Non-Linear Optimization for Parameter Estimation for Flood Forecasting

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Keywords: Flood Forecast, Flood Modelling, Rainfall-Runoff, Non-Linear Optimization

EXTENDED ABSTRACT

Floods are the response of a catchment area to severe rainfall events. Each catchment will have its unique response which is dependent on its own characteristics and the temporal and spatial distribution of the oncoming rainfall event. A non linear optimization technique has been applied to historical data for rainfall and river flows of the Kakanui catchment in North Otago, New Zealand, to estimate the parameters of a model based on the transfer function concept. The non linear optimization is based on Powell algorithm. Powell algorithm has been widely used in the literature, and it is more efficient and faster than the Simplex method (Press et al., 1989)

Observed rainfall events at two locations in the Kakanui catchment, along with the corresponding observed flows of the river have been utilized to estimate the transfer function which represents the response of the Kakanui catchment to rainfall events. An adjusted form of Philip's equation for infiltration was used to estimate the abstraction of the rainfall event and obtain the effective rainfall which will contribute to the river flow. Weighing factors were assigned to each of the rainfall sites to obtain the best fit between observed and forecasted flows. Nine flood events were used for the calibration process, while two events were utilized for the validation of the derived model. The model has 19 parameters for the transfer function, 2 parameters for the hydrologic abstractions model, and 2 parameters for the weighing factors of the rainfall sites. This results in a total of 23 parameters for the developed model. The ratio of observed cumulative rainfall at Clifton Falls to the corresponding rainfall at the Dasher for historical events is not consistent, and varies significantly from one event to another. This indicates the high variability of the spatial distribution of rainfall events over the Kakanui catchment. As these rainfall events were used in the model calibration, it was difficult to obtain the correct transfer function without proper accounting for the spatial distribution of rainfall over the whole watershed. However, the model, in general, performed satisfactory, given the difficulty in representing the spatial variability of the rainfall events. The model was capable of simulating the flood hydrographs of several events which were incorporated in its calibration, but did not perform well with others. The model was able to simulate well the flows of a flood event which was not included in its calibration. Moreover, in applying the derived model for a real case event which occurred most recently on 30 July 2007, the model was able to forecast very closely the peak flow, but the whole flow hydrograph was not forecasted as good.

1. INTRODUCTION

Floods can have catastrophic impact on our life, and can cause widespread damage over affected regions. Flood forecasting is an essential tool for flood warning. A proper flood warning could mitigate the impact of a flood event by giving people/authorities enough time to evacuate, take stock away, or prepare a temporary flood protection scheme.

Available mathematical models in the literature can be categorized into two main approaches. The first approach simulates the associated hydrologic processes and utilizes hydraulic or hydrologic routing to estimate river flows. The second approach incorporates the concept of a transfer function (step or simultaneous) to relate effective rainfall to river flow. These techniques are mainly dependent on the simulation of rainfall losses to obtain the effective rainfall, and consequently use hydraulic/hydrologic routing or a response function to estimate the river flow (Chow et al 1988, Maidment et al 1993, Jowitt 1999, Thompson 2002, Tripathi et al, 2003, Chen-ShenHsien et al, 2006, Chang-Fi John et al, 2007).

This paper is based on the second approach and has focused on establishing a proper transfer function between hourly rainfall and flow data. Due to the non-linearity of the objective function to minimize, in order to estimate the model's parameters, a non derivative optimization technique had to be applied. The Powell algorithm for non-linear optimization has been utilized to estimate the model's parameters which describe the infiltration losses, the weighted average between rainfall sites, and the parameters of the transfer function. The Powell algorithm (Reklaitis et al, 1983; Press et al, 1989), which is an expanded variation of the univariate gradient search, has been widely applied to water resources problems. The Kakanui catchment in North Otago has been selected for the development and application of this model.

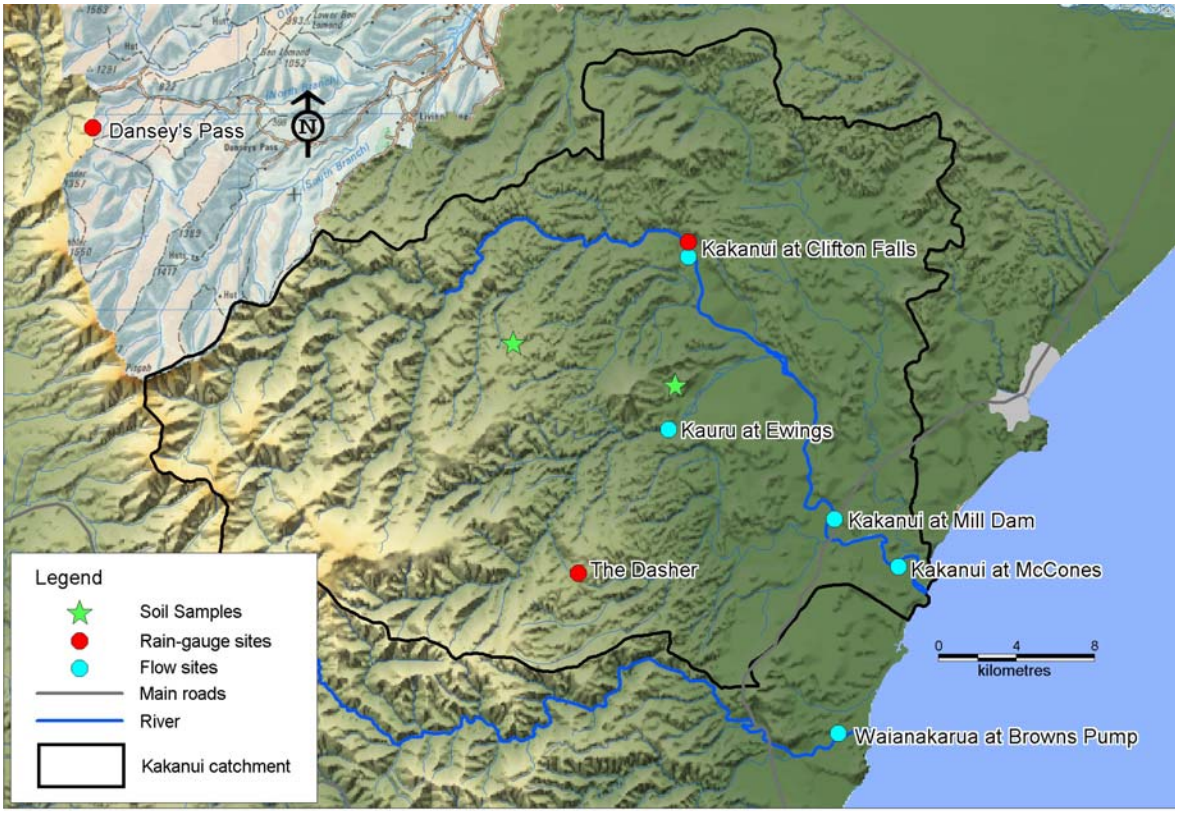

Figure 1 Location of soil sampling sites, rain-gauges, and flow monitoring sites for the Kakanui catchment.

2. CATCHMENT DESCRIPTION

The Kakanui catchment is contained by the Kakanui Mountains and Pisgah Spur to the west and south, and by hill country to the north dividing it from the Waitaki Basin. The Kakanui River flows into the Pacific Ocean 10km south of Oamaru. With a catchment of 894km^2, the catchment consists of about 35% river valley and 40% of rolling hills or downland of less than

600m elevation. The remaining 25% of the catchment is mountainous, reaching heights of some 1640m (Otago Regional Council, 2000).

The main tributaries of the Kakanui River are the Kauru River, Island Stream and Waiareka Creek. The Kakanui and Kauru Rivers both rise in the western mountainous region and flow through gorges incised in rolling or downland country. The Kakanui River flows out of the gorge at Clifton Falls to be joined further down the widening valley by the broad, gravel bedded Kauru River. Island Stream and Waiareka Creek drain the lower downland areas in the south-east and north-east of the catchment respectively.

3. FLOODING HISTORY

The Kakanui Valley has a long history of flooding, with known records extending as far back as 1868 (ORC 2000). More recently large floods were recorded in 1968, 1980, 1986 and 1993. These floods inundated large areas of the floodplain and at times the river and tributaries have broken out of their natural channels.

Table 1 summarises rainfall statistics at rain-gauge sites used for this model, while Figure 1 shows the location of telemetered rainfall and flow monitoring stations in and near the Kakanui catchment.

Table 1 Summary of Rainfall Data

	Clifton Falls	The Dasher
Record Begins	July 1987	January 1952
Elevation	95	540
Mean Annual Rainfall (mm)	483	814
Max. 24 hr Rainfall (mm)	128.5	217
Max, 12 hr Rainfall (mm)	95.5	123.5

The floods of the Kakanui River can be fast and quick, with a 60 m^3/s rise in 15minutes during the flood event of April 2006, as shown in Figure 2.

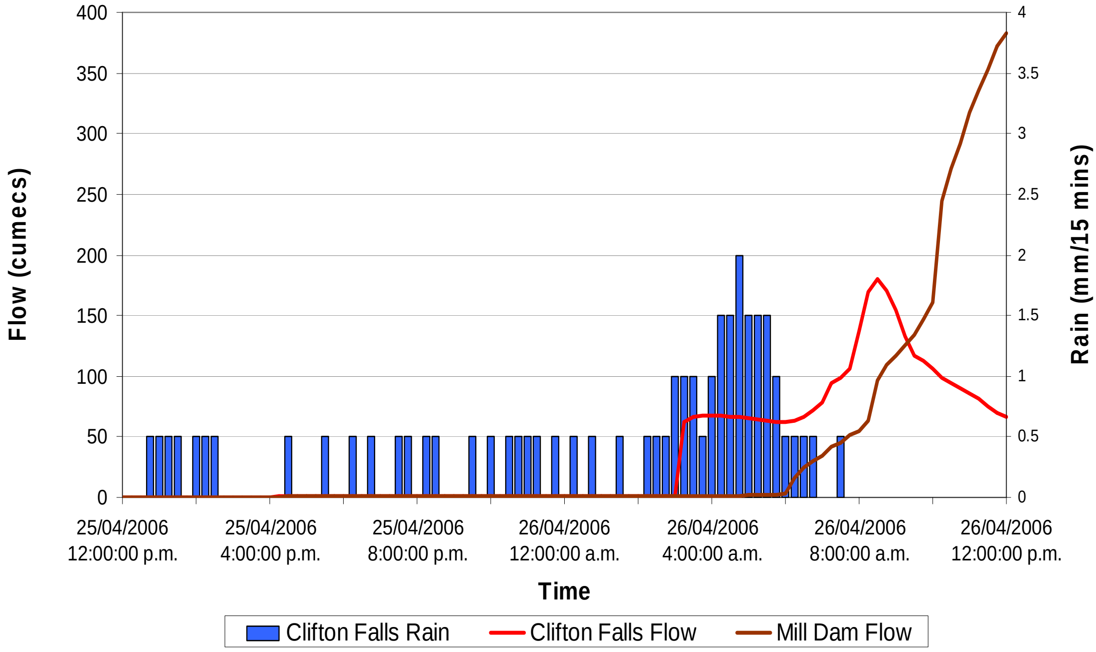

Figure 2 Kakanui Flood event of April 2006

4. MODEL DEVELOPMENT

4. 1 Transfer Function

The concept behind this research is that each catchment should have its own "unique" hydrologic characteristics which will impact on its response to a rainfall event. This transfer function is assumed to be invariant with time, and the case which is presented here considers a linear transfer function. A rainfall-runoff model based on a transfer function to translate rainfall into river flows has been developed for the Kakanui River. The parameters of the transfer function have been estimated by using Powell algorithm to minimize the objective function which describes the relation between observed and modelled flows. The objective function F_x is defined as the sum of squared errors as follows:

$$F_x = \sum_{j=1}^{m} \sum_{i=1}^{n_j} (Q_{i,j} - \widehat{Q}_{i,j})^2 \qquad (1)$$

Where m is the number of events included in the calibration process, n_j is the number of intervals of event j, $Q_{i,j}$ is the observed flow at interval i of event j, and $\widehat{Q}_{i,j}$ is the forecasted flow for event j at time interval i. The forecasted flow $\widehat{Q}_{i,j}$ is calculated from the transfer function $\underline{X}$ as follows:

$$\widehat{Q}_{i,j} = \sum_{k=1}^{i \le L} R_k X_{i-k+1} \qquad (2)$$

Where L denotes the number of the rainfall intervals for event j, R_k is the rainfall depth at time interval k, and X_i is the i^{th} parameter of the transfer function. It should be noted that the objective function F_x is highly nonlinear in so many parameters, and thus a numerical non-linear optimization technique had to be carried out.

4.2 Hydrologic Abstractions

Philip's equation to simulate infiltration through a soil (Singh, 1992) was used to estimate the hydrologic abstractions from a rainfall event in the model. The infiltration rate is defined as:

$$f_t = a + \frac{b}{2\sqrt{t}} \text{ mm/hr} \qquad (3)$$

while the cumulative infiltration is:

$$F_t = \int_0^t f_t = at + b\sqrt{t} \quad \text{mm} \qquad (4)$$

It is the cumulative infiltration after a time interval "t" which we are interested in for this model. However, this model assumes ponding conditions apply since the start of the event, which is not the case. To account for this, cumulative infiltration every time step has been calculated and compared to the cumulative rainfall up to this time step, then the estimated infiltration depth is taken as the smaller value of the cumulative observed rain or the cumulative infiltration depth from equation (4). The same procedure was carried out to define the ponding time, after which equation (4) is applicable. The parameters for the infiltration model were estimated also through the optimization process by Powell algorithm. Thus, equation (4) was used to estimate the abstraction losses, and in turn the effective rainfall of the event.

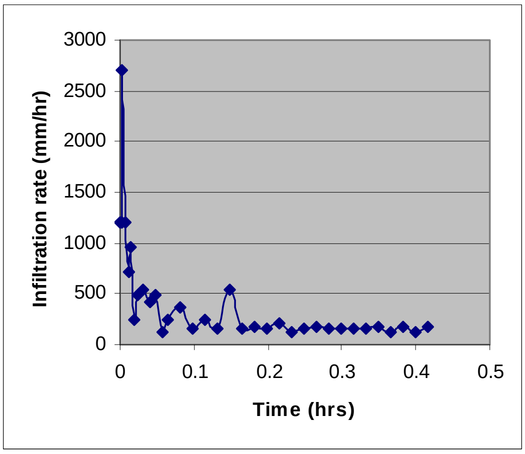

Figure 3 Infiltration test at the Dasher

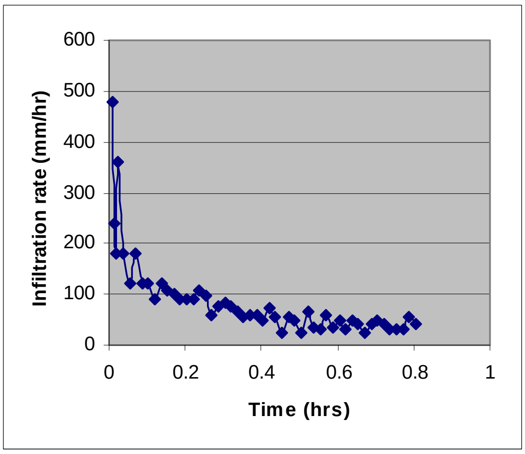

Figure 4 Infiltration test at Upper Clifton Falls

Moreover, an infiltration test, using the double ring infiltrometer, was carried out at two sites in the catchment to investigate the infiltration capacity of the Kakanui catchment. The locations of these two sites are at the upper of Clifton Falls, and at the Dasher, as illustrated in Figure (1). Figures (3) and (4) show the results of these tests. The average "stable" observed infiltration rates at upper Clifton Falls and the Dasher were 40 mm/hr and 150 mm/hr, respectively. This is a very high value, and will exceed the rate of any rainfall event. This suggests that most of the flow in the river is coming from hilly areas as "through flow". However, these infiltration tests were carried out for less than one hour, and of course the response of the catchment due to a rainfall event which lasts much longer than one hour could be different. Moreover, a rainfall runoff experiment was carried out in the field in the upper Clifton Falls catchment, and confirmed that the "steady" infiltration capacity of the soil is higher than 30mm/hr.

In addition to at site field experiments, two soil samples of the Kakanui catchment at upper

Clifton Falls and the Dasher have been obtained for further experiment in the hydraulic lab of Lincoln University. Each sample is 700x700x200 mm, and a sprinkler was used to simulate rainfall events over these samples. Surface runoff and through flows were collected and measured for several simulated rainfall events. Again, the results confirmed that the infiltration capacity of the soil is high, and most of the river flows would be from the through flow and not the surface runoff.

The above analysis leads to the fact that the infiltration model is used to estimate the temporal hydrologic abstractions from a rainfall event, rather than the actual infiltration to the soil, as most of this infiltrated water will contribute to the river flow as a through flow. The optimized values for parameters a and b were estimated at 0.5 mm/hr and 1.5mm/hr$^{0.5}$, respectively.

4.3 Model Calibration

Nine observed flood events at Clifton Falls, along with their corresponding rainfall events at Clifton Falls and the Dasher, were used for model calibration and the estimation of the model parameters. Powell's algorithm for multidimensional minimization has been used in this research to estimate the model parameters during the calibration process. Powell's approach is more efficient and faster than the Simplex Method (Press et al, 1989). Table (2) presents a summary of these flood events, in addition to the two flood events which were used for model validation.

The selected floods for model calibration cover a wide variety of flood events with regard to the duration, total rainfall, peak flow and the total runoff volume, as shown in table (2). Moreover, it is obvious from the table that usually the Dasher receives more rain than Clifton Falls. Unfortunately, the ratio between the rainfalls of the two sites is not the same for all rainfall events, and in some cases, such as event 2, Clifton Falls received more rain than the Dasher. Of course this is dependent on the rainfall event, its direction, and its spatial distribution as it hits the catchment. Such discrepancy is expected to confuse the model. Figures (5) and (6) show that the model did not simulate well the flood events of 20 November 1996 "event 2" and 20 July 2001 "event 7". This would have been expected for event 2 which deviates from the pattern of the other events, but It should be noted that also for event 7 the rainfall at Clifton Falls was more than half the rainfall at the Dasher. However, Figure (7) presents a good match between the forecasted and observed flows using the developed model for events 8 and 9 "January 2002". This could be attributed to the fact that Clifton Fall's rain was less than 1/3 the rainfall at the Dasher for events 8 and 9, which is the case for most of the events. The good performance of the model for the flood events of January 2002 indicates the ability of the model of reliably forecasting the flows of flood events which preserve the pattern of the significantly lower rainfall at Clifton Falls than the Dasher.

Table 2 Flood Events for Model Calibration and Validation

Event	**Date**	**Duration (hrs)**	**Total Rain (mm)**		**Peak Flow(m^3/s)**	**Total Runoff (10^6 m3)**
			Dasher	Clifton falls	Clifton Falls	Clifton Falls
1	13/06/1995	85	128.0	37.5	77.1	9.0
2	20/11/1996	43	31.0	37.0	82.1	3.1
3	4/02/1997	56	73.8	22.5	44.3	3.4
4	31/08/2000	129	217.0	67.0	148.0	19.0
5	11/02/1997	60	51.1	47.5	93.1	4.0
6	18/07/2001	50	40.0	15.0	12.0	0.8
7	20/07/2001	72	149.0	77.0	96.0	7.9
8	10/01/2002	68	182.5	58.5	256.6	11.9
9	12/01/2002	63	265.0	62.5	256.0	16.7
10	18/01/2002	28	26.5	10.0	45.2	2.3
11	30/07/2007	51	154	79	215	Not available

The final forecast model for the Kakanui catchment has 19 parameters for the transfer function, 2 parameters for the hydrologic abstraction model and 2 parameters for the weights of the rainfall sites, totalling 23 parameters.

4.4 Model Validation and Application

The fitted forecast model for the Kakanui River was applied to several flood events which were not included in its derivation. Figures (8) and (9) present the application of the transfer function model to the observed rainfall events at Clifton

Falls and the Dasher, then comparing the forecasted flows versus the observed ones. The model performed well in forecasting the flood flows of event 5 (11 February 1997), and did well in forecasting the peak flow for the most recent event on 30 July 2007 (event 11). It is the peak flow which usually is important for the flood forecast, and despite the whole hydrograph for the July 2007 was not forecasted as good, still the model's performance was satisfactory as it simulated very good the peak flow. It is noted that the Dasher rain gauge stopped sending signals for its rainfall after about 20 hours from the start of the event, and estimated values for the Dasher rainfall were estimated based on Clifton Falls. This could have an impact on the hydrograph for the forecasted flows.

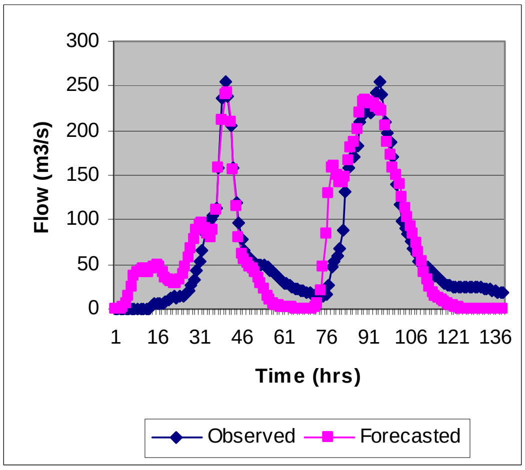

Figure 7 Flood Events Jan. 2002 (events 8 and 9)

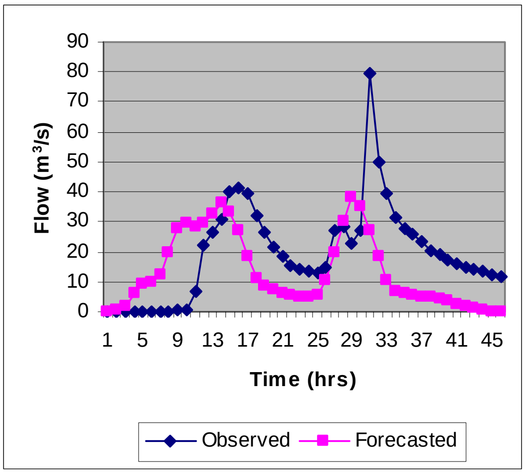

Figure 5 Flood Event 20 Nov. 1996 (Event 2)

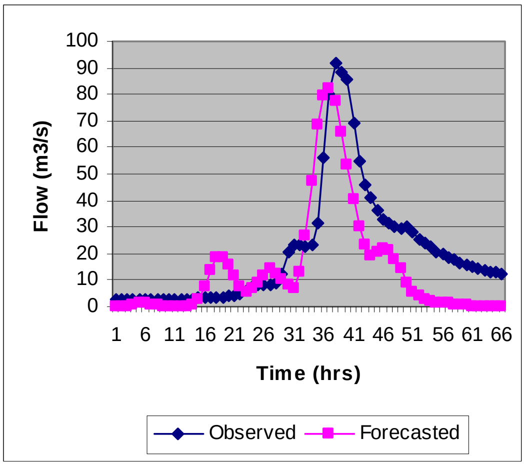

Figure 8 Flood Event 11 Feb. 1997 (Event 5)

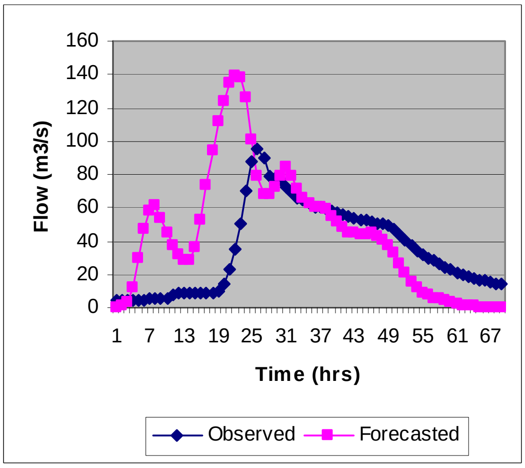

Figure 6 Flood Event 20 July 2001 (Event 7)

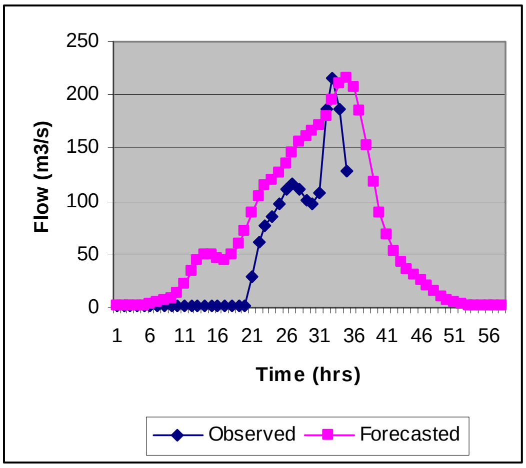

Figure 9 Flood Event 30 July 2007 (Event 11)

5. CONCLUSIONS

The derived model by using non-linear optimization to estimate the parameters of a transfer function was capable of forecasting the Kakanui River flows satisfactory. The technique made use of the observed flood events and of the basic concept that each catchment should have its unique response to rainfall events. However, the high variability of rainfall events, and the difficulty of representing the spatial variability of a rainfall event makes it difficult to exactly estimate the perfect transfer function. Moreover, the derived transfer function was linear, while adding non linearity could improve the performance of the derived model. In addition, it is recommended to include the direction or the type of the rainfall event as this could incorporate the spatial variability in the modelling process.

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