

# A 1-D Inversion Method for Permittivity Calculations Using a Non-Invasive TDR

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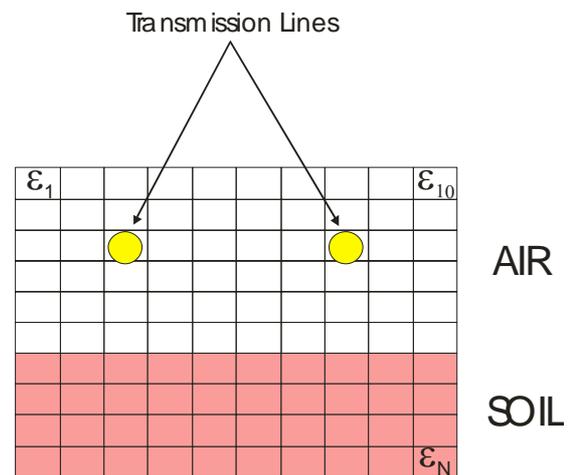
## Abstract

The non-invasive time domain reflectometry (TDR) technique uses the method of moments to determine the polarized electric field induced by dielectric cells under the action of an impressed field. The contribution from this field to the total field between the transmission lines enables an estimate to be made of the cell's dielectric permittivity. The geometry currently used for this method of moments integration can be simplified considerably by eliminating redundant cells, either by invoking symmetry or changes in the way the field between the transmission lines is calculated. Here we provide details of the simplification method together with the reduction in complexity that can be achieved.

**Keywords:** TDR, dielectric, permittivity, moisture

## 1 Introduction

Time Domain Reflectometry (TDR) is becoming an increasingly popular method for determining the moisture content contained within various materials. The process works by inferring the relative dielectric permittivity,  $\epsilon_r$  from the propagation time,  $t_p$  down some form of transmission line. A pulse inserted at the head of the transmission line will travel down its length and return back to the head after being reflected from its end. The electric field,  $E_i$  generated by this pulse causes the dielectric to generate an opposing polarization electric field (whose magnitude is determined by  $\epsilon_r$ ) which will in turn act to decrease the velocity of the pulse. Once the permittivity is determined by this method a modelled transform is used to convert it into the volumetric water content,  $\theta_v$  (e.g. [1]). Most methods use wire probes that are inserted into the material, usually soil, with the pulse generator and receiving equipment (reflectometer) attached via a coaxial cable. Woodhead *et al* [2] however, developed a method whereby parallel transmission lines could measure the time difference and thus permittivity by placing the probe near the material to be measured. This non-invasive technique has several advantages, namely 1) the ability to perform measurements on material that is not amenable to invasive probes (e.g. timber, concrete), 2) it does not affect the moisture distribution of the material in any way (e.g. inserted probes may form "wells" of moisture around them, and 3) the ability to readily measure the moisture distribution since the relative geometry between the material and probes can be easily changed thus providing a tomographic view.



**Figure 1** The 2-D arrangement of discrete cells

The method uses a 3 dimensional grid of cells that contain the transmission lines, surrounding air and anomalous material. Woodhead *et al* [2] showed how this 3-D problem could be approximated by the 2-D case if it assumed that the change in permittivity along the axis parallel to the transmission line was assumed to be invariant (figure 1). The relationship between each of the cells in this 2-D system form a set of coupled simultaneous equations which can be solved iteratively to provide the values of the permittivity in each of them. The success of any iteration technique depends upon the "shape" and dimensionality of the function being solved together with prior information about constraints and initial values. It is the purpose of this paper to point out some changes in the cell geometry that will offer considerable reduction in formulation complexity and thus the iterative solution.

## 2 Background Theory

The method devised by Woodhead *et al.* [2] discretises the sampled material into N finite dimensional cells (figure 1) so that the polarization electric field ( $E_p$ ) contributions from each cell can be summed by the method of moments. This results in an equation relating the electric field components between each of the cells [3]:

$$\begin{pmatrix} l_{11}^{xx} + \frac{1}{\Delta\epsilon_1} & \dots & l_{1N}^{yy} \\ \vdots & \ddots & \vdots \\ l_{N1}^{xx} & \dots & l_{NN}^{yy} + \frac{1}{\Delta\epsilon_N} \end{pmatrix} \begin{pmatrix} P_1^x \\ P_1^y \\ \vdots \\ P_N^y \end{pmatrix} = - \begin{pmatrix} E_{i_1}^x \\ E_{i_1}^y \\ \vdots \\ E_{i_N}^y \end{pmatrix} \quad ..(1)$$

Where

$E_i$  are the x, y components at each cell of the transmission line generated field (impressed field) and can be explicitly determined for specific cell and transmission line geometries.

P are the components of each cells polarization.  
l relates the geometric gradient of the polarization P to the electric field  $E_p$ , between each of the N cells and can be calculated explicitly for any particular geometric arrangement of the cells.

$\Delta\epsilon = \epsilon_0 (\epsilon_r - 1) = \epsilon_0 \chi$ , where  $\epsilon_r$  is the relative permittivity and  $\chi$  is the susceptibility.

Note: the overall matrix containing the components l and  $\Delta\epsilon$  will be often be referred to as **L**.

From transmission line theory the total electric field  $E_t$  between the lines can be related to the pulse propagation time t by:

$$v = \frac{2l}{t_p} = \sqrt{\frac{\pi E_t}{\rho \mu \cosh^{-1}(b/a)}} \quad (2)$$

Where

$$E_t = \int E \cdot dl$$

v is the velocity of the propagating pulse.

l is the length of the transmission line

$t_p$  is the total propagation time of the pulse from the beginning of the line to its reflection at the end and back to the beginning.

$\rho$  is the line charge density

$\mu$  is the magnetic permeability

b is the spacing between the parallel lines,

a is the diameter of each of the lines.

Rearranging equation (2) gives the total electric field between the parallel transmission lines as:

$$\int E \cdot dl = \frac{4l^2 \rho \mu \cosh^{-1}(b/a)}{\pi t_p^2} \quad (3)$$

The usual method of determining the permittivity expression,  $\Delta\epsilon$  is to first reduce equation (1) into explicit expressions for the polarization components. This is currently achieved by using a Singular Value Decomposition inversion for the matrix L. The values of the electric field of those cells *between* the transmission lines are then calculated by using the relationship between P and E:

$$\bar{E} = \bar{P} / \Delta\epsilon \quad (4)$$

The summed field of each of these cells is then used in an approximation to the integral on the left hand side of equation (3). In this way the measured propagation time can be related directly to each of the individual cell values of  $\Delta\epsilon$ . The procedure now becomes an inverse, or iterative, process whereby values of  $\Delta\epsilon$  are tested until the measured propagation times are matched. Looking closely at the form of equation (1) in relation with figure (1) several points are noteworthy:

1. If, as figure (1) suggests, N = 100 the L of equation (1) becomes a 200 X 200 matrix..
2. The values for  $\Delta\epsilon$  of air are known, while those of the soil are not (for brevity the anomalous material will hereafter be assumed to be soil). In theory the same number of independent measurements of time,  $t_p$  are required as there are individual values of  $\Delta\epsilon$  (40 in the cases of figure (1)).
3. Since air between the rods has a value of  $\Delta\epsilon = 0$ , equation (4) is actually singular. To avoid this problem an artificial, very small value, is added to  $\epsilon_0$  at the start of the procedure.

## 3 The One Dimensional Case

As noted above, for a fixed geometry between the cells and transmission lines, the values of the components l are known as are the components of the impressed field, E. The components of P and the values  $\Delta\epsilon$  however, are not. Equation (1) thus describes an underdetermined system of 2N equations and 3N unknowns (2 X N Polarization components and 1 X N  $\Delta\epsilon$  values).

There are two possible simplifications to the geometry shown in figure (1) and thus to physical relationships described in equation (1). These are:

1. In the case where the probed medium can be considered as plane stratified in a region parallel to the transmission lines the permittivity values in each layer can be set as the same. Thus for instance, there will be one value of  $\Delta\epsilon$  for each of the four soil layers depicted in figure (1).
2. The cells representing the air are of known permittivity (i.e.  $\Delta\epsilon = 0$ ) and need not be included as components within the set of simultaneous equations. This reduces the complexity of the equations considerably.

### 3.1 Eliminating Known Cells

If the cells representing air are eliminated from the construction, clearly the number of terms in equation (1) is reduced considerably. The polarization vector for each soil cell is determined as before and extrapolation of these components to the required positions between the transmission lines results in the calculated total electric field. The electric field calculated in this manner at point  $p$  between the rods takes the form:

$$E_t^x = \begin{pmatrix} l_{p1}^{xx} & l_{p1}^{xy} & \dots & l_{pM}^{xy} \end{pmatrix} \begin{pmatrix} P_1^x \\ P_1^y \\ \vdots \\ P_M^y \end{pmatrix} \quad (5)$$

Where

There are  $M$  soil cells.

$l$  is the dyadic component between each soil cell and the point  $p$ , and is of the same form as those given in equation (1).

Note that between the transmission lines the  $y$ -component of the total electric field is zero. Following this procedure, the summed field between the transmission lines can be chosen with any resolution, though of course the true resolution is still a function of the soil cell size.

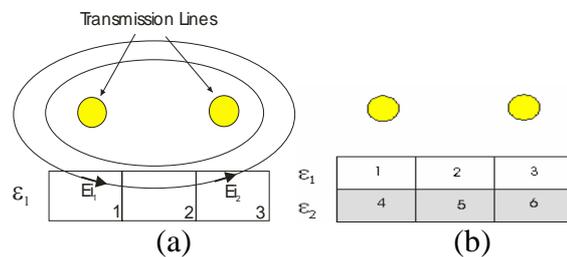
### 3.2 Using Symmetry

When the region of interest can be regarded as being plane stratified a number of simplifications in the field components of each cell can be exploited. To consider the situation in more detail, consider a system with a reduced number of cells as shown in figure 2(a). In this case, the geometry may be chosen so that the cells are symmetric about the transmission lines and the components of the imposed electric

field,  $E_i$  and the Polarization,  $P$  can be reduced so that:

$$\begin{aligned} P_1^x &= P_3^x & E_{i_1}^x &= E_{i_3}^x \\ P_1^y &= -P_3^y & E_{i_1}^y &= -E_{i_3}^y \\ E_{i_2}^y &= -P_2^y = 0 \end{aligned} \quad (6)$$

Any number of cells mirrored in the vertical plane around the transmission lines will conform to these simplifications. The number of coupled equations is now reduced by a factor of nearly two, since mirrored cells contribute only degenerate equations.



**Figure 2:** Cell and transmission line geometry, (a) for a single stratified layer with all cells having the same permittivity value and, (b) two stratified layer, halved in height from (doubled in resolution) from that in (a). Each strata has a different value of permittivity.

Extending this procedure to  $M$  soil cells, reduces the number of equations from  $2M$  to  $M + L$ , where  $L$  is the number of stratified layers. There are now only  $L$  unknown values of  $\Delta\epsilon$  and so the number of required independent measurements is reduced to this value.

Using figure (1) as the example again, the reduction in complexity by adopting the above two methods may be summarized as in table (1). Clearly there is a considerable reduction in system complexity resulting from implementing the changes described above and this increases both the speed and accuracy of the iterative procedure.

The efficiency of any iteration procedure is also dependant on how close the initial (*prior*) values of the iterate are to the actual values. With this in mind the stratified formulation can be made to start with oversized cells (i.e. cells larger than the required resolution) to which a solution for  $\Delta\epsilon$  can easily be found using the formulation resulting from the geometry of figure 2a. The results from this are then used for the prior values of cells with half the resolution in height (figure 2b). In this fashion the stratified values of  $\Delta\epsilon$  can be calculated down to layers with much finer resolution.

**Table 1:** Reduction in system complexity by using simple geometry considerations for the special case where the layers may be considered as plane stratified.

|                                  | All Cells | Eliminated Cells | Eliminated + symmetric reduction |
|----------------------------------|-----------|------------------|----------------------------------|
| System Dimension                 | 200       | 80               | 43                               |
| Number of independent time Meas. | 40        | 40               | 3                                |

#### 4 Conclusion and Future Work

The stratified method described here provides a quick and robust means for determining the one dimensional permittivity of a sample for cases when stratified, or nearly stratified geometry can be assumed.

To characterize the usefulness of the process more fully however requires an evaluation of:

1. The form and magnitude of errors in  $\Delta\epsilon$  when the sample deviates from the 1-D case (i.e. not plane stratified).
2. The ability to extend the process to the 2-D case either by considering a perturbation method or by using the full 2-D integration by moments method with the *a priori* value for each cell layer set by the stratification method.

To test the of the entire non-invasive TDR system, moisture panels have been constructed with known percentages of water content. Since the probes do not require physical contact with the sample, these panels can be fully enclosed in a low permittivity material thus ensuring that there is no change of water

distribution. The percentage water content can be varied by the placement of small spacers within the unit and the panels may be stacked vertically to provide a horizontal stratified sample with precisely known volumetric water content values at each level. Clearly the stratification assumption is appropriate here and will be used to calibrate the system by:

1. Accurately determining the relationship between the pulse propagation time and the relative permittivity.
2. Assessing and perhaps modifying the transforms between the relative permittivity  $\epsilon_r$  and the volumetric moisture content  $\theta_v$ .

#### 5 Acknowledgements

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#### 6 References

- [1] Topp, G. C., Davis, D. L. and Annan, A. P., "Electromagnetic determination of soil water content: Measurement in coaxial transmission lines", *Water Resources Research*, Vol. 16, No. 3, June, 1980, pp. 574-582.
- [2] Woodhead, I. M., Buchan, G. B and Kulasiri, D., "Pseudo-3-D Moment method for rapid calculation of electric field distribution in a low-loss inhomogeneous dielectric", *IEEE Transactions on Antennas and Propagation*, Vol. 49, No. 8, August 2001, pp 1117-1122.
- [3] Harrington, R. F., "Field Computation by Moment Methods", R. E. Kreiger, 1987.