Working the blue gold: a personal journey with mathematical tools for water management

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Outline

1. Engineers, tools, and hydrology
2. Water quantity stories
3. We have control
4. Water quality stories
5. We still manage
6. Conclusions
Engineers, tools, and hydrology

- Hydrological process knowledge
- Simple concepts
- Application tools
- Mathematics

Scientists → Engineers
Simple concept: water flow and storage

Inflow - $I$

Storage - $S$

Outflow - $O$

Outflow is proportional to storage:

Mathematical tool for calculating daily flows:

$$O_k = a \cdot O_{k-1} + b \cdot I_k$$

Storage time $T = \frac{S(t)}{O(t)}$

Coefficients:

$$a = \exp(-1/T)$$
$$b = 1 - \exp(-1/T)$$
Scientific complexity

- Hydrological science can be complex, in terms of biophysical processes.

- Some complex processes can be described by complex concepts.

- Some complex concepts can be considered as systems of simple concepts.
A system of conceptual linear water storages

Inflow - $I$

Storage - $S$

Outflow - $O$

$O_k = a \ O_{k-1} + b \ I_k$

Mathematical tool

Conceptual catchment model (Nash, 1957)

$O_k = a_1 O_{k-1} + a_2 O_{k-1} + \ldots + b_1 I_k + b_2 I_{k-1} + \ldots$
Stochastic in Colorado (1970–72)
with Vujica Yevjevic (“Dr Y”) and his graduate students

- **Stochastic** hydrology refers to random influences
- A principal source is inherent randomness of climate dynamics
- Water resource modelling requires many realisations of likely hydrological time-series
Stochastic in Colorado (1970–72)
with Vujica Yevjevic (“Dr Y”) and his graduate students

Stochastic refers to random influences
Input $I$ can contain random components


\[ O_k = a_1 O_{k-1} + a_2 O_{k-1} + \ldots + b_1 I_k + b_2 I_{k-1} + \ldots \]

Economic & industrial applications

Auto-Regressive, Moving Average

ARMA model

can be a “black box”
Flood forecasting in Kuala Lumpur (1972-74)

FIG. 1 The catchment of the Klang River at Kuala Lumpur

Flood forecasting in Kuala Lumpur (1972-74)

Cardboard template placed over real-time record of rainfall & river stage observations, sent by telephone.

ARMA model calculations by electronic calculator

Cardboard mathematical tool - ARMA model

FIG. - 2 The template used for flood forecasting
Guided model for flood forecasting
with George Griffiths, 1994

Real-time flood stage forecasts for Waimakariri River, by simple model:

\[ O_k = a_t O_{k-1} + b_t I_k \]

with time-varying parameters.

Real-time, adaptive forecasting by use of the **Kalman Filter** algorithm

Kalman Filter algorithm developed by engineers. Initial applications in aerospace and guided missile control.

**FIG. 1**—Comparison of 4-hour forecast and observed stage, and time variation of model parameters, at Waimakariri Gorge and Old Highway Bridge, for a flood on 31 August 1970.

*NZ Journal of Hydrology* 32(2):1-15
Discovery at Well 92
with Peter Callander and Catherine Moore, 1991

Monitoring well L36/0092

Land surface recharge

Central Canterbury
Discovery at Well 92
with Peter Callander and Catherine Moore, 1991

Simple ARMA relationships between monthly rainfall recharge and groundwater level at L36/0092

- initially a “black box” approach!

- concept building came later.

FIG. 2—A simple physical model of the Greenside aquifer.

FIG. 4—The (a) water level and recharge series, (b) (1, 1, 2) simulation, and (c) (2, 1, 1) simulation, for monthly data January 1968–December 1996.
Buckets of groundwater
with Matthew Morgan, for the Canterbury Strategic Water Study, 2002

Alluvial aquifers
200 – 500 m thick
Area ~ 2300 km²

The resulting **Eigenmodel** is a system of conceptual linear water storages.

\[
\frac{\partial}{\partial x} \left( T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( T_y \frac{\partial h}{\partial y} \right) + R = S \frac{\partial h}{\partial t}
\]

Mathematical theory
(Sahuquillo, 1983)
Forecasting groundwater level
with ECan groundwater staff
We have control  
– mathematics for water management

• Many water resource quantity issues can be described by systems of conceptual linear water storages

• The mathematical basis for analysis, prediction, and control of linear systems is a well developed engineering science

• This discipline includes algorithms, such as Kalman filter, which take account of knowledge uncertainty as well as imperfect monitoring
Waves in the vadose zone
with Hugh Thorpe, 1998-2001

Non-linear kinematic wave in macropores with sorption to micropores

Fig. 3. Water fluxes and volumes for the macropore and micropore regions of a typical cell in the model.

Fig. 5. Result of calibration of the kinematic cell model to the data from Lysimeter 4 for the irrigation of 9 May 1996.

Old water at Maimai
with Mike Stewart, 1997
Old water at Maimai
with Mike Stewart, 1997

Oxygen-18 transport through a steep hillslope during rainstorms

Figure 1: The hydrometric and oxygen-18 tracer data for the stream M8 during the storm of 18-19 January 1994.

Figure 4: The model structure for testing the “old water” hypothesis

MODSIM 97, 42-47
Mixing cell is a simple concept for water quality

System of mixing cells simulates advective-dispersive solute transport

Peclet number = 2 x (number of mixing cells)

\[ C_{out}(k) = a \ C_{out}(k-1) + b \ C_{in}(k) \]
Mixing cells in the soil with soil scientists at Lincoln University

Conceptual model of advective-dispersive solute transport and transformations in soil, designed for prediction and management with use of Kalman filter.

Fig. 1. Conceptual structure of the state–space mixing cell (SSMC) model.
Managing land treatment of wastewater with meat processing industry

Physical processes

Mixing-cell concept

Waste treatment site

Target region

Vadose zone

Aquifer

Nitrate-N concentration (g/m³)

Groundwater surface (forecast)

Target region (forecast)

Maximum allowable value

Monitored (0.7 m depth)

Cumulative drainage (mm)

- Monitored (0.7 m depth)
- Groundwater surface (forecast)
- Target region (forecast)
- Maximum allowable value
Managing effects of land use on water quality
Streamlined groundwater flow

River recharge

Land surface recharge

200 - 500 m

Pumped abstraction

40 - 50 km

Groundwater discharge to streams, lake and sea
Mixing cells in the aquifer
Simulate horizontal & vertical, advective-dispersive transport, and transformations
Streamlines + mixing cells = nutrient transport

River recharge
Nitrate-N < 1 mg/L

Discharge to surface waters
Nitrate-N = 3 mg/L

Nitrate-N concentration (mg/L)
Groundwater age is a contaminant that grows in the mixing cells.

Aquifer is 300 m thick
Porosity is 0.15
We still manage - water quality tools

- Mixing cells don’t exist in the bio-physical world
- Stream functions are a mathematical construct that can’t be directly observed
- Mathematics links these concepts to processes
- Mathematics describes systems of concepts and enables management
Conclusions from the journey

- “Scientists” pursue knowledge
- “Engineers” apply knowledge to decisions
- Concepts are the meeting points of these minds
- Mathematics links knowledge to solution of real problems by means of abstract concepts
- Mathematical tools are the expressions of these linkages
Mathematics enables human benefit from hydrological science