>
<tr>
<td>Fat at &quot;C&quot; (mm)</td>
<td>14.38</td>
<td>13.5</td>
</tr>
</tbody>
</table>

* Based on data from Lodge, et al. (1972)

The growth rate is slightly faster and this is reflected in the marginally better feed conversion. Fat and lean are marginally less than actual and this is reflected in the fat measurement at 'C'. Undoubtedly such differences would occur for any set of comparison data due to such factors as feed variability both in quantity and quality.

The results are not only an indication of the accuracy of the model itself but also of the nutrient requirement estimates which form the basis for estimating the state movement costs.

6. PERSPECTIVE

Pig growth and nutrition is undoubtedly more complex than suggested in the discussion. Current knowledge on the bio-chemical pathways, efficiencies, de-amination, and so on indicate this but, for economic purposes, a test of any model must rely on how well it predicts response and the nutrient requirement information needed. In that the models used provide reasonable predictions they are regarded as being adequate for testing the planning systems proposed. Before they can be used in a practical situation they will require adjustment to suit locally used pig strains and conditions. The models used rely on data
collected from a range of sources and therefore a range of genotypes and conditions, particularly with respect to the nutrient contents of locally produced ingredients.

Since the ideas presented here were developed, Whittemore & Fawcett (1974) have published a response model. Their model is very much less detailed as it does not include body component relationships, intake limitations or allow an energy balance system through fat mobilisation. De-amination is assumed to occur at a constant rate of efficiency and the division between essential and variable fat is ignored.
CHAPTER VII

DETAILS OF THE LINEAR PROGRAMMING MODEL AND DATA USED

1. INTRODUCTION

Due to the large number of possible pen states and the extensive range of possible state movements that are embodied in the models outlined, the computational costs involved in implementing the planning system are extensive. However, a number of transformations and simplifications are possible. The reason for developing the models in their basic form is to enable an objective consideration of possible simplifications. This chapter contains a discussion of simplification methods as well as other factors leading to a reduction in the computational requirements. The estimation of genotype differences, outcome distributions and actual state variable values to be used are also considered. The objective is to develop a model with the minimum number of constraints but which is sufficiently similar to the previously defined models as computational costs in a L.P. model are related to the number of constraints.

As the study is primarily concerned with the development of a planning system, the stress is placed on the principles and methods that should be used in deriving the relevant data. Accordingly an exhaustive search of all data sources would be necessary to apply the model in any specific area.

2. TRANSFORMATIONS AND SIMPLIFICATIONS

2.1 Genotype Classes

The number of different genotype classes to consider must depend on the extent of variability due to genotype and the correlations between
the various traits of economic importance. If the traits are independent it is necessary to define a number of classes to allow any independence between the traits to be expressed.

The important economic traits are the potential growth rate of bone and muscle, the maintenance requirement expressed as a feed intake requirement (and thus allowing for both digestibility and metabolic efficiency variations), the production feed requirements and the appetite limit. A number of other factors are conceivably important but are either direct components of the listed factors, or exhibit non-significant variability. Included in these categories are disease resistance (affecting potential growth and feed requirements) and body conformation (the genetic variability in improved breeds is small). Potential maximum variable fat is not regarded as being important as most grading systems do not encourage feeding systems giving maximum fat.

To indicate the extent of phenotypic variability and the level of genetic correlations that are possible, data derived by Lucas (1968) is presented in Table 7.1. Lucas used progeny testing data for 'white pigs' as the source material.

**TABLE 7.1**

<table>
<thead>
<tr>
<th>Trait</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Correlations (genetic)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Daily Gain Conversion</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ratio</td>
</tr>
<tr>
<td>Daily gain (kgs)</td>
<td>0.67</td>
<td>0.054</td>
<td>1.0</td>
</tr>
<tr>
<td>Kgs feed/kg LW. gain</td>
<td>3.46</td>
<td>0.23</td>
<td>-.76</td>
</tr>
<tr>
<td>Fat at 'C' (mm)</td>
<td>21.2</td>
<td>3.86</td>
<td>-.15</td>
</tr>
<tr>
<td>Killing out %</td>
<td>73.9</td>
<td>1.62</td>
<td>-.19</td>
</tr>
</tbody>
</table>

* Based on Lucas (1968), (progeny testing data).
The significant correlation is the 76% between daily gain and feed conversion efficiency. While this is partially explained through the lower maintenance requirements of faster growth rate pigs it indicates that if potential growth rate differences are recognised it is likely that differences in feed utilisation will follow a similar pattern. Lucas does not give appetite figures as controlled feeding was used. However, Bichard (1968) has noted that considerable phenotypic appetite variability exists but that it is positively correlated with mature weight and that mature weight is positively correlated with growth rate. Thus, variability in growth rate appears to explain a large percentage of the feed efficiency variation as well as being related to appetite variability so, at least for the figures quoted, it is the important trait to consider.

For the pigs used by Lucas the extent of daily gain and feed conversion phenotypic variability is not great as indicated by the standard deviations (Table 7.1). For a four-week growth period, assuming a normal distribution, approximately 70% of growth observations would fall within the range 17.25 to 20.27 kgs. Similarly, feed conversion would lie within the range 3.23 to 3.69 kgs feed/kg LW. These ranges must be reduced when considering the effect of genotypic variability only. The A.R.C. (1967) report that for feed conversion, 50% of the variability is due to genetic effects (14% to litter effects and 36% unexplained. This suggests that the ranges above are likely to be halved when only the genotype induced variability is considered. For the general case, where it is assumed a number of genotype classes can be defined, the following relationships dividing total variance can be derived:

(i) Variance due to genotype
\[ \sum_{i} \sum_{j \neq j} \frac{\sigma_{i}^{2}}{p_{i}} (p_{i} - p_{j}^{2}) - \sum_{i} p_{i}p_{j}y_{i}y_{j} \]

(ii) Variance due to environment
\[ \sum_{i} p_{i} \sigma_{i}^{2} \]
where
\[ y_i = \text{mean value of trait for the } i^{th} \text{ genotype}, \]
\[ p_i = \text{probability of the } i^{th} \text{ genotype}, \]
\[ \sigma_i^2 = \text{variance of trait for the } i^{th} \text{ genotype}. \]

The division of total variance depends on the differences between the \( \sigma_i^2 \) and the \( y_i \) but with the dispersion of the \( p_i \) also being of some importance. In any particular situation all these variables must be considered to enable a decision on the number of genotype classes to recognise. In that the example data quoted suggests that growth rate explains much of the variability, and that the variability due to genotype is not great, it is unlikely that more than two to three distinct genotype classes need be recognised. In this study two were recognised, one representing a higher maximum potential growth in bone and muscle as well as a slightly greater food conversion efficiency than the other. The determination of the specific differences is considered later.

Recognising two genotypes implies each class represents a range of actual genotypes so that even after initial growth observations the real case of non-certainty regarding genotype must be recognised. (Observational errors and the existence of micro-environments within a pen are other reasons.) This implies that any batch of weaners must be taken to be either one genetic class or the other as if both classes were assumed to occur the initial growth observations would indicate with certainty the genetic class of each pig. Accordingly, any one pen must be assumed to consist of one class. These necessary assumptions mean that pen reformation decisions do not need to be actively incorporated though it is implied that pen reformation is always carried out giving pens of pigs of a similar state (though their genotypic class is non-certain).
2.2 Transformations Resulting from Limits on the Number of Feed Mixes

As a pen is assumed to contain pigs of a similar state, the possibility of only having to recognise individual pig states rather than pens exist. Assuming the entity starting state consists of a feasible pen structure, where only individual pig states are recognised a limit on the maximum number of mixes that can be used means pigs in one pen are likely to remain in that pen even though individual pens are not recognised. Furthermore, an economically optimal action will tend to preclude the division of a particular state group into separate feeding groups. The reasons are discussed below.

Whether a group of pigs of a given state can potentially be divided into different treatment groups depends on the number of different state groups on hand in relation to the potential maximum number of treatment activities possible. These depend, in part, on the maximum number of different mixes possible as several treatments may be made up from the same mix (using different quantities). If the maximum number of activities is less than, or equal to, the number of state groups then group division is not possible so pigs in one pen can be assumed to remain in the pen. However, if the opposite holds one state group can potentially be divided into more than one treatment group. In such cases pen numbers may mean the division can occur along pen lines, or at least very similar to pen lines.

To examine the possibility of infeasible pen divisions occurring, consider the economic reasons why it may be profitable to divide a particular state group into several treatment groups. For any state group of pigs the future prices and costs give an optimal set of period by period treatments which terminates at either the maximum assumed weight or at the point beyond which expected marginal cost exceeds
expected marginal return. As the objective is maximum expected cash return, there is no advantage in using a range of treatments as outcome variability is not relevant. However, the limited fattening shed space may mean it will be optimal to use several treatments. The possible reasons are:

(a) if the space can be more profitably used by competing stock either all or a proportion of the group will be sold rather than the selection of a 'sub-optimal' treatment.

(b) if available space is insufficient to allow the group to be carried through to the profit maximising time period either a proportion of the pigs will be sold or a proportion will be treated to give a slower growth rate and thus spread the space requirement through time.

It is only where it is necessary to select a slower growth rate that a state group division can occur. Where this occurs, the division may be along pen lines or at least relatively close to it. If this is not the case the numbers involved may enable an additional, separate, pen to be formed. Thus, the L.P. model was set up to recognise state groups rather than individual pens. Where infeasible divisions occur in the solutions it is necessary to re-solve the problem with additional constraints. (Experience showed that divisions of any kind seldom occurred.)

Where it is necessary to recognise more than two genotypes the same system can be used. The only difference is each state group must be defined as a combination of physical and genetic probability characteristics representing the pen structures possible.

2.3 The Feed Mixing Problem

A feature of the least cost model defined in chapter VI was
the restrictions placed on the maximum change on ingredient proportions 
in sequential periods. Use of the model requires estimates of 
ingredient costs for all future periods within the planning period 
and a multi-period solution for all the possible state movement 
combinations for each state group. This means a large number of 
solutions, which also need repeating for each re-run of the total model, 
are necessary.

In view of this computational burden it is important to consider 
possible simplifications. The principles to use must rely on the fact 
that in continuous planning it is only in the first period that detailed 
solution information is required. For the second and following periods 
the model uses treatment cost information to assess the first period 
actions. Provided any adjustments to the system do not distort the 
cost ratios, and therefore the slope of the objective hyperplane, they 
are acceptable. Where confidence in the detailed predictions of 
ingredient costs for future periods is not high it may be adequate to 
use projections of treatment costs based on the detailed least cost 
solutions for the initial periods. Similarly, as the model is re-run 
in each period, re-solving all the feed mix problems may not be justified 
compared with a simple adjustment to all treatment costs based on general 
ingredient cost changes.

Another potential problem is that the formulated mixes may all 
be slightly different in terms of ingredient proportions so that a common 
mix cannot be used for several groups of pigs, even at different 
quantities per day. Given a limit on the number of mixes that can be 
used it may be impossible to feed all groups. To overcome this 
problem it is also necessary to formulate a number of 'sub-optimal' 
mixes for each treatment activity so that the opportunity of selecting
common mixes which satisfy the mix number limit is increased.

Considering the points raised the treatment costs for inclusion in the objective function of the linear programming model used were determined using a slightly amended version of the originally specified least-cost feed problem. The amendments were:

(a) to ensure a number of common mixes exist, a number of protein-energy-intake combinations were defined and least cost solutions on a one period basis obtained for each. These mixes were then used as the candidates for use in the first period of the least cost model but with integer restrictions to ensure only one was used. As the mix may not exactly provide the nutrient requirements the prediction model discussed in Chapter VI was used to predict the outcome and thus to give the end of period state.

(b) the limit on the maximum number of mixes that could be used in the profit maximising problem was only imposed every four weeks.

3. ACTIVITIES, DOMINANCE AND REDUNDANCY

The general form of the required treatment activities has been outlined in Chapter VI. This section will include a discussion on the general principles used in selecting the particular activities to include.

The need to develop adequate weaner sources and finished stock outlets requires some limits to be placed on the range of purchase and sale weights assumed possible. In this study the range of possible purchase weights was taken as 20-22.5 kgs LW. and the range of possible sale dressed weights as 37-51 kgs. This is based on the local grading
system weights for what are commonly referred to as porker pigs. These ranges give a total liveweight range of approximately 20-68 kgs, and a total bone plus muscle range of approximately 10-29 kgs.

Within the general price and cost ranges that are likely to be encountered in any one situation, many state movements will always be dominated and so can be ignored. Given the competition for space, an optimal system will tend to use only the higher growth rate treatments. The reason for selecting a slower growth rate could be the possibility that the less concentrated feed mixes required will enable a feed cost reduction which offsets the increased use of space and lower conversion efficiency. Typical prices and costs mean that this will tend to only occur for small reductions in the growth rate. Similarly, if the cost of variable fat deposition is less than the marginal return it will be optimal to deposit maximum levels of variable fat (energy requirements per unit of deposition are constant).

In the example problem, for a given state group three state movements, and therefore treatment activities, were defined for each genotype. One represented maximum growth in bone plus muscle with zero variable fat growth, the second represented 80% of maximum bone plus muscle growth and zero variable fat growth, while the third represented maximum growth in both bone plus muscle as well as variable fat. Over the fattening life of a pig these three alternatives enable a large number of possible weekly sequences to be selected.

For any particular set of prices, costs and genetic class probabilities that occur, it may be possible to isolate and remove dominated activities from within the generally applicable set and thus reduce the computational burden. Cases where potential dominance occurs are listed below:
(a) activities representing variable fat growth will tend to be dominated particularly where:

(i) the current payout system heavily discounts a high fat content, and

(ii) variable fat growth can be achieved at a lower cost in the periods immediately preceding sale and thus reducing maintenance requirements in the early periods. In this case activities representing fat deposition in the initial periods will be dominated.

(b) activities representing a treatment designed for a particular genotype where the probability of the state group actually being the genotype is low. There will be a critical probability below which the same treatment designed for the other genotype will clearly dominate in terms of expected net return.

(c) for any given entity starting state many treatment activities, particularly in the first period, will be irrelevant, and therefore dominated, as the state group to which they apply is empty. This will apply to states which cannot be purchased, and to states in the second and later periods which must remain empty as the starting state groups from which it is technically possible to provide such pigs are empty.

Following from (c) many constraints will be redundant for a particular entity starting state. These will be rows representing empty state groups in the first period, and which cannot be purchased, and rows in subsequent periods which represent states which cannot be given a positive quantity.
4. TIME AND NON-CERTAINTY CONSIDERATIONS

In planning to determine the optimal first period action the number of periods (in this case weeks) included in the model influences the inherent evaluation of alternative actions. While the determination of the required number of periods is considered in detail in the following chapters, at this stage it is assumed the model includes twelve weeks. This is based on the maximum time a pig can be held at the slowest growth rate without exceeding the maximum sale weight assumed possible.

Within this twelve week total planning period it is necessary, in principle, to recognise non-certainty. For any problem a decision is required on the number of states of nature to recognise and to decide whether it is necessary to construct a model which incorporates the non-certain nature of the problem in every period. In effect, it may not be necessary to incorporate the outcome distributions in each week. An over-riding concern is the computational and physical operational difficulties of solving large scale problems. This is a potential disadvantage of using linear programming in solving stochastic decision problems.

As only the first period solution is actually implemented, the criteria for deciding on how detailed the recognition of non-certainty must be is that of ensuring the subsequent periods give an adequate estimate of the effects of first period actions. Further, as the slope of the objective hyperplane largely determines the members of the solution vector it is the relative values of first period actions that are the major concern.

The three non-certainty factors requiring active recognition in the valuation of alternative initial actions are the space requirements (space feasibility problem), sale value estimation and genotype probability
revisions. This latter factor is relevant as actual outcomes lead to updated probability estimates which consequently influence the evaluation of alternative treatments. The more outcomes, or states of nature, that are recognised the greater the number of probability combinations can be recognised and therefore the more accurate will the selection of optimal treatment activities be.

In terms of obtaining correct sale value estimates the relationship between expected dressed weights and expected prices is important. In that the New Zealand payout system maintains relatively constant prices for wide ranges of dressed weights, the potential bias resulting from using the product of the expected dressed weight and expected price per kg is not great. With respect to space feasibility, an important factor is the relationship between response and variations in the space provided per animal of a given liveweight. In that inadequate data exists on such relationships an objective assessment is not possible. Further, the chance of total physical infeasibility occurring is not significant where the entity starting state is feasible.

In assessing the possibility of exceeding, or being below the recommended space allowances, the extent of liveweight variability is important. Using the growth variability figures quoted by Lucas (1968) and assuming a starting LW. of 20 kgs, a growth period of 8 weeks, and a normal distribution, the 70% range of possible ending LWs. is approximately 54.5 to 60.5 kgs where both genetic and environmental variability are included. Where initial responses are used to update genotype knowledge, the expected range for a group of pigs would be less as the figures quoted are based on all pigs sampled. Using standard space requirement figures (discussed below) the difference in requirement between the upper and lower weights of the 70% range is only 186 cm².
In view of the factors discussed, in the example problem, the number of nature states recognised was limited to two and the total number of periods was divided into blocks of four weeks with the non-
certain outcomes being recognised at the end of each block. The model therefore has the following general structure for a total of twelve weeks (the notation is of the same general form defined in Chapter V):

$$\text{find vectors } x_{j}^{i,k} \text{ which maximise}$$

\[(i) \quad z = c_{1}^{0,0} x_{1}^{0,0} + p_{1} c_{2}^{1,0} x_{2}^{1,0} + p_{2} c_{2}^{2,0} x_{2}^{2,0} + p_{2} c_{3}^{1,1} x_{3}^{1,1} + p_{2} c_{3}^{2,2} x_{3}^{2,2}$$

$$= p_{1} p_{2} c_{3}^{1,2} x_{3}^{1,2} + p_{1} p_{2} c_{3}^{2,1} x_{3}^{2,1}$$

and satisfy:

\[(ii) \quad A_{1}^{0,0} x_{1}^{0,0} \leq b_{1}^{0,0}$$

\[(iii) \quad A_{2}^{1,0} x_{2}^{1,0} \leq b_{1}^{1,0}$$

\[(iv) \quad A_{2}^{2,0} x_{2}^{2,0} \leq b_{2}^{2,0}$$

\[(v) \quad A_{3}^{1,1} x_{3}^{1,1} \leq b_{3}^{1,1}$$

\[(vi) \quad A_{3}^{1,2} x_{3}^{1,2} \leq b_{3}^{1,2}$$

\[(vii) \quad A_{3}^{2,1} x_{3}^{2,1} \leq b_{3}^{2,1}$$

\[(viii) \quad A_{3}^{2,2} x_{3}^{2,2} \leq b_{3}^{2,2}$$

and

\[(ix) \quad x_{j}^{i,k} \geq 0$$

The subscript refers to a period of four weeks' duration and the superscripts refer to the preceding periods' state of nature. A zero is used to indicate that a preceding period does not exist. Each
'A' matrix represents a multi-period non-stochastic model of $a_{ij}$ coefficients having four one-weekly periods. Similarly the vectors $x$ represent the solution vectors.

The 'A' matrices were formulated so that the minimum number of rows possible were necessary. This results in the schematic form presented in Table 7.2. In this table each named block represents submatrices in which some of the coefficients are non-zero.

**TABLE 7.2**

The Structure of a Section of the Linear Programming Model*

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>F</td>
<td>G</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Legend: The numberals represent equation groups while the letters represent activity blocks. These are described in the text.
The numerals represent the different equation groups. They are:

1. State group reconciliation equations.
2. Mix limit equations.
3. Space reconciliation equations - one for each week.
4. State group reconciliation equations for the following period assuming one of the nature states has occurred.
5. Same as 4 for for the alternative nature state.

The letters represent the different activity or coefficient blocks. All purchasing activities are combined with treatment activities and similarly selling activities except for state groups that have reached an immediately saleable weight. The different groups are:

A. Sale activities.
B. Activities representing purchase and feed actions for a four-week period. Some are simple treatment activities and some combined purchase and treatment activities. They are for states which cannot reach a saleable weight in four weeks.
C. Purchase and feed activities for the last three weeks. Purchaseable weights cannot reach a saleable weight in three weeks.
D. Same as C except for the last two weeks.
E. Same as C except for the last week.
F. Feeding for three weeks and sale activities for state groups that can reach a saleable weight in the last three weeks.
G. Same as F except for the last two weeks.
H. Same as F except for the last week.
I. Integer activities associated with the limit on the number of mixes that can be used.
J. Coefficients recording the type of mix used by each treatment activity.

K. Space requirement coefficients.

L. Coefficients recording the type of state group occurring at the end of the four-week period resulting from the first state of nature.

M. Same as L except for the second state of nature. L and M are null for $A_3$ matrices.

5. THE PARAMETERS OF THE GENOTYPES

5.1 The Outcome Distributions

Whatever the number of genotype classes that are recognised, each one represents a range of actual genotypes and so will have a particular average potential growth rate, feed requirement and so on. Further, they will exhibit an expected outcome distribution which will be dependent on the environmental variability. The problem is to determine the values of the various parameters for each genotype.

There are a number of factors which potentially determine the growth distribution of a genotype class. The mean weight increment will influence the distribution range as the greater the total growth the greater will be the effects of variations in feed quality and shed environment including disease factors. The starting state of a pig in any period may also influence the distribution parameters as pigs of a different weight can have different disease resistance and similarly different fat levels can affect the influence of climatic variability.

For use in the example problem, data presented by the National Research Council (N.R.C.) (1968) was analysed to test the importance of these factors. This analysis indicated that the standard deviation of pigs classified into two groups on the basis of their total gain for
a given period was dependent on the mean growth increment but that the starting weight was not correlated with the deviation. Thus, in deriving the genotype parameters it was assumed the standard deviations were dependent only on the mean growth increment.

The only data from which to estimate the particular parameters are observed frequency information resulting from a given feed treatment for a randomly selected group of pigs drawn from the weaners used. From this information two normal distributions (one for each genotype) must be constructed so that the predicted outcome distribution is similar to the observed frequency distribution. The parameters required are the genotype mean growths \( u_1 \) and \( u_2 \), the standard deviations \( \sigma_1 \) and \( \sigma_2 \) (this also leads to the relationship between \( \sigma_i \) and \( u_i \) when the \( \sigma_i \) and \( u_i \) are determined for a range of growth periods), and the probability of a pig being a particular genotype \( p = \text{probability of genotype 1}; \ 1-p = \text{probability of genotype 2}. \). With five unknowns, five relationships are required. This leads to the following requirements:

(i) \[ u = pu_1 + (1-p)u_2 \]

(ii) \[ s = u_1 - 1.5\sigma_1 \]

(iii) \[ t = u_2 + 1.5\sigma_2 \]

(iv) \[ \sigma_2 = p\sigma_1^2 + (1-p)\sigma_2^2 + u_1(p-p^2) + u_2(p-p^2) - 2u_1u_2(p-p^2) \]

(v) predicted distribution must not be significantly different from the observed distribution.

where

\[ u = \text{mean growth of total sample}, \]
\[ \sigma^2 = \text{variance of growth exhibited by sample}, \]
\[ s \ & t = \text{lower and upper observed range}. \]

Relationships (ii) and (iii) recognise that normal distributions are asymptotic whereas in reality, while the genotype distribution
approaches normality, it is clearly not asymptotic. In order to use relationship (v) the genotype parameters for a range of $p$ values must be estimated and then the predicted distribution tested against the observed to select the $p$ value giving a non-significant difference. A difficulty is that where two genotypes are assumed, the predicted distribution for a randomly selected pig will be bi-modal as it is a combination of two uni-modal distributions. In reality this is not the case so an adjustment procedure to recognise this was developed, the details of this are discussed in a later section. Further, relationship (iv) leads to two sets of parameter values. Use of local data indicated one set was usually illogical (e.g. gave a negative $\sigma_i$).

Data from a local boar testing station were used to quantifiably develop the model. While such data are not ideal, they were the only source of detailed weekly recorded information for a standard feed type. The environment is more closely controlled compared with a commercial operation, though this is offset in part by the greater variety of pig sources. Data were collected for several time periods so that differences in external environment and feed sources are reflected, again partially offsetting the controlled system. Further, the data were used to obtain relative rather than absolute information. This was used to adjust the growth rate and other data discussed in Chapter VI. Using the method defined gave the following information:

<table>
<thead>
<tr>
<th>Genotypic probability</th>
<th>Percent of mean</th>
<th>Value of $x$ in: $\sigma_i = x_{\text{growth}}$ (kgs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genotype 1:</td>
<td>0.4</td>
<td>82</td>
</tr>
<tr>
<td>Genotype 2:</td>
<td>0.6</td>
<td>112</td>
</tr>
</tbody>
</table>

The probabilities reflect that the observed distribution was negatively skewed. Using the observed distribution for a growth period of ten weeks (the long period enables a full expression of variability),
there was a 4% chance (using $\chi^2$) of the predicted distribution being
different from the observed. Using the body component relationships
the liveweight growth percentages lead to the following percentages of
mean growth in bone plus muscle:

Genotype 1: 80%
Genotype 2: 116%

5.2 Efficiency of Feed Use

Besides the potential growth rate differences between the genotype
classes, the other factor to be considered is the difference in efficiency
of feed use. (This must be estimated after removing the direct effects
of growth rate on conversion.) To obtain this information it is necessary
to isolate pigs from each genotype class. This can be achieved by using
historic information for an extensive growth period and taking only pigs
which exhibit a growth increment within the range $u_i \pm \sigma_i$ as this gives
a high probability that the pigs will be correctly identified. (For
the local data given above, the probability was 0.98.)

As the efficiency of feed use for growth tends to be constant
between pigs (Bichard, 1968), the differences between the classes is due
to variability in the maintenance requirements. Knowing the feed type,
intake and growth for pigs identified as belonging to one class the growth
requirements can be deducted giving the maintenance consumption. Through
estimating the sum of daily metabolic weight an estimate of the feed
requirement per unit of metabolic weight is obtained for each class and
for all pigs taken together. Using the local data the following factors
were derived:

Daily Maintenance Requirement

Genotype 1: 1.11 x mean requirement for all classes.
Genotype 2: 0.93 x mean requirement for all classes.
6. GROWTH OUTCOMES UNDER NON-CERTAINTY

6.1 Approximating a Normal Distribution

The normal distribution's asymptotic feature and its manipulative difficulty mean it is not entirely satisfactory as a genotype distribution for use in determining the outcome distribution. In choosing between the alternative distributions that could be used a major consideration was the work involved in estimating cumulative probabilities and similar calculations for use in updating genotype class probabilities and selecting outcomes to provide data for the linear programming model. Of the alternatives the triangular distribution was selected as it is easy to manipulate and it gave a good approximation of the outcome distribution. This is shown by the results of tests which are presented in a following section (6.3). The triangular distribution has three parameters, the minimum variable value \( a \), the maximum value \( c \) and the modal value \( b \). The expected value, variance, probability of a given value of the variable, and the modal probability are given by:

\[
E(x) = \frac{1}{3} (a+b+c)
\]

\[
V(x) = \frac{1}{18} [(b-a)^2 + (b-a)(c-b) + (c-b)^2]
\]

\[
p(x) = \frac{2(x-a)}{(c-a)(b-a)} \quad \text{if} \quad a < x < b
\]

or

\[
p(x) = \frac{2(x-c)}{(c-a)(b-c)} \quad \text{if} \quad b < x < c
\]

\[
p(b) = \frac{2}{(c-a)}
\]

The distribution can be positively or negatively skewed. This is a requirement as, depending on the updated genotype probabilities, the outcome distribution can take on either form.

Experimentation indicated that the best fit, assuming a mean growth increment equal to the level of growth likely in a four week period, was obtained by setting \( b \) equal to the normal distribution mean,
and \( a \) and \( c \) equal to \( u \pm 2.2\sigma \) respectively. Taking a thousand
observations from each distribution, a goodness of fit test showed,
assuming the triangular distribution set was an experimental set, that
there was a greater than 95\% chance that the two distributions were the
same. While this is not a correct use of the test, it indicates the
accuracy of the approximation method.

6.2 Estimating Outcomes and Updated Genotypic Probabilities

To estimate the possible liveweight outcomes a method of
formulating the outcome distributions based on the individual genotype
distributions and their probabilities is required. This composite
distribution can then be used to select any number of possible outcomes
and their probabilities. The updated genotypic probabilities can also
be derived from this information.

The method is based on representing the individual genotype growth
distributions with a triangular distribution derived using the system
defined above. Each distribution is weighted by its probability to
give the combined distribution. As with the normal distribution case
this distribution will be bi-modal whereas in the real multi-genotype
situation it is uni-modal. An adjustment or smoothing procedure is
therefore required.

A number of smoothing procedures were explored and tested.
The method selected together with the system for obtaining the outcomes
for two states of nature and the updated probabilities is presented in
Appendix III. The accuracy of the methods are discussed in the following
section.

6.3 Accuracy of the Method

The important consideration in testing the method is the
accuracy of the derived outcome and posterior probability estimates.
The actual distribution shape over the entire range is only important in that it leads to these estimates. Accordingly the testing method selected compares 'true' (see next sentence) observations with the predicted as a percentage accuracy \((100 \pm \frac{\text{True-Predicted}}{\text{True}} \times 100)\). As the use of two genotypes is a representation of the multi-genotype case, the 'true' situation was assumed to be represented by six genotype classes, each with a normal outcome distribution. To cover the range of possible liveweight growth distributions, three genotype probability cases (A, B & C) were considered for a growth period of four weeks. These represented both a positively and a negatively skewed distribution as well as an approximately symmetric distribution. The details of the six individual genotype distributions, the a priori probabilities on each genotype to give the cases above, and the results of the accuracy tests where two growth observations are selected are given below. The two observations were selected by dividing the distribution using the median and selecting the value from each half so that it represented the 'expected value' of the area to the right or left of the median. (Appendix III contains the details.) This procedure ensures the significance of the possible outcomes are adequately reflected.

A. Details of the Liveweight Growth Distributions for the Six Genotypes

<table>
<thead>
<tr>
<th>Genotype</th>
<th>Mean LW. Growth (kgs)</th>
<th>Standard Deviation</th>
<th>Probabilities on each Genotype to give case:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>1.36</td>
<td>.15</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>1.44</td>
<td>.35</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
<td>1.52</td>
<td>.25</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>1.6</td>
<td>.15</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>1.68</td>
<td>.08</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>1.76</td>
<td>.02</td>
</tr>
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</table>
B. Percentage Accuracy of 'The Method'*

<table>
<thead>
<tr>
<th>Observations</th>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation 1 (First nature state)</td>
<td></td>
<td>98.23</td>
<td>97.64</td>
<td>95.92</td>
</tr>
<tr>
<td>Observation 2 (Second nature state)</td>
<td></td>
<td>99.81</td>
<td>99.93</td>
<td>99.84</td>
</tr>
</tbody>
</table>

Posterior Probabilities

<table>
<thead>
<tr>
<th></th>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given Observation 1</td>
<td></td>
<td>97.92</td>
<td>96.82</td>
<td>96.24</td>
</tr>
<tr>
<td>p (genotype 1)</td>
<td></td>
<td>73.81</td>
<td>82.14</td>
<td>86.09</td>
</tr>
<tr>
<td>p (genotype 2)</td>
<td></td>
<td>99.91</td>
<td>97.35</td>
<td>75.71</td>
</tr>
<tr>
<td>Given Observation 2</td>
<td></td>
<td>99.88</td>
<td>98.57</td>
<td>93.47</td>
</tr>
<tr>
<td>p (genotype 1)</td>
<td></td>
<td>99.29</td>
<td>99.15</td>
<td>98.06</td>
</tr>
<tr>
<td>p (genotype 2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E (liveweight growth)</td>
<td></td>
<td>99.29</td>
<td>99.15</td>
<td>98.06</td>
</tr>
</tbody>
</table>

* Details of the calculational steps are given in Appendix III.

The average accuracy for the observations is 98.56% while for the posterior probabilities on the genotypes it is 91.49%. This latter lower level is due to the second genotype probabilities for observation one. These probabilities are relatively small as given observation one, the less favourable outcome, the change of the pig being the more efficient genotype is less than for observation two. Overall, the level of accuracy is acceptable.

While the outcome estimates are primarily required for predicting the entity starting state for the start of each future four week period, they are also required for estimating the sale value of any group. For activities representing the sale of a group at the end of a week not coinciding with a 4-week period, estimates must be made of the outcome distribution which, together with the grading system, provides the expected value. Similarly, at the end of the total twelve week period it is
necessary to value all pigs on hand and this requires an outcome distribution for each group.

7. **STATE VARIABLE VALUES AND SPACE REQUIREMENTS**

The relevant state variables are the weights of bone plus muscle and variable fat as well as the genotypic probabilities. Any set of values for these three variables defines a group of pigs that will respond similarly. There is clearly an infinite range of possible combinations so it is necessary to isolate the relevant total ranges and the groupings in order to have a feasible number of possible combinations.

For the example problem, bone plus muscle levels were represented using one, and one and a half, kilogram ranges. These intervals were based on minimum growth rates possible for the defined activities. For each bone plus muscle level four combinations of variable fat and genotypic probabilities were defined. It was only necessary to have two levels each of variable fat and genotypic probabilities as in any particular sub-matrix in a future period and for any particular bone plus muscle level, the possible variable fat and probability levels are limited to a sub-set of the total set of possible levels. These levels vary with the particular situation. The particular state groups which the state constraints represent therefore vary with each four weekly sub-matrix in the total model.

The appropriate genotypic probability groups in the first period depend on the state of nature occurring and actually observed in the previous period. Similarly, the appropriate ranges defined for future periods depend on the sequence of nature states preceding the particular period. Thus, for example, if the previous two four-weekly periods are both the first state of nature, the appropriate probability groups to
cover the possible values should be different compared with a sequence of second state of nature outcomes. The full range of possibilities are therefore not required in any one sequence. Similarly, possible variable fat levels depend on the bone plus muscle levels so that for any one weight it is unnecessary to allow for the full range. Further, classification of the pens relies on visual and external physical observations\(^2\) so confidence limits do not warrant defining many variable fat groups.

The only constraints not yet defined are the space constraints. For the planning system demonstrations the space requirements were based on recommendations given by Marten (1971). Over the liveweights of interests these are:

\[
Y = 0.3098 + 0.0034x
\]

where

\[
Y = \text{space requirement in } m^2,
\]

\[
x = \text{liveweight (kgs) at start of weekly period}.
\]

8. **USING THE L.P. MODEL**

In using the model to explore various planning systems it is necessary to simulate possible outcomes at the end of the first period as it is assumed that the solution vector for the period is implemented. The selected outcomes become the updated entity starting states for the continuous planning process.

In simulating the planning process over a number of periods it is implied that continuous updating of price and cost expectations is carried out and the results are used in the continuous planning system each period. The difficulties associated with detailed predictions may

\(^2\) In some units ultrasonic equipment may be available thus increasing the accuracy of type casting.
mean that no more than estimates of proportionate increases or decreases are warranted. For the first period of the model at each planning point it is possible to make accurate forecasts as the prices may already be known with certainty.

The computational burden involved in running the model means it is important to utilise any procedure with reduces the costs involved. The use of providing subjectively estimated optimal solutions as a starting basis is an example. With continuous planning, previous solutions provide reference points for this procedure. Similarly, for any entity starting state many constraints are redundant and activities irrelevant so that if the model, or a modification of it, should prove to be commercially viable, it would be important to develop matrix generator programmes which construct the relevant matrix for the particular case so that row numbers are minimised. Such systems also enable price and cost changes to be simply included with a minimum of hand manipulation required.

The model is still incomplete with respect to the planning horizon problem. The model developed has been based on an example twelve-week period. This may be excessive or insufficient, though the time period necessary will vary with the starting state and price conditions. The following chapters contain an exploration of this problem.
CHAPTER VIII

THE CONCEPT OF A PLANNING HORIZON

1. INTRODUCTION

To determine an optimal first period decision for implementation in a continuous planning system it is necessary to include several periods in the planning model used. The minimum number to ensure that the first period decisions are optimal is defined as 'the planning horizon'. This horizon must be known every time planning occurs as it will tend to change as conditions change.

It is only in recent years that any real recognition has been given to the question of a planning horizon in theories of a firm. This has stemmed from the non-acceptance in the formal theories of the dynamic (both in a non-certain and time sense) nature of the planning and implementation situation. Even currently it is only in specialised cases (Hillier & Lieberman, 1967) that the question of a planning horizon is discussed in detail. Usually modern texts make (Naylor & Vernon, 1969; Baumol, 1972) only passing reference to the problem indicating the need for a general theory in this area.

This Chapter contains a discussion of the various planning horizon concepts that have been proposed in order to give perspective and a proof that a planning horizon, as generally defined here, can exist. Further, as prediction of the planning horizon is essential to the proposed system some thought is given to the factors that may influence the length of a planning horizon. Finally, a review is given of past attempts at determining a planning horizon.
2. PLANNING HORIZON CONCEPTS

2.1 The Total Horizon

Any planning horizon concept must be contained within the bounds of what can be termed the total horizon of the firm. This is defined as the total future time over which the firm is expected to be maintained as a producing unit in one form or another. Commonly the total horizon will be either an extensive period, which can be regarded as infinity, or of a defined length. Most public company organisations tend to fall in the first category whereas single proprietor firms may fall in the second category where asset transference to offspring is not possible. For both cases there will be non-certainty attached to the total horizon.

Assuming the total horizon is greater than the planning horizon, two basic concepts of a planning horizon can be defined. One relates to a positive and the other a normative approach. For continuous planning these two approaches need to be reconciled.

2.2 The Normative Approach

The normative planning horizon is defined as the minimum number of periods that need to be included in the planning model in order to solve for the optimal first period decision set. It is assumed that the entrepreneur is rational with respect to a defined objective function and that parameter values used in planning are the best available. This definition does not assume that the second, and later period decisions obtained from the model are optimal. In a practical situation it may be necessary to use a planning horizon giving apparent optimality of the decisions for several of the initial periods as a means of reducing planning costs. In this case continuous planning becomes semi-continuous planning.
Modigliani and Cohen (1961) discuss in some detail the conceptualisation of a normative planning horizon. They note that planning must consider future periods, as first period decisions may affect the opportunities open to a firm in later periods. Accordingly definitions of relevant and irrelevant parameters are made which then lead to defining a planning horizon. Their sufficient conditions for a parameter to be irrelevant are:

"A parameter \( p \) of a given future constraint is conditionally irrelevant within some stated range if and only if the optimal value of every component of the first move (first period decisions) is unchanged, no matter what value \( p \) might take in the stated range; it is unconditionally irrelevant if the state range includes all \textit{a priori} admissible values of \( p \)."

Conversely, a relevant parameter is one which does effect the first period decisions. While the definition is in a constraint context, Modigliani and Cohen also discuss relevance with respect to the objective function. Essentially their second theorem states that where the total payoff can be expressed in terms of two sub-outcomes and the first largely depends on first period decisions and the second only on later decisions, then decisions relating to the second pay-off component can be ignored provided decisions relating to the first component do not effect opportunities of obtaining the second component. Thus, a separable objective function is a pre-requisite.

Besides defining conceptual relevance they also introduce practical irrelevance. A parameter is said to be practically irrelevant where either implementation inaccuracies mean the parameter is not significant, or where the additional gains occurring from allowing
for the parameter are small, or where the solving costs associated with its inclusion do not warrant its inclusion.

Noting that "plans are not decisions about future courses of action", the ideas on irrelevance lead Modigliani and Cohen to define a planning horizon through stating "the latest time period for which plans are made can be called the relevant planning horizon". Effectively they are saying that decisions must be made regarding which parameters are relevant so that the nearest future period in which all parameters are conditionally irrelevant can be isolated. The time up to and including the previous time period then becomes the relevant planning horizon.

In terms of an operational system for determining the planning horizon, the ideas of relevancy provide only broad principles and are only capable of direct use in a limited number of particular cases. Such cases occur where resources available for productive uses at some future period are in no way affected by any possible actions during the preceding periods or, through initial simple exploration, it is clear that the possible optimal decision set will not be influenced by previous actions. These cases seldom occur. Cases where the objective function is separable are common but the problem of later period physical actions being effected by first period actions means this separability cannot be exploited. In pig production, first period decisions affect future opportunities through determining fattening shed space availability and the number and type of pigs on hand (and possibly the types of ingredients that can be used where there are constraints on the degree of mix flexibility). While actions in one period do not influence the form of objective function components in other periods, these physical relationships mean irrelevance cannot be determined simply. Thus, while Modigliani and Cohen's principles provide a general conceptualisation of the problem, particularly the idea of conditional irrelevancy, other methods must be
determined for solving the planning horizon problem.

2.3 **The Positive Approach**

The positive planning horizon is defined as the period of time over which an entrepreneur is prepared to plan. It is largely determined on the basis of whether it is considered that useful estimates of future conditions can be made. This approach can embody the use of subjective probabilities in quantifying non-certainty.

The normative horizon ideas of first period decision optimality do not enter the concept. The positive horizon is the time period over which an entrepreneur actually makes plans and this may largely depend on intuitive decisions. Shackle (1961) has attempted to rationalise why entrepreneurs appear to have a distinct planning horizon. He suggests they estimate what they regard as being the maximum loss possible from a system. As future periods get 'less certain', this will increase. The estimate of maximum loss is supposed to be based on what the decision maker would be 'very surprised' could occur in any eventuality (called the 'Focus of Loss'). Once this maximum possible loss, which increases with time, reaches a level equal to the maximum loss the decision maker is prepared to accept, the horizon is cut off for planning purposes. Similar concepts are defined for minimum gain situations. While the ideas were developed specifically for capital investment situations they clearly have possible implications to all forms of forward planning (Haring, 1971). Subjectively estimated distributions may be given a greater range with time so that maximum losses are exceeded and minimum gain requirements not met.¹

The existence of a positive horizon is a reaction to non-certainty. This is stressed by Svennilson (1938) and Naylor and Vernon (1969). Svennilson uses a slightly different approach to Shackle by suggesting

¹ The 'focus-loss' concept has also been applied to static planning. An example is given by Boussard and Petit (1967).
that at some future time the decision maker loses confidence in his own ability to anticipate outcomes and so cuts off the horizon. Naylor and Vernon make the point that alternative investments may have different 'degrees of non-certainty' and so in estimating their worth different horizons are used for each. Similarly, Brownlee and Gainer (1949) noted that farmers had greater confidence in their ability to predict technical outcomes rather than price outcomes so that plans were based on technological considerations. In these cases the period over which plans are made could well be that necessary, in the farmers opinion, to ensure the technical success of the operation.

2.4 A Reconciliation

Neither of the two planning horizon concepts per se lead to a conclusion on the number of periods to consider. Assuming a farmer has rationally formed his positive planning horizon, the planning period to use should be the horizon that contains the smallest number of periods provided this is less than the total horizon. The reason is that the farmer bears the responsibility of decisions implemented so that he must act on the basis of his subjective estimates. However, the estimates of future conditions used by the farmer may have been formulated on the basis of inadequate information and of a poor understanding of the problem. An adviser must ensure the farmer is fully informed so that, following discussion, the farmer may alter his positive horizon. \(^2\)

Similarly, the normative horizon may have been incorrectly formed due to a mis-specification of the farmer's objective function. It must also be recognised that the best estimates of future conditions are largely based on historic information. This means it is not possible to prove that the normative horizon based on these estimates is the correct planning horizon since it is seldom possible to prove that the relationships used in deriving the estimates will occur in the future.

---

\(^2\) Winkler (1968) discusses methods of obtaining consensus subjective estimates.
Assuming that the farmer is fully informed, the normative planning horizon can be re-defined as the number of periods it is necessary to include in the planning model to ensure first period optimality where the farmer's estimates of future conditions are used to derive the horizon. This implies that if the farmer's positive horizon involves a smaller number of periods than the previously defined normative horizon, then the derived horizon will be the positive horizon as beyond this period the farmer is not prepared to estimate prices and costs so that these periods become irrelevant in the planning model.

Given this reconciliation the concept of an optimal planning horizon can be introduced. If the first period decisions are estimated using a model containing less periods than the planning horizon, then sub-optimal actions will occur. If more periods than are necessary are used then the planning costs may exceed the minimum necessary.

Finally, the optimal planning horizon is a dynamic concept. Due to changing conditions the length of the horizon may change from period to period in a continuous planning situation.

3. THE EXISTENCE OF A PLANNING HORIZON
3.1 Introduction

In order to develop a method of determining the planning horizon it is necessary to objectively define the general situations under which a planning horizon can exist and can be isolated. It is assumed subjective estimates of all information required are used in both the certainty and non-certainty situations. Accordingly, where a decision maker is not prepared to make any estimates of future conditions beyond a certain time it will be clear where the maximum potential planning horizon lies.
3.2 The Certainty Case

Strictly, in this case the determination of a planning horizon is not necessary. Given the unlikely occurrence of a complete certainty situation, only one planning operation is required if the total horizon can be included in the model. However, it is useful to consider this case to derive the non-certainty conditions.

A convenient method of representing the general planning problem is through the use of dynamic programming. For exploring the problem define the following terms:

\[ S_i^j = \text{the } i^{th} \text{ possible state the business entity can take on at the end of the } j^{th} \text{ period; } i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \]

\[ h_i^j = \text{the within period return resulting from a decision which gives a state movement from states } s_h^{j-1} (h = 1, 2, \ldots, m) \text{ at the beginning of period } j \text{ to state } S_i^j \text{ at the end of period } j. \]

\[ x_i^j = \text{the total return from following an optimal policy from the end of period } j \text{ to the end of the total period given the state of the system at the end of period } j \text{ is state } S_i^j \text{ (Note that } x_i^j = 0 \text{ for } j > \text{ positive horizon).} \]

\[ b_i^j = \text{the total return from following an optimal policy from the current time and starting state to the end of period } j \text{ given the state at the end of period } j \text{ is state } S_i^j. \]

Thus, given the current state \( (h = s) \), the optimal first period decision is the decision giving the state movement which maximises:

\[ \max_i [s_i^1 + x_i^1] \]
However, this calculation requires a knowledge of $x^1_i$ and this involves analysing all periods within the total horizon. For a planning horizon less than the total horizon to exist it must be possible to arrive at the same first period decision without calculating $x^1_j$. Consider the conditions which will give rise to such a condition. If for some $j < n$;

(i) $b^j_f \geq b^j_i$ all $i$ except $i = f$ (if one of 1, 2, ..., $m$)

and

(ii) $b^j_f + \max \left[ f, j \right] + x^j_i \geq \max \left[ b^j_f + h^j_i + x^j_i \right] \left( \text{all } i, h \right)$

then, a myopic search of considering only the first $j-1$ periods will provide the optimal decision set for each of the $j-1$ periods. These conditions are sufficient conditions for a planning horizon less than the total horizon to exist. But, to determine whether these conditions exist requires a knowledge of $x^j_i$ all $i$ and $j$.

Assume $s^1_g$ is the optimal first period ending state. For some period $d < n$, if a myopic search gives $b^d_k > b^d_i$ and for $b^d_k$ the first period optimal ending state is $s^1_g$, then the horizon up to and including period $d$ gives the planning horizon. But, again, a myopic search involving only $d$ periods does not indicate whether $s^1_g$ will in fact be the optimal first period ending state. However, this leads to a possible method of satisfying necessary and sufficient conditions. If a method of determining the possible ranges of the $x^j_i$ that does not require consideration of the total horizon can be found, then necessary and sufficient conditions for the planning horizon are:

---

3 A myopic search is a search involving a limited number of sequential periods from within the total horizon.
If for some period \( d < n \), where \( x_i^d(y) \), \([y = 1, 2, \ldots, z,]\) represents the possible \( x_i^d \) values, \( z < \infty \), then if optimal \( s_i^1 \) has the same \( i \) value in
\[
\max_{i, h} \left[ b_h^{d-1} + h_i^d + x_i^d(y) \right]
\]
for all \( y \) values, then \( d \) provides a planning horizon.

The planning horizon may involve less periods than \( d \). The planning horizon can therefore be defined as the smallest \( j \) for which the necessary and sufficient conditions hold. In some cases \( j \) may equal \( n \), particularly where \( n \) is small. Using a greater number of periods than \( d \) still provides the optimal first period decision.

Modigliani and Cohen's (1961) concept of parameter irrelevancy can be objectively interpreted using the definition of the existence of a planning horizon. For all parameters, or what are effectively the decision variables, to be unconditionally irrelevant in some future period \( d \), then optimal \( s_i^d \) must be the same in both optimal decision set paths given by:

\[
(i) \quad \max_{i, h} \left[ b_h^{d-1} + h_i^d + x_i^d \right]
\]
and

\[
(ii) \quad \max_{i, h, t} \left[ b_h^{d-1} + h_i^d(t) + x_i^d(t) \right]
\]

where
\( h_i^d(t) \) and \( x_i^d(t) \), \( t = 1, 2, \ldots \), represent all the a priori permissible values of these variables.

The most likely situation for unconditional irrelevance to exist is where, a priori, it is known that a particular \( s_i^{d-1} \) is optimal under all future conditions. For example, in an independent feed
inventory problem where there is a single state variable of feed on hand, if known feed and storage costs are such that a positive inventory should not be held at some point in time, then it can be predicted that the planning horizon will be no greater than this period.

For all decision variables to be conditionally irrelevant in some future period $d$ then optimal $S_i^d$ must be the same in both optimal decision set paths given by:

\[(i) \max_{i,h} \{ b_{i}^{d-1} + h_{i}^{d} + x_{i}^{d} \} \]

\[\text{and}\]

\[(ii) \max_{i,h,t} \{ b_{i}^{d-1} + h_{i}^{d} (u) + x_{i}^{d} (u) \} \]

where $h_{i}^{d} (u)$ and $x_{i}^{d} (u)$, $u = 1, 2, \ldots$, represents possible sets of these variables derived from a search of periods which does not involve a complete evaluation of the total horizon. The more limited the search can be the more efficient will be the system. The set of $r(u)$ and $X(u)$ will contain less elements than the set of $r(t)$ and $X(t)$.

3.3 The Non-Certainty Case

When non-certainty must be accounted for, the conditions necessary for a planning horizon to exist are similar to the certainty case. The adjustments involve using expected payoffs and recognising that a particular set of decisions through time will not give a particular state at the end some period with certainty. Thus, the optimal first period decision cannot be defined by giving the optimal $S_i^1$. Accordingly, the sufficient conditions for the existence of a planning horizon are:

Let $d_j^h e_i$ = the $e$th possible decision (set) in period $j$ given a period starting state of $S_i^{j-1} h$, $e = 1, 2, \ldots$.

$h e_i = S_i^j$ occurring given $d_j^h e_i$. 

\( c_i \) = the expected total utility of following an optimal strategy from the start of period 2 to end of period \( d \) assuming the starting state is \( S^1_i \).

\( b^j_i \) = the expected total utility of following an optimal strategy from period 1 to the end of period \( j \) assuming an ending state of \( S^j_i \). (Calculated by weighting the total \( E(u) \) from each optimal strategy by the chance of \( S^j_i \) occurring for each optimal policy.)

\( r(h^d_{i,e}) \) = the expected within period utility from taking action \( h^d_{i,e} \).

Then, if a period \( d \) exists such that optimal \( h^d_{i,e} \) is the same for the maximum of the following statements, then \( d \) forms a planning horizon:

\[
(i) \quad \max_{e} \left[ r(h^1_{i,e}) + \sum_i (C_i)(p^1_{h^1_{i,e},i}) \right]
\]

\[
(ii) \quad \max_{\text{all } h,e} \left[ b^{d-1}_{h,e} + r(h^d_{i,e}) + \sum_i (X^d_i)(p^d_{h^d_{i,e},i}) \right]
\]

While this is a sufficient condition it may not be necessary. Provided, for some period \( d \), \( d^d_{i,e} \) is constant for all \( X^d_i \) sets within a bounded range, the bounds being determined from a limited search of the total horizon, the necessary condition exists.

If the future sequence of nature states could be predicted the case reduces to the certainty situation. For each possible sequence there will be a planning horizon which may involve a different number of periods in each case. Each sequence has a given a priori probability. Thus, the planning horizon is a random variable. The greatest number of periods, however, determines the planning horizon for planning purposes. Conceptually, after allowing for planning costs, the optimal planning horizon may be less than the greatest number of periods as the marginal
gain from considering this number of periods may not exceed the marginal increase in planning costs.

4. FACTORS AFFECTING THE PLANNING HORIZON LENGTH

4.1 Principles

In searching for the planning horizon it is important to understand the potential influence of various factors on the length of the planning horizon. In most cases it is not possible, a priori, to state with certainty where the planning horizon will fall so that factor variations can only be used to indicate a trend. As factors change, the $X^j_i$ may change thus possibly varying the planning horizon. If the set of possible $X^j_i$ is reduced, particularly the ranges, the tendency is for the planning horizon to be shortened as the differences between alternative $X^1_i$ are reduced. Using this principle the major components of any decision problem can be assessed. Appendix IV contains a discussion on such trends. The conclusion is that the higher the rate of time preference, the greater the risk aversion, the more restrictive the starting state and other constraints, the smaller the number of production and investment opportunities, then the tendency will be for the planning horizon to be closer to the present compared with a lower rate of time preference, risk preference and so on, as these changes tend to reduce the ranges on the possible $X^j_i$.

4.2 The Effect of Non-Certainty Again

An increased range and variance of important variables tends to lengthen the planning horizon as favourable outcomes lead to new opportunities which would otherwise not be possible. A popular conception however, is that non-certainty shortens the horizon.4 These are different effects. The horizon lengthening effect of increased variances assumes

4 See, for example, Graaf (1957), p. 95.
the farmer is prepared to make estimates and act on them whereas if sufficient doubt exists a positive horizon may be created by the producer and therefore effectively places an upper limit on the potential horizon. (Where uncertainty exists as to the time period at which truncation should occur and subjective probabilities are placed on this uncertainty, these estimates can be incorporated into a quantitative analysis. White (1965) outlines such a method where it is assumed planning takes into account all periods up to the non-certain ending period.)

These aspects raise the general question of whether in some cases available data and forecasting methods for some future period can be sufficiently accurate to warrant their use in planning in a strictly normative situation. Consider, for example, a price forecasting model with independent variables which are, \textit{ex ante}, random variables. While the information available may give a distribution for each independent variable, the confidence limits on these may be wide, effectively giving a range of price distributions each with its own chance of occurrence. Further, the potential of unforeseen factors being introduced by governments and others must be recognised but can only be estimated using historic information and this, it would seem, may not repeat itself. Effectively, the probabilities on each possible price distribution must be regarded as being 'random' variables. This is where 'degrees of belief' or subjective probabilities must enter the system. Ignoring the knowledge that the information obtained from past data may be invalid, the confidence limits on distribution probabilities may be so wide and their distribution of a shape that the weighted price distribution has a very high variance and exhibits no modal tendencies. This extreme may lead to alternative actions having nearly equal expected payoffs which indicate that under some circumstances an arbitrary choice will provide the same results as an objective analysis.
It is possible that cases exist in which some alternatives dominate all others and that a simple analysis clearly shows which are the dominating activities. If forecasting accuracy is such that these activities have similar expected payoffs then planning must concentrate on technical efficiency attainment so that detailed economic planning models become irrelevant.

5. METHODS USED IN DETERMINING A PLANNING HORIZON

5.1 Introduction

Modigliani and Cohen's (1961) concepts do not provide specific myopic search techniques enabling the planning horizon to be isolated. However, a number of workers have developed specialised myopic search techniques for a variety of cases. Essentially the requirement is for techniques giving the $X_i^j$, or at least narrow bounds on the $X_i^j$, without the need to consider the total horizon. In some cases the $X_i^j$ may be such that one dominates all others and this may be discernable without actually calculating the $X_i^j$. The purpose of this section is to consider the various cases for which solutions are obtainable as they may have implications to the general case. Also, there will be problems in which justifiable simplifications mean the use of methods that are conceptually incorrect will be warranted. In reviewing the cases it is useful to categorise the types of planning problems and to consider each in turn. In all cases it is assumed the total horizon is known and that the total horizon for planning purposes is the shorter of the total horizon or the positive horizon. Further, it is assumed there is at least one resource that is not disposable which effectively means the business is to continue.

Cases are classified according to two factors, the first being, whether certainty exists, or is assumed to exist, with respect to all future prices, costs and technology. Any case in which at least one
variable is non-certain is classified as a non-certainty case provided use of expected values does not effectively reduce it to a certainty case. The second factor relates to whether conditions in each future period are changing relative to previous periods. If prices, costs and technology, whether the certain values or the density functions, change from period to period the case is called the non-stationary case and vice-versa.

Within some combinations of these two factors it is also necessary to introduce two sub-classifications. The first relates to whether the maximum time span of any investments possible enables particular investments to be repeated within the effective total horizon. The possibility of repeatability depends on the type of investments possible where an investment in this context refers to the purchase of any resource no matter what its expected maximum life is. Thus, the purchase of a weaner is classified as an investment. Repeatability is also related to the concept of a stable policy. A stable policy is defined as a set of decisions which are sequentially implemented in identical form. The length of period to which the set of decisions apply can vary and effectively can involve any number of sub-periods. Thus, for repeatability the time span of any investments must be short enough to enable the stable policy to be applied several times within the total horizon. If repeatability is not possible a stable policy cannot exist. Further, as one year is frequently used as the accounting period, in assessing investments with a time span greater than a year the form of the consumption function is important, particularly with respect to time preference (the particular objective function used in any case implies a specific consumption function). The second relates to whether it is physically possible to instantaneously adjust the system to any other state within the total set of possible states. Whether a non-constant cost
or return is associated with the change is also important.

5.2 The Case of Certainty and Stationarity

If repeatability exists and a large number of repetitions are possible it is not necessary to consider the total horizon since a simple myopic search system is possible (e.g. at the start of the growing season on a cropping farm where previous crops do not influence future responses). The problem is to find the optimal stable policy and then to immediately implement it. Determining the optimal policy involves the use, for example, of a static linear programming model where the length of the production systems are fixed. In a variable time system (the replacement time problem referred to in Chapter III is an example) the optimal policy consists of repetitions (replacement) at the time maximising net present worth.

If time and cost is associated with changing from the current state to the starting state of the optimal stable policy then initial period decisions may not be the stable policy decisions. Thus it is necessary to consider a number of periods. The problem consists of determining the optimal stable policy and then, given the current state, setting up a model with a sufficient number of periods such that the decision set of the last period is the same as the stable policy. To test for this condition it is clearly necessary to pre-determine the stable policy. Effectively, the stable policy provides $x^*$ values which give rise to resource valuations. Once the resource valuations obtained for the last period of the planning model are the same as those in the optimal policy sufficient periods have been considered. Grinold (1971) rigorously develops a proof of this system based largely on a linear model.

Where repeatability is not possible the concept of an optimal
stable policy cannot be used. Included in this case are problems where repeatability can occur but the number of repetitions are insufficient for the optimal stable policy in a long total horizon to be, in fact, optimal. This will occur where the starting state is sufficiently different from the starting state of the stable policy to make it sub-optimal to use the policy.

Attempts to find a myopic search system in this case have rested on the turnpike idea. Tsukui (1966) explains the turnpike concept in the following statement:

"... suppose a balance growth path (the turnpike) of the stock of goods is uniquely determined in a closed reproduction system. Then the efficient time-path of stocks, starting from any given common initial stocks and attaining in the terminal period (N) a Pareto-optimum in the possible set of stocks, will have the following properties:

(a) If N is sufficiently large, all efficient paths of stocks stay outside of a properly selected neighbourhood of the turnpike for at most a certain period $N_o$ determined independently of N.

(b) All efficient paths of stocks remain consecutively in the neighbourhood of the turnpike except for certain period at the start and the termination."

Radner (1961) was one of the first workers to prove the existence of a turnpike. With respect to the planning horizon problem, the turnpike concept says that, in a growth situation, no matter what the starting state and the desired ending state, the optimal policy consists of making for the turnpike and staying on this until somewhere near the total horizon. Thus, provided a sufficient number of periods are considered to ensure that the system has reached the turnpike, the optimal first period decision will have been determined.
Boussard (1971) has applied the concept to farm planning. The assumptions necessary, however, are very limiting. Boussard assumes there are no constraints other than the initial cash supply and that a linear consumption function exists. This means a given proportion of each period's income is invested in capital stocks giving a growth situation. Using this simple function the objective is to maximise the total value of assets held at the end of the total horizon since if terminal assets are maximised so will consumption in each period. (In that the only capital good used was land, problems of depreciation are not considered.) To overcome the problem of determining the correct valuations, Boussard used the turnpike concept as if the total horizon is long enough it is not necessary to actually know the optimal final state. The problem is to ensure that a sufficient number of periods are included in the model (a linear programming formulation) so that it can be guaranteed that the turnpike has been reached. However, the number of periods necessary may give matrix size problems so Boussard notes that provided the correct $X^j_1$ values are placed on the possible ending states then the model can be truncated. His procedure was, therefore, to set up the model for a limited number of periods and to solve for the optimal solution with the land held at the end of the last period valued at subjectively estimated maximum and minimum values. If the first period solution was the same for both solutions it was assumed the first period action must be the optimal first move in approaching the turnpike. In that the linear consumption function, non-constrained case is seldom encountered the approach has little practical use. Further, it relies on subjective valuations so it cannot be proved that the first period decision is the optimal first period move to attain the turnpike.

Boussard attempts to introduce non-certainty through using Shackle's (1961) 'focus of loss' concept. In that some of the
alternative within period actions may not satisfy the minimum loss requirement, he notes that the action set is reduced and therefore it may take longer to reach the turnpike. The conclusion is that the planning horizon will be shorter as with fewer alternative actions less periods will be needed in the model to give the same first period decision for the maximum and minimum valuations. However, cases where the only effect of non-certainty is to reduce the number of possible actions are limited.

5.3 The Case of Certainty and Non-Stationarity

Given the expectation that conditions will continually change the concept of a stable policy cannot be used in a myopic search technique. In this case a common approach is to solve the problem with a range of planning horizons and, if the first period solution is constant, to accept this as being optimal. The work of Rae (1970) and Byrne & Healy (1969) are two examples of this approach. (This approach can be used in any of the cases.) Use of the approach cannot, however, guarantee an optimal first period decision as there is no proof that adding additional periods will not, at some stage, lead to a change in the first period decision.

The only objective myopic search techniques developed involve the general inventory planning problem. While this only involves one product it will be discussed as it is an example of how specific conditions can lead to an easily identified planning horizon using a limited search. It is also the problem for which the first objective planning horizon rules were developed (Modigliani & Hohn, 1955). Of the work in this area, that of Charnes et al. (1966) is used as it also introduced a non-certainty case and is therefore referred to later.

For the deterministic case Charnes et al. consider the problem of determining the optimal quantity \( X_j \) of a good to purchase and sell
(Y,

) in time period j given a warehouse of known capacity (B) and


known future prices and costs (p,


and c,


independent of quantity)


over a total horizon of N periods. The initial inventory is designated


h,


N


. Where \( b \) is the per unit value of any inventory on hand at the


total horizon, the problem is to find \( X \) and \( Y \) values which maximise.


\[
Z = \sum_{j=1}^{N} (p_j Y_j - c_j X_j) + \beta_N \left[ h_0 + \sum_{j=1}^{N} (X_j - Y_j) \right]
\]

subject to:


(i) \( Y_j \leq h_0 + \sum_{i=1}^{j-1} (X_i - Y_i) \)


(ii) \( h_0 + \sum_{i=1}^{j} (X_i - Y_i) \leq B \) for \( j = 1, 2, \ldots, N \)


(iii) \( X_j, Y_j \geq 0 \)


Thus, sales occur from inventory and must not exceed inventory at the


end of the previous period and inventory at the end of a period must


not exceed warehouse capacity. If accumulated return \( (f_j) \) and period


inventory \( (h_j) \) are defined as:


\[
f_j = \sum_{i=1}^{j} (p_i Y_i - c_i X_i) = f_{j-1} + p_j Y_j - c_j X_j
\]


\[
h_j = h_0 + \sum_{i=1}^{j} (X_i - Y_i) = h_{j-1} + X_j - Y_j
\]


then the problem is to:


maximise \( f_N + \beta_N h_N \)

subject to


(i) \( Y_j \leq h_{j-1} \)


(ii) \( h_j \leq B \) \( j = 1, 2, \ldots, N \)


(iii) \( X_j, Y_j \geq 0 \)
Considering the last period, the problem is to find $X_N$ and $Y_N$ which:

$$\text{Max.} \quad f_N + \beta_N h_N = f_{N-1} + P_N Y_N - C_N X_N + \beta_N (h_{N-1} + X_N - Y_N)$$

subject to:

(i) $X_N, Y_N \geq 0$

(ii) $Y_N \leq h_{N-1}$

(iii) $X_N - Y_N \leq B - h_{N-1}$

Due to the linear price, cost and terminal value functions, it is profitable to go to the extremes, depending on the conditions, of buying and selling that the inventory level and the warehouse capacity allow.

For example, if $P_N < C_N < \beta_N$ it is not profitable to sell any inventory on hand and purchase to build up the terminal inventory. Thus, given these conditions, the optimal decision is to set $Y_N = 0$ and to purchase sufficient to make up the inventory to $B$. Similarly, the possible relationships between $P_N, C_N$ and $\beta_N$ lead to distinct decision rules.

Charnes et al. summarize these in a table which is given below:

<table>
<thead>
<tr>
<th>Event</th>
<th>Optimal Decision</th>
<th>Terminal Assets</th>
<th>Value of Terminal Assets</th>
<th>$f_N + \beta_N h_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_N &lt; P_N, C_N$</td>
<td>$X_N = 0, Y_N = h_{N-1}$</td>
<td>$h_{N-1}$</td>
<td>$f_{N-1} + P_N h_{N-1}$</td>
<td>$f_{N-1} + \beta_N h_{N-1}$</td>
</tr>
<tr>
<td>$C_N &lt; B, P_N$</td>
<td>$X_N = B, Y_N = h_{N-1}$</td>
<td>$B$</td>
<td>$f_{N-1} + P_N h_{N-1} - C_N B$</td>
<td>$f_{N-1} + \beta_N h_{N-1}$ + $B (\beta_N - C_N)$</td>
</tr>
<tr>
<td>$P_N &lt; C_N &lt; \beta_N$</td>
<td>$X_N = B - h_{N-1}, Y_N = 0$</td>
<td>$B$</td>
<td>$f_{N-1} - C_N (B - h_{N-1})$</td>
<td>$f_{N-1} + \beta_N h_{N-1}$ + $B (\beta_N - C_N)$</td>
</tr>
<tr>
<td>$P_N &lt; \beta_N, C_N$</td>
<td>$X_N = 0, Y_N = 0$</td>
<td>$h_{N-1}$</td>
<td>$f_{N-1}$</td>
<td>$f_{N-1} + \beta_N h_{N-1}$</td>
</tr>
</tbody>
</table>


It will be noted that in all four cases the value \( f_N + \beta_N h_N \) is a linear function of initial assets \((f_{N-1}, h_{N-1})\) and of \( B \). Thus, using \( \beta_{N-1} \) and \( q_{N-1} \) as the coefficients of \( h_{N-1} \) and \( B \) respectively, the following equality holds:

\[
f_N + \beta_N h_N = f_{N-1} + \beta_{N-1} h_{N-1} + q_{N-1} B
\]

The table indicates that \( \beta_{N-1} \) and \( q_{N-1} \) will have specific values depending on the price, cost and value relationships. Effectively, \( \beta_{N-1} \) is the implicit value of a unit of inventory carried into period \( N \) and \( q_{N-1} \) is the 'evaluator' of a unit of warehouse capacity. Examination of the table indicates

\[
\beta_{N-1} = \max \left[ P_N, \min \left( C_N, \beta_N \right) \right]
\]

Furthermore, this relationship holds recursively for \( N-2, N-1, \ldots, j, \ldots, 2 \). Noting that \( q_{N-1} B \) is constant with respect to possible \( X_N \) and \( Y_N \) values, the optimal second to last period decision is given by:

\[
\max_{X_{N-1}, Y_{N-1}} (f_{N-1} + \beta_{N-1} h_{N-1}) = f_{N-2} + P_{N-1} Y_{N-1} - C_{N-1} X_{N-1} + \beta_{N-1} (h_{N-2} + X_{N-1} - Y_{N-1})
\]

Thus, the table can be used to give the optimal decision provided \( N \) is replaced by \( N-1 \). Further:

\[
f_{N-1} + \beta_{N-1} h_{N-1} = f_{N-2} + \beta_{N-2} h_{N-2} + q_{N-2} B
\]

so that for some period \( j \):

\[
f_N + \beta_N h_N = f_j + \beta_j h_j + B \sum_{i=j}^{N-1} q_i
\]

where

\[
\beta_j = \max \left[ P_{j+1}, \min \left( C_{j+1}, \beta_{j+1} \right) \right]
\]

and

\[
q_j = \max \left[ 0, (\beta_{j+1} - C_{j+1}) \right]
\]
The decision problem at period $j$ is therefore to find $X_j$ and $Y_j$ which maximise:

$$Z = f_j + \beta_j h_j$$

Given the $\beta_j$, the table indicates the optimal values of $X_j$ and $Y_j$. However, $\beta_j$ depends on future periods so to develop a myopic search technique it is necessary to see whether $\beta_j$ can be determined from a limited search. The linear nature of the problem has already been used to show that $\beta_j$ will have specific values depending on the relationships. Thus, from

(a) $\beta_j = \max \{p_{j+1}', \min \{C_{j+1}', \beta_{j+1}'\}\}$

and

(b) $\max \{C_{j+1}', p_{j+1}'\} > \beta_j > p_{j+1}$

(that is unit value of inventory lies within the bounds of the purchase and sale prices in the following period)

if:

(i) $p_{j+1} > C_{j+1}'$ then $\beta_j = p_{j+1}$

or

(ii) $p_{j+1} > \max \{C_{j+2}', p_{j+2}'\}$, then $p_{j+1} > \beta_{j+1}'$ and $\beta_j = p_{j+1}$

or

(iii) $p_{j+1} < C_{j+1}' < p_{j+2}'$, then $p_{j+1} < C_{j+1}' \beta_{j+1}'$ and $\beta_j = C_{j+1}$

In all other cases $\beta_j$ cannot be uniquely determined without reference to many more periods. These results occur as, given specific conditions, $\beta_j$ depends only on the next period's conditions in the case of (i), and on the next two periods in the case of (ii) and (iii). These conditions could be further extended to see whether $\beta_j$ can be uniquely determined.

The planning horizon significance of the $\beta_j$ determining rules is that if some period $j$ exists such that $\beta_j$ is uniquely determined by
examining the next two periods, then an optimal first period decision can be made without recourse to periods beyond \( j + 2 \). If the above conditions do not occur further periods must be considered.

This inventory model has been considered in detail as it clearly demonstrates the principle that in some cases a limited search may uniquely determine \( x_j^* \) and so lead to a planning horizon. Whether this will occur depends on the nature of the problem. Necessary and sufficient planning horizon conditions have been developed for other types of inventory problems within the certainty-non-stationarity case. They all depend on the various price relationships, which are relatively simple to explore for the single product situation. Examples are (a) Eppen et al. (1969) who consider the problem of deciding on period production levels of a single product to meet known demands where set-up costs, holding costs and production costs vary with time and (b) Kunreuther et al. (1973) who consider a similar problem but with an additional cost being related to the differences in production levels in each period (production smoothing) and (c) Lieber (1973), who considers the problem where backlogging can occur at a known cost in each period.

5.4 The Case of Non-Certainty and Stationarity

If the total horizon is long enough the concept of an optimal stable policy can be used as repeatability exists. With non-certainty that cannot be reduced to a certainty equivalent form, an optimal stable policy consists of a set of decision rules rather than a single policy. For each of the condition sets that can occur, one of the decision rules applies. Given instantaneous adjustment is possible, the myopic procedure is to determine the optimal stable policy and to immediately implement this in the first period. Shapiro (1968) considers such a
problem for cases where an 'unique optimal stationary strategy' exists. He uses the turnpike concept by noting that, provided the total horizon is long enough, the infinite horizon stable policy can be used in the initial periods.

Where instantaneous adjustment is not possible, the more likely case, the transition problem of reaching the optimal stable policy must be considered. Essentially, the myopic system of determining the stable policy and then setting up a model with a sufficient number of periods to give a final period solution the same as the stable policy can be used. Thus, a priori knowledge of the stable policy indicates when a planning horizon has been reached. Due to the non-certainty, the actual solution to the stable policy problem is not simple. Burt et al. (1963) provide a typical example of the methods used.

They consider the problem of deciding whether to plant a crop of wheat or to leave the field fallow in a semi-arid region. Actions and events in the immediately preceding year (wheat or fallow and rainfall) determine the soil moisture level at planting. Response in the current period depends on this moisture level and the rainfall. The decision on whether to plant or fallow depends on soil moisture and the rainfall probabilities as well as the prices and costs. For a given starting soil moisture level and decision there are a set of probabilities for the possible end of season soil moisture levels. Therefore, for a particular soil moisture level decision rule, a matrix of transition probabilities is defined, each row of which holds for one of the starting soil moisture levels. If \( R \) is defined as a column vector with components being the one period return for each starting state, \( \beta \) the discount factor, \( P \) the matrix of transition probabilities and \( f(n) \) a column vector with components being the present worth of following the given policy for each starting state to infinity, then:
\begin{align*}
(i) \quad f(n) &= R + \beta P (n-1) = R + \beta PR + \beta P (n-2) \\
&= R + \beta PR + \beta^2 P^2 R + \beta P (n-3) \\
&= (I + \beta P + \beta^2 P^2 + \ldots + \beta^{n-1} P^{n-1}) R \\
\text{and} \\
(ii) \quad \text{for } n \to \infty \\
f(n) &= (I - \beta P)^{-1} R \\
\end{align*}

Thus, the present worth vector of a constant policy over infinity can be determined by solving this single matrix equation. Essentially, this gives the $X^j_i$ values in a dynamic programming formulation assuming a particular policy is followed. In order to solve the transition problem, and so obtain the first period decision, Burt et al. set up a dynamic programming formulation for the actual starting state for a limited number of total periods. This was solved and re-solved with additional periods until a constant policy was implemented over the last few periods. Then using the suggested constant policy to obtain $f(n)$ above, and therefore the $X^j_i$, the original problem was effectively resolved using the derived $X^j_i$ as the ending state valuations. If the same constant policy occurs, the first period decision is optimal no matter how many additional periods might be used and thus a planning horizon determined.

Where the problem features do not permit repeatability the optimal stable policy cannot be used. The only methods used in this case have been to either use the total horizon or to use a subjectively determined system. One is to add periods to the model until it appears the first period solution is stable. Another is to argue that the market determines correct marginal value products of resources making up the ending state and, therefore, to use a limited number of periods with the ending $X^j_i$ being determined from the market values. Trebeck et al. (1972) use this latter approach. But, clearly, the market valuations will not reflect the true marginal value products to an individual as objectives,
fixed factors, total horizons, and managerial ability vary. Further, many factors are priced on a 'cost plus a percentage' basis. Thus, in neither case can it be proved that the first period decision is optimal.

5.5 The Case of Non-Certainty and Non-Stationarity

This is likely to be the most common case assuming the model used represents reality. The non-stationarity means an optimal stable policy will not exist assuming cases where the extent of change between periods and over the total horizon is sufficiently minor so that effectively they become non-certainty/stationary cases.

Solving approaches must either use the total horizon or use planning horizon rules which guarantee a planning horizon can be isolated. Due to the non-stationarity, workers involved in such problems have had to search for special conditions giving a planning horizon. The only cases where necessary and sufficient conditions have been isolated are the single product inventory problems. In other cases the subjectively assessed approximate methods of Trebeck et al. (1972) and Rae (1970) have been used.

The inventory problem discussed by Charnes et al. (1966) is a case where limited planning horizon rules can be proved given specific forms of non-certainty are introduced into the problem. The method used in devising the rules is similar to that used in the certainty case. It is assumed that the product purchase and sale prices in any period follow a joint density function but that once a particular period is reached the prices are known with certainty. They also consider the case where future price distributions depend on current prices (serial dependence). Effectively, the system involves taking a limited number of periods with the terminal $\beta_j$ set at either plus or minus infinity. If the two $E(\beta_j)$ derived in each case give values which
satisfy conditions similar to those in their certainty decision rule table (section 5.3), then the optimal first period purchase and sale decisions are uniquely determined. If not, the number of periods is increased. With price non-certainty the simple certainty rules cannot be used as they rely on known prices. Other examples of inventory type problems are given by Veinott (1968), who considers a production scheduling problem where demands are stochastic and in which more explicit horizon rules can be determined, and Symonds (1962), who considers a similar problem in which backlogging can occur (but at a cost).

5.6 Combinations of the Cases

Cases may exist, or at least be closely allied with, in which there are groups of periods in which any one of the cases defined may occur. It is possible, for example, for a decision maker to make estimates of future conditions which reflect non-certainty and non-stationarity for the initial periods and non-certainty and stationarity for the remaining periods. Depending on the case it may be possible to use the stable policy concept. Hopkins (1971), in solving the equipment replacement and capacity expansion problem found in a single-product firm provides an example. He considers the certainty case in which non-stationarity eventually gives way to stationary conditions. Thus, the method is to determine the solution to a finite horizon problem which includes a valuation of terminal stocks. The valuations are then revised using stationary estimates to obtain first period optimality.

6. PERSPECTIVE

The survey of solving methods indicates that under stationarity it is possible to determine the optimal first period decision without explicitly determining 'the' planning horizon. For the more generally realistic non-stationarity case planning horizon rules have been developed
for a limited number of relatively simple problems. As Charnes et al. (1966) note (p. 308), "no generally applicable methodology has yet been devised .... for locating horizons". As many problems, including the pig problem fall within the non certainty-non-stationarity case there is a need for exploring objective methods of determining the planning horizon. Furthermore, even in cases where stationarity enables an optimal first period decision to be uniquely determined, a method of determining the planning horizon may remove the need for estimating the stable policy where this is a complex problem. Faced with any particular problem, however, a decision is required on whether simplifications are justifiable which will then enable, for example, the stable policy concept to be used where its determination is relatively simple. (Thus the importance of reviewing all cases.)
CHAPTER IX

DETERMINING A PLANNING HORIZON

1. INTRODUCTION

The pig fattening problem frequently falls in the non-stationarity case. Even where stationarity exists, or is assumed to exist at some stage, continuous planning may require a model containing many periods so that a method of determining a planning horizon can simplify the decision process. This Chapter contains the development of a method for determining a planning horizon that does not rely on stationarity.

The system relies on determining a number of periods which ensure the first period decision set is optimal rather than explicitly finding 'the' planning horizon. From an operational view this tends to be a simpler approach. The general approach is to find the boundary of the set of possible \( x_{ij}^{j} \) (optimal return resulting from the system being in state \( i \) in period \( j \)) for some period \( j \) and to test whether the optimal first period decision for each member of the set gives a common policy. If this occurs the first period decision is optimal so that the periods up to period \( j \) provide 'a planning horizon'.

The method is discussed by first proving the existence of a myopic search technique and then showing how the technique can be used to put explicit bounds on the set of possible \( x_{ij}^{j} \). It is then proved that it is possible to test for first period decision optimality without an exhaustive search of the \( x_{ij}^{j} \) set. This leads to a continuous planning system under non-stationarity and a conclusion on the conditions under which a planning horizon can be determined without the need for experimentation.
2. THE BASIS OF A MYOPIC SEARCH TECHNIQUE

2.1 The Certainty Case

In section 3.2, Chapter VIII, it was noted that the following situation gives a planning horizon:

'If for some period \( d < n \), and \( X_i^d(Y) \), \( Y = 1,2, \ldots, z \), represents the possible \( X_i^d \) values, \( z < \infty \), then if optimal \( S_i^1 \) has the same \( i \) value in

\[
\max_{i, \forall i} \left[ h_i^{d-1} + h_i^d + X_i^d(Y) \right]
\]

for all \( Y \) values, then \( d \) provides a planning horizon.'

A similar statement was given for the non-certainty case in section 3.3. It was accepted, however, that the \( X_i^j(Y) \) could be estimated without recourse to considering the total horizon. It is necessary, therefore, to show how a bounded set can be determined that is sufficiently restricted to give a possibility of the above condition being satisfied.

For the pig fattening problem the \( S_i^j \) represent the possible combinations of quantities and types of pigs that can be on hand. The \( h_i^j \) define the period feed costs and in some cases the sale return or weaner purchase costs as well. The \( b_i^j \) are the net return resulting from an optimal policy for periods 1 to \( j \) and therefore include all pig purchase and feed costs as well as sales. They do not include the potential returns resulting from the pigs on hand at the end of period \( j \). These returns are included in the \( X_i^j \).

For this situation it can be shown that if the return from following an optimal policy from period \( j \), given a starting state of zero pigs on

\[ S_i^j \] is the \( i \)th possible state at the end of period \( j \), \( h_i^j \) is the within period return resulting from a decision given a state movement from \( S_i^{j-1} \) to \( S_i^j \) and \( b_i^j \) is the total return from an optimal policy from the starting period and state to \( S_i^j \).
hand, is subtracted from all \( x^j_1 \) the optimal decision set is unaltered. This means bounds can be placed on the possible \( x^j_1 \) after deducting this constant. The minimum values will be given by valuing the pigs giving rise to the \( x^j_1 \) at their market prices. The maximum values will be given by valuing the pigs using the maximum cash return that can be obtained from running each class of pig to an optimal weight without regard to any fattening space constraints. The true adjusted \( x^j_1 \) fall within these bounds so that a range of possible combinations exist. The details of these developments are given in Appendix V.

Assuming \( x^j_1 \) is subtracted from each \( x^j_1 \), let each combination of the possible values for all \( i \) be defined by \( \overline{x}^j_1 (u) \), \( u = 1, 2, \ldots \). Thus, given the \( b^j_i \), the optimal decision path can be calculated for each \( \overline{x}^j_1 (u) \) set.

This is given by

\[
\max_{i} \left[ b^j_i + \overline{x}^j_1 (u) \right], \quad \text{all } u
\]

The result gives which end of first period state (\( S^1_i \)) should be obtained, and therefore a particular first period decision, for each \( u \). If \( i \) on \( S^1_i \) is the same for all \( u \) then the first period decision must be optimal and \( j \) forms a planning horizon. If the optimal \( S^1_i \) varies with \( u \) the planning period must be extended until this occurs. In the extreme \( N \) is attained and the set \( \overline{x}^j_1 (u) \) has only one value. Alternatively, reducing the bounds on the \( \overline{x}^j_1 (u) \) may be more productive. This is achieved at some period \( j \) by obtaining the \( \overline{x}^j_1 \), where \( \overline{x}^j_1 = x^j_1 - x^j_1 \), from:

\[
\overline{x}^j_1 = \max_{i} \left[ x^j_1 + \overline{x}^{j-1}_1 \right] - x^j_1
\]

The bounds will be reduced as actual \( x^j_1 \) are used.

The number of possible \( S^j_1 \) in the pig fattening case precludes the use of dynamic programming for actual solving. However, this broad
myopic search technique has a direct counterpart where the problem is set up as a linear programming problem. This is done by formulating the L.P. model for a number of periods and giving pigs on hand at the end of the final period (ending pigs) a zero value. The \( b_i^j \) can be obtained by a series of solutions. If constraints are included to force state \( S_i^j \) to occur, then the objective function value equals \( b_i^j \). Similarly \( b_i^j \), all \( i \), can be obtained by altering \( b \), the requirements vector, in successive solutions. Some \( S_i^j \) will be infeasible for a given \( S_i^0 \).

If, however, the maximum, or minimum values as defined are placed on the ending pigs and a solution obtained, this gives:

\[
(i) \quad \max_i [b_i^j + w_{ Xi }], \text{ where the superscript } w \text{ refers to the minimum values.}
\]

and

\[
(ii) \quad \max_i [b_i^j + m_{ Xi }], \text{ where the superscript } m \text{ refers to the minimum values.}
\]

Similarly, solutions can be obtained giving

\[
\max_i [b_i^j + \overline{X}_i^j (u)] \quad \text{all } u.
\]

If the first period solution is the same in each case it is the optimal decision and a planning horizon has been attained. The set of possible \( \overline{X}_i^j (u) \), however, is infinite. While a grid approach, or the use of variable price programming, will overcome this problem it is shown later that this is not necessary.

2.2 The Non-Certainty Case

In the pig fattening case, where the objective is the maximisation of expected profit, the certainty case reasoning directly applies given modifications in the definitions of the variables. These involve
re-defining the $b^j_i$ and $X^j_i (u)$ as expected values. In determining these values feasible state movements will differ from the certainty case.

$E(X^j_i (u))$ is given by:

$$\max \left[ \sum_{i,j} X^j_i (u) + \sum_{i,j} E(X^{j+1}_i (u)) \right] \sum_{i,j} P_i$$

where

$P_i = \text{the probability of attaining } S^j_i \text{ given the action resulting in } i \times i'$

$i r^j_i = \text{the expected return (cost)}.$

$E(b^j_i)$ is given by:

$$\sum_{y} b^j_i (y) P_y$$

where

$b^j_i (y) = \text{the return from following an optimal set of decision to attain state } S^j_i \text{ assuming that the set of period nature states leading to the } j \text{th period is given by } y, y = 1, 2, \ldots.$

$P_y = \text{probability of the nature state set } y \text{ occurring.}$

Similarly the problem can be set up as a stochastic L.P. model.

3. AN EFFICIENT MYOPTIC SEARCH TECHNIQUE

3.1 Introduction

The exhaustive testing required by the system given above makes it impractical. However, using the fact that the $X^j_i (u)$ are in effect the sum of the possible marginal value products multiplied by the number of pigs of each type represented by a $S^j_i$ enables an efficient approach to be developed. Effectively, the maximum and minimum values defined form bounds on the true marginal value product of a pig of any given
type. The true marginal value is defined as the net addition to the total horizon objective function value that a pig of a given type (conformation, weight and genetic class probabilities) would contribute if available at the margin in a particular period. Reducing the range of the set of possible $X_1^j (u)$ revolves around exploring the determinants of the marginal value products (M.V.P.).

If the true M.V.Ps. of pigs at the end of the first period were known it would only be necessary to solve a one period model to obtain the optimal first period decision. However, if the model is extended to encompass a number of periods, M.V.Ps. are in part endogenously determined thus effectively providing prices to place on the pigs on hand at the end of the first period. These M.V.Ps. are unlikely to take on their true values as they still depend on the prices placed on the pigs on hand at the end of the last period. Ranging through possible price combinations on ending pigs (end prices) will provide sets of mutually applying end of first period M.V.Ps. But this again is impractical.

The M.V.P. of any pig depends on its possible net cash profit minus the opportunity cost of any fixed resources it uses. The primary resource used is fattening shed space in each period. Potentially, the other major factor affecting the M.V.P. of pigs on hand at the end of the first period is the starting state ($S_1^1$). Given a model of arbitrary length, ranging through end prices effectively gives a unit of shed space in each period a range of values (space values) as well as directly effecting the end of first period pig's M.V.Ps. It is the link between end prices and space values, however, which provides a method of determining the set of possible end of first period M.V.Ps. without the need to range through all combinations of ending prices.
The approach is to find the boundaries of the set of possible end of first period M.V.Ps. and then to test whether members of the set give the same first period decision. If this occurs a planning horizon is attained. Further, it will be shown that it is frequently only necessary to find a limited number of extreme points of the set of possible M.V.Ps. as if the first period decision is the same for each of these extreme points, it will also be optimal for any other member of the set. With continuous planning it is irrelevant whether the second and later period decisions are constant.

To develop the system the certainty case will be considered first. It will then be extended to allow for non-certainty and the complications introduced by imposing limits on the number of feed mixes that can be used. The model used is based on the block diagonal form of the polyperiod L.P. model rather than the partially triangular form defined in Chapter VII. The triangular form can, however, be converted to the block diagonal form without affecting the system represented so that the results are quite general.

3.2 The Determinants of the Marginal Value Products

The optimal first period decision depends on, in part, the endogenously determined end of first period M.V.Ps. The features of the pig fattening problem means it is relatively easy to isolate the M.V.P. determinants. Value relationships stemming from the features of the problem are developed in some detail and presented in Appendix VI.

The features lead to the conclusion that in the final period of the model only one production activity for each pig class need be included in the model because for any set of final period prices one activity will dominate. For the same reasons only one weaner purchase activity is
required. Similarly, in any period of the model one activity of those using a particular class will dominate depending on the endogenously determined values so that only one activity from each group utilizing a particular pig type will tend to be basic in any period. This occurs as the basic activity selection depends on their net revenues \((C_j)\) and \(a_j\) vectors assuming any set of basic vectors can produce a feasible solution. This tends to occur where pig numbers can be adjusted through selling operations. These features also mean that all of the final period activities will be basic.

The important result stemming from these relationships is that where only the dominating activities are basic the value of fattening space in any period is dependent on the M.V.P. of the pig type produced by the weaner purchase activity in the period. This M.V.P. is in turn dependent on the space value in subsequent periods. Due to the nature of the requirements vector \((b)\), in some cases the space values will also depend on the M.V.P.s of pigs which displace the weaner purchase activities. Similarly, the M.V.P. of pig classes in any period which are obtained from pigs on hand in the first period are dependent on the space values and on the prices given to the end of final period classes which can be provided by the particular type.

For any set of final period prices taken from within the maximum and minimum bounds a mutually operative set of end of first period M.V.P.s is obtained so that as these prices are varied a bounded set of mutually operative M.V.P.s are determined. While in general it is necessary to find the extreme points of this bounded set to test for first period optimality, it is shown in the following section that in the pig problem it is frequently only necessary to find a limited range of these extreme points.

\(^2\) See section 5 of Appendix VI.
The value relationships which are presented in Appendix VI lead to the following general conclusions on the first period M.V.Ps. where it is assumed an initial solution is obtained for all ending prices set at their minimum value and then subsequent solutions obtained for increases in the prices which determine the space values.

(a) The value of a pig that cannot be carried through to the period in which the space value increases will increase.

(b) The value of a pig that can be carried over the period in which the space value increases will decrease and possibly eventually increase if it is sold earlier. If it is not carried over in the initial solution, it will increase.

(c) The value of a pig which must be carried over the period in which the space value increases will decrease.

All the end prices which do not influence the space value in any period are directly related to the M.V.P. of pigs on hand at the end of the first period. This means a variation in any of these end prices will either directly affect the M.V.P. or have no effect if the maximum price is not great enough to ensure the relevant pig class is held over to the last period. The detailed reasoning leading to these statements is given in Appendix VI.

Given a solution for a given set of prices for the variable end prices, the conclusions indicate that if the prices giving space values are varied some of the extreme points of the set of values can be found. This assumes that all prices other than the space value determining prices are taken to be constant. Where all pig values increase or decrease at a constant rate the maximum increase or decrease forms an extreme point valuation vector. Where the rate of value change varies or the direction of change alters, there will be other
extreme points that can only be located by a systematic search.

A simplification for solving purposes arises from the features of the problem. Where more than one activity can determine the space value in any period only the dominating one is effective. Thus, only one such activity is necessary provided the end prices effecting its value are varied through the total range for all such activities. Where an activity may also directly effect the value of a 1st period pig this simplification cannot be used.

3.3 First Period Optimality

For each of the pig groups on hand at the start of the first period, one of the treatment activities will tend to dominate. Which activities are selected depends on the values derived by the valuation component of the total model. If the dominating activity (or activities) in each case does not change as end prices are changed the first period decisions are optimal. The basis of the system is therefore to select a set of ending prices and to observe which first period activities dominate. Thus, utilising the value relationships, an assessment of whether these can change is made.

Assuming a solution is obtained with all prices set at their absolute minimum, consider the activities in the first period which utilise a given starting class. Any change in the non-basic activities' price (value) only influences their \( Z_j - C_j \) \( (Z_j = c_B A^{-1} a_j) \). Any change in the basic activities price, however, effects the \( Z_j \) of the non-basic activities through affecting \( c_B \). However, for a unit change in the price there will be a unit change of the same direction in all \( Z_j \) of those activities using the same pig class as they have the same \( a_j \) components except for the negative coefficient representing the supply of a pig to the following period. Thus, if a change to the first period decision can occur, it will occur when the end prices are set so that the basic activity has its minimum price and the non-basic activities
have their maximum price. Thus, it is necessary to vary the end prices to explore whether such a change can occur. If a change does occur the planning horizon needs extending.

The combinations of first period activity price or value changes that can occur for the activities related to a particular pig class are:

<table>
<thead>
<tr>
<th>Type</th>
<th>Basic Activity</th>
<th>Non-Basic Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Increase</td>
<td>Decrease</td>
</tr>
<tr>
<td>(ii)</td>
<td>Decrease</td>
<td>Increase</td>
</tr>
<tr>
<td>(iii)</td>
<td>Increase</td>
<td>Increase</td>
</tr>
<tr>
<td>(iv)</td>
<td>Decrease</td>
<td>Decrease</td>
</tr>
</tbody>
</table>

If change type (i) occurs the basic activity will remain basic so such changes can be ignored. Change type (ii) can clearly lead to a change in the first period basis. However, if space value changes lead to this form of movement, the change in basis will occur at the extreme movement, if at all. Thus, intermediate changes need not be considered so that it is only necessary to set the end prices at their extremes to give this kind of movement to the non-basic activity prices. Using the general space price-value relationships it can be determined which prices need to be set at their maximum and which at their minimum.

If either of change types (iii) and (iv) occur the basis can change at other than the extremes of any movement as if the rate of change should vary the basic activity may become non-basic at an intermediate point but not at the extreme. In these cases, where the rate of change varies, it is necessary to parameterise the space price changes.

Some simplifications are possible. While there are a wide range of pig classes, any movement in their prices depends on the
periods in which they can, or must, be carried, due to the space value effects. For price purposes a pig type is therefore dependent on its possible future time span. Further, the relevant first period decisions relate to actions involving feeding a positive number of pigs. Depending on the starting state, some variables may be basic at zero level. Whether such activities remain basic or not is irrelevant to the first period actions. Effectively, in exploring price changes the only relevant prices are those for pig types which can potentially be transferred to the second period at a level greater than zero. Examining these indicates whether some of the classes produced should be increased in quantity, and whether some should not be produced.

3.4 Determining a Planning Horizon

Utilizing the M.V.P. relationships, the total system consists of the following steps:

(a) set all prices at their absolute minimum and solve,
(b) observe which end of first period pig types are produced, and which are not of those that potentially can be produced at a positive level,
(c) devise a set of solutions designed to test for 1st period stability and solve these with the end prices directly affecting the non-basic activities set at their absolute maximum.
(d) if the first period solution with respect to the activities representing a positive number of pigs changes, the total planning period must be extended.

This system must terminate in a finite number of steps. If the planning period is extended sufficiently the pigs on hand at the start of the valuation period will not be carried through to the last period (length will depend on starting state) and any last period price changes will have insignificant effects. However, a computationally more efficient system
exists and is discussed below.

In setting the bounds on the true M.V.Ps. the absolute minimum and maximum have been taken as the expected market price and maximum net cash return. While the minimum is always the market price it is possible to reduce the maximum M.V.Ps. The value of space used by any pig has a minimum level as determined by the market price of pigs purchased in a particular period. This means that the maximum value of any pig will be no greater than the maximum cash return less the minimum value of space used in each period. In some cases the marginal net return from carrying a pig for an extra period will be less than the minimum value of space used so that it must be sold. In calculating the maximum values in this way some cases will give rise to a value equal to or less than the minimum value. Their true M.V.P. is therefore the market price.

Rather than extend the total planning period where a planning horizon does not occur it is computationally more efficient to use a multi-phase system. Having set up the L.P. model for a number of periods this can be used to reduce the bounds on the M.V.P. estimate for use in determining the optimal first period decision. Given, for example, an eight week model a two phase system would initially consist of using the model to obtain the maximum and minimum prices holding at the beginning of week nine (phase I) and then using these in the search for first period optimality over the first eight weeks (phase II). The second phase would use the same basic model but with appropriate price changes. If necessary a third, or more, phase could be used. The first phase is solved with all direct prices at their maximum and a parametric routine used to set the space value determining prices such that each type of pig is given its maximum value. In some cases these will be the market price. The maximum values are then used in the second
phase.

The multi-phase system may not terminate in a finite number of steps as each phase is solved independently. In such cases it is necessary to extend the initial planning model.

3.5 Introducing Limits on the Number of Feed Mixes Used

As the system developed ignores the additional requirement of limiting the number of mixes used it is necessary to consider whether any modifications are required.

Introducing mix limits effectively gives a number of sub-problems. Each one represents the opportunities where activities are defined assuming only an acceptable number of alternative mixes are used. For any one of these sub-problems the system developed is applicable since each one does not contain mix limit constraints. To interpret the effect of solving the sub-problems concurrently it is necessary to consider the nature of the mix limit constraints involving integer requirements.

Including the mix limit constraints gives the following set of typical dual constraints for one of the periods where only one mix can be used out of two available.

\[
\begin{align*}
C_1 &= V_1 + a_1 V_3 + V_4 - d_1 \\
C_2 &= V_1 + a_2 V_3 + V_3 - d_2 \\
C_3 &= V_2 + a_3 V_3 + V_4 - d_3 \\
C_4 &= V_2 + a_4 V_3 + V_3 - d_4 \\
C_5 &= a_5 V_3 + V_4 - d_5 \\
C_6 &= a_6 V_3 + V_5 - d_6 \\
0 &= -mV_4 + V_6 - d_7 \\
0 &= -mV_5 + V_6 - d_8 
\end{align*}
\]
Primal activities with net revenues \( C_j \), \( j = 1-6 \), are pig production activities. The value of the pig class used is given by \( V_1 \) and \( V_2 \). The coefficients representing the pig supply to the following period are not included as they can be regarded as being included in the \( C_j \). The zero coefficients are the net revenues of the primal integer variables. In the primal, one of these variables must equal one while the other must be zero. The variables \( V_4 \) and \( V_5 \) represent the value of the alternative mixes while \( V_6 \) is the value on the constraint limiting the mixes used to one. Finally, \( V_3 \) is the space value and the \( \delta_j \)'s the slack activity levels. Given a solution to the integer problem either \( \delta_7 \) or \( \delta_8 \) will be zero. Assuming \( \delta_7 \) is zero and noting that, due to the \(-m\) coefficient, \( V_4 \) must be zero leads to the conclusion that \( V_6 \) must also be zero. However, \( \delta_8 \), the \( Z_j - C_j \) of the other integer variable will usually be negative as removing the mix limits will increase the objective function value. If it is positive it indicates the mix limits do not reduce profitability. While \( \delta_8 \) is negative the solution is still optimal given the integer requirement. The negative value of \( \delta_8 \), however, does not have a direct significance due to the \(-m\) component of the activity's \( a_j \). More importantly, the negative \( \delta_8 \) means \( V_5 \) will reflect the value of restricting the mixes.

To understand the significance of \( V_5 \), it is necessary to recognise the solution form under the mix limit situation. Due to the additional constraints added, those primal activities that would normally be basic, if it was not for the mix limit requirement, will continue to be basic, but at zero level. The additional constraints provide sufficient basic locations for this to occur. Thus, the non-basic primal disposal activity with a \( Z_j - C_j \) giving \( V_5 \) has \( y_j \) components (\( y_j = B^{-1}a_j \)) reflecting that its introduction to the basis would decrease the level of the basic activity at zero level. Accordingly, \( V_5 \) indicates the true value of
allowing the additional mix to be used at unit level (one or more additional pigs). Further, as those activities that would normally be produced are basic as well as those at positive levels, the $Z_j - C_j$ ($d_j$) of non-basic production activities reflect not only their normal net opportunity cost but also a component reflecting the mix limit effect. This occurs as the marginal rate of substitution between the activity and the, possibly, two basic activities using the pig class for which they compete are positive. If only one activity is basic the $Z_j - C_j$ has the conventional interpretation.

The value $V_5$, or $V_4$ if the other mix is used, indicates the difference in value of a pig ($V_1$ and $V_2$) that occurs in the mix limit solution compared with that which would occur if the mix limits were not imposed. Thus, the values obtained from the valuation section of the planning model are a true reflection of the M.V.Ps. under the mix limit situation given the optimal integer solution for any set of end prices.

With the above background it is possible to interpret whether any modification to the first period optimality testing system are required. The final period with mix limits added still has a solution in which at least one activity using each of the possible pig classes will be basic. Further, the $x + 1$ constraints added, where $x$ is the number of possible mixes, enables more than the dominating production activities to be basic so that a change in one pig price will influence more than one activity in some cases. If $y$ is the maximum number of mixes that can be used, $x - y$ of the integer variables will be non-basic but this is immaterial as it does not affect the direct effect of price changes on values. Further, in any earlier period all the dominating activities will be basic as well as the 'second best' activities due to the addition of the mix limit constraints. Thus, any variation in
the end prices has the same effect on first period pig values as in the non-mix limit case except that the values will include an allowance for the constraining mix limits. This includes the space value effects.

In exploring whether price changes will change the first period solution, the branch and bound integer solving algorithm used in experiments with the model repeats the sub-problem comparison system. If a price change, and therefore a potential pig value change, means either a non-basic activity should be introduced, or, one of the zero basic variables made positive, the solving procedure will indicate this.

The fact that some zero basic activities can, potentially, be positive means that one adjustment in the testing system is necessary. If the price on the pig supplied to the second period increases sufficiently it will be marginally profitable to change the mixes used and increase the zero variable to a positive level. Thus, space value effects which give such increases need to be explored. This assumes the two basic activities utilising a given pig class in the first period supply a different class. Further, for the same reason it is necessary in the testing system to increase the prices directly effecting the values of such basic activities to their maximum. In effect such activities must be treated as non-basic activities.

3.6 Introducing Non-Certainty

As in the mix limit case, introducing non-certainty into the pig fattening planning model specified necessitates only minor modifications to the planning horizon system developed. The inclusion of non-certainty requires all prices and costs to be replaced by their expected values and for the model to be amended to allow for the variable physical outcomes. The criteria by which to decide on the necessary amendments is whether the value relationships are affected as the first period model structure is the same as in the certainty case.
To appreciate the effect of introducing non-certainty consider the following representative valuation model constraint set for a two period situation.

\[
\begin{align*}
C_1 &= V_1 + a_1 V_5 - V_6 - V_{12} - d_1 \\
C_2 &= V_1 + a_1 V_5 - V_7 - V_{13} - d_2 \\
C_3 &= V_2 + a_2 V_5 - V_8 - V_{14} - d_3 \\
C_4 &= V_2 + a_2 V_5 - d_4 \\
C_5 &= V_3 + a_3 V_5 - d_5 \\
C_6 &= V_4 + a_4 V_5 - d_6 \\
C_7 &= a_5 V_5 - V_6 - V_{11} - d_7 \\
\end{align*}
\]

Primal activities with net revenues \( C_j \) \((j = 1-7)\) represent the first period activities so that \( V_i \) \((i = 1-4)\) are the value of the starting pig classes and \( V_i \) \((i = 6-9)\) are the value of pigs supplied to the following period given the first state of nature while \( V_i \) \((i = 11-14)\) are the values for the second state of nature. To obtain the expected
value of pigs supplies to the second period these sets must be added
as they already allow for the chance on the alternative outcomes. The
first period space value is \( V_5 \). Net revenues \( C_j \) (\( j = 8-12 \)) are the
expected values of a particular pig class in the final period. These
appear twice as there are two possible outcomes from first period
actions. They are the variable prices. The \( p_j \) (\( j = 1,2 \)) are the
probabilities on each state of nature. The only other variables are
\( V_{10} \) and \( V_{15} \), the spaces value in each of the second period sub-models.

The features of the typical problem are that when an ending price
is varied two prices require changing as any one pig class appears in
each sub-model. With more periods they would appear more times. Each
first period activity supplies two different pig classes for use in the
second period sub-models representing the outcomes under the alternative
nature states. As discussed below, this is the major difference
requiring an adjustment to the certainty system.

Each last period sub-model has a unique solution and in any period
dominance occurs where the requirements vector does not influence the
optimal solution. The activity dominating will depend on, in the
example problem, the sum of the values of each of the 'two' pigs all the
activities using a particular pig class supply.

These features indicate that any change in end prices will have
the same effect on first period values as the certainty case. A change
in \( C_{12} \), for example, changes \( V_{10} \) and \( V_{15} \), the space value, so that all
pigs coming from the previous period have their values changed. A change
in \( C_8 \) affects both \( V_6 \) and \( V_{11} \) thus affecting both \( V_5 \), the first period
space value, and \( V_1 \). As \( V_5 \) is affecting all other 1st period values,
\( C_8 \) affects many pig values. This brings out the difference between the
certainty and non-certainty case. A change in one end price will usually
affect more starting pig values, either directly or through the space value effects than the certainty case. Accordingly, when testing for first period optimality, any end price affecting space values must be treated as such even though it may also directly affect the value of a starting pig. Further, prices directly affecting first period pig values may affect both basic and non-basic activity prices requiring therefore the same treatment as the space prices.

Where a multi-phase operation is used, each phase provides ranges of expected prices. These provide the maximum and minimum expected prices for the following phase. These will frequently be the same value.

3.7 Pre-Determining a Planning Horizon

In the pig fattening case pre-determination of a planning horizon depends on conditions existing such that true M.V.Ps. can be determined without experimentation. Such cases are unlikely to occur though initial use of the multi-phase system may indicate that the maximum and minimum M.V.Ps. coincide. (In general, there will be a limited number of cases where true M.V.Ps. can be pre-determined. Some inventory problems in which the cost and return functions indicate that a positive inventory should not be held at some point are examples.)

In the pig problem the only general case where pre-determination may be possible is where an anticipated price drop, and possibly a cost increase, indicate that all pigs should be sold in some period. Such prices are unlikely to occur. Special cases may occur in which a fixed state must be adhered to at some period. Where contracts have been made necessitating a fixed state means the problem is one of achieving the state at least cost. Thus, the maximum planning horizon is the total period up to the period at which the fixed state must be achieved.
Application of continuous planning requires that these conditions be checked for before experimentation is used.

In an applicational sense, a form of pre-determination is the use of a representative model to indicate the true M.V.P.s. These estimates can then be used in applicational models for individual farms. This concept will be discussed in a later chapter.

4. A GENERAL APPROACH TO THE PLANNING HORIZON PROBLEM

While some problems give unconditional irrelevancy of all variables or use the stationarity concepts, in general determining a planning horizon must rely on finding the bounds of the M.V.P. vector set followed by a testing system to ensure a common first period decision. If this occurs the optimality of the decision is proven. (Note also that the use of stationarity gives the true M.V.P.s.)

Testing for optimality does not require all vectors within the bounded set to be assessed in linear programming formulations. This would be an impractical requirement. If all the extreme points of a convex set containing the bounded set of price vectors are found, then if the optimal first period decision is common to all extreme points it will be common for all members of the set. Appendix VII contains the proof.

In many cases it will not be necessary to find all the extreme points of the valuation set. The pig fattening case is an example. In this case, knowing the general nature of value changes as prices change enables isolating the required points. In others there will only be a limited number of extreme points. In the pig problem, if all pigs had to be kept for a fixed time the number of extreme points would be reduced. Whatever the case it will always be important to examine the technical and economic nature of the problem to find possible simplifications.
As the M.V.Ps. depend on the assets' eventual realisation price, if any, and the value of the fixed resources utilised, finding the valuation set extreme points must rest on exploring the possible resource values. The simplest problems will involve small numbers of short lived assets and have a small number of decision points within the assets' life. Similarly, if the number of resources is small, value exploration is simplified.

Actively determining a planning horizon has implications in attempts at solving multi-period complex decision problems where stationarity occurs. It may be computationally impossible to actually use the existence of stationarity. If a planning horizon can be found, however, it may not be necessary to include as many periods as using the stationarity concept requires. Thus, the problem becomes tractable. Similarly, a multi-phase approach enables decomposition. Another possible simplification, which follows from the planning horizon concepts, occurs under non-certainty cases. As it is only necessary to find the bounds on the price set there will be cases where considering only the most and least favourable outcomes will provide bounds giving an optimal first period decision. This has implications in stochastic linear programming in that this approach would enable the matrix size to be reduced.

In many realistic problems determining a planning horizon is complex and expensive. This is an inescapable feature of applied problems. In applied continuous planning it is likely that using a research model to determine horizon rules of thumb for different conditions will be adequate. These can then be used in continually determining first period actions for applied problems.
CHAPTER X

THE APPLICATION OF THE MODELS DEVELOPED

1. INTRODUCTION

A study and development of a planning system does not provide specific conclusions. The usefulness of the system developed, both in pig production and in other production systems to which the general system might be applied, depends on the particular conditions applying in any area. However, a number of experiments were carried out using local price information to explore the potential value of continuous planning and to indicate further simplifications that might be possible in operating the system.

On the technical side of the study, the construction of the pig growth model has led to a better understanding of response and has provided a base from which further experiments aimed at providing technical information for management purposes can be designed. An example of how an understanding of the nature of response is directly useful comes from a fuller recognition that the nutrient requirements for a particular type of growth constantly changes. Feeding recommendations frequently suggest that only two or three feed types should be used over the life of a pig (A.R.C., 1967). Due to the significance of feed costs, more frequent changes are likely to lead to significant economies where the physical facilities make this possible.

The development of the conceptual planning model also provides a background to consider planning systems. For example, the inclusion of the planning horizon concept in continuous planning indicates that the development of long term plans and budgets may only have marginal
benefits. Control systems which predict physical and financial outcomes for periods beyond the planning horizon only provide secondary benefits. These include the demonstration of future profit trends to financial institutions for borrowing purposes and to promote the confidence of a manager and workforce.

The information obtained from the planning experiments is discussed by initially considering the determination of a planning horizon, giving examples of the value of continuous planning and then discussing possible approaches to the application of continuous planning in an advisory context. Finally, further developments that are necessary to the models developed are briefly reviewed.

2. DETERMINING A PLANNING HORIZON

Historically, local pig prices have exhibited considerable variability over short periods. Ten years' records showed that the maximum increase in any one year was 37 per cent for fat prices and 80 per cent for weaner prices while over any single month the maximum increases were 8 and 80 per cent respectively. Similarly, feed prices have varied markedly. For example, milk powder has recently increased from $170/tonne to $300. In view of these price changes an experiment was carried out to determine a planning horizon assuming the gross margins were anticipated to increase constantly at a rate of 3 per cent compound every two weeks over sixteen weeks (23% in total) and then to decline at the same rate. In contrast to this a further experiment assumed anticipations were static.

In the case of static prices a two phase operation was required to obtain first period stability where an eight week planning model was used.\(^1\) The two phases were required despite a starting state consisting

\(^1\) Using the Burroughs Corp. Tempo System on a B6700 computer, an initial solution for the problem (approx. 1000 variables) required 111 seconds of C.P.U. time while testing for first period stability working from saved bases required 311 seconds C.P.U. time.
of pigs which could not be held for longer than eight weeks. In table 10.1 some examples of the effect of using the model to reduce the range on the possible marginal value products of different pig types are given (Phase I).

**TABLE 10.1**

Marginal Value Product Ranges for Some Example Pig Types

<table>
<thead>
<tr>
<th>Example Pig Type</th>
<th>Range Using Absolute Maximum and Minimum Values</th>
<th>Range Obtained from Phase I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$3.63</td>
<td>$0.0</td>
</tr>
<tr>
<td>2.</td>
<td>$1.29</td>
<td>$1.28</td>
</tr>
<tr>
<td>3.</td>
<td>$6.34</td>
<td>$1.68</td>
</tr>
<tr>
<td>4.</td>
<td>$7.8</td>
<td>$5.87</td>
</tr>
</tbody>
</table>

The differences in the reduction of the ranges are quite variable and are clearly critical to the success of a multi-phase system. The variability is in part due to the maximum number of periods a pig can be held and therefore the total space used.

For the changing price situation, using the same starting state that was used in static price case, it was necessary to use three phases in order to obtain first period stability. This involved a total of twenty four weeks which meant the planning operation covered all of the period in which prices were rising as well as eight weeks of anticipated decreasing margins. It was clear the planning operation would have required more phases if the prices had been assumed to continue increasing. The falling prices meant the maximum and minimum marginal value products
(M.V.Ps.) were either equal or covered a small range. Using only two phases gave a wide range in the possible M.V.Ps.

The experiments demonstrated the importance of anticipated profit margins in determining a planning horizon. With increasing margins planning must consider a longer total time to assess the value of current actions. It was also clear that the general profit margins are more important than the relativity between the prices of different meat grades and of feed ingredients. Grade prices and ingredient costs affect the first period decisions but the anticipated margins determine the value of space in any period and therefore influence the planning horizon.

Similarly, the nature of the starting state tends not to influence the planning horizon though it does effect the maximum number of periods pigs on hand can be held and therefore may increase the period over which price anticipations must be considered. Where prices are expected to fall after several periods and the starting state is made up of largely light weight pigs, the starting state can be important since it may be profitable to carry the pigs onto a heavier weight than usual rather than purchasing further weaners which would eventually be sold at lower prices. This means the value of space would be determined by the pigs displacing the weaners so that less periods may be required to reduce the bounds on the M.V.P. estimates.

3. THE VALUE OF CONTINUOUS PLANNING

Assuming continuous planning is carried out at each decision point with a knowledge of the planning horizon, the marginal value of using the system compared with alternative approaches depends on a range of factors. These include:

(a) the variability of prices, costs and physical

2 Market prices for live animals were assumed to be correlated with meat prices. The relationship used was based on an analysis of local data.
outcomes,

(b) the accuracy with which future conditions can be predicted,

(c) the opportunities for adjusting the production systems through time, and

(d) the nature of the objective function.

If prices and costs are relatively static and physical outcomes certain there will only be small gains in continual reviews. Where considerable variability through time occurs, the ability to predict trends is a major factor in determining the usefulness of continuous planning. Methods of determining subjective estimates are important in this respect (Smith, 1967) as are the availability of forecasting models. In cases where predictive success is reasonable there must still be the opportunity to adjust actions to take advantage of the anticipations. In pig production, feeding and selling policies can be changed regularly (within limits) but in some farming systems this flexibility is not available. Crop production systems, for example, provide little scope for adjustments once the crops have been planted. Overriding all these factors is the relevant objective function. Where the objective is primarily satisficing in nature, the use of a stable policy despite changes in anticipations may provide satisfactory outcomes under many condition sets. Effectively, the added management complexity of continuous planning may not be considered worthwhile. Furthermore, even where all the conditions potentially necessary for continual reviews to provide a net gain exist, a limited number of the alternative products and production systems may tend to economically dominate under a wide range of conditions so that continual reviews will provide only marginal gains.

Use of forecasts must allow for the actions of other producers. See Dixon (1967).
Continuous planning can be operated without a knowledge of the planning horizon. A model which uses market prices as the M.V.Ps. of resources potentially on hand at the end of the last period of the model can obviously be used at each planning point. Another example not requiring formal planning is the use of subjectively estimated revisions based on an understanding of the nature of the problem. The accuracy and value of these methods must depend on the factors discussed above. Effectively, methods of determining a planning horizon constitute prediction techniques which will have a value in any particular case (Byerlee & Anderson, 1969).

To test the importance of some of the points raised a number of continuous planning experiments were carried out for the pig problem. The tests involved simulating production over twelve weeks (assumed maximum time a pig can be held) using a randomly selected series of environmental outcomes. Two price anticipation cases were simulated. The first assumed anticipations did not change with time (the same series as used in the changing price planning horizon experiment) and the second assumed anticipations changed every two weeks. To consider an extreme case it was assumed anticipations changed from the static prices to the prices assumed in the first case and vice-versa in each sequential period. For both cases the net cash gross margin over the twelve weeks plus the value of pigs on hand at the end of the twelfth week (using a multi-phase system to give the M.V.Ps.) was calculated for the following three planning methods:

(a) continuous planning using a multi-phase system to give the M.V.Ps. ('First Period Optimality').

(b) Continuous planning using a four week model with ending pigs valued at their market price. ('Four

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4 Based on the two week outcome periods.
Week-Market Prices').

(c) Using standard extension advice. This assumed feeding was organised to give maximum fat free growth and that pigs were sold at their maximum weight under all price conditions. ('Standard Policy').

While the planning models developed assumed that first period prices were known with certainty in some cases this may be unrealistic. To indicate the effect of non-certain first period prices, the comparison was also carried out assuming actual meat prices were either plus or minus 10 per cent of the expected price. Two sequences were used, the first being randomly selected (Series I) and the second having the same percentage change but with the signs reversed (Series II). The results of all cases are given in table 10.2 where the standard policy is used as a comparative base.

TABLE 10.2

A Comparison of Planning Methods

Percentage Difference in Net Return

Over a Standard Policy as a Base

<table>
<thead>
<tr>
<th>METHOD</th>
<th>CASE 1</th>
<th>CASE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Constant Anticipations)</td>
<td>(Changing Anticipations)</td>
</tr>
<tr>
<td>A: Known First Period Prices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. First Period Optimality</td>
<td>+51%</td>
<td>+70%</td>
</tr>
<tr>
<td>2. Four Week-Market Prices</td>
<td>+44%</td>
<td>+55%</td>
</tr>
<tr>
<td>B: Non-Certain First Period Prices - Series I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. First Period Optimality</td>
<td>+69%</td>
<td>+102%</td>
</tr>
<tr>
<td>2. Four Week-Market Prices</td>
<td>+53%</td>
<td>+56%</td>
</tr>
<tr>
<td>C: Non-Certain First Period Prices - Series II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. First Period Optimality</td>
<td>+32%</td>
<td>+40%</td>
</tr>
<tr>
<td>2. Four Week-Market Prices</td>
<td>+36%</td>
<td>+54%</td>
</tr>
</tbody>
</table>
Where the first period prices are known with certainty there was a marked advantage in using the four week model over the standard policy. With constant anticipations (Case 1) the increased gain from determining a planning horizon is not great but where price anticipations change (Case 2) the marginal gain is significant. This would be expected as theoretically correct continuous planning is designed to account for information updating. The considerable gains from using either of the continuous planning methods are in part a result of the inflexible policy used as the comparative base. A farmer using a standard policy would in reality tend to make marginal changes on a subjective basis according to the current conditions.

The two experiments where the first period prices were assumed to be non-certain demonstrate the importance of predictive accuracy (and the possible effect of the shape of the price distributions). In series I, the gain from determining a planning horizon is greater than the first period certainty case. In series II, however, determining a planning horizon actually has less value than using a four week model.

It is clear from the experiments that in any particular area all the factors discussed must be carefully considered before setting up a management system. Besides the price variability and forecasting situation, the experiments stress that the decision flexibility available is a critical consideration.

4. THE PRACTICAL APPLICATION OF CONTINUOUS PLANNING

Where the formal use of the systems developed provide a significant payoff compared with alternative approaches, methods of simplifying the procedures need to be considered to reduce the costs. A major question is whether the results from representative cases can
be extrapolated from or whether individual farm planning is warranted. Methods of reducing costs include:

(a) the use of a research model to derive M.V.P. estimates for use in applicational models,

(b) the use of treatment cost predictors,

(c) the use of matrix generators and an overall control programme to automate the total system,

(d) the use of representative model experiments to indicate whether re-planning is necessary every period.

A major cost is the estimation of a planning horizon and the compilation of feed mixes at every planning point for all the alternative actions. The use of a basic model to derive narrow M.V.P. bounds which could then be used in a, say, four period model for individual farms would lower costs considerably. The M.V.Ps. could be derived for a range of technical efficiencies and starting state configurations and the results used to derive a number of functions to act as M.V.P. predictors. This system would introduce inaccuracies but would allow individual farm problems to be solved inexpensively. Similarly, as it is only for the first period that the details of feed mixes are required, initial experimentation could be used to derive treatment costs for a range of ingredient prices and the results used to derive feed cost tables or functions. At any planning moment these tables would be used to update treatment costs for the second and later periods rather than completely solve all the least cost problems.

A major cost component is the setting up of all the models and the sequential re-solving necessary in determining whether first period stability has been obtained. The development of matrix generators would enable individual farm matrices to be constructed with redundant
rows and clearly dominated activities removed. This system could utilise M.V.P. and treatment cost predictors. Given the reduced model, an overall control programme could be used to obtain the initial solution and test for first period stability if fixed M.V.P. estimates from a basic model are not used. Similarly, the determination of M.V.P. estimates for use in individual farm models could be automated.

Finally, under periods of anticipated static prices a constant policy may be optimal. The continuous use of representative models could be used to indicate when revisions are required on an individual farm basis though individual farmer estimates of prices to be used must be allowed for.

5. FURTHER DEVELOPMENTS

Throughout the discussion on pig growth, areas where further work and data is required were noted. Major examples are the need to have detailed information on the individual sex response relationships, on the interactions between space available per pig and productivity, on the nature of variable fat deposition and its relationship with essential fat levels and the derivation of genotype class data.

Comments were also made about the development of planning systems with particular reference to pig fattening. An example is the need for an exploration of the value of allowing for non-certain outcomes in the model under different conditions though this must depend on the nature of the objective function. Models may need to be developed for a range of objective functions but this aspect must rely on assessment of common objectives in any area.

For providing general extension information, the models developed can be used to compare a wide range of basic management policies. Possible policies for assessment include the use of various input and output contracts,
the profit losses resulting from maintaining a constant weaner purchase and fat pig sale programme and the effect of using the same feed throughout the life of a pig.

Finally, while economically viable planning systems similar to the system developed could be developed for many farming situations, the question of whether farmers are prepared to utilise such intensive systems needs to be explored. Farmers have traditionally been slow to accept management aids (Harrison & Rades, 1973) and where the system provides most of the decisions they have historically made themselves, a sceptical attitude may occur. Training courses are likely to be essential and the presentation of output information would need to be carefully designed. In some cases working through extension personnel rather than directly with farmers may be necessary.
CHAPTER XI

ON DEVELOPING PLANNING SYSTEMS

The primary objective of the study was to consider the nature and structure of dynamic planning through the development of a model for pig fattening. The objective originally conceived was the development of a general theory on dynamic planning including the determination of a planning horizon. It is evident, however, that the development of planning systems must be closely allied to the technology of the relevant problems. Effectively, the idea that 'economics without a technology' is but half an art must be stressed. The structure of the pig fattening problem, for example, dictated the basis of the methods adopted in the study. Somewhat similarly, in studying the technology of a problem it is important to determine the nature and form of the determinants of physical output. This is basically the systems approach in contrast to the simple trial approach. In the pig problem, a general theory on response provides the interface between the technology and the economics of pig production.

There is a danger, however, in developing planning systems based on a systems simulation approach as it is commonly used. Conclusions are largely formed through using a simple comparison of alternative policies rather than considering the basic economic structure of the problem. In the context of the 'comparative analysis and farm standards' diagnostic and planning system, the potential problems and inefficiencies of simple

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1 Technology in this context can refer to social group responses as well as physical relationships.

2 See, for example, Dalton (1971).
comparisons have been fully demonstrated (Candler & Sargent, 1962).

In Chapter VI it was noted that the growth in the use of systems simulation has been due in part to a failure to develop analytical and numerical methods capable of representing the complex nature of farm planning problems. Many economic theories and solution methods developed rely on over simplified problems to demonstrate their application (Boussard, 1971 & Cocks, 1968). In future work, emphasis must be placed on the formulation of methods capable of solving large problems. This requires a careful and detailed statement of the problem enabling the economic structure of the problem to be conceived and exploited. It also leads to a decision on justifiable simplifications as well as clearly indicating areas where available data is inadequate. In some cases the cost of acquiring the data may not be warranted compared to using subjective estimates. This is one area where economics becomes an art. The difficult problem of developing applied systems may not lead to elegant theories but in many cases will give an improved efficiency in sectors of the agricultural industry.

The secondary objective of the study was to consider the application of sophisticated planning systems at an individual farm level. It is clear that the determination of a planning horizon is important in this respect. Given methods of determining representative marginal value products, the computational costs of applying continuous planning are reduced. This demonstrates the value of examining the economic structure of a problem. It also implies that the number of periods that must be included in individual farm models can be reduced. This allows the periods that are included to be more realistic within the

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3 Though systems simulation techniques are not being used at the farm level to any extent. See Blackie & Dent (1976).
bounds of a fixed computational capability or cost.

"Progress, therefore, is not an accident, but a necessity."

Herbert Spencer
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APPENDIX I

OPTIMAL REPLACEMENT TIME

The work of Chisholm (1966), Winder & Trant (1961) and Faris (1960 & 1961) are examples of attempts at formulating the principles involved in an optimal replacement time. The form of the conclusion drawn depends on whether time preference is regarded as being important. The consensus is that the objective is to maximise the present value (the discount factor may be unity) of future profits over the total time span of interest to the entrepreneur. With no time preference this is achieved by replacing at the time at which (following Chisholm)

\[
\frac{dv}{dn} - \frac{dc}{dn} = \frac{P}{n}
\]

where

\[ v = \text{gross revenue/period} \]
\[ c = \text{variable costs/period} \]
\[ n = \text{duration of production before replacement} \]
\[ P = \text{total net pay off when replacement occurs at } n. \]

That is, marginal net profit per unit of time should equal the average net profit per unit of time. The assumption is that the system will be repeated in identical form.

Where time preference is important the criteria for maximising total present value is to replace such that (following Chisholm)

\[
PNV^* = PNV + \frac{PNV}{(1+i)^{n-1}}
\]

is a maximum where

\[ PNV^* = \text{present value from a perpetual sequence of production units,} \]
\[ PNV = \text{present value from a single production period,} \]
\[ i = \text{discount rate.} \]

While these concepts provide the general principles they cannot be directly applied as the assumptions are unrealistic. Changing conditions mean the system cannot be identically repeated and in animal production the total animal production is not necessarily sold at one
point though each group of similar animals can be treated as the decision unit. There are, however, interactions between groups particularly with respect to the competition for fattening house space (Dent 1971). Further there are likely to be side conditions which must be met such as a requirement for a minimum level of cash income per period. In a pig fattening situation such a requirement is not particularly restrictive as the production period per pig is relatively short.

With time preference included a major problem is the selection of the correct discount rate. This requires a clear understanding of the distinction between time preference proper and opportunity cost. Such an understanding is not always exhibited in the literature. In devising a decision model the requirement of the entrepreneur must be allowed for and this will, in general, not be one of simply maximising the present value without regard to the consumption pattern. It has been pointed out that the system must provide at least some minimum level of cash flow. Further, a component of a typical farmer's objective is the requirement that he remains in farming so that there is a limit to any off-farm investment he might consider. In a simple pig fattening case without alternative farm investments the opportunity cost of cash is given by these off-farm opportunities. This means that once a system is devised to ensure the minimum cash requirement, surplus funds should be allocated with due allowance to the competition between consumption and investment. Investment off-farm requires that the return be at least as great as the marginal on-farm investment return from pigs and that the level of the farming activities are at the required level. Otherwise farm investment will occur. However, such investment should not occur where consumption will provide greater utility. This is where time preference becomes a factor. Return from investments is not instantaneous so that unless the return is sufficient to compensate for delaying consumption the funds should be consumed. Thus, in considering on-farm decisions the discount rate should be the greater of the opportunity cost of off-farm investment and the rate of time preference. The within farm opportunities and associated opportunity costs should be handled within the model. It must be noted, however, that the ratio of the two will not be constant as the rate of time preference will be a function of the current consumption level and in some cases the return from off-farm investments will vary with the level of investment contemplated. It should also be noted that arguments about whether to use borrowing or
lending rates are irrelevant for an individual farm investment problem as interest paid is a cash expense and the farmer is concerned with net cash flows.

Investment funded from borrowing is not a cash expense to the farmer. The cash costs of such an investment are the interest and principal payments. Thus, the farmer should make decisions within the confines of limits on borrowing such that the present value of his operations are maximised where the present value is calculated using a discount rate as defined above. This rate may vary with the cash level of the operations. Furthermore, where the model endogenously allows for off-farm investment opportunities then the discount rate should simply be the rate of time preference. Consumption will occur if the return occurring within the model is less than the rate of time preference given the minimum cash requirements are satisfied (the rate of time preference for cash income levels less than the minimum requirement is infinity so that cash will be consumed rather than invested in the system).
APPENDIX II

PREDICTING RESPONSE

To predict the growth response from a given feed mix the following calculations are carried out on a daily basis in sequential order. Growth during a day gives the commencing state variable values for the following day.

(i) Estimate the energy and protein surplus to maintenance requirements.

(ii) Estimate whether there is sufficient protein to enable the maximum potential muscle growth to occur. If this is not possible, the calculations in section B below are followed. Otherwise, the steps follow through section A.

A. (i) Estimate whether the surplus energy combined with any energy from protein in excess of growth requirements is sufficient for the protein deposition and associated essential fat requirements.

(ii) If the energy available exceeds the total muscle growth requirement, estimate the addition to the weight of variable fat that is possible. Thus, the end of day state variables can be estimated from the growth in muscle and variable fat.

(iii) If the energy available is less than the total muscle growth requirement, either: the muscle growth for the day must be reduced to ensure that energy supply equates with energy use, or; stored energy in the variable fat must be released to give energy balance, or; some combination if the stored energy is insufficient. Which case occurs depends on the level of variable fat. These calculations lead to an updated set of state variables. Mobilisation of variable fat provides 39.61 M.J.M.E./kg.\(^1\)

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\(^1\) The combustion energy of fat is 39.61 M.J./kg. This is all available for use as M.E. as fat mobilisation involves hydrolysis (requires no energy).
B. (i) Estimate the muscle growth that is feasible with the surplus protein.

(ii) Determine whether the surplus energy is sufficient for the potential muscle growth and the associated essential fat growth requirements.

(iii) Depending on whether an energy surplus exists, the calculations follow the same steps as in A (ii) and A (iii) above. These lead to updated state variable values.
APPENDIX III

SAMPLING OUTCOMES AND DETERMINING POSTERIOR PROBABILITIES

The method used to formulate the growth distribution, based on two genotype classes, and to obtain the necessary data for the linear programming model from these distributions is presented below in stepwise form.¹

A. Determining the liveweight growth distribution

(i) Determine the parameters of the triangular distribution for each genotype. The mode is given by the expected liveweight growth and the ranges by ±2.2σ.

(ii) Determine the parameters of the combined liveweight growth distribution through:

(a) Modal value = mode of constituent distributions with the greatest probability, where the probability is given by:

\[ P_j = \sum_i P_i q_i \]

where

\[ P_i = \text{probability of genotype } i, \]
\[ q_i = \text{probability of the modal value of the } i^{\text{th}} \text{ genotype distribution if } i = j, \]

or

the probability of the modal value in the \( i^{\text{th}} \) genotype's distribution if \( i \neq j \).

(b) The range \((a \& c)\) is given by:

\[ a \text{ = } a \text{ from genotype 1's distribution} \]
\[ c \text{ = } c \text{ from genotype 2's distribution} \]

provided this gives a modal probability equal

¹ A listing of the programme used is available on request.
to that derived in (a). If not the range is adjusted so that the modal probability is maintained by altering \(a\) or \(c\). If the range must be increased, \(a\) or \(c\) is increased depending on which constituent genotype has the greatest probability. For a decrease \(a\) or \(c\) is decreased, the choice being based on the genotype with the lowest probability. This system is based on, (1) the importance of the modal probability, and (2) the least important genotype in defining the outcome distribution being the one with the lowest probability.

B. Estimating the two liveweight outcomes

The appropriate outcomes to select are those that will give the most accurate representation of the future value of possible actions. Dividing the range into two equal intervals and selecting mid-points is unlikely to achieve this as their chance may be small (depending on skewness). To ensure equal significance they should be selected to give a probability of 0.5. Further, given the median is used to divide the distribution it is necessary to ensure the values selected from each half reflect the probabilities on individual values. Thus, half distribution expected values were used. The steps involved are:

(i) estimate the median by solving for the liveweight giving the cumulative probability equal to 0.5.

\[ \text{median} = \int_a^x f(x)\,dx \]

(ii) estimate \(Z = \int_a^x f(x)\,dx\)

where

\(f(x)\) is the density function for liveweight \((x)\).

(iii) Outcome 1 = 2Z

Outcome 2 = 2[E(x) - 2Z]

The probabilities for each outcome are therefore 0.5.

(iv) For each liveweight growth outcome estimate the total liveweight and from these estimate the levels of bone plus muscle and variable fat. In doing this it is assumed the ratio of variable fat to bone plus muscle occurring at the mean is maintained. The
estimation of the bone plus muscle level requires an iterative procedure due to the complex relationship giving the body component structure.

C. Calculate the updated genotypic probabilities

As each of the two outcomes represent a range of possible events the posterior probabilities must be determined through assessing the proportion of the distribution area within the range being presented by the particular outcome contributed by each genotype after allowing for their \textit{a priori} probabilities. Thus, the method is:

(i) estimate the distribution area on either side of the median for each genotype.

(ii) for both outcomes, weight the areas by the \textit{a priori} genotypic probabilities. The posterior probabilities are then the ratio of the weighted area for one genotype to the weighted area for both genotypes.

The advantage in using the triangular distribution is that distribution area and similar calculations involve relatively simple manipulations. Such calculations are frequently required in the defined systems.
APPENDIX IV

FACTORS AFFECTING THE LENGTH OF THE
PLANNING HORIZON

1. INTRODUCTION

To consider the effect of factor variations it is assumed a set of possible \( S^j_i \) values (the total return from following an optimal policy given starting state \( i \) and period \( j \)) for some period \( j \) for which the first period decision variables values are the same for all components of the set. Further, it is assumed that limiting the myopic search does not produce this condition and nor does basing the search on period \( j-1 \). Thus, a case of conditional irrelevancy exists. Given this situation, if a factor within the arbitrarily selected set of conditions change the bounds on the set of possible \( X^j_i \) values may change. If the bounds are extended the optimal first period decision variable values may no longer be constant for the components of the \( X^j_i \) set. If this occurs the myopic search must be extended to reduce the bounds. The extension must continue until the first period decision variable values become common to all possible \( X^j_i \) values. This extension could involve increasing \( j \) on \( X^j_i \) and using the same search procedure at this period. If the bounds are reduced by the factor variation, for the new conditions it may only be necessary to use the myopic search at some period \( d < j \) to give possible \( X^d_i \) for which the first period decision variable values are constant. At worst, the planning horizon will not increase. Alternatively, it may be possible to reduce the extent of the myopic search at \( j \).

Where the myopic search produces a set of specific \( X^j_i \) values (in effect a number of possible vectors with components \( X^j_i, x = 1, 2, .. .. m \)), rather than a bounded set within which all vectors are possible, the effect of changing factors cannot be isolated without an explicit objective analysis of any particular case.

While the conditional irrelevancy case is the most likely, cases of unconditional irrelevancy leading to the planning horizon may occur. An extreme example in the pig case is where, for some future period, anticipated feed and weaner costs are such that variable costs
cannot be covered so that it is clear that at this period no pigs should be held. Given such a case, variations in relevant factors may mean the time at which this condition occurs is changed but there is no way of indicating the general direction of the change unless specific parameters are quantitatively tabulated.

2. FACTORS AND THEIR EFFECT

Any factor which affects the bounds on the $X_i^j$ values can potentially effect the horizon. These will depend in part of the nature of the myopic search method used. The approach taken is therefore of isolating major groupings and giving specific examples within each to indicate to trend that will occur. This is done within the context of a given total and positive horizon. Changes in the total horizon, if the positive horizon is not shorter, may affect the planning horizon depending on whether the additional periods add extra opportunities and therefore change the relativity between the $X_i^j$ estimates. If the relativities change the optimal $j^{th}$ period decisions may change and resultingly the first period decisions may no longer be common to each possible $X_i^j$ set.

The factor groupings and potential change effects are listed below.

(a) The Objective Function

Changes in the objective function compared with that assumed will clearly not have a definite trend in general. However, specific changes will. Where the number of periods considered make time preference a significant factor, an increased rate will shift the bounded set downwards so that some components may become negative thus reducing the effective bounds and potentially reducing the horizon. Attitudes to change, as portrayed in the objective function, if they become more restrictive, will tend to reduce the possible values. Day (1963) discusses the effect on growth of change limits. Similarly, the consumption function will influence potential future opportunities through an investment funds availability effect and thus tend to narrow the bounds of the set. Hutton (1966) implicitly shows the effect of the consumption function on opportunities in a growth situation. The other major factor in the objective function is the attitude to risk. In a simple multi-period portfolio Mossin (1968) considers the effect of attitude on what is, in effect, the unconditional irrelevancy situation.
Extreme risk aversion will reduce the bounds on the set as the upper values in any one vector are likely to involve considerable risk.

(b) **The Starting State**

The starting state, which is largely the current resource structure and the quantities of the various production activities together with the stage at which they are at, tends to effect the planning horizon through effecting future opportunities. The level of funds on hand and the potential access to borrowed funds, for example, determine in part future investment and production opportunities. (See Mossin, 1968). If the starting state is changed such that opportunities are increased the bounds on the possible \( x^j_i \) tend to widen so that, if a change occurs, the planning horizon will increase. Similarly, a flexible resource structure (multi-purpose buildings etc.) will tend to lengthen the horizon. Some components of the starting state may, however, tend to shorten the horizon. The existence of quantity and price contracts, and of lease agreements, are two examples which make up the starting state which can decrease the future opportunities.

(c) **Constraints on Choice**

The objective function and the starting state improve limits on future actions. However, there are likely to be other constraints resulting from Governmental and other institutional requirements. Any change which makes such constraints more restrictive, or introduces new constraint areas, will tend to reduce the planning horizon through limiting opportunities or at least shift downwards the possible \( x^j_i \)'s and potentially make some negative and thus effectively tightening the bounds on the \( x^j_i \).

(d) **The Form of the Physical Production Functions, Price and Cost Functions**

Any change in these factors may lead to changing the possible \( x^j_i \). The trend resulting, however, can be in either direction depending on the circumstances. Simple overall shifts may simply move all \( x^j_i \) whereas a change in a function for a particular product will potentially affect some \( x^j_i \) and therefore alter the bounds. For example, if prices decrease the bounds can be reduced as if the decrease is large some \( x^j_i \) may become negative. In the extreme it may be clear that a carryover from one period to another of variable resources is not profitable so that a case of unconditional irrelevancy occurs. Anticipated technological change will effect physical relationships and as such may be significant
(Butcher & Whittlesy, (1966) discuss technological change effects on firm growth). Similarly, the existence of size economies will tend to lengthen the horizon through increasing the range of possible $X_i^j$ values. Other important considerations will be marketing and purchasing opportunities (possible contracts) through their effect on prices and costs and the existence of set up costs associated with investment opportunities. These factors may be a component of a size economy effect.

(e) **Variable Variance**

Increased variance can embody an increased possible event range for any random variable. Thus, while the $X_i^j$ are expected values, the bounds on the possible values will tend to widen as favourable outcomes may enable investment and productive opportunities that would not otherwise be possible. Hart (1942) discusses these effects though not in a horizon context. Increased variance therefore tends to increase the planning horizon where the objective function does not embody extreme risk aversion. Similarly, increased non-certainty regarding the correct objective function to use, if a range of possibilities are included, will widen the bounds. Reduction in variance to the extreme case of certainty will reduce the planning horizon though enabling a myopic search to isolate an optimal equilibrium plan. This is discussed in detail in Appendix V.

(f) **Management Ability**

Management ability, as reflected in technical coefficients and planning efficiency (e.g. frequency of considering replanning) will influence the planning horizon largely through affecting the production opportunities. If increased ability means more opportunities occur, (frequent replanning tends to provide this) the horizon will tend to lengthen through widening the bounds. If a simple shift in technical efficiency is the only effect the bounds will tend to move together. Patrick & Eisgruber (1968), in a different context, clearly demonstrate the importance of ability on growth.

(g) **Production and Investment Opportunities**

Many of the trends discussed rely on the effect of factors on opportunities. Therefore any change in opportunities *per se* may influence the planning horizon. An increased level will tend to lengthen the horizon though in some cases the new opportunities may have no effect as they are inferior.
APPENDIX V

DEFINING BOUNDS ON THE TOTAL RETURN FROM FOLLOWING AN OPTIMAL POLICY FROM A PARTICULAR PERIOD

If $S_j^j$ represents a state consisting of zero pigs on hand at the start of period $j$, and $X_j^j$ defines the optimal return given this starting state, then

$$X_j^j < X_i^j \quad \text{all } i \quad (i \text{ is the subscript for other states})$$

as any other state represents a positive inventory of pigs and these can at least be sold at market prices to give $S_j^j$. More likely a greater return will be possible from running at least some of the pigs for a number of periods. This holds for all $j$.

Now consider some period $t$. If $X_t^j$ is deducted from $X_j^j$, all $i$, so that $X_t^t$ is equal to zero, the optimal decision path is the same as if

$$b_q^t + x_t^t \geq b_i^t + X_i^t \quad \text{all } i$$

then the inequality still applies if the constant $X_t^t$ is deducted from both sides. Similarly for any period if a constant equal to $X_j^j$ is deducted from $X_j^i$, all $i$, the optimal decision path is not changed. Thus, if

$$b_q^j + x_j^j \geq b_i^j + X_i^j \quad \text{all } i, \quad j = 1, 2, \ldots, N$$

then

$$b_q^j + x_j^j - \sum_{j=1}^{N} x_j^j \geq b_i^j + X_i^j - \sum_{j=1}^{N} x_j^j.$$  

Note that $X_j^j$ will probably vary for different starting dates in the continuous planning process for constant $j$ due to the non-stationary conditions.

With this background consider the last period in the total horizon with particular reference to the maximum and minimum possible values $X_1^{N-1}$ can take on. $N$ can be any number of periods ranging up to a very large number representing infinity. The lowest possible value $X_1^{N-1}$ can

---

1 The terms used are defined in section 2.1, Chapter IX.
take on will be no less than the return from selling off all pigs on hand plus \( x_{1}^{N-1} \) as if in fact this was the optimal policy it would be embodied in all of the \( x_{1}^{N-1} \). That is:

\[
N-1 \sum_{f} k_{f} v_{f} + x_{1}^{N-1} = \sum_{f} k_{f} v_{f} + x_{1}^{N-1}, \quad \text{all } i
\]

where

\( w_{x_{1}^{N-1}} \) represents the minimum (worst) possible value,

and

\( k_{f} \) = the number of pigs on hand of type \( f \) giving some state \( s_{1}^{N-1}, \)

\( V_{f} \) = the current market price per head of pigs of type \( f \).

As subtracting \( x_{1}^{N-1} \) from all \( x_{1}^{N-1} \) does not affect the selection of the optimal decision, define:

\[
w_{x_{1}^{N-1}} = \sum_{f} k_{f} v_{f} + x_{1}^{N-1} - x_{1}^{N-1}
\]

\[
= \sum_{f} k_{f} v_{f}
\]

The greatest possible \( x_{1}^{N-1} \) for any \( i \) (define as \( m_{x_{1}^{N-1}} \)) will be no greater than the net return from running the pigs using a feeding policy which maximises the eventual realisation return minus the feed costs without regard to the time it takes, plus \( x_{1}^{N-1} \). In fact the period return must be somewhat less as this assumes that both these pigs and any purchases giving rise to \( x_{1}^{N-1} \) can be run in the limited fattening shed space. Thus,

\[
x_{1}^{N-1} < m_{x_{1}^{N-1}} = \sum_{f} k_{f} c_{f} + x_{1}^{N-1}, \quad \text{all } i
\]

where

\( c_{f} \) = the maximum net cash return obtainable from a pig of type \( f \) without regard to space requirements or time.

As subtracting \( x_{1}^{N-1} \) from all \( x_{1}^{N-1} \) does not alter the optimal decision, define:

\[
m_{x_{1}^{N-1}} = \sum_{f} k_{f} c_{f} + x_{1}^{N-1} - x_{1}^{N-1}
\]

\[
= \sum_{f} k_{f} c_{f}
\]
Now consider the second to last period. Using the same reasoning as above, the minimum possible \( x_{i}^{N-2} \) will be:

\[
x_{i}^{N-2} = \max_{i} \{ r_{i}^{N-2} + w_{i}^{N-1} \}
\]

Thus, \( x_{i}^{N-2} \leq w_{i}^{N-2} = \sum_{f} k_{f} V_{f}^{N-2} + x_{i}^{N-2} \) all \( i \)

A superscript is placed on \( V_{f} \) as the market prices may vary between periods. Subtracting \( X_{i}^{N-2} \) from all \( X_{i}^{N-2} \) defines:

\[
w_{X_{i}}^{N-2} = \sum_{f} k_{f} V_{f}^{N-2} + X_{i}^{N-2} - X_{i}^{N-2}
\]

Similarly, the maximum possible values \( (m_{X_{i}}^{N-2}) \) that the \( X_{i}^{N-2} \) can take on will be:

let \( X_{i}^{N-2} = \max_{i} \{ r_{i}^{N-2} + m_{i}^{N-1} \} \)

thus \( X_{i}^{N-2} \geq m_{X_{i}}^{N-2} = \sum_{f} k_{f} C_{f}^{N-2} + X_{i}^{N-2} \)

Subtracting \( X_{i}^{N-2} \) from all \( X_{i}^{N-2} \) defines:

\[
m_{X_{i}}^{N-2} = \sum_{f} k_{f} C_{f}^{N-2} + X_{i}^{N-2} - X_{i}^{N-2}
\]

But, the minimum and maximum \( X_{i}^{j} \) values are calculated in the same way in both periods. Their actual values may be different due to changes in the prices and costs. Similarly, the process can be continued back through the time periods so that at any period \( j \) the maximum and minimum possible values on any \( X_{i}^{j} \) will be:

\[
m_{X_{i}}^{j} = \sum_{f} k_{f} C_{f}^{j}
\]

\[
w_{X_{i}}^{j} = \sum_{f} k_{f} V_{f}^{j}
\]

In reality the actual values are likely to lie well below the \( m_{X_{i}}^{j} \).
APPENDIX VI

THE DEVELOPMENT OF PIG MARGINAL VALUE PRODUCT RELATIONSHIPS

1. INTRODUCTION

To develop the relationships it is necessary to consider the features of the problem so that the nature of optimal solutions can be determined. This leads to value relationships enabling the effect of price changes to be observed and to conclusions on methods of determining the bounds of the set of possible marginal value products (M.V.P.).

2. FEATURES OF THE PROBLEM

Consider a representative model conceptually involving four periods. As the M.V.Ps., or prices, on the end of first period pigs on hand are the important variables it is only necessary to consider the last three periods. For the same reason using the dual form of the linear programming (L.P.) model is appropriate.

To set this model up it is important to recognise that the pig fattening problem, as outlined in Chapter VII, exhibits the following features:

(a) some of the pig classes potentially on hand at the start of the second period cannot be sold by the end of, in this case, the fourth period as they will not have attained the minimum saleable weight.¹

(b) similarly, some of the pig classes cannot be carried through to the end of the fourth period as they would have exceeded the maximum saleable weight.¹

(c) each class can be utilised in a number of different ways. An activity is defined for each and represents rapid growth, slower growth, and so on for each of the genetic classes. Each activity provides a pig of a defined class for use in the following period and these are all different. If this was not the case the activities would be technically identical

¹ While most producers would plan within these constraints it is always possible to adjust stock numbers by selling to other producers through local markets.
and therefore the dominated one could be removed.

(d) in every period some of the classes must be sold off while others cannot be sold.

(e) in every period it is possible to purchase at least one weaner class.

(f) the space requirement of activities representing feeding actions to pigs of the same class are equal.

(g) as some of the last period activities represent the feeding and holding over of pigs, their prices or M.V.Ps. are not known. Some of the final period activities, however, have a fixed price as they represent feeding a pig that cannot be carried over.

These features do not allow for pigs to be sold off at other than certain weights. However, while a general policy of producing pigs within a range is common, it is always possible to adjust the numbers on hand through selling pigs to other fatteners. Thus, the first period of the model should include sale activities allowing such adjustments to be made. In effect the minimum M.V.P. on a given class of pig at any time is at least the market price.

Accordingly it is also necessary to include selling activities in the last period of the model to reflect disposal at the beginning of the period. Alternatively sale activities can be defined for the end of the previous period.

The following dual form of the second, third and fourth periods is representative of these features:

$$\text{Minimise } Z = \sum_{i=1}^{12} b_i v_i$$

subject to:

$$C_1 = v_3 + 7v_4 - d_1$$
$$C_2 = v_1 + 5v_4 - v_6 - d_2$$
$$C_3 = v_1 + 5v_4 - v_7 - d_3$$
$$C_4 = v_2 + 6v_4 - d_4$$
$$C_5 = v_2 + 6v_4 - v_7 - d_5$$
$$C_6 = 5v_4 - v_5 - d_6$$
\[ \begin{align*}
c_7 &= v_1 + 5v_4 - v_5 - d_7 \\
c_8 &= v_5 + 5v_8 - v_{10} - d_8 \\
c_9 &= v_5 + 5v_8 - v_{11} - d_9 \\
c_{10} &= v_6 + 6v_8 - v_{11} - d_{10} \\
c_{11} &= v_6 + 6v_8 - d_{11} \\
c_{12} &= 5v_8 - v_9 - d_{12} \\
c_{13} &= v_7 + 7v_8 - d_{13} \\
c_{14} &= v_5 + 5v_8 - d_{14} \\
c_{15} &= v_9 + 5v_{12} - d_{15} \\
c_{16} &= v_{10} + 6v_{12} - d_{16} \\
c_{17} &= v_{11} + 7v_{12} - d_{17} \\
c_{18} &= 5v_{12} - d_{18}
\end{align*} 

and subject to \( v_i \geq 0 \)

where

\[ \begin{align*}
v_i &= \text{value of the } i\text{th resource,} \\
b_i &= \text{availability of the } i\text{th resource,} \\
c_j &= \text{the net revenues of the } j\text{th primal activity,} \\
\hat{c}_j &= \text{the net revenue of the } j\text{th primal activity,} \\
d_j &= \text{the slack activities' values.}
\end{align*} \]

The constants reflect typical space requirements (in square feet). This unit was used for ease of understanding later manipulations. There are three pig classes whose values in the first period are given by \( V_1, i = 1, 2, 3 \). The space value in each period is given by \( V_i, i = 4, 8, \) and \( 12 \). It will be noted that each pig can only be utilised in one way in the last period. The reason for this is discussed below. Further, it should be noted that the requirements vector (b) consists of all zeros except for the components \( b_4, b_8 \) and \( b_{12} \) which represent the space available in each period. The primal activities whose net revenues are \( c_6, c_{12} \) and \( \hat{c}_{18} \) represent weaner purchase activities.
In the last period of the model it is only necessary to include one primal activity for each pig class. For any given set of ending prices one of the activities using a defined class will dominate all the others as all such activities represent vectors which are a simple linear combination of each other. This occurs as last period actions do not provide pigs for use in the non-existent following period. Similarly, only one weaner purchase activity is required.

In exploring the effect of variations in \( C_{15}, C_{16} \) and \( C_{18} \) it must, however, be recognised that such activities will provide a range of classes for use in the following period.

The set of variable prices (M.V.Ps.) on all potential last period activities has the broad bounds defined by the market price of each class produced and the maximum net cash return that can be obtained from carrying the pig on for a number of periods. For any particular set, or vector, of prices taken from within these bounds one of the activities using a given class on hand at the start of the final period will dominate. When a different price vector is selected the dominance may change. Thus, given only one primal activity is defined for each class, where its price is ranged through the minimum price any of the activities using the particular class can take on to the maximum price, using only one such activity allows for the full range.

Somewhat similarly, in any other period one of the primal activities of those using a particular class of pig will dominate the others. To see this consider a subsection of the model in table form:

<table>
<thead>
<tr>
<th>Variable:</th>
<th>( V_i )</th>
<th>( V_{i+s} )</th>
<th>( V_{i+t} )</th>
<th>( V_{i+u} )</th>
<th>( d_j )</th>
<th>( d_{j+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function value:</td>
<td>( b_i )</td>
<td>( b_{i+s} )</td>
<td>( b_{i+t} )</td>
<td>( b_{i+u} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
C_j = 1.0 \quad s \quad -1.0 \quad -1.0 \\
C_{j+1} = 1.0 \quad s \quad -1.0 \quad -1.0 \\
\]

where \( s \) = space requirement per pig.

For a given set of end pig prices \( V_{i+t} \) and \( V_{i+u} \) will take on particular values. The same applies to \( V_{i+s} \), the value of space (space value). Depending on these values and \( C_j \) and \( C_{j+1} \), \( V_i \) will take on a particular value so that either \( d_j \) or \( d_{j+1} \) will be equal to zero. Which one is
zero gives the dominating primal activity through the complementary slackness relationships between the primal and dual forms of a L.P. problem. If both are zero an arbitrary choice can be made though this does not preclude both activities from being basic. However, it is shown below that this will not occur in most cases. It will mean the existence of an alternative optimal solution.

Taking the total four period model for a given set of end prices, if it is solved in primal form the optimal solution has \( c_B B^{-1} a_j - C_j \geq 0 \), all \( j \), where \( c_B \) is the vector of basic activity net revenues, \( B \) is the basis matrix, \( a_j \) the vector of input-output coefficients for the \( j \)th activity, and \( C_j \) the net revenue of the \( j \)th activity. The requirements vector, \( b \), does not directly influence an optimal solution except through determining whether \( x \), the solution vector, is feasible. Thus, if the activities are such that a feasible solution is always possible for any \( b \), an optimal basis is not influenced by the components of \( b \). They do, however, determine the total profit. The pig fattening problem tends to be such a case as after the initial buying and selling actions the amended \( b \) tends not to influence the optimal solution. Accordingly, the value relationships are initially developed under this assumption and later amended to allow for the influence of \( b \). Where \( b \) does not influence the optimal solution, the M.V.Ps. of pigs on hand at the end of the first period depend on only the \( C_j \) and \( a_j \). Accordingly, of the activities in any period using a particular pig class, only one will be basic due to dominance. This means that, in general, all but one of the association \( d_j \)'s in the dual will be non zero. Where two or more are zero, one or more of the activities will be non-basic. A feature of this situation is that any group of starting pigs will never be split into sub-groups so that pens will remain as a single pen.

The significance of the feature above is that all the final period activities will be basic as there is only one for each starting class and one purchase activity. That is, all their \( d_j \)'s will be zero. Similarly, where there are \( S \) possible pig classes in any period, there are \( S + 1 \) constraints (allowing for space) in the period so that, as there will be only one activity basic for each class, the other basic activity in a period will be a weaner purchase activity (possibly at zero level). Thus, the value placed on the pig class which such an activity provides in the following period determines the value of space in the purchase period. Further, such values will depend on the end prices. This can be seen from examining the sub-section of the dual model representing the last
period. As these dual constraints have their $d_j$'s equal to zero it has the following form in the example problem:

\[
\begin{align*}
\hat{C}_{15} &= V_9 + 5V_{12} \\
\hat{C}_{16} &= V_{10} + 6V_{12} \\
\hat{C}_{17} &= V_{11} + 7V_{12} \\
\hat{C}_{18} &= 5V_{12}
\end{align*}
\]

Given $C_{18}$, the value of space in the last period will be $\hat{C}_{18}/5$ so that:

\[
\begin{align*}
V_9 &= \hat{C}_{15} - \hat{C}_{18} \\
V_{10} &= \hat{C}_{16} - \frac{6}{5}\hat{C}_{18} \\
V_{11} &= \hat{C}_{17} - \frac{7}{5}\hat{C}_{18}
\end{align*}
\]

Further $V_i$, $i = 9, 10$ and 11, the values of the pigs that can be provided by actions in the previous period, are uniquely determined and directly observable for any set of $\hat{C}_j$, $j = 15, 16$ and 18.

4. THE DEVELOPMENT OF VALUE (PRICE) RELATIONSHIPS

While, for any set of $\hat{C}_j$, solving the dual provides the prices on the end of first period pigs (in the example case, $V_i$, $i = 1, 2$ and 3), the features of the problem enable relatively simple relationships to be developed giving these values as a function of the $\hat{C}_j$. Given the final period solution, the second to last period activities which dominate can be determined and similarly the value of space in this period. This in turn determines the values of the pigs potentially on hand at the end of the previous period. Following through all periods gives $V_i$, $i = 1, 2$, and 3. Similarly, values for the $d_i$ that are not zero are uniquely determined through:

\[
\begin{align*}
C_j &= V_i + a_{ij}V_s - V_k - d_j \\
\therefore d_j &= V_i + a_{ij}V_s - V_k - C_j
\end{align*}
\]

To demonstrate the procedure consider the derivation of the relationship giving $V_3$. This variable represents the M.V.P. or price of a pig class which must not be carried beyond the first period of the valuation model. In the example problem the primal activity with $C_1$ is the only activity using this pig class. There can, of course, be several such activities but simple dominance gives the one to be
included in the model. Thus, \( d_1 \) will equal zero. Therefore:

\[
C_1 = V_3 + 7V_4
\]

\[\therefore V_3 = C_1 - 7V_4\]

But \( V_4 = \frac{1}{5}C_6 + \frac{1}{5}V_5 \) as \( d_6 = 0 \) due to the primal activity having \( C_6 \) always being basic.

From

\[
\begin{align*}
C_8 &= V_5 + 5V_8 - V_{10} - d_8 \\
V_5 &= C_8 + 5V_8 + V_{10} + d_8
\end{align*}
\]

\[\therefore V_4 = \frac{1}{5}C_6 + \frac{1}{5}C_8 - V_8 + \frac{1}{5}V_{10} + \frac{1}{5}d_8\]

\[\therefore V_3 = C_1 - \frac{7}{5}C_6 - \frac{7}{5}C_8 + 7V_8 - \frac{7}{5}V_{10} - \frac{7}{5}d_8\]

But, from

\[
C_{12} = 5V_8 - V_9 \text{ (} d_{12} = 0 \text{ as activity 12 is a weaner purchase activity)}
\]

\[
V_8 = \frac{1}{5}C_{12} + \frac{1}{5}V_9
\]

(i). \( \therefore V_3 = C_1 - \frac{7}{5}C_6 - \frac{7}{5}C_8 + \frac{7}{5}C_{12} + \frac{7}{5}V_9 - \frac{7}{5}V_{10} - \frac{7}{5}d_8 \)

Thus, as \( V_3 \) depends on \( d_8 \) consider the value it will take on. Further, as \( C_1, C_6, C_8, C_{12} \) are all constants, combine then into one constant, \( Q_1 \). Similarly, for constants appearing in the following relationships combine then to form a new constant, \( Q_1 \). Also substitute for \( V_9 \) and \( V_{10} \).

\[
\text{If } d_8 = 0 \quad V_3 \text{ has the form:}
\]

(ii) \[
V_3 = Q_1 + \frac{7}{5}C_{15} - \frac{7}{5}C_{16} + \frac{7}{25}C_{18}
\]

\[
\text{If } d_{14} = 0,
\]

from \( C_{14} = V_5 + 5V_8 - d_{14} \)

\[
V_5 = C_{14} - 5V_8
\]

from \( C_8 = V_5 + 5V_8 + V_{10} + d_8 \) and substituting for \( V_5 \),

\[
d_8 = C_{14} - V_{10} - C_8
\]
Thus, substituting for \( v_{10} \) in the \( d_8 \) relationship and using this in (i) gives:

\[
(iii) \quad v_3 = q_2 + \frac{7}{5} c_{15} - \frac{7}{5} c_{18}
\]

If \( d_9 = 0 \)

From \( c_9 = v_5 + 5v_8 - v_{11} - d_9 \)
\( v_5 = c_9 - 5v_8 + v_{11} \)

From \( c_8 = v_5 + 5v_8 + v_{10} + d_8 \) and substituting for \( v_5 \),
\( d_8 = c_9 + v_{11} - v_{10} - c_8 \)

Thus, substituting for \( v_{10} \) and \( v_{11} \) in this relationship and using the result in (i) gives:

\[
(iv) \quad v_3 = q_3 + \frac{7}{5} c_{15} + \frac{14}{25} c_{18}
\]

Now consider the relationships determining which of \( d_8', d_9 \) or \( d_{14} \) equal zero. (That is, which of the primal activities having net revenues of \( c_8', c_9 \) and \( c_{14} \) will dominate.)

From \( c_{14} = v_5 + 5v_8 - d_{14} \)
\& \( c_9 = v_5 + 5v_8 - v_{11} - d_9 \)
\& \( c_8 = v_5 + 5v_8 - v_{10} - d_8 \)
as \( v_5 \) and \( 5v_8 \) are common the dominating activity depends on \( v_{11} \) and \( v_{10} \).

Thus, substituting for \( v_{11} \) and \( v_{10} \), if:

(a) \( c_{14} \geq c_9 + c_{17} - \frac{7}{5} c_{18} \)
\& \( c_{14} \geq c_8 + \frac{c_{16}}{6} - \frac{5}{5} c_{18} \)
then \( d_{14} = 0 \)

(b) \( c_9 \geq c_{14} - c_{17} + \frac{7}{5} c_{18} \)
\& \( c_9 \geq c_8 - c_{17} + \frac{c_{16}}{6} + \frac{1}{5} c_{18} \)
then \( d_9 = 0 \)
(c) \[ \hat{C}_8 = \hat{C}_{14} - \hat{C}_{16} + \frac{6}{5} \hat{C}_{18} \]

\[ \hat{C}_9 \leq \hat{C}_{17} - \hat{C}_{16} - \frac{1}{5} \hat{C}_{18} \]

then \( d_8 = 0 \)

An inspection of (a), (b) and (c) indicates that as \( \hat{C}_{18} \) increases the tendency is for \( d_{14} \) to become zero. If \( d_{14} \) is initially zero, it will remain so. As \( \hat{C}_{16} \) increase up to its maximum possible value the tendency is for \( d_8 \) to become zero.

From the three \( V_3 \) relationships, (ii), (iii) and (iv), it can be seen any increase in \( \hat{C}_{15} \) increases \( V_3 \) in a continuous and constant direction. This movement does not depend on the dominating activities (that is, the \( \hat{C}_{16} \) and \( \hat{C}_{18} \) values). As \( \hat{C}_{16} \) increases \( V_3 \) is not effected if \( d_8 > 0 \). If \( \hat{C}_{16} \) increase sufficiently to make \( d_8 = 0 \), \( V_3 \) declines at a constant rate as \( \hat{C}_{16} \) continues to increase. In contrast, as \( \hat{C}_{18} \) increases \( V_3 \) may decline or increase if \( d_8 > 0 \) but it will eventually decline and continue to do so if \( d_{14} \) becomes zero.

The significant features of the variables are:

(i) \( V_3 \) is the value of a pig that cannot be carried beyond the 1st period of the valuation model.

(ii) \( \hat{C}_{18} \) determines \( V_{12} \), the value of space in the last period, and therefore directly effects \( V_9 \), \( V_{10} \) and \( V_{11} \).

(iii) \( \hat{C}_{16} \) determines \( V_4 \), the value of space in the first period of the valuation model, through its effect on \( V_{10} \) and \( V_5 \).

(iv) \( \hat{C}_{15} \) determines \( V_8 \), the value of space in the second period of the valuation model, through its effect on \( V_9 \).

Thus, as the second period space value increases, \( V_3 \) increases in a constant and continuous direction. With increasing 1st period space value, \( V_3 \) is either unaffected, constant for a range and then declines at a constant rate, or continuously declines. Increasing final period space value always affects \( V_3 \) by either initially reducing it and then increasing it or continuously increasing it. In order to appreciate the complete significance of variations in the variable prices the \( V_1 \) and \( V_2 \) relationships need to be considered.

If more than one \( d_i \) (i = 8, 9 and 14) is zero for any \( \hat{C}_j \) combination, two, or three, of the \( V_3 \) relationships will give the same
value. Given a marginal price change, depending on the change combination, it is likely only one of the $d_i$ will remain constant.

Using the same approach $V_1$ and $V_2$ relationships can be derived. These depend in part on the $V_5$ relationship and therefore which $d_i$ ($i = 8, 9$ and 14) is zero as well as other dominance relationships. For $V_2$, the value of a pig which can be carried for a maximum of two periods within the valuation model, the relevant $d_i$ are $d_4$ and $d_5$. The following condition gives the zero $d_4$:

$$\begin{align*}
\text{if} & \quad C_4 - C_{13} - C_5 + \frac{7}{5} C_{12} > \frac{7}{5} (\hat{C}_{18} - \hat{C}_{15}), \text{ then } d_4 = 0 \\
\text{if} & \quad C_4 - C_{13} - C_5 + \frac{7}{5} C_{12} < \frac{7}{5} (\hat{C}_{18} - \hat{C}_{15}), \text{ then } d_5 = 0.
\end{align*}$$

For the combination of $(d_{10}, d_9$ and $d_{14})$ and $(d_4$ and $d_5)$ the value of $V_2$ is given by:

$$\begin{align*}
d_8 = d_4 = 0 &: V_2 = Q_4 + \frac{6}{5} \hat{C}_{15} - \frac{6}{5} \hat{C}_{16} + \frac{6}{25} C_{18} \\
d_9 = d_5 = 0 &: V_2 = Q_5 - \frac{1}{5} \hat{C}_{15} - \frac{6}{5} \hat{C}_{16} + \frac{41}{25} C_{18} \\
d_9 = d_4 = 0 &: V_2 = Q_6 + \frac{6}{5} \hat{C}_{15} - \frac{6}{5} \hat{C}_{16} \\
d_5 = d_5 = 0 &: V_2 = Q_7 - \frac{1}{5} \hat{C}_{15} - \frac{12}{5} \hat{C}_{16} + \frac{7}{5} \hat{C}_{18} \\
d_{14} = d_4 = 0 &: V_2 = Q_8 + \frac{6}{5} \hat{C}_{15} - \frac{12}{5} \hat{C}_{16} + \frac{42}{15} \hat{C}_{18} \\
d_{14} = d_5 = 0 &: V_2 = Q_9 - \frac{1}{5} \hat{C}_{15} - \frac{12}{5} \hat{C}_{16} + \frac{71}{25} \hat{C}_{18}
\end{align*}$$

The conditions indicating the zero $d_i$s show that as $\hat{C}_{15}$ increases $d_4$ tends to become zero and as $\hat{C}_{16}$ increases the tendency is for $d_8$ to become zero. As $\hat{C}_{18}$ increases $d_{14}$ and $d_5$ tend to become zero. But the value relationships show that $V_2$ increases or is unaffected as $\hat{C}_{18}$ increases in all cases. Similarly, as $\hat{C}_{16}$ increases, $V_2$ decreases in all cases. However, $V_2$ either increases or decreases, depending on the case, as $\hat{C}_{15}$ increases.

For $V_1$, the value on a pig that can be carried through to the final period, the $d_i$ values of relevance are $(d_8, d_9, d_{14}), (d_2, d_3, d_7)$ and $(d_{10}, d_{11})$. The conditions giving the member of the pair $(d_{10}, d_{11})$ which will be zero are:
\[
\begin{align*}
\text{if} & \quad C_{11} - C_{18} - C_{10} \geq -\frac{7}{5} \hat{C}_{18}', \quad \text{then } d_{11} = 0 \\
\text{if} & \quad C_{11} - C_{18} - C_{10} < -\frac{7}{5} \hat{C}_{18}', \quad \text{then } d_{10} = 0
\end{align*}
\]

The conditions for determining the \((d_2, d_3, d_7)\) values are:

(a) \textbf{If} \(d_{10} = 0\) and:

(i) \(Q_{10} - Q_{11} \geq -\frac{1}{5} \hat{C}_{15} + \frac{1}{5} \hat{C}_{18}\)

\(\&\) \(Q_{10} - Q_{12} \geq \frac{1}{5} \hat{C}_{15} + \hat{C}_{16} - \frac{7}{5} \hat{C}_{18}'\) \quad \text{then } d_2 = 0

or (ii) \(Q_{11} - Q_{10} \geq \frac{1}{5} \hat{C}_{15} - \frac{1}{5} \hat{C}_{18}\)

\(\&\) \(Q_{11} - Q_{12} \geq \frac{2}{5} \hat{C}_{15} + \hat{C}_{16} - \frac{8}{5} \hat{C}_{18}'\) \quad \text{then } d_3 = 0

or (iii) \(Q_{12} - Q_{10} \geq \frac{1}{5} \hat{C}_{15} - \hat{C}_{16} + \frac{7}{5} \hat{C}_{18}\)

\(\&\) \(Q_{12} - Q_{11} \geq \frac{2}{5} \hat{C}_{15} - \hat{C}_{16} + \frac{8}{5} \hat{C}_{18}'\) \quad \text{then } d_7 = 0

(b) \textbf{If} \(d_{11} = 0\) and:

(i) \(Q_{10} - Q_{11} \geq -\frac{1}{5} \hat{C}_{15} + \frac{2}{5} \hat{C}_{18}\)

\(\&\) \(Q_{10} - Q_{12} \geq \frac{1}{5} \hat{C}_{15} + \hat{C}_{16} - \frac{7}{5} \hat{C}_{18}'\) \quad \text{then } d_2 = 0

or (ii) \(Q_{11} - Q_{10} \geq \frac{1}{5} \hat{C}_{15} - \frac{8}{5} \hat{C}_{18}\)

\(\&\) \(Q_{11} - Q_{12} \geq \frac{2}{5} \hat{C}_{15} + \hat{C}_{16} - \frac{8}{5} \hat{C}_{18}'\) \quad \text{then } d_3 = 0

or (iii) \(Q_{12} - Q_{10} \geq -\frac{1}{5} \hat{C}_{15} - \hat{C}_{16}\)

\(\&\) \(Q_{12} - Q_{11} \geq -\frac{2}{5} \hat{C}_{15} - \hat{C}_{16} + \frac{8}{5} \hat{C}_{18}'\) \quad \text{then } d_7 = 0

For the combinations of \((d_2, d_3, d_7), (d_{10}, d_{11})\) and \((d_8, d_9, d_{14})\),
the value of \(V_1\) is given by:

\[
\begin{align*}
\text{if } d_{10} = d_2 = 0 : \quad V_1 &= Q_{13} + \frac{7}{3} \hat{C}_{15} - \hat{C}_{16} - \frac{17}{15} \hat{C}_{18} \\
\text{if } d_{10} = d_3 = 0 : \quad V_1 &= Q_{14} - \frac{4}{15} \hat{C}_{15} - \hat{C}_{16} + \frac{22}{15} \hat{C}_{18} \\
\text{if } d_{10} = d_7 = 0 : \quad V_1 &= Q_{15} + \frac{2}{15} \hat{C}_{15} - \frac{7}{15} \hat{C}_{18}
\end{align*}
\]
\[ d_8 = d_{11} = d_2 = 0 : V_1 = Q_{16} + \frac{7}{3} \hat{c}_{15} - \hat{c}_{16} - \frac{2}{3} \hat{c}_{18} \]

\[ d_8 = d_{11} = d_3 = 0 : V_1 = Q_{17} - \frac{4}{15} \hat{c}_{15} - \hat{c}_{16} + \frac{8}{15} \hat{c}_{18} \]

\[ d_8 = d_{11} = d_7 = 0 : V_1 = Q_{18} + \frac{2}{15} \hat{c}_{15} - \hat{c}_{16} + \frac{16}{15} \hat{c}_{18} \]

\[ d_9 = d_{10} = d_2 = 0 : V_1 = Q_{19} + \frac{7}{3} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} - \hat{c}_{18} \]

\[ d_9 = d_{10} = d_3 = 0 : V_1 = Q_{20} - \frac{4}{15} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} + \frac{8}{15} \hat{c}_{18} \]

\[ d_9 = d_{10} = d_7 = 0 : V_1 = Q_{21} + \frac{2}{15} \hat{c}_{15} + \frac{2}{3} \hat{c}_{16} - \frac{1}{3} \hat{c}_{18} \]

\[ d_9 = d_{11} = d_2 = 0 : V_1 = Q_{22} + \frac{7}{3} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} - \frac{8}{15} \hat{c}_{18} \]

\[ d_9 = d_{11} = d_3 = 0 : V_1 = Q_{23} - \frac{4}{15} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} + \frac{2}{3} \hat{c}_{18} \]

\[ d_9 = d_{11} = d_7 = 0 : V_1 = Q_{24} + \frac{2}{15} \hat{c}_{15} + \frac{2}{3} \hat{c}_{16} - \frac{14}{15} \hat{c}_{18} \]

\[ d_{14} = d_{10} = d_2 = 0 : V_1 = Q_{25} + \frac{7}{3} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} - \frac{29}{15} \hat{c}_{18} \]

\[ d_{14} = d_{10} = d_3 = 0 : V_1 = Q_{26} - \frac{4}{15} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} + \frac{2}{3} \hat{c}_{18} \]

\[ d_{14} = d_{10} = d_7 = 0 : V_1 = Q_{27} + \frac{2}{15} \hat{c}_{15} + \frac{2}{3} \hat{c}_{16} - \frac{19}{15} \hat{c}_{18} \]

\[ d_{14} = d_{11} = d_2 = 0 : V_1 = Q_{28} + \frac{7}{3} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} - \frac{22}{15} \hat{c}_{18} \]

\[ d_{14} = d_{11} = d_3 = 0 : V_1 = Q_{29} - \frac{4}{15} \hat{c}_{15} - \frac{1}{3} \hat{c}_{16} - \frac{4}{15} \hat{c}_{18} \]

\[ d_{14} = d_{11} = d_7 = 0 : V_1 = Q_{30} + \frac{2}{15} \hat{c}_{15} + \frac{2}{3} \hat{c}_{16} - \frac{28}{15} \hat{c}_{18} \]

Examination of the relationships giving the zero \( d_1 \)'s and the \( V_1 \) equations leads to the following conclusions:

(a) As \( \hat{c}_{18} \) (last period space value) increases, the tendency is for \( d_{14}, d_{10} \) and \( d_3 \) to become zero. Thus, \( V_1 \) continuously decreases or decreases followed by a possible increase.

(b) As \( \hat{c}_{16} \) (3rd to last period space value) increases, \( d_{10} \) and \( d_{11} \) dominance is unaffected but \( d_8 \) and \( d_7 \) tend to become zero. Thus, \( V_1 \) continuously decreases or decreases followed by a possible increase.

(c) As \( \hat{c}_{15} \) (2nd to last period space value) increases \( d_{10}', d_{11}, d_8, d_9 \) and \( d_{14} \) dominance is unaffected but either \( d_2 \) or \( d_7 \) may become zero. Thus, \( V_1 \) continuously increases or decreases followed by a possible increase.
A major feature of these relationships is that if \( d_8 = 0 \), as \( \hat{C}_{16} \) increases, \( V_1 \) either decreases by one unit for each unit change in \( \hat{C}_{16} \) or is unaffected. The significance is that if the \( V_1 \) pig is carried through to the last period its price is given by \( \hat{C}_{16} \) so that a direct relationship occurs. For the other cases \( \hat{C}_{16} \) affects \( V_1 \) due to its influence on \( V_4 \) as well, the first period space value. If a more complex example had been used in which such a price did not influence the value of a purchased weaner, then the effect in all cases would be the same as the \( d_8 = 0 \) case. Further, the effect would be a direct increase.

The total significance of all the valuation relationships is discussed in the next section.

4. TOWARDS FINDING THE EXTREME POINTS OF THE VALUATION OR PRICE SET

Recall that for any given combination of last period prices, taken from within the absolute maximum and minimum bounds, a set of mutually operative values occur for each pig class on hand at the end of the first period of the total model. These form the prices for determining the optimal first period decision. As the end prices are varied a new set of values may occur so that all combinations map out a bounded set of values within \( n \) dimensional space where \( n \) equals the number of pig classes. An end price combination gives a vector \( V \) with components \( V_i \), \( i = 1, 2, ..., n \), representing the price of a pig of a given class at the end of the first period. In general, to test for first period optimality it is necessary to find the extreme points of the set of \( V \). However, for many dynamic planning problems this may be impractical. In the pig fattening case this would be achieved by setting up the valuation model, in the final period with two sets of constraints. One set for the absolute minimum and the other for the maximum prices. The first set are formed as minimum constraints while the second are formed as maximum constraints. Then, if an objective function constraint is also included and parameterised the extreme points can be located as it will form an extreme point at every point on the boundary of the bounded space formed by the maximum and minimum price constraints. But, with \( X \) variable prices there will be approximately \( 2^X \) extreme points.

It will be shown, in the pig fattening case, that it is frequently only necessary to locate a limited number of the extreme points. In
order to prove this it is necessary to consider the nature of value changes as ending prices are changed. This is achieved by considering the value relationships for the example problem.

The value relationships show that the effect of any variation in the variable prices on the value of pigs on hand at the start of the valuation model depends on whether a particular price is the value of pig transferred through to the final period or whether it affects the value of space in some period. Pigs on hand cannot affect space values except where a class that it can form in a future period can also be purchased. These cases will be treated as space value determinants and discussed below.

Any starting pig can be classed into one of three groups. Those that must be transferred through to the final period, those that may be transferred through and therefore can be sold in a prior period, and those that cannot be transferred through to the final period. In this later case variation in the ending prices can only effect their values through space value effects. In the other two cases, assuming all pigs are transferred through to the final period, any variation in the prices of pigs that can be supplied from the starting pigs will have a direct effect on their values. This was pointed out above with respect to C_{16}.

If a particular class is not transferred through to the last period, any variation of the end prices of pigs which the starting pig can potentially supply will have, initially, no effect on its value. If the change is an increase, this may be great enough for it to be profitable to carry the pig through. If this occurs there will then be a direct increase in its value to further end price increases.

As the example problem shows, any change in a price affecting a space value can effect all, and in most cases will, effect the values of all starting pigs. To provide the conclusions on the general nature of the effects that are given in the main text, consider the value movements as the space prices (C_{16}^{1}, C_{15}^{1} and C_{18}^{1}) are increased. Assume a solution is initially obtained with these prices set at their minimum. Inspection of the value relationships leads to the following observations:

(a) Increase 1st period space value (C^{1}_{16}).

All values (V_{1}, V_{2} & V_{3}) are decreased or unaffected if the complication of the primal activity with C_{7} net revenue is removed. This occurs simply because the value of resources used increases.
(b) Increase 2nd period space value \( (\hat{C}_{15}) \).

1. One period pigs' value \( (V_3) \) increases as the first period space value declines through the value of the pig determining this space value declining. This results from the increase in the 2nd period space value.

2. Pigs using both the 1st and 2nd periods either increase or decrease and then possibly increase in value \( (V_1 & V_2) \). The reason is that when a space value increases all previous space values decline as the increase in space value decreases the value of pigs determining the space values. But, more importantly, not only do the space values decline but also the value of the pigs which are carried over the period in which the space value increases. Thus, an increase in \( \hat{C}_{15} \) decreases \( V_1 \) & \( V_2 \). But if the decrease is sufficient to make it more profitable to switch to selling off the pig earlier, further increases, because it decreases the value of space in earlier periods, will then increase the value of the pig. This occurs as the increasing space value no longer directly influences the value. Thus, the decline terminates. If the pig is not carried through to the period in the initial solution, any increase in \( \hat{C}_{16} \) will always increase its value.

(c) Increase 3rd period space value \( (\hat{C}_{18}) \).

1. The value of \( (V_2 & V_3) \) pigs that must terminate prior to this period increase in value as the earlier space values decline though the effect on \( V_3 \) is affected by the \( C_7 \) primal activity. Alternatively,

2. The value of pigs using this period \( (V_1) \) decrease due to the increase in resource use value but may eventually increase if the pig is sold sooner. This occurs as space values in earlier periods decline.
5. THE EFFECT OF THE REQUIREMENTS VECTOR (b)

The conclusions developed rely on an optimal solution being independent of b. However, there will be cases where the optimal solution, as determined by $C_j$ and $a_j$, will not be feasible. This will occur where the available space prevents a group of pigs being carried through to their optimal weight and the best alternative is not to initially sell all, or some, of the pigs. In this case the alternatives are to:

(a) sell earlier (part or all) if this is a possible action,

(b) sell off (part or all) another group if this is possible,

(c) use a slower growth rate to spread the space requirement through time either for the particular group or a competing group.

Which of these actions dominates clearly depends on their relative profitabilities. Where selling is possible and is the best alternative, the M.V.P. of the relevant groups are given by the market prices. As the ending prices are changed so that the space values change these actions may change. Where the dominating alternative is to use a slower growth rate and this involves dividing a pig group into several treatment groups this can lead to a different basic solution form compared to the case where b does not effect the selection of activities. With more activities being basic than just the dominating activities, the weaner purchase activities may become non-basic. In this case it is more profitable to allocate the space to existing pigs rather than purchase a new group. Whether this occurs depends on the available space in relation to the number and type of pigs in the first period and on subsequent purchasing.

If the weaner purchase activity in a period is non-basic the space value is dependent on the activities replacing such activities. The development of value relationships similar to the relationships presented in the previous sections show that the space value is directly related to the M.V.Ps. of the pig types resulting from the two or more activities which replace the single dominating activity. (This would be expected as the space is utilised, at the margin, by the new group.) This means, in determining the end of first period valuation set, it is necessary to include in the list of space value determining prices


previously defined all ending prices which lead to the M.V.Ps. of pigs resulting from the division of a pig group.

The significance of all the value relationships in determining a planning horizon is discussed within Chapter IX.
APPENDIX VII

THE SET OF MARGINAL VALUE PRODUCTS AND
OPTIMAL SOLUTION STABILITY

For problems that can reasonably be approximated using a linear
programming model the set of possible valuation vectors form a convex
set. This occurs as the set is bounded by the final period dual
constraints with the prices set at both their maximum and minimum
values. Each such constraint defines a hyperplane giving a closed
half space and this is a convex set. Further, the intersection of
several convex sets is also convex.

For other problems to be tractable it will be necessary to
find supporting hyperplanes to the set such that the set is within
the set defined by the supporting hyperplanes. The vectors representing
all the hyperplane intersections must then be tested.

To prove that if the extreme point vectors give a common first
period decision, the decision will be the same for any price vector,
consider a problem in which the extreme points are denoted by \( \mathbf{v}_i \),
i = 1, ..., m. Take any two of these vectors, say \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) and take
an arbitrary point from the line joining these two points (the set of
points defining the line are the vectors \( \mathbf{q} = \lambda \mathbf{v}_1 + (1 - \lambda) \mathbf{v}_2 \), \( 0 < \lambda < 1 \)).
Let this be \( \mathbf{q}_1 \). Define a vector \( \mathbf{e} \) as:

\[
\mathbf{e} = \mathbf{q}_1 - \mathbf{v}_1
\]

Thus, some scalar \( \alpha \) exists such that

\[
\mathbf{v}_2 = \mathbf{v}_1 + \alpha \mathbf{e}
\]

Given the first period decision, let the prices on the assets supplied
by the basic activities be \( \mathbf{v}_j' \), \( j = 1, 2, ..., k \), and generally be
denoted by the vector \( \mathbf{v}_{B,i} \). Note that the components of all \( \mathbf{v}_{B,i} \) are
assumed to be the value for the same assets. As each first period
solution for the extreme point valuation vectors are optimal:

(i) \( \mathbf{v}_{B,1} \preceq \mathbf{v}_1 \)

and

(ii) \( \mathbf{v}_{B,2} \preceq \mathbf{v}_2 \)
where
\[ Y = B^{-1} A \]
and
\[ A \] is the matrix of input-output coefficients.

Further, \( Y \) is the same for both solutions as \( B \) is the same.

Now,
\[ V_{B,2} = V_{B,1} + \alpha e_B \]
and
\[ V_2 = V_1 + \alpha e \]
where
\[ e_B = \text{the vector of } e \text{ components associated with the basic activities.} \]

Substitute in (ii) above:

(iii) \( \therefore (V_{B,1} + \alpha e_B) Y \geq V_1 + \lambda e \)

Replace \( \alpha \) with \( \lambda \)
\( \therefore \) if \( \alpha = \lambda \),

(iv) \( (V_{B,1} + \lambda e_B) Y \geq V_1 + \lambda e \)

But, if \( \lambda \) varied between zero and \( \alpha \) the line joining \( V_1 \) and \( V_2 \) is mapped out.

Also, for \( \lambda = 0 \), (iv) holds as it reduces to (i).

Rearrange (iv) assuming \( \lambda = \alpha \),
\[ \therefore V_{B,1} Y + \lambda e_B Y \geq V_1 + \lambda e \]
\[ \therefore V_{B,1} Y - V_1 \geq \lambda (e - e_B Y) \]
\[ \therefore \frac{(V_{B,1} Y - V_1)}{(e - e_B Y)} \geq \lambda, \text{ all } j \text{ where } j \text{ refers to the } j \text{th component.} \]

Now consider other values of \( \lambda \).

As \( \lambda < \alpha \),
\[ \therefore \frac{(V_{B,1} Y - V_1)}{(e - e_B Y)} \geq \lambda \text{ all } j, \ 0 \leq \lambda < \alpha \]
\[ \therefore V_{B,1} Y - V_1 \geq \lambda (e - e_B Y) \text{ all } \lambda \text{ values.} \]
\[ \therefore (V_{B,1} + \lambda e_B) Y \geq V_1 + \lambda e \text{ all } \lambda. \]
That is, any valuation vector on the line joining $V_1$ and $V_2$ gives the same optimal first period solution.

Similarly, valuation vectors on the lines joining any two extreme point vectors give the same optimal solution. Further, any vector on a line joining any two vectors taken from lines joining the extreme points will have the same optimal solution for the same reasons. Thus, the whole bounded set will give the same first period solution if the extreme points have the same solution.
DYNAMIC PLANNING AND PIG FATTENING

ADDENDA

p 2, line 1:
The use of techniques...should read...Techniques...
p 4, line 7:
...facilities exist...should read...facilities exists...
p 5, line 13:
...options exist...should read...options exists...

p 5. Add the following footnote:
The arrows indicate the directions in which resources, inputs and outputs flow.

p 6, line 14:
...options exist...should read...options exists...

p 8, line 1:
...largely effect...should read...largely affect...

p 8, line 7:
...involves both quantity...should read...involves quantity...

p 8, lines 8 & 9:
...these two groups in...should read...the feed demanding and supplying units in...

p 11, line 14:
...the marginal monetary return of decisions.
Should read
...the marginal cash return of actions...

p 12, line 26:
...(Rao, 1968)...should read...(Rao and McConnell, 1968)...
p 18, line 10:
... ad libitum... should read... ad libitum...

p 24, line 3:
... used in this way... should read... used as the basic
framework for a sophisticated model...

p 24, line 28:
... (Computer Systems... should read... (College of Agriculture)...

p 36, line 6:
Similarly pigs do not have to be all sold... should read...
Similarly all pigs do not have to be sold...

p 36, Section 3.3:
Delete the 2nd and 3rd sentences in this section and replace
them with the following sentence:-
This can be seen when it is understood that in each
decision period (which could well be a week), an updated
value for most decision variables is required.

p 38, line 14:
... periods if for no other reason than the... should read
... periods as, if for no other reason, the...

p 41, line 1:
... \( V_T \sum_{t=1}^{T} \ldots \) should read... \( V_T = \sum_{t=1}^{T} \ldots \)

p 41, line 9:
... \( e^{-yt} \ldots \) should read... \( e^{-y^t} \)

p 42, line 19:
... level if \( j \) th activity... should read... level of \( j \) th activity...

p 44, line 12:
... = \( \frac{52}{t+2} \ldots \) should read... = \( \frac{52}{t+2} \)

p 45, line 3:
... should read... \( \Pi_t = \ldots \)
P 46, line 10:
...is synonymous...should read...synonymous...

P 46, line 24:
\[ \sum_{i=1}^{S-1} \ldots \text{should read} \sum_{i=1}^{S-1} \ldots \]

P 50, line 14:
...energy source...should read...energy source...

P 51, line 17:
Maximise \( y = \ldots \text{should read} \) Maximise \( y = \ldots \)

P 61, line 19:
\( x_t \) should read \( x_t \)

P 67, line 12:
...confirmation...should read...conformation...

P 72, line 7:
\( p(\gamma^d_o / \gamma^q) \) should read \( p(\gamma^d_o / \gamma^q) \).

P 72, line 18:
...non a-symptotic. should read...non asymptotic.

P 76, line 24:
...this nature does...should read...this nature do...

P 79, line 11:
...weaners are...should read...weaners is...

P 79, line 27:
Handley...should read Hanley,...

P 94, line 20:
...Loftgard...should read...Loftsgard...

P 95, line 16:
...above allow...should read...above allows...

P 97, line 5:
...implies...should read...assumes...

P 103, Section 5.3:
Delete the 2nd sentence.
p 106, line 6:

...moevements... should read... movements...

p 116, line 11:

...this later estimate... should read... this latter estimate...

p 120, line 17:

...jth ingredient, should read... jth ingredient (excluding protein energy),

p 137, line 17:

is presented... should read... are presented...

p 152, line 20:

...$\sigma_2 = ...$ should read... $\sigma_2^2 = ...$

p 153, line 1:

...not asymptotic. Should read... not asymptotic (the value of 1.5 in the relationships was obtained from experimentation).

p 163, line 11:

For both cases there... should read... For the former case there...

p 167, line 15:

...the horizon that... should read... the horizon of either the positive or normative horizons that...

p 168, line 13:

... containing less periods... should read... containing fewer periods...

p 170, line 9:

$... \max\ [b_{ij}^{-1} \text{ should read } \max\ [b_{ih}^{j-1} \text{ all } i, h \text{ all } i, h ...}$

p 173, line 2:

... period $\not\in ...$ should read... period 2...

p 176, line 12:

... variety... should read... variety...

p 180, line 14:

... periods are... should read... periods is...

p 190, line 11:

Cases may exist, or at least be closely allid with, in which... should read Cases may exist in which...
p 194, line 24:
\[ \cdots + \frac{X_{i}^{j-1}}{X_{i}^{j}} \ldots \text{should read} \cdots + \frac{X_{i}^{j+1}}{X_{i}^{j}} \ldots \]

p 201, line 22:
\[ \cdots = c_{b}A_{i}^{-1} \cdots \text{should read} \cdots = c_{b}A_{j}^{-1} \cdots \]

p 228, line 3:
ALEXANDER, R. H. (1957) \ldots \text{should read} ALEXANDER, R. H. & HUTTON (1957) \ldots

p 232, line 41:
HARLEY, \ldots \text{should read} HANLEY, \ldots

p 234, line 9:
JOHNSTON, J. (1966) \ldots \text{should read} JOHNSTON, J. (1965) \ldots

p 237, line 16:
RENBORG, W. \ldots \text{should read} RENBOR, U. \ldots

p 248, line 7:
\ldots \text{condition and nor} \ldots \text{should read} \ldots \text{condition, nor} \ldots

p 248, line 10:
\text{conditions change the} \ldots \text{should read} \text{conditions changes the} \ldots
FURTHER ISSUES*

I. INTRODUCTION

In some areas of a study there is frequently a number of approaches that can be taken in providing an answer or in presenting ideas and theories. There are even topics on which opposing views are held by many people, often due to the impossibility of setting up completely objective analysis on which all commentators can agree. Some of the approaches used and topics covered in this thesis are cases in point. In order to give perspective to these areas the following discussion is included as part of the thesis. The specific points discussed include the interpretation of the neo-classical model of the firm, the form of the objective function used in the models, the question of obtaining probability estimates for use in decision making, and the potential for commercial use of the models developed.

II. THE NEO-CLASSICAL MODEL OF THE FIRM

The neo-classical theory or model of the firm is usually regarded as being the set of concepts originally developed by Cournot in 1838. ¹ These theories were not generally considered until the 1920s and 1930s, but over this period they were re-discovered and developed by people such as Robinson ² and Hicks. ³ These developments were

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¹ Cournot A. (Trans. by N. T. Bacon, 1897), Recherches sur les Principes Mathematiques de la Theorie des Richesses, N.Y., Macmillan & Co.


* New references are indicated by a superscript and their details given in a footnote.
designed to provide a theoretical base from which the reaction of firms to economic stimuli could be predicted. Hicks, in particular, considered in some detail the dynamic problems facing a firm. The direct planning use of these models was not stressed for a number of years and it was not until 1952 that they were put together in a text book designed to represent the farm planning problem (Heady, 1952). This book stimulated the use of the theory of the firm as a framework on which to base experimental work. Frequently, only components of the theory were used due to the complexity of the problem when all the components were taken together. An example (Heady et al. 1961) of the use of components of the overall model was used in this study as the basis on which to discuss the adequacy of the model.

Conceptually, the theory as expounded by Hicks does allow for some of the criticism introduced in Chapter 3, particularly the dynamic planning questions. However, due to the mathematical difficulties and the imprecise nature of some of the ideas, the theory is not applied in this way. Naylor (1969) makes this clear when he lists the deficiencies of the model from a predictive point of view and in doing so assumes the model implies the idea of static equilibrium.

Since the 1930s many workers have improved the theory of the firm. Some of this work has been brought together by Dillon (1968). The models discussed indicate that many attempts have been made to introduce the dynamic nature of planning. However, computational difficulties exist due to the extreme complexity of dynamic bio-economic systems. Naylor (1971, p 8) notes:

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Although it may be conceptually possible to formulate a mathematical model describing the behaviour of a dynamic, multi-process firm operating under uncertainty, present day mathematics is simply incapable of yielding solutions to a problem of this magnitude."

Due in part to these problems, linear programming and simulation (Dent and Anderson 1971) have been used in attempts to formulate solvable models. Indeed, linear programming has been used as a basis for formulating an alternative theory of the firm. This means that when discussing theories of the firm it might be regarded that linear programming should be included in these discussions. The criticism of the theory of the firm outlined in Chapter 3 did not take this point of view. Thus, the criticism did not allow for later developments.

III. THE FORM OF THE OBJECTIVE FUNCTION

The planning model developed was based on an objective of maximising expected total gross margin. If an alternative objective function had been used the conclusions and models developed may have been different and so it is important to introduce these possible effects.

The theories developed for determining a planning horizon depend on the ability to define bounds on marginal value products (M. V. Ps.). Provided, therefore, that a M. V. P. can be formulated for resources on hand then the same concepts can be applied. To formulate a M. V. P. requires a quantifiable objective function.

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Objective functions can generally be classified into lexicographic or continuous type functions. Conceptually, any form of continuous objective function can be used to provide M.V.P. estimates. Similarly, lexicographic objectives can provide a basis for estimating M.V.P.s where it is assumed the marginal utility for a lexicographic system is initially a positive constant but, at a cut-off point, it becomes zero. Thus, in theory M.V.P. estimates can be made for any form of objective. The difficulty, however, lies in obtaining satisfactory methods of obtaining measures of utility, even on an ordinal basis, that are simple to implement. In a practical sense, no more than intuitive adjustments to conclusions based on a simple objective, such as that used in this study, may be warranted at this stage.

The actual objective used will influence the structure of the models used. For example, where risk is an important component of the objective, the models can no longer be based on expected prices as their use will lead to a biased estimate of expected utility. The general form that a programming model designed to allow for risk attitudes must take has been discussed by Rae. Similarly, including other components of utility, such as leisure, would require the models to be modified to allow for the several factors contributing to utility and the, in general, non-linear nature of the relationships. Such modification would invariably add to the complexity of the models.

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IV. PROBABILITIES AND DECISION MAKING

The models developed incorporate the concept of probability. In the discussion stress was not laid on the alternative concepts of probability and, therefore, the alternative estimation methods available. The solution to the estimation problem is clearly critical to the successful use of the models, both in an intuitive way and in their direct application.

Dillon (1971) discusses the alternative concepts of subjective and objective probability. He defines subjective probability as the measure of the degree of belief the decision maker has about a particular outcome occurring. On the other hand, objective probability is the chance of an outcome occurring where the chance is calculated from relative frequency data or from a logical analysis. Dillon argues that because it is the decision maker who must accept the responsibility for the eventual outcome of any decision, the probability to use should be subjective probability. However, it should be noted that there is no reason why objective probability cannot form the basis of the probabilities accepted by the decision maker. Once accepted, they become both the objective and subjective probabilities. A real difficulty is that in many cases objective probabilities cannot be formulated as a guide due to lack of information. In these cases there is no choice but to use subjective probability where a probability based decision analysis is used.

Many contemporary workers regard all probability estimates as being subjective once they are used, and therefore accepted, by the decision maker. An implication of this personalized approach is that

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results will only apply to one decision maker. To overcome this problem the idea of stochastic dominance has been developed. This concept leads to the derivation of sets of efficient plans from a risk attitude point of view.

Within this study, methods were developed for calculating objective probabilities on genetic classes and physical outcomes. It must be stressed that where subjective probability is accepted as a rational base for decision making, the decision maker should use his subjective estimates and these may or may not be similar to the objective probability estimate derived from the techniques developed.

V. THE POTENTIAL FOR COMMERCIAL APPLICATION OF THE SYSTEMS DEVELOPED

The planning experiments performed using the models and methods developed indicate that total gross margin can be increased as much as 100 per cent over a simple policy of maximising growth rates under extreme price and cost conditions typical of the Christchurch, New Zealand area. For large enterprises with net returns of, for example, around $50,000 a 100 per cent increase in return means considerable expenditure can be devoted to the operation of a management scheme similar to the systems developed. Whether such gains could be realised will depend on the features of the environment in which the unit is currently operating. Where there is a history of rapidly changing prices and these changes are difficult to predict and, further, this situation is expected to continue, the potential gains from utilising the formal continuous planning system are likely to be realised. If there are many large (3000 plus animal throughput annually) farms

operating in one area under these conditions, it would be worthwhile exploring the demand for a management scheme based on the systems developed.

There are many factors that need to be assessed in considering the demand and potential success of such a scheme. The major factors are listed below.

(i) The inclination of farmers to operate an adequate recording scheme and to submit all the planning data required.

(ii) The attitude of farmers to accepting and acting on detailed weekly decision advice.

(iii) The desire of farmers to improve their decision making and the amount of improvement possible. This must rest on their objectives. Further, the ability to incorporate their objective in the decision model is critical to its success.

(iv) The ability of farmers to relate the change in income (and utility in general) to the contributing factors. In particular, distinguishing between the effect of using management aids compared with other factors such as price changes.

(v) The attitude of extension personnel to the use of automated management aids and their level of training.

(vi) The availability of farm management experts capable of developing the systems for use in a specific area and of developing the necessary programmes for the matrix generator and report writers (as these are essential to minimize costs).
(vii) The availability of computing facilities and a system of promptly returning output information.

Assuming the desire to utilise sophisticated management systems exists, problems in any of the above factors can be overcome given adequate training facilities, time and the necessary financial resources. Thus, the real criteria for deciding whether the systems can be used commercially must be the number of large farms in one area (the area may be a whole country provided the technology is the same throughout) that can use the same basic model and the desire of the farmers to change. This assumes conditions (prices, costs etc.,) change frequently so that a large fraction of the potential 100 per cent increase over current income is a real possibility. Where prices are fixed the gains from continuous planning are less and are unlikely to warrant the use of the models developed.

While each case must be specifically assessed, the following figures give an indication of possible costs and therefore the necessary benefits. Development costs are likely to be approximately $60-80,000 and running costs approximately $20-30 per use of the system plus the involvement of extension personnel. Assuming a ten year life of the system (in fact it would be continuously improved over the years and last for many more years) and a discount rate of 10 per cent, an annual payment of approximately $13,000 would be required to cover development costs. Allowing for an extension personnel contact amounting to 5 days per year at a cost of $70/day, the running costs would amount to $800 per farm for 18 uses per year.

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10 This figure is based on the experience of people developing management aids in such institutions as Purdue University, Indiana, Michigan State University, Michigan and BOCM Silcocks. It is based on 1977 costs.
If 100 farms used the system the break even charge would have to be approximately $930/farm, for 200 farms the charge would have to be $865. These are likely to be acceptable costs for large farms so that provided, say, 20 per cent\(^{10}\) of large farmers used the system, a population of 500-1000 large farms would be necessary to support a scheme.