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Abstract

Conventional time domain reflectometry (TDR) for measurement of water content typically uses a parallel waveguide buried in the material under test, and provides a measure of the mean permittivity and hence water content of a volume surrounding the waveguide. Here we describe an alternative arrangement whereby the waveguide is positioned outside the material. With the altered arrangement, the mean near-surface water content may be deduced using a modeling technique. However, by making measurements at several different positions of the waveguide relative to the material, we show that combining forward modeling with an inversion technique enables the near-surface water content profile to be determined. Using laboratory measurements, we demonstrate that for a coarse-grained soil, an accuracy of 2% volumetric water content is feasible when using an accurate time domain reflectometer, a self-calibration technique and the Topp¹ equation for translation of real permittivity to soil water content. We then outline two improvements on the modeling and inversion techniques. The first is a simplification of the forward problem that reduces the size of the field matrix and the number of unknown variables. We then describe how combining the simplification with alternative approaches to the inverse calculation can yield improvements in calculation time.

Keywords  soil, moisture measurement, profile, non-invasive

Introduction

Determination of moisture content $\theta$ is of vital interest to a wide range of disciplines and industries. The range of materials is also diverse and includes soil, cereals, dairy products and timber. For soil, electrical methods now dominate $\theta$ measurement and time domain reflectometry (TDR) using an open waveguide inserted into the soil, is the method of choice for most soil scientists. TDR, by measuring the propagation velocity $v_p$ of an electromagnetic signal, provides a measure of soil permittivity $\varepsilon_s$, which is related to volumetric soil moisture content $\theta_v$ via a suitable dielectric model, such as that described by Topp et al., 1980¹. Waveguide installation generally disturbs the soil profile and depending on orientation, may provide preferential paths for water transport. Since soil disturbance is an issue for researchers investigating water movement in soil, efforts have been made to establish techniques for non-invasive measurement of $\theta$. Although Selker (1993)² described the use and calibration of a non-invasive TDR surface probe, it was only useful for assessing surface moisture content. In many instances, $\theta$ profiles add further useful information to measurements of the mean $\theta$ since they strongly affect moisture and heat transport, solute movement, and the activity of biological organisms, and are influenced by drainage, surface evaporation, plant water uptake and capillarity.
The few non-invasive methods for measuring \( \theta \) profiles include unguided reflectometry in the frequency domain, also referred to as ground penetrating radar (GPR) (Keam et al., 1999[3]; Lee et al., 2002[4]) and electromagnetic (EM) induction, Sheets and Hendricks, 1995[5]). The former is an immature technique, but seems likely to provide a depth range of approximately one metre, and may typically resolve moisture content in 100 mm layers. EM induction is dependent on soil type and conductivity, but can provide useful indications for non-critical applications. We have previously described a method for measuring the 2-D \( \varepsilon_r \) distribution (Woodhead et al., 2001)[6] by ‘time domain reflectometry imaging’ (TDRI). Here we first describe a 1-D method for the measurement of near-surface (0-100 mm), \( \varepsilon_r \) profiles, and then show results from measurements within an artificially layered soil.

The TDRI probing signal comprises the transverse field of a parallel waveguide. A set of TDR measurements for different heights of the waveguide above the underlying soil is inverted to obtain a representation of its \( \varepsilon_r \) and distribution. In the present case \( \varepsilon_r \) is assumed invariant in the axial (z) and horizontal transverse direction (x), so the waveguide is moved along the y-axis (normal to the soil surface) to resolve \( \varepsilon_r \) of flat sheets of soil in which \( \theta_v \) is assumed uniform. Having obtained the \( \varepsilon_r \) profile of the soil, a dielectric model is used translate the \( \varepsilon_r \) profile to \( \theta_v \).

Use of the evanescent field as the EM probing signal contrasts with the more conventional microwave imaging method, and provides two important advantages. Scattering does not significantly attenuate the signal as occurs with microwave imaging (Jofre et al., 1990[7]), and the use of an evanescent field provides a spatial resolution that is not inherently limited by the wavelength (\( \lambda \)) as in GPR. However, TDRI requires a very high time measurement resolution which in turn determines the depth resolution. Since transverse electromagnetic mode (TEM) propagation is assumed, the waveguide separation must be small compared with \( \lambda \), so a practical penetration depth limit is approximately 80 mm with current geometry, instrumentation and measurement frequencies.

Here we describe some theory of the TDRI technique, and present results from a laboratory trial using a physical model of a layered soil profile.

**Theory**

Fundamental to TDRI is a forward solution to provide a prediction of \( v_p \) on a waveguide surrounded by a prescribed, discretised, inhomogenous \( \varepsilon_r \) distribution. In this application the waveguide remains in the air space above the soil, and we formulated a generalised model to provide the prediction of \( v_p \). A suitable numerical procedure using a moment method (MM) has been described (Harrington, 1968[8]), and is used in conjunction with a solution to the inverse problem (Woodhead et al., 2001[6]), to quantify the \( \varepsilon_r \) distribution given a set of \( v_p \) representing different positions of the waveguide above the soil. The assumption of lossless soils is common in the use of TDR for soil moisture measurement and although the forward model described here is currently limited to real \( \varepsilon_r \), we expect to add a component to the method to include complex values of \( \varepsilon_r \). The MM is a volume integral EM modelling technique but was adapted to provide rapid calculation in 2-D (Woodhead et al., 2001[6]).
Given an impressed electric field $\mathbf{E}_i(r)$ in a region, the total resultant electric field distribution $\mathbf{E}_{tot}(r)$, for a material of arbitrary permittivity distribution $\varepsilon(r)$ is:

\[
\mathbf{E}_{tot}(r) = \mathbf{E}_i(r) + \mathbf{E}_{pol}(r)
\]

so that

\[
-\mathbf{E}_i(r) = \mathbf{E}_{pol}(r) - \frac{\mathbf{P}(r)}{\varepsilon_0 \chi(r)} = K(\mathbf{P})
\]

$K$ is a linear operator acting on the polarization $\mathbf{P}$, $\chi(r)$ is the electric susceptibility and $r$ is the space dimension ($x,y,z$). The resultant polarisation field $\mathbf{E}_{pol}(r)$ is obtained from a numerical integral of the individual polarisation fields from all cells within the region (the soil cells of Figure 1).

\[
\mathbf{E}_{pol} = -\frac{1}{4\pi \varepsilon_0} \nabla \left( \sum_r \frac{\mathbf{P} \cdot \mathbf{r}}{r^3} \Delta r \right)
\]

where $\Delta r$ defines the size of a cell. The polarisation region may now be discretised, and following the MM [8] and using Eqn 2, we calculate the matrix of polarisation vectors $[\mathbf{P}]$:

\[
[\mathbf{P}] = -[K]^{-1} [\mathbf{E}_i]
\]

Fig. 1. The 2-D arrangement of discrete cells in the transverse ($x$-$y$) plane showing waveguide rods and air and soil regions discretised into 10mm square cells. The $z$-axis is into the page.
Next, the electric field strengths are extracted.

\[ \mathbf{E}_{tot}(r) = \frac{\mathbf{P}(r)}{\varepsilon_0 \chi(r)} \]  

(5)

Finally, to complete the solution for the case of a parallel waveguide, the voltage between the waveguide rods is obtained from a line integral over a suitable integration path \( l \) between the waveguide rods to obtain the line capacitance \( C \)

\[ C = \frac{\rho}{\int \mathbf{E}(r) \cdot dl} \]  

(6)

where \( \rho \) is the linear charge density used to define \( \mathbf{E} \). \( v_p \) for a pulse reflected on the waveguide is then calculated using the telegrapher’s equation for a lossless waveguide.

The inverse solution may be described by:

\[ m = g^{-1}(d) \]  

(7)

where \( m \) is a set of model parameters or values of \( \varepsilon_r \), \( g \) describes the forward transfer function or forward model, and includes reliance on \( \varepsilon_r \) and other parameters such as waveguide geometry and fundamental constants, and \( d \) is a set of observations or readings of \( v_p \), in this instance converted to propagation time \( t_p \). The recurrence relation is

\[ m_{i+1} = m_i - \alpha_i J_i^{-1} \Delta t_i \]  

(8)

Here the new model \( m_{i+1} \) is derived from the previous permittivity model \( m_i \), corrected by the weighted product of the sensitivity \( J_i^{-1} \) which in this case used \( \mathbf{E}_{pol}(r) \) as a surrogate for the Jacobian, and \( \Delta t_i \) which is the difference between observed and predicted values of \( t_p \).

Weighting factor \( \alpha_i \) controls the step size. We chose the Polak-Ribière variant (Press et al., 1992\(^9\)) of conjugate gradient (CG) optimisation which alters the correction factor to explore new regions of the solution space, and employed an earlier 2-D inversion method, reduced to 1-D by using the mean value of \( \varepsilon(y) \) at each iteration.

**Materials and Methods**

Evaluation of TDRI for measurement of soil moisture profiles used a physical model of a soil profile. The model comprised a perspex housing 400 by 200mm in the \( x-z \) plane, containing five, 20mm thick layers of fine sandy loam, separated by 1mm polycarbonate separator sheets. To obtain similar dry bulk densities in each layer, one layer was filled with oven dry screened soil while the housing was gently shaken, and then the soil was removed and weighed. In several sub-layers, the measured mass (1509g) of soil was used to fill each 20mm layer. Each sub-layer was interspersed with a predetermined quantity of water sprayed on the soil surface to obtain the target moisture content for that layer. The final step in the process was to sprinkle an additional quantity of dry sieved soil to fill the layer, hence
compensating for the slight differences in packed volume. As a result, there were minor
differences in the dry bulk density between layers. The weight of the model was monitored at
regular intervals throughout the trial.

Fig. 2. Arrangement of the waveguide and the physical model of a layered soil profile. The
apparatus at the left enables adjustment of the y-axis (vertical position) of the waveguide.

The waveguide was constructed from 6 mm diameter brass rods spaced 60mm apart, with an
active length of 300mm. Although stainless steel waveguide rods have been used for
conventional TDR measurement of $\theta_v$, degradation in the risetime ($t_r$) of the voltage step
from the balun was approximately 100 ps, hence leading us to use a lower loss material. The
brass waveguide had a negligible effect on $t_r$.

The beginning of the active portion of the waveguide was defined by a shunt diode. The
diode, a HP5082-3188 PIN diode (on-resistance $R_s = 0.6 \Omega$ at a diode current $I_d = 10$ mA,
and reverse bias capacitance $C_r < 1 \mu F$ at 20 V) was employed in a manner similar to that of
Hook et al., 1992$^{[10]}$, and a bias network enabled the diode to be switched on to provide a
reference measurement. However, contrary to their findings when applied to the less
demanding measurements using a buried waveguide, we detected the effect of diode
impedance changes up to a reverse bias on the diode of 20 volts, although the forward current
had little effect once the diode was forward biased. Consequently, a forward current of 10
mA was chosen for the reference $t_p$ and minus 20 V to measure total $t_p$.

Measurements were made with a Hewlett Packard (since renamed Agilent) HP54121T
digitising oscilloscope which incorporates a step generator for TDR measurements. The
waveguide assembly was connected to the HP54121T by a semi-rigid coaxial cable, a balun
and the diode bias network. A Guanella type balun was constructed similar to the 1:4 balun
described by Spaans and Baker (1993)$^{[11]}$ but used 3.5 mm diameter, grade S3 ferrite toroids
with three turns of 0.125 mm enameled wire for both isolating and splitter baluns.
Waveguide positioning was achieved by a 1 mm pitch lead screw driven from a $7.5^\circ$ stepper controlled from a standard PC printer port and power interface (Figure 2).

The measured and reference waveforms (Figure 3) were retrieved from the HP54121T and the difference waveform was smoothed and differentiated using 25 point least squares fitting routines (Savitzky and Golay, 1964$^{[12]}$). Both reference and measured reflection times on the difference waveform were determined from the intersection of tangents to the maximum slope of the returned edge and the preceding plateau in a manner similar to that used by van Gemert (1973)$^{[13]}$. The time difference provided a measurement of $t_p$ for the active section of the waveguide. The end effect, which increases mean $t_p$ due to end capacitance, was ignored.

Next, the vector of measured $t_p$ was used in the inversion process outlined above to provide a predicted $\theta_v$ profile, but for each iteration, $\varepsilon_r$ of all cells in each horizontal layer was reset to their common mean value. A plane, transverse to the waveguide, was discretised into a 16 by 16 array of 10-mm square cells. The cells in the six upper rows were assigned values of $\varepsilon_r = 1$ and represented the space above the soil surface in which the waveguide was positioned, and the lower 10 rows represented 5 layers of soil, each 20 mm thick. The cells in the soil layers were all assigned an a priori value (used as the starting distribution in the iterative inversion procedure), of $\varepsilon_r = 20$, corresponding to $\theta_v = 0.33$. Conversion between $\varepsilon_r$ and $\theta_v$ used the equation of Topp (1980)$^{[1]}$.

The single calibration point used a reading of $t_p$ with the waveguide far (>100 mm) above the soil layer. The measured $t_p$ (with the waveguide far above the physical soil model) was compared with the predicted reading for a uniform distribution of $\varepsilon_r = 1$, and the difference (46 ps) was used to offset the predicted $t_p$. 

![Fig. 3. Difference waveform and tangents for HP54121T measurements.](image-url)
Results and Discussion

Table 1 shows the constituents of each soil layer. The mass of water was calculated to provide the given profile of $\theta_v$. Although not used, the dry bulk density has been calculated for each layer since it does affect the Topp dielectric model, most notably at low values of $\theta_v$.

Table 1. Results showing quantity of fine sandy loam and water in each layer, the volumetric water content, and the dry bulk density

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth (mm)</th>
<th>Mass dry soil (g)</th>
<th>Mass water (g)</th>
<th>Soil volume (cc)</th>
<th>Moisture content (V/V)</th>
<th>Dry bulk density (g/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-20</td>
<td>1554</td>
<td>136</td>
<td>1360</td>
<td>0.1</td>
<td>1.143</td>
</tr>
<tr>
<td>2</td>
<td>20-40</td>
<td>1594</td>
<td>204</td>
<td>1360</td>
<td>0.15</td>
<td>1.172</td>
</tr>
<tr>
<td>3</td>
<td>40-60</td>
<td>1566</td>
<td>272</td>
<td>1360</td>
<td>0.2</td>
<td>1.151</td>
</tr>
<tr>
<td>4</td>
<td>60-80</td>
<td>1584</td>
<td>408</td>
<td>1360</td>
<td>0.3</td>
<td>1.165</td>
</tr>
<tr>
<td>5</td>
<td>80-100</td>
<td>1596</td>
<td>476</td>
<td>1360</td>
<td>0.35</td>
<td>1.174</td>
</tr>
</tbody>
</table>

Six measurements were made at distances from the soil surface of 5 to 55 mm. Errors could reasonably be expected due to the influence of the polycarbonate ($\varepsilon_r = 3$) sheets separating the layers, so the first measurement positioned the waveguide rods in line with the centres of the cells used for the forward problem model, i.e. with 5 mm spacing between each rod centre and the soil surface (half the cross-sectional dimension of each cell). Hence the upper polycarbonate sheet was included in the row of cells immediately above the soil surface.

Measured and predicted (using the forward model) readings for $t_p$ are listed in Table 2. The inversion process stopped after 60 iterations and errors in $t_p$ (between the measured values and those predicted from the solution $\theta_v$ profile) then ranged from 0.06 ps to 0.3 ps. Although Table 2 indicates a very close correlation between measured $t_p$ and that predicted from the known $\theta_v$ profile, no adjustment has been included for the effect on the forward model of the polycarbonate separator sheets. Fig. 2ure 4 shows the predicted and actual (as defined in Table 1) $\theta_v$ profiles.

Table 2. Simulated and measured values of $t_p$ (ns) and their difference (ps). No correction was made for the 1 mm polycarbonate sheets separating the soil layers.

<table>
<thead>
<tr>
<th>Simulated</th>
<th>Measured</th>
<th>Error (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.721</td>
<td>2.714</td>
<td>-7</td>
</tr>
<tr>
<td>2.248</td>
<td>2.241</td>
<td>-7</td>
</tr>
<tr>
<td>2.134</td>
<td>2.128</td>
<td>-6</td>
</tr>
<tr>
<td>2.090</td>
<td>2.084</td>
<td>-6</td>
</tr>
<tr>
<td>2.070</td>
<td>2.064</td>
<td>-6</td>
</tr>
<tr>
<td>2.060</td>
<td>2.054</td>
<td>-6</td>
</tr>
</tbody>
</table>
Fig. 4. Comparison of actual and measured \(\theta_r\) at different depths using a physical model of layered soil.

For the current configuration, we envisage a practical depth limit of 60 mm. The technique may provide some indication of moisture content beyond that depth, but is constrained by the rapidly decreasing influence of \(\theta_r\) on \(t_p\), as depth increases. Current work is focussed on measurements using wider-spaced waveguides, which is expected to provide measurements at greater depths, although bandwidth reduction will increase soil-type dependence. The forward model assumes a uniform moisture distribution within each layer, violated in this trial by the polycarbonate separators. As noted earlier, accuracy is dependent on the measurement accuracy of \(t_p\), but is also dependent on the accuracy of the forward model and on the inversion technique.

We have already refined the forward model to provide improved modelling accuracy (Woodhead et al., 2005[14]) and now describe our approach for providing improvements to the inverse problem solution to improve execution time. There are two possible simplifications to the geometry shown in Figure 1 and thus to physical relationships described in Equation (1).

Since the cells representing air are known (\(\varepsilon_r \approx 1\)) and need not be included as contributing to \(E_{pol}\) of Eqn 3, they may be eliminated from the calculation of \(P\). This significantly reduces the size of matrix \(K\) and hence of the entire solution and execution time. To calculate \(E\) between the rods, cells are positioned on a line between the rods, but due to symmetry, they are only required between the mid point and one rod. Since these cells may be of arbitrary size, interpolation as described by Woodhead et al., 2000[15] is not required, the cell size is chosen to achieve the resolution required for accurate integration of \(E\) between the waveguide rods. Use of this simplification means a fundamental change to the manner in which the MM is formulated, but early results indicate that existing assumptions are sufficient and will not be violated.

In the case where the soil can be considered as plane stratified in a region parallel to the waveguides, \(\varepsilon_r\) for each soil layer can be made uniform so there will be one value of \(\varepsilon_r\) for each of the five soil layers shown in Figure 2, further decreasing the number of cells required.
to define the MM problem. For the 16 by 16 cell problem depicted in Figure 1, the full 3-D method requires matrix $K$ of eqn 4 to be $3n^3$ by $3n^3$ where $n$ is the number of cells in each dimension, or 12,288 by 12,288. In the pseudo-3D method described in Woodhead et al., 2001[14] and used for the above procedure, matrix $K$ is $2n^2$ by $2n^2$, or 512 by 512 entries in this case. For the simplifications described above, the cell size within the soil region would be 20 by 20 mm leading to 40 cells for each of the $x$ and $y$ electric field vectors. Using symmetry, $K$ then becomes a 40 by 40 matrix leading to polarisation vector $P$ of Eqn 4, which is used to calculate field strength $E$ in the cells between the waveguide rods. The increase in cell size to match the selected depth of each soil layer is now feasible since the original choice of 10 mm cells was to ensure a representative number of discrete cells between the waveguide rods for integration of $E$. The result is a two order of magnitude reduction in the number of cells from the pseudo 3-D method described in Woodhead et al., 2001[16].

The efficiency of the inverse iteration procedure is also dependant on how close the initial or a priori model is to the actual values. In principle, the stratified formulation may begin with oversized cells that are larger than the selected depth resolution using a simplified search method rather than the full inversion technique deployed for the solution described above, and then this interim solution used to form the a priori model for the full resolution inversion. This approach is expected to provide more rapid convergence than the full procedure used to date, and we expect to report on the results from this trial shortly.

**Conclusions**

A novel TDRI method has been developed to non-invasively measure the near-surface moisture profile of soil in the range 0 to 100 mm. Measurements were made using precision TDR equipment, and reconstruction of the soil moisture profile used a moment method solution to solve the forward problem, and a conjugate gradient approach for the iterative inverse solution. An instrument based on the approach described here should have broad application in both research and commercial applications where a non-invasive method of measuring near-surface soil moisture profiles is required.

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**References**


Biographical Information

Ian Woodhead received the BSc degree from Canterbury University in 1985, and PhD from Lincoln University in 2001. He was employed as a scientist at the Agricultural Engineering Institute, Lincoln University from 1985 to 1992 whereupon he was appointed research engineer with Lincoln Technology, a division of Lincoln Ventures Ltd, and in 1998 appointed manager of Lincoln Technology. Ian has directed the research and development of several technologies that have led to commercial products. He is programme manager for an ongoing government-funded research programme investigating new sensing and data analysis techniques and their application to industrial processes and products. His research interests include dielectric modelling and inversion of electromagnetic data.

Ian Platt was awarded a PhD in Theoretical Physics from La Trobe University Australia in 1987. He has considerable experience in both the academic and commercial environments, including research into Over the Horizon Radar Networks (JORN), Propagation analysis for HF communication systems, upper atmospheric structure and Ionospheric Modification. He is a research engineer with Lincoln Ventures Ltd and is investigating new techniques for use in Time Domain Reflectometry Imaging (TDRI) and Optical Strain Sensing.

Graeme Buchan received a BSc and a PhD degree in theoretical physics (quantum statistical mechanics) from the University of Aberdeen in 1972 and 1976, respectively. From 1981-1986 he was a lecturer in Applied Physics (environmental physics) at Strathclyde University, Glasgow. Since 1986 he has been in the Soil & Physical Sciences Group at Lincoln University, New Zealand, where he is now Associate Professor. His research spans soil physics, environmental and plant physics, moisture measurement, and environmental education. Graeme is a Fellow of the Royal Meteorological Society (UK), Chartered Physicist (UK), and a Member of the Institution of Environmental Sciences (UK).