REAL VALUE VALUATION FOR PROPERTY IN THE 21ST CENTURY?
- A COMPARISON OF CONVENTIONAL AND REAL VALUE MODELS

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ABSTRACT

An old idea transforms into a simplified dynamic application for valuing investment property in the 21st Century. This paper presents a new all risks real yield (ARRY) valuation model for the 21st Century valuer’s use. It requires a paradigm shift in the minds of valuers from 20th Century thinking in nominal value terms to thinking in real value terms. This affects the methodology, valuation models and techniques required to value income producing property. It is compared to and distinguished from the conventional all risks yield (ARY) model used in the UK and previous real value valuation models developed in the UK and USA in the 1970-80’s. This ARRY model is an advance on generic real value valuation models of the 1990’s. A sales analysis and common valuation examples compare the ARRY model with conventional valuation methodology.

Keywords: All risks real yield; conventional and traditional valuation theory; discounted cash flow; dynamic capitalisation; income valuation models; investment real estate; real value valuation.

INTRODUCTION AND HISTORY

The “old idea” on which this paper is based is not a traditional property valuation model, but related to one that first appeared in relation to forestry valuations accredited to the German engineer Martin Faustman in 1849. The Faustmann Formula is well known in forestry economics (Viitala, 2006) as a real value model for valuing forests and land expectation value (LEV) for forest land (Bullard & Straka, 1998; Straka & Bullard, 1996) adopted in New Zealand forestry valuation standards (NZIF, 1999).

The quest for a real value valuation model that would meet the needs of 21st Century valuers takes a positive problem and value definition following the approach suggested by Whipple (1990, 1992, 1995; 2006, pp. 78-117). It challenges the
normative implication of the widely held view that explicit DCF valuations are a superior valuation technique that gives the “right” answers.

**Historical setting**

A history of real value valuation models over the last four decades was presented in Jefferies (2009a). These income property valuation models are set in the context of a longer history over the last four centuries as described in Jefferies (2009b).

A real value valuation model for income property valuation was first conceived in the United Kingdom (UK) in the early 1970’s by Wood (1972) as the *valuation technique for the seventies*.

Wood’s real value model introduced the term *inflation free real yield* (IFRY) as a “true yield” and in his thesis (Wood, 1972), which he promoted in a series of articles in the *Estates Gazette* (Wood, 1973). The RICS *Property Valuation Methods: Research Interim Report* (Trott, 1980) however, described it as suffering from being “too esoteric and the use of the complex formulae” and was rejected by the professional body in favour of the equated yield (EY) model. When his real value model was criticised and rejected in the RICS *Final Report* (Trott, 1986), Wood stoutly defended it in a two part series in the *Journal of Valuation* (Wood, 1986).

To bridge the divide between Wood’s real value and the conventional EY capitalisation model, Crosby (1985) promoted his *real value equated yield hybrid* model, later changed to *real value/short-cut DCF hybrid* model. Despite extensive descriptions in valuation teaching texts (Baum & Crosby, 1988, 1995, 2008), it has suffered its own demise. It appears the old habits of the UK chartered surveyors’ reliance on the EY model are very intractable.

Despite the ARY’s and EY’s models’ inaccuracies (Crosby, 1986), the EY income property valuation model has proved very endemic as the mainstay of UK practice and teaching. Its resistance to change or even being dislodged by short-cut or explicit DCF techniques is reflected in recent UK texts that ignore the real value approach (Blackledge, 2009; Sayce, Smith, Cooper, & Venmore-Rowland, 2006; Wyatt, 2007). Others briefly mention the real value approach as an historical technique (Scarrett, 2008) or amongst growth explicit alternatives (Armatys, Askham, & Green, 2009).

In the UK, explicit DCF techniques have not been adopted for market valuation purposes. Little has changed to date as most recently opined at the ERES 2010 Conference “There is an institutionalised comfort with the capitalisation rate approach within industry and the need for change is not accepted” (Crosby & Henneberry, 2010, p. 16). Explicit DCF techniques are used for investment analysis or calculation of worth, but in the most of the rest of the world, they are now used ubiquitously for any investment grade property market or asset valuation.
Quite independently, a dynamic capitalization model was promoted in the USA by an actuary/appraiser Blackadar (1984, 1986, 1989), but was virtually ignored by the North American appraisal profession. Blackadar described his real yield as a “force of interest, δ” i.e. a continuous compounding real rate of return using international actuarial notations (IAN) and his model as "an income approach in real dollars at real interest”. Backadar does not use a capitalisation rate, but a function not a specific formula, that establishes an "equation of value". That produces a “varying annuity factor”, in effect an income multiplier (YP in UK terminology). It is applied to a stabilised effective gross income, with property expenses (both annual or occasional) not deducted but adjusted for in the income multiplier.

A generic net of growth yield real value valuation model was developed in the 1990’s quite independently in New Zealand (Jefferies, 1997a, 1997b). This was formulated in a term & reversion model format, but with modifications, and improvements on previous real value models.

The quest for a better real valuation model was aided by computer technology; especially spreadsheet modelling that enabled rapid sales analyses, to extract implied yield or growth rates from sales evidence. Prior to that, manual trial and error type re-calculation was very tedious, relatively inaccurate relying on pre-computed valuation tables. Even with the advent of electronic calculators enabling valuers to do faster calculations, it was still tedious (Jefferies, 2009b).

In current practice, Excel™ spreadsheet models using a goal seek function are used for solving these yields and/or growth rates, since computer spreadsheet programs became available on PCs ubiquitously since the late 1980’s. Advances in technology has gone hand-in-hand with the search for more pragmatic inflation and real growth market based inputs, e.g. the development of computerised net of growth real value models in the 1990’s (Jefferies, 1997a, 1997b).

The ARRY model is yet a further advance on these prior models incorporating the best of their concepts, but introduces new techniques providing enhanced simplicity and ease of use. It corrects some of these predecessors’ deficiencies. It maintains, but is not dependent on, a term & reversion model format, with its main advancement being in the method of valuing the term component enabling a fully real value approach.

OUTLINE OF PAPER

Firstly, this paper introduces the seminal concept of the all risk real yield (ARRY) real value valuation model.
Secondly, conventional valuation models are contrasted with the real value models by means of twelve conventional versus real value valuation concepts.

Thirdly, the ARRY model is specified and illustrated by the process of analysis of comparable sales and application valuing a typical investment property. The difference between a conventional nominal term and reversion capitalisation approach and the ARRY real valuation approach is demonstrated using a common real estate income valuation problem.

Fourthly, advanced applications of the ARRY model are outlined. Further research and empirical testing requirements are discussed, with a view to its potential adoption as a new generic income valuation approach by the valuation profession.

THE SEMINAL ALL RISKS REAL YIELD (ARRY) CONCEPT

The all risks real yield (ARRY) is the market determined expected annually in arrears real return from a property investment. It includes all risks in future changes in rents, expenses, capitalisation rates as reflected in real dynamic market values and property investment risk profiles. Fundamentally, it is based on the grounded property and land economy principle that the present value (as reflected in real prices and current real yields) is what the market determines as the present real value (PRV) of all future expectations.

The definition of rates, terms, symbols, and notations are consistent with those used in the United States and Australian latest texts (Appraisal Institute, 2008; Reed, 2007) and as defined by Fisher (1995), or otherwise defined herein.

The ARRY \((Y_A)\) is defined, where \(Y = \text{yield}, \) subscript \(A = \text{a notation indicating “all risks real” as:}\)

The real value annually in arrears yield - the real internal rate of return that discounts the real values of the term to run and the real reversionary value to the present real value (PRV) or sale price/current value.

This is distinguished from the ARY, which is a capitalisation rate that reflects the rental review terms of the lease and frequency of rental payments. It is also distinguished from the equivalent yield \(E_qY\) as once used in the UK, only able to be analysed after a valuation is completed or from a sale. The \(E_qY\) is both the internal rate of return and used also as the capitalisation rate (an inherent error) to calculate the reversionary value, but is usually unique to that property’s lease term to run (Jones, 1983; Rich, 1999; Worthington, 1979, 1990).
It also differs from the equated yield \( EY \) (Clark, 1978; Marshall, 1976) as adopted in the conventional UK valuation methodology since the mid 1980’s RICS Research Project into Property Valuation Methods reports (Trott, 1980, 1986).

The ARRY real value valuation model uses a the real reversionary capitalisation rate \( R_A^F \) that is based on the \( Y_A \) but takes into account both the market’s implied forecast real growth rate \( G_r \) and the rental review frequency \( F \). It will usually be lower than the \( Y_A \), where there is a positive expected real growth. Expected inflation \( I_e \) is exogenously derived from independent reliable econometric forecasts. Overall growth \( G_O \) is the combination of expected nominal currency inflation \( I_e \) and real growth \( G_r \), on a nominal annual basis where \( G_O = (I_e + G_r) \), where also \( Y_A \approx Y_O - G_O \) or \( Y_O \approx Y_A + I_e + G_r \). The ARRY model is described in detail with examples of sales analyses and valuation applications later in this paper.

All property rentals, operating expenses (OPEX), capital expenditure (CAPEX), values, both current and future, are dealt with in current PRVs. All future expectations and changes including nominal and real growth, unexpected currency inflation, changes in future yields and thus capitalisation rates, due to uncertainty and risk inherent in a property investment are reflected in the \( Y_A \).

An ARRY valuation is a snap-shot market expectations assessment, based on current market data at the date of recent sales analysis, adjusted to the date of valuation. Reliance on the valuer’s subjective adjustments is limited to short-term trends for differences in the property’s location and intrinsic risk characteristics to adjust the \( Y_A \) to be applicable to the individual property’s characteristics.

The \( Y_A \) being sourced from current market transactions is the model’s greatest advantage, in that it does not require dubious future explicit growth forecasts, except the minor but important element of real growth. It avoids explicit period by period nominal value future forecasts, such as in explicit discounted cash flow (DCF) techniques, thus greatly simplifying the methodology. The ARRY model provides market determined accuracy of the same reliability as reflected in the reliance on current market rental assessments, costs and the market’s reflection of its changing dynamics, i.e. property transaction rentals, prices, implied real growth and yields, as reflected in the market’s PRVs. Updating market analyses for new transactions, as they are available incorporating that dynamic market data change and trends into updating valuations become a relatively simple exercise.

The potential challenge to valuation and appraisal professionals is that adoption of the valuation model requires a change of thinking from a nominal value approach, particularly in not forecasting future rents, cost, values etc., but relying on current real value ones. This changed paradigm will be resisted by conventional valuation methodologists who rely on what they were taught and what works for them.
Resistance will also come from vested interests in explicit DCFs, due to the commercial interests of DCF program software suppliers and client’s accountancy focussed expectations that major income property valuations will be based primarily on explicit nominal value DCF cash flow forecasts presented in the valuation report.

Any widespread promotion of ARRY models and their methodology will need to be empirically tested and shown to be of significant advantage to valuers and their clients in terms of equivalent or superior accuracy and reliability. Significant savings in time input to complete valuations will provide efficiency and savings in costs of valuation services. However, at best, the use of ARRY models will be critically reliant on education of both valuation students and practitioners through CPD education. The ARRY model will also need to serve its apprenticeship as a check valuation method alongside traditional and conventional valuation methodology until ARRY valuation proves its worth, superiority and acceptability. This process happened with the introduction of DCFs before the technique gained its ubiquitous adoption as a method of prime choice and reliability over conventional methods in the USA, Australia and New Zealand (Jefferies, 2009b), but not for market valuations in the UK (Crosby & Henneberry, 2010).

Currently, international valuation standards are written in a nominal currency valuation paradigm, with the International Valuation Standards Guidance Note 9 (IVGN 9) Discounted Cash Flow (DCF) Analysis for market and non-market based valuations having only passing reference to “real rates of return” (IVSC, 2008). However, there is one reference to “real terms” valuation and “real rates” in the IVSC 2010 Proposed New International Valuation Standards Exposure Draft, in para. 26 of IVS 301.01, Valuations of Businesses and Business Interests under Income Approach: “When the forecasted income or cash flow is expressed in real terms, i.e., level prices, real rates should be used.” (IVSC, 2010, p. 76). This opens the door to the ARRY’s acceptable use, but provides no guidance on the implementation or methodology to be employed in using real terms or real rates in valuations. It is noted that there is no continuation proposed of IVGN 9 or any equivalent DCF valuation guidance.

CONVENTIONAL VERSUS REAL VALUE MODELS

To illustrate the two different paradigms, the conventional and real value models are contrasted in terms of their respective concepts.

The fundamental point of commonality is that the present or current value of a property when expressed in nominal terms is also its value in current real terms. The simple difference between the conventional capitalisation models and real value valuation models are conventional direct capitalisation models presume implicitly or explicitly, that future income will be received in future nominal currency, including growth in income giving rise to increased future growth in capitalised value.
Conventional yield capitalisation models (such as DCF valuations) presume future cash flows – both income (net cash flows) and reversion (net terminal values) including any resulting growth will be received in future nominal currency. Whilst real value valuation models presume implicitly or explicitly, that all income is in current real value terms; allowing implicitly or explicitly for growth in real terms. Real value yield capitalisation models (such as real value DCF valuations) presume future cash flows – both income (net cash flows) and reversion (net terminal values) will be received including any resulting real growth only, in future real terms, as expressed in discounted current (real) values.

The disadvantages of conventional models are conventional capitalisation models require implicit or explicit allowance for inflation incorporated in the overall nominal growth assumption, and discount future cash flows at an overall inflation prone nominal yield or discount rate. The advantages of real value models are that generic real value capitalisation models require implicit or explicit allowance only for real rental growth, inflation is not allowed for and future real cash flows, including real growth, are discounted at an inflation free real yield (IFRY) or net of inflation yield or discount rate. ARRY – all risk real yield real value capitalisation models allow implicitly for all risks and expectations, including inflation and real growth, and future real cash flows are discount at the ARRY yield as a discount rate.

In terms of operating expense (OPEX), in conventional capitalisation models, any unrecovered OPEX are allowed for by capitalising their current costs at a nominal overall capitalisation rate that implies future nominal increases at the rental growth rate. In real value capitalisation models, any unrecovered OPEX are allowed for by capitalising the current real costs at a real capitalisation rate that implies no future real (but only inflationary) cost increases.

In terms of capital expenditure (CAPEX), in conventional capitalisation models by discounting their future forecast nominal CAPEX costs at nominal discount rate that implies future nominal increases at the implied rental growth rate. In real value capitalisation models by discounting their current real CAPEX costs at a real discount rate, implies no real (only CPI inflation or an expected construction cost index) increases in the future.

**COMPARING CONVENTIONAL AND REAL VALUE VALUATION MODELS**

**Valuation problem**

To compare the conventional nominal value term & reversion valuation model with the ARRY real value model, a simple example will be used:
The property being valued has $F = 5$ year rent reviews, the lease has $T = 4$ years to run to the next review at the existing contract rent $C = $39,500 p.a. The current market contract rental assuming it was reviewed at the sale date is $C_C = $40,685 p.a. It is in a location where growth in the market rental supported by other recent reviews and new leasing indicates an approx $G_O \approx 3\%$ p.a. nominal growth in rentals is sustainable.

A chart of the conventional nominal forecast cash flows and term & reversion structure follows:

**Comparable sale and its analysis**

There is good comparable recent market evidence, the most comparable being Sale 1 of a similar type, property characteristics, location, asset class and investment risk profile.

This Sale 1 property sold for $570,000 and has a lease with $F = 3$ years rent reviews and sold (to make the calculation simpler) on an anniversary of a rent review date with $T = 2$ years to run till the next rental review. The existing contract rental $C = $40,000 p.a., the current contract market rental (if reviewed at sale date) $C_C = $41,200 showing $1,200/40,000 = G_O = +3\%$ p.a. nominal growth in one year and in line with the general market evidence of growth rates. Independent evidence of expected inflation is $I_e = 2\%$ p.a. and based on wider sales analysis and trends in real rentals, real growth $G_r = 1 \%$ p.a.

In a conventional direct capitalisation approach, the sale analyses to an *initial yield or capitalisation rate* (but not an $R_O$ or $ARY$ as the date of sale is between rent reviews) is:
$40,000 \text{ p.a.} ÷ $570,000 \Rightarrow 0.070175 \text{ or } 7.0175\% \text{ p.a.} \quad (1)

Intuitively, the valuer will know that this initial yield is lower than it would be if recently let, < R_o or ARY shown later in Equation (9) as \( R_o^2 = 7.2137\% \) p.a., as the rent is low compared to current market, but this reflects the pending rental review in two year’s time. It is not indicative of the capitalisation rate to apply, due to quite different term to run \( T = 4 \) years and review term \( F = 5 \) years. Note that the symbol \( \Rightarrow \) is used in a formula for \( \text{results in} \) and \( \Leftarrow \) for \( \text{resulting from}, = \text{for an equality} \) in an equation, and \( \cong \) for \( \text{equivalent to} \).

**Calculation of “correct” capitalisation rate**

To analyse the sale to derive the implied overall yield, given evidence of the rental growth rate for a more accurate valuation, the valuer might apply a term and reversion technique. This short-cut discounted cash flow (DCF) technique requires the three inputs of \( Y_O, G_O, \) and \( F \) to calculate the market contract rental capitalisation rate \( R_o^F \).

Sourced from Whipple (2006, p. 232), the capitalisation rate formula is based on the present value factor applied to $1 of an annual in arrears annuity in perpetuity that increases at a constant growth rate at regular frequency of time intervals = \( F \). Whipple attributes this formula to Rose (1979), but it is only postulated in that paper and also in *Rose’s Valuation Tables* (Rose, 1976). He also refers to a derivation and proof provided in Worthington (1979, pp. 368-369), later published in Worthington (1990, p. 42).

The capitalisation rate formula using the foregoing symbols and notations is:

\[
R_o^F \Leftarrow Y_O - Y_O \left[ \frac{(1+G_O)^F - 1}{(1+Y_O)^F - 1} \right] \quad (2)
\]

The 2\textsuperscript{nd} term: \( Y_O \left[ \frac{(1+G_O)^F - 1}{(1+Y_O)^F - 1} \right] \) represents the adjustment to the overall required yield rate \( Y_O \) to allow for the deferred timing of the growth over \( F = \) rent review frequency, to derive the capitalisation rate. Where there is no growth, i.e. \( G_O = 0 \); then this 2\textsuperscript{nd} term collapses to = 0 and thus the capitalisation rate = \( Y_O \).

Formula (2) can be simplified to:

\[
R_o^F \Leftarrow Y_O \left[ 1 - \frac{(1+G_O)^F - 1}{(1+Y_O)^F - 1} \right] \quad (3)
\]

or expressed as an in-line formula:

\[
R_o^F \Leftarrow Y_O \left[ 1 - \frac{(1+G_O)^F - 1}{(1+Y_O)^F - 1} \right] \quad (4)
\]
Extracting the expected overall yield $Y_O$ from the sale analysis

In a conventional nominal term & reversion valuation model, analysing Sale 1, given the market evidence of $G_O = 3.0\%$ p.a. and $F = 3$ yearly, the valuer needs to solve by trial and error for $Y_O$ and this will also calculate $R_O^3$.

The term & reversion short-cut DCF calculation is where:

$$CMV \leftarrow [PV\ of\ term\ T\ to\ run\ to\ next\ review] + [PV\ of\ forecast\ market\ value\ as\ at\ period\ T]$$  \hspace{1cm} (5)

$$CMV \leftarrow [PV\ of\ C\ as\ an\ annuity\ in\ arrears\ discounted\ @\ Y_O\ over\ T\ periods]\ +\ [PV\ of\ forecast\ C_C\ capitalised\ in\ T\ periods\ @\ R_O^3,\ disc.\ @\ Y_O\ over\ T\ periods]:$$  \hspace{1cm} (6)

$$CMV \leftarrow C \frac{1 - (1 + Y_O)^{-T}}{Y_O} \times (1 + Y_O)^{-T}$$ \hspace{1cm} (7)

Substituting the known data in Formula (7), the valuer solves by trial and error for $Y_O$:

$$\$570,000 \leftarrow \$40,000 \frac{1 - (1 + Y_O)^{-1}}{Y_O} + \{ \$41,200 + Y_O \left[ 1 - \frac{(1 + G_O)^{F - 1}}{(1 + Y_O)^{F - 1}} \right] \} \times (1 + Y_O)^{-1}$$ \hspace{1cm} (8)

which results in $Y_O = 10.0147\%$ p.a. and $R_O^3 = 7.2137\%$ p.a.  \hspace{1cm} (9)

**Conventional application in valuing**

The property is valued using the analysed market overall yield rate $Y_O$ together with other similar sales analyses and in applying judgement that a $Y_O = 10.0\%$ p.a. together with the market evidence of $G_O = 3.0\%$ p.a. is applicable to this property.

Applying Formula (7), where $T = 4, F = 5, C = \$39,500$ p.a., $C_C = \$40,685$ p.a.:

$$CMV \leftarrow \$39,500 \frac{1 - (1.10)^{-4}}{1.10} + \{ \$40,685 \times (1.03)^4 + 0.10 \left[ 1 - \frac{(1.03)^{5 - 1}}{(1.10)^{5 - 1}} \right] \times (1.10)^{-4} \}$$ \hspace{1cm} (10)

$$CMV \leftarrow \$39,500 \times 3.169865 + \{ \$40,685 \times 1.125509 + 0.073911 \times 0.683013 \}$$ \hspace{1cm} (11)

$$\$548,367 \leftarrow \$125,210 + \$423,157$$ \hspace{1cm} (12)
The ARRY $Y_A$ valuation model approach

This model is presented in a simpler formulation than prior real valuation models. As defined, the ARRY $Y_A$ is the annually in arrears required real return or discount rate, which in this model is used to discount all future real values to bring them to present real values (PRVs).

This ARRY $Y_A$ rate, where the "-A" subscript denotes “all risks real yield” is also used in calculating the all risks real yield capitalisation rate $R_A^F$, which equates to the overall capitalisation rate $\approx R_O^F$ as in Formula (2). This is adjusted for expected inflation $I_e$ and the real growth $G_r$, and for the frequency of the rent review terms in years $F$ to estimate the real reversionary value.

$I_e$ is sourced from economic data, i.e. The Treasury, the NZ Institute of Economic Research Inc (NZIER), or other similar economic forecasting service in other countries. Real growth is where rentals and thus values grow in monetary terms at a greater rate than expected inflation, i.e. where $G_O > I_e$, and thus $G_r = G_O - I_e$.

To calculate the all risks real yield capitalisation rate $R_A^F$ requires it to be based on the total yield as the sum of the all risks real yield $Y_A$ plus expected inflation $I_e$ and real growth $G_r$, shown as $(Y_A + I_e + G_r)$, replacing the over-all required yield $Y_O$ in the standard capitalisation Formulae (1, 3, or 4). This is because in the ARRY model, $Y_O$ is not used or derived from the market, but built-up from its components of $I_e + G_r$, replacing $G_O$.

The all risks real yield capitalisation rate used for calculating the real value of the term to run in the ARY model at the valuation date is $R_A^T$, where $T$ = the term to run\(^1\), on the assumption that it continues in perpetuity using Formula (4) to give the following capitalisation rate equation:

$$R_A^T = (Y_A + I_e + G_r) - (Y_A + I_e + G_r)[((1 + I_e + G_r)^T - 1)/((1 + Y_A + I_e + G_r)^T - 1)]$$

(13)

The all risks real yield capitalisation rate used for calculating the real reversion value at the date of commencement or at a rent review or renewal date is $R_A^F$, where $F$ = the rent review period:

$$R_A^F = (Y_A + I_e + G_r) - (Y_A + I_e + G_r)[((1 + I_e + G_r)^F - 1)/((1 + Y_A + I_e + G_r)^F - 1)]$$

(14)

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\(^1\) Where the valuation date coincides with the commencement or review/renewal date is the $R_A^T = R_A^F$, i.e. the full rental term is yet to run until the next review date. In these circumstances the valuation simply collapses to the new contract rental = current market contract rental, capitalised at $R_A^F$ there is no term to run and reversion is current not deferred.
Where this is applied to annual rent reviews, i.e. \( F = 1 \);

\[
\begin{align*}
R^A_1 &= (V_A + I_e + G_r) - (V_A + I_e + G_r)[((1 + I_e + G_r)^{-1})/(1 + Y_A + I_e + G_r)^{-1})]
\end{align*}
\]  

(15)

then this capitalisation equation collapses to:

\[
\begin{align*}
R^A_1 &= (V_A + I_e + G_r) - (V_A + I_e + G_r)[(1 + I_e + G_r-1)/(1 + Y_A + I_e + G_r-1)]
\end{align*}
\]  

(16)

\[
\begin{align*}
R^A_1 &= (V_A + I_e + G_r) - (I_e + G_r) = Y_A
\end{align*}
\]  

(17)

Thus, given this definition of \( R^A_1 \) as the all risks real yield annual review capitalisation rate, this is equivalent to the ARRY \( Y_A \) as defined. Thus, this equivalency \( R^A_1 \equiv Y_A \) makes the determination of \( Y_A \) or its implied rate by extraction from sales simple and unique to this ARRY model.

**Specifying the ARRY \( Y_A \) valuation model**

As defined, ARRY \( Y_A \) is the real value annually in arrears required return or discount rate, which in this model is used to discount all future real values to bring them to present real values (PRVs).

This basic ARRY valuation model is presented in a simpler term & reversion formulation than prior real valuation models.

i. *The real value of the term to run*

The term is treated in a three-step calculation in real terms. Firstly, capitalising in perpetuity the current (existing and real) contract rental \( C \), as if it was currently reviewed at that rental at the valuation date. It is capitalised at the all risks real capitalisation rate \( R^T_A \) in Equation (13), based on rental review terms equal to the term to run \( T \), and on future reviews (not the actual contract rental review term \( F \)), giving the PRV of current contract rental on that assumption as:

\[
\text{PRV of current contract rental} \leftarrow \frac{C}{R^T_A} \quad (18)
\]

Secondly, the PRV of current contract rental is deferred until the next review date \( T \) at \( Y_A \) as the discount rate, giving the PRV of the (future real value (FRV) of the real) current contract rental reversion as:

\[
\text{PRV of current contract rental reversion} \leftarrow \frac{C}{Y_A}(1 + Y_A)^{-T} \quad (19)
\]

Thirdly, the PRV of current contract rental reversion is then cut-off, by deducting it from the capitalised contract rental (i.e. PRV current contract rental – PRV current
contract rental reversion). This difference gives the PRV of the real contract rental term to run as:

\[
PRV \text{ of real contract rental term to run} = \frac{C}{R_A^F} - \frac{C}{R_A^F} \left(1 + \frac{1}{Y_A}\right)^{-T} \tag{20}
\]

This three-step calculation can be further simplified which makes the calculation easier. Simplified to:

\[
PRV \text{ of contract rental term to run} = \frac{C}{R_A^F} \left(1 - \left(1 + \frac{1}{Y_A}\right)^{-T}\right) \tag{21}
\]

This is a lot simpler than trying to treat the contract rental as an annuity to run discounted to also allow accurately for the effects of inflation and lack of real growth over the term to run (requiring a triple discounting for \(Ie\), \(G_r\), and \(Y_A\)).

This key difference in concept, or paradigm shift required for valuers, is to think and make calculations in truly real terms. This distinguishes the model from all conventional and previous real value models. It calculates present values, in real terms using real rentals as at the valuation date, giving the PRV of the term to run, plus similarly the PRV of the reversion (as shown hereunder).

\(ii. \ The \ real \ value \ of \ the \ reversion\)

The reversion is treated in a two-step calculation in real terms by firstly capitalising in perpetuity the current real contract market rental \(C_C\), as if it was reviewed at that market rental at the valuation date. It is capitalised at the all risks real capitalisation rate \(R_A^F\), using Equation (14), based on the contract rental review terms \(F\), giving the PRV of the current market rental as:

\[
PRV \text{ of current contract market rental} = \frac{C_C}{R_A^F} \tag{22}
\]

Secondly, the PRV of current contract market rental is deferred until the next review date \(T\) at \(Y_A\) as the discount rate giving the PRV of the contract market rental reversion as:

\[
PRV \text{ of contract market rental reversion} = \frac{C_C}{R_A^F} \left(1 + \frac{1}{Y_A}\right)^{-T} \tag{23}
\]

\(iii. \ The \ total \ real \ current \ market \ value\)

The CMV of the property is the sum of the PRV of contract rental term to run plus the PRV of the contract market rental reversion, combining Formulas (21) and (23):
CMV = PRV of Term + PRV of Reversion

\[ CMV = \frac{C}{R_a} (1 - (1 + Y_a)^{-T}) + \frac{C_c}{R_a} (1 + Y_a)^{-T} \]  \hspace{2cm} (24)

The above simplified formula is the basic ARRY \( Y_A \) real value valuation model.

The initial yield to apply to the current contract rental can also be calculated simply as:

\[ R_0 = \frac{C}{\left( \frac{C}{R_a} (1 - (1 + Y_a)^{-T}) + \frac{C_c}{R_a} (1 + Y_a)^{-T} \right)} \]  \hspace{2cm} (25)

or as a passing contract rental income multiplier (YP):

\[ YP = \frac{1}{C} \left( \frac{C}{R_a} (1 - (1 + Y_a)^{-T}) + \frac{C_c}{R_a} (1 + Y_a)^{-T} \right) \]  \hspace{2cm} (26)

ARRAY sale analysis

The model relies on fewer assumptions by the analyst/valuer than the other conventional and real value models. The key \( Y_A \) all risks real yield and the real growth rate \( G_r \) are extracted from market sales evidence, but still requiring judgement in applying these in the model.

Given reliable exogenous data available for \( I_e \) and given a supportable expectation of \( G_r \), the \( Y_A \) is extracted from comparable market sales by solving for it in the ARRY valuation model, or vice versa, given \( Y_A \), the \( G_r \) can be extracted from comparable market sales.

Assuming there is good comparable recent market evidence represented by Sale 1 of similar type, property characteristics, location and asset class and investment risk profile to the above example.

Sale 1 sold for $570,000 and has a lease with \( F = 3 \) years rent reviews and sold (to make the calculation simpler) on an anniversary of a rent review date with \( T = 2 \) years to run till the next rental review. The contract rental is \( C = $40,000 \) p.a., the current contract market rental (if reviewed at sale date) is $41,200 in line with the general market evidence of growth rates. Independent evidence of expected inflation \( I_e = 2\% \) p.a. and based on wider sales analysis and trends in real rentals, real growth \( G_r = 1 \% \) p.a.

This is applied to analyse Sale 1 by substituting the known lease data and market evidence in Formula (24) and solving for the unknown ARRY \( Y_A \) to give the sale price:
The capitalisation rates \( R_A^3 \) and \( R_A^4 \) are in turn dependant on \( Y_A \) and are solved concurrently with solving for \( Y_A \). This has been done using Goal Seek analysis in a spreadsheet version of the model as shown in Schedule 1 (including a sales analysis on a nominal value model) and gives a \( Y_A = 7.0147\% \) p.a.

The step by step calculations are completed follows:

$$ $570,000 \left( 1 - (1.070147)^{-2} \right) + \frac{41,200}{0.071150} (1.070147)^{-2} $$ \hspace{1cm} (28) $$

$$ $570,000 \left( \frac{579,062 \times 0.873198}{571,133 \times 0.126802} \right) $$ \hspace{1cm} (29) $$

$$ $71,288 + $498,712 $$ \hspace{1cm} (30) $$

### Schedule 1 – Analysis of Sale 1

<table>
<thead>
<tr>
<th>Conventional Direct Capitalisation Model Nominal Value Calculations - annually EOP</th>
<th>All-risks Real Value Yield - ( Y_A ) - Valuation Model Calculations - annually EOP - allowing real growth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal value over-all Growth rate:</strong> ( G_0 )</td>
<td><strong>Nominal value Growth rate:</strong> ( G_0 = G + I_e )</td>
</tr>
<tr>
<td><strong>Over-all Required Nominal Yield (Disc. Rate):</strong> ( Y_0 )</td>
<td><strong>Expected (Nominal) Inflation Rate:</strong> ( I_e )</td>
</tr>
<tr>
<td><strong>Review Term Frequency in yrs:</strong> ( F )</td>
<td><strong>All-risks Real Yield Rate:</strong> ( Y_A = (Y_0 - G - I_e) )</td>
</tr>
<tr>
<td><strong>Capitalisation Term - All Risk Yield rate:</strong> ( ARY = R_A^3 )</td>
<td><strong>All-risks Real Capitalisation rate:</strong> ( R_A^3 )</td>
</tr>
</tbody>
</table>

**Sale Analysis**

**Term & Reversion: I.e. Valuation not at Review or Commencement**

<table>
<thead>
<tr>
<th><strong>Term - Nominal Value:</strong></th>
<th><strong>Term - Real Value:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Term to run in yrs:</strong> ( T )</td>
<td>2.00 years</td>
</tr>
<tr>
<td><strong>Contract Rent:</strong></td>
<td>$ 40,000</td>
</tr>
<tr>
<td><strong>Current Contract Market Rent:</strong></td>
<td>$ 41,200</td>
</tr>
<tr>
<td><strong>PV Contract Rent $40,000 for 2 years, disc @ ( Y_0: 10.0147% ) p.a.:</strong></td>
<td>$ 69,408</td>
</tr>
<tr>
<td><strong>Reversion - Nominal Value:</strong></td>
<td><strong>PRV of 2 years term to run (by deduction):</strong></td>
</tr>
<tr>
<td><strong>Contract Market Rent $40,000 with growth over 2 yrs @ 3% p.a.:</strong></td>
<td>$ 43,709</td>
</tr>
<tr>
<td><strong>Contract Market Rental $41,200 capped @ ( R_A^3: 7.2137% ) p.a.:</strong></td>
<td>$ 605,915</td>
</tr>
<tr>
<td><strong>PV of Reversion in 2 yrs disc @ ( Y_0: 10.0147% ) p.a.:</strong></td>
<td>$ 500,622</td>
</tr>
<tr>
<td><strong>Total PV: Term + Reversion:</strong></td>
<td><strong>Total PRV: Term + Reversion:</strong></td>
</tr>
<tr>
<td><strong>Current Market Value (Rounded):</strong></td>
<td><strong>Initial yield or over-all capitalisation rate:</strong></td>
</tr>
</tbody>
</table>

**Application of ARRY real valuation model**

This sales evidence supports the market data inputs (previously assumed) and is applied by substituting the known lease data and market based \( Y_A \) which is rounded to 7.0% p.a. in Formula (24):
Recalling that the above example is based on \( C = \$39,500 \text{ p.a.}, \) \( C_C = \$40,685 \text{ p.a.}, \) the term to run \( T = 4, \) rental review terms \( F = 5 \) and checking the interrelationship between rates:
\[
Y_A = Y_O - I_e - G_r = 0.10 - 0.02 - 0.01 = 0.070 \text{ or } 7.0\% \text{ p.a.}
\]

\[
CMV = \frac{\$39,500}{0.072957} \left(1 - (1.07)^{-4}\right) + \frac{\$40,685}{0.073911} \left(1.07\right)^{-4} \tag{31}
\]

\[
CMV = (\$557,661 \times 0.762895) + (\$550,457 \times 0.237105) \tag{32}
\]

\[
\$548,314 = \$128,373 + \$419,941 \Rightarrow \text{Say } \$548,000 \tag{33}
\]

The capitalisation rates \( R_A^4 \) and \( R_A^5 \) are dependant on \( Y_A \) and are calculated concurrently (which has been done using a spreadsheet version of the valuation model as shown in Schedule 2 (including a nominal value model for comparison)).

<table>
<thead>
<tr>
<th>Schedule 2 – Valuation of Example 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional Direct Capitalisation Model</td>
</tr>
<tr>
<td>Nominal value over-all Growth rate: ( G_0 )</td>
</tr>
<tr>
<td>Over-all Required Nominal Yield (Disc. Rate): ( Y_0 )</td>
</tr>
<tr>
<td>Nominal Value Calculations - annually EOP</td>
</tr>
<tr>
<td>All-risks Real Yield Rate: ( Y_A = (Y_0 - G - I_e) )</td>
</tr>
<tr>
<td>Review Term Frequency in yrs: ( F )</td>
</tr>
<tr>
<td>Capitalisation Rate - All Risk yield rate: ( ARY = R_0^4 )</td>
</tr>
<tr>
<td>Sale Analysis</td>
</tr>
<tr>
<td>Term &amp; Reversion: I.e. Valuation not at Review or Commencement</td>
</tr>
<tr>
<td>Term - Nominal Value:</td>
</tr>
<tr>
<td>Term to run in yrs: ( T )</td>
</tr>
<tr>
<td>Contract Rent; ( $39,500 )</td>
</tr>
<tr>
<td>Current Contract Market Rent; ( $40,685 )</td>
</tr>
<tr>
<td>PV Contract Rent $39,500 for 4 years, disc @ ( Y_0 ); 10% p.a.</td>
</tr>
<tr>
<td>Reversion - Nominal Value:</td>
</tr>
<tr>
<td>Contract Market Rent $39,500 with growth over 4 yrs @ 3% p.a.</td>
</tr>
<tr>
<td>Contract Market Rental $40,685 capped @ ( R_0^5 ); 7.3911% p.a.</td>
</tr>
<tr>
<td>PV of Reversion in 4 yrs disc @ ( Y_0 ); 10% p.a.</td>
</tr>
<tr>
<td>Total PV: Term + Reversion =</td>
</tr>
<tr>
<td>( $548,314 )</td>
</tr>
<tr>
<td>Current Market Value (Rounded)</td>
</tr>
</tbody>
</table>

Note that \( R_A^F \cong R_O^F \), i.e. \( R_A^5 \cong R_O^5 \), as this must be the case as they are direct capitalisation rates relating current real and nominal rents with current real and nominal capital values as defined, i.e. assuming the lease commenced, or was reviewed or renewed as at the valuation date. This is distinguished from the overall capitalisation rate \( R_O \), which is a generic term for the actual relationship between the
contract rental (passing rent) and the value at the valuation date which commonly is between the rental review dates. It will (normally) be at a % p.a. logically less than the $R_{OF}$, due to the pending review in a situation where $T < F$ and growth is expected in the reviewed rental since it was last reviewed.

The methodology used in the ARRY model is shown diagrammatically in the following charts where the real cash flows have been forecast to show the break-up of the term to run and reversionary present and future values.

The first Chart combines both sets of real cash flows assumed $F = 4$ year reviews up to and beyond the reversionary date, and the $F = 5$ year reviews from the next review (reversionary) date. The second Chart shows ARRY model methodology used in the valuation, cutting-off the PRVs of the $F = 4$ year review FRVs of the reversionary real cash flows after the next review; and adding only the PRVs of the reversionary $F = 5$ year review FRVs of the reversionary real cash flows from the next review date.

Example 1: Real Value Cash Flows and PRVs - Term & Reversion Methodology
Combined $F=3$ year and $F=5$ years Rental Reviews

[Image of a graph showing real market rentals, real contract market rentals, and present values of future cash flows for different periods.]
APPLICATIONS AND FURTHER RESEARCH

It is beyond the scope of this paper to describe the fully developed model and progressively remove the provisos and bring the model into a more practical application to be used by valuers in their day to day professional work.

The model is being developed and empirically tested to deal with the following:

- The future expiry of a current lease, and re-leasing on a different rent review market terms

- Vacancies and rent-up periods

- Inducements and leasing costs

- Cash flow timings in advance and in arrears (BOP and EOP)

- Monthly and quarterly or other rental payment frequencies

- Multi-tenanted properties with different tenancies having different lease terms

- OPEX recoveries with net or gross leases and mixed in multi-tenanted properties
• CAPEX, refurbishment and contingent deferred maintenance liabilities
• Costs of purchase (including due diligence) and costs of sale at termination of the investment
• Terminating leases and interests
• Leaseholds both lessor’s and lessee’s interests
• Applications to ground lease tenures and real value ground rental valuation models
• Tax effects, i.e. where capital gains tax (CGT) applies
• After finance and after tax real value models for investment analysis.

The techniques employed are similar to those in the generic real value valuation model and real value lessor’s and lessee’s interest valuation model (Jefferies, 1997a, 1997b).

CONCLUSION

This paper has introduced a new ARRY real value valuation model and demonstrates its use in both sales analysis and market valuations in a simple common sales analysis and valuation example. Its potential adoption amongst the valuation and appraisal fraternity will depend on the results of its empirical testing and promotion.

Hopefully, it will not suffer the rejection of previous real value models and the author is conscious of the need to engage with practitioners to test and promote its use, as it will depend not only on showing it to be a simpler technique, but also with commercial advantages. The observation from recent research on the history of changes in and adoption of new income valuation techniques presented at the ERES 2010 Conference is pertinent advice regarding the future of the ARRY model in the 21st Century:

“We feel that this ... (history)... applied to property investment has provided useful insights into the process by which mathematical formulations evolve from ideas to mainstream application in practice. It identifies some of the drivers for change including the need for the inventors and supporters to engage with practice if they are to convince users of the merits of their formulation” (Crosby & Henneberry, 2010, p. 17).
REFERENCES


Wood, E. (1972). *Property investment, a real value approach*. Unpublished PhD, University of Reading, Reading, UK.


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