Realistic Forecasting of Groundwater Level, Based on the Eigenstructure of Aquifer Dynamics

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What is eigenstructure?  
- an analogy

Music:
- Sound of a drum
  is a mixture (eigenvector)
  of the modal frequencies (eigenvalues)

Aquifer:
- Groundwater level response to recharge
  is a mixture (eigenvector)
  of water storage modes (eigenvalues)
Water storage modes

$k_i$ - discharge coefficient (eigenvalue)

discharge = $k_i \times \text{storage}$

$g_i$ - weighting coefficient (eigenvector)
Aquifer eigenstructure properties

• Water storages are conceptual, not physical
• Discharge coefficients (eigenvalues) are the same everywhere in the aquifer, and are related to aquifer properties
• Weighting coefficients (eigenvectors) depend on observation location and spatial pattern of recharge
• Piezometric response to land surface recharge is the most time-variable, and is usually dominated by the smallest eigenvalues
Conceptual model of piezometric response to land surface recharge

Land surface recharge (LSR) $k_v(x,y)$

Vadose zone storage $g_1(x,y), g_2(x,y), g_3(x,y)$

Aquifer storage $k_1, k_2, k_3$

Piezometric effect of LSR relative to river recharge effect $d(x,y)$
Model assumptions

• Piezometric effects of river recharge are steady, but spatially variable
• Spatial pattern of land surface recharge is fixed, but the magnitude is unsteady
• Groundwater abstractions are unknown, and considered to be part of the time-varying model error
From conceptual model to forecast equation

- Conceptual model of piezometric response to land surface recharge is a linear dynamic system.
- System structure is described by z-transforms.
- Z-transform model, with a noise term, is expressed as an ARMAX (Box-Jenkins) stochastic difference equation for forecasting groundwater level.
- Forecast equations have previous forecast errors as an additional input.
Model calibration

• Parameters of the **conceptual** model are calibrated, because of structural independence and physical realism
• Noise term has one autocorrelation parameter
• Objective function is minimisation of mean-square forecasting error
• Calibration with the “solver” tool in excel
• Avoids some identification and calibration issues in conventional use of ARMAX equations
Application example

- Observation well in a 2000 km² aquifer, Central Canterbury Plains, New Zealand
- Land surface recharge (one, monthly series) calculated from daily water balance model
- Forecasts of groundwater level convert to forecasts of low flow in river supplied by aquifer, under drought conditions
- Effect of unknown groundwater abstraction is incorporated into the forecast equation as inputs of previous forecast errors
Summary

• Eigenstructure approach expresses the dynamic behaviour of an aquifer as a simple linear system
• System parameters are physically realistic, related to aquifer properties, and structurally independent
• System can be expressed as a stochastic difference equation for real-time forecasting
• Calibration based on system structure avoids identification issues with ARMAX approach