INCOME TAX EVASION
AND BRIBE CHAINS

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Abstract

Bribe chains arise when a hierarchy of corruptible auditors audits taxpayers. We investigate fine and reward structures for breaking bribe chains and establishing income-revealing equilibria. We show that no super-auditing is necessary in one-off audits if auditors' rewards are related to the amount of tax evasion detected by them. However, such equilibria do not survive repeated encounters, even with super-audits. We then show that if rewards are based on tax collection, income-revealing equilibria can be sustained over repeated encounters. In such structures, the number of levels in the hierarchy is immaterial for income revelation as such, but it determines the net revenue of the government.

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Key words: tax evasion, bribe chain, hierarchy, auditing, optimal mechanism, repeate
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1. Introduction

In the early literature on tax evasion, policy implications were generally based on the comparison of gains from evasion with expected penalty costs (Allingham and Sandmo, 1972; Srinivasan, 1973). However, imposing an expected penalty cost on an offending taxpayer involves positive resource expenditure by the revenue authority. This latter cost increases significantly if the revenue administration is corrupt. A corrupt administration leads to collusion between taxpayers and auditors making detection highly costly\(^1\).

Collusion in tax administration has been noted as a problem even in pre-modern administrations. Some economic historians believe that the practice of auctioning off the tax office in sixteenth and seventeenth century Europe and variants of this practice in the Ottoman, Mughal, Quajar and Manchu empires was a response to collusion between taxpayers and collectors (Braudel, 1983, pp 540-542; Swart1980, and Theobald, 1990, pp 27-29). In the context of modern tax administrations, two suggestions are made to resolve this problem. The first, the auditing of officials by an external agency, is practiced in many countries today and is often thought to be the most potent remedy against bureaucratic corruption (Palmier, 1983). However, the more successful examples of external auditing come from public services outside of tax administrations\(^2\). The second suggestion is departmental or hierarchical auditing of auditors; our paper is concerned with this procedure. The generic problem of monitoring a corrupt agent by a supervisor has been explored in Bac (1996a, 1996b) and Bag (1997). In the context of tax administration, Chander and Wilde (1992) analyzed a model with corrupt auditors under a team of super-auditors. However, if corruption is widespread, super-auditors are likely to be corruptible so that the collusion between taxpayers and auditors may extend through the higher levels in the audit hierarchy. Examples of such collusive chains along a supervisory hierarchy are fairly common and have been discussed in the literature\(^3\). Modelling the problem with a single level of super-auditors appears \textit{ad hoc} unless we can argue that it generates the best payoff for the principal. Two obvious questions arise. Is hierarchical monitoring or audit necessary at all for revealing


\(^2\) The most successful example cited in the literature is that of the breaking up of syndicated corruption in police administration in Hong Kong by the Independent Commission Against Corruption (ICAC) established in 1974. For details, see Palmier (1983). The record of the Central Bureau of Investigation (CBI), a similar agency in India, is not as spectacular. The number of convictions of corrupt public servants has trailed far behind the rate at which corruption has allegedly grown. See Theobald (1990), pp 140-41, and also \textit{Guardian Weekly}, January 10, 1988.

\(^3\) For an interesting account of chains of corruption and bribes, see Wade (1988), and The Policy Group (1985).
private information? Is there an optimal level of hierarchy that generates the best payoff for the principal?

We address both issues in the context of income tax evasion. We analyze a model of taxpayers audited by a hierarchy of auditors, using a construction similar to that developed in Basu, Bhattacharya and Mishra (1992). Such structures are shown to sustain bribe chains, formally defined below, from taxpayers to the highest audit level. We then explore possible reward schemes that may induce income-revealing equilibria by snapping the bribe chain. We show that, if rewards are related to the evasion detected by an auditor, no hierarchy is necessary for income revelation in one-off audits, i.e., a single layer is sufficient. However, we argue that this mechanism can not be sustained under repeated encounters between taxpayers and auditors. The practical problem of tax evasion, auditing and corruption relates to repeated year after year encounters between taxpayers and auditors and this problem has to be analyzed in that institutional context.

Turning to repeated encounters, we show that no level of hierarchy, however large, can sustain truth-revealing equilibria, if rewards are related to the detection of evasion. Such mechanisms are very likely to be undermined by collusion between taxpayers and the entire audit hierarchy. We then try to model an alternative mechanism that can sustain revelation in repeated encounters. We try to establish several propositions in this connection. First, rewards dependent on tax collection can sustain income-revealing equilibria in repeated encounters. Secondly, we show that the number of levels in the hierarchy is not important to the elimination of bribe chains. At the same time, the analysis brings out an important role of hierarchy. We show that the authorities can concentrate the rewards in the highest level of the hierarchy. Given that hierarchies are pyramidal, reward money payed out for chain breaking is smaller if the hierarchy contains more levels. Finally, this gain from increasing the number of levels is weighed against the marginal cost of setting up an extra layer of audit; we derive the conditions for an optimal level of audit hierarchy.

The paper is organized as follows. Section 2 develops a model of hierarchical audit and bribe chains. It then proposes a fine and reward scheme that can produce income-revealing equilibrium in single encounters. Section 3 constructs a repeated game to indicate that the above scheme is likely to be undermined by collusion between taxpayers and the audit

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4 This result supports the observation made in Basu, Bhattacharya and Mishra (1992) that a small reward can stop a large revenue leak because individuals in a chain get only a fraction of total bribes.
hierarchy if there are repeated tax returns and audit decisions. Section 4 looks at the problem from an optimal mechanism design perspective. Using the results of that literature, we present some intuition about a possible design that might work. Section 5 uses this intuition to develop a mechanism that can sustain income revelation in repeated games. Section 6 explores the conditions of the existence of an optimal level of hierarchy in that mechanism. Finally, section 7 contains some concluding observations.

2. A Model of Hierarchical Audit and Bribe Chains

This section develops a model of hierarchical audit, characterizes bribe chains, and examines a possible structure of fines and rewards that can generate income-revealing equilibrium. Consider an audit hierarchy in which taxpayers are audited by a group of auditors, who are in turn audited hierarchically by another group and so on. Make the following assumptions:

1. There are \( m \) levels of audit.
2. Let \( p_1 \) denote the probability that a taxpayer is audited, \( p_2 \) that a level 1 auditor is audited, \( \ldots \), and \( p_m \) that the \((m-1)\) level auditor is audited, \( 0 < p_i \leq 1 \), for all \( i \).
3. Let \( f_2 \) denote the proportional fine rate for first level auditors, \( f_3 \) that for the second level, \( \ldots \), and \( f_m \) that for the \((m-1)\)th level. These rates apply to the amount of undisclosed income an auditor has been caught condoning. For symmetry of notation, we denote by \( f_1 \) the rate of tax plus fine, or \( t + F \), to be levied on a taxpayer’s undisclosed income, if he is caught evading.
4. The highest, i.e., the \( m \)th level, is not monitored. Instead these authorities are offered a reward if they detect dishonest reporting at lower levels. The reward is proportional to the income involved in the offense. The reward rate is \( R > 0 \).
5. Auditors at all levels are potentially corrupt.
6. Auditors and taxpayers act honestly if they are indifferent between cheating and honest action.
7. All auditors and taxpayers are risk neutral.

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5 If we use a nonlinear penalty function, the Nash solution discussed below, is difficult to characterize. In the most general case, doubts arise about using the Nash solution, which was developed for convex feasible sets. See Basu, Bhattacharya and Mishra (1992) and Anant, Basu, and Mukherji (1990).
(8) In negotiations, possible fines and rewards alone describe potential losses or gains. There is no hidden asymmetry in the bargaining position of any agent.

(9) The proportional tax rate, $t$, is fixed by fiscal authorities, while the revenue administration can choose audit probabilities, $p_i$ ($i = 1, 2, \ldots, m$), fine rates $f_i$ ($i = 1, 2, \ldots, m$), and the reward rate $R$. Thus, the revenue administration chooses $(2m + 1)$ variables.

The decisions of taxpayers and auditors constitute an $(m+1)$-stage game, with stages denoted by 0 to $m$. Stage 0 comprises the no action strategy for all players except taxpayers, who decide to either evade taxes or not evade. In stage 1 all players, except the taxpayers and level 1 auditors, have the no action strategy. Furthermore, if taxpayers had played not evade in stage 0, they and the auditors at level 1 have the no action strategy. Otherwise, the taxpayers’ strategy is to collude, while the auditors’ strategy is to either report or collude. The report strategy is simply to report the offense. Collude is taken to mean accept the Nash bargain solution value of bribe, if it exceeds the reservation level payoff.

In stage 2, all players, except level 1 and level 2 auditors, have the no action strategy. If level 1 auditors had played report in stage 1, they and the level 2 auditors have a singleton strategy, no action. Otherwise level 1 plays collude, while level 2 auditors can either report or collude. Continuing similarly, in stage $m$, all players except auditors of level $(m-1)$ and $m$ have the no action strategy. If $(m-1)$ level auditors had played report in stage $(m-1)$, both groups of auditors have a singleton strategy of no action. Otherwise the $(m-1)$ level auditors collude, while $m$ level auditors can play either report or collude. The game tree is shown in figure 1.

We define a bribe chain as a sub-game perfect equilibrium of this game in which taxpayers evade taxes and auditors at all levels earn positive bribes. In figure 1, the strategy profile (evade, collude, collude, \ldots, collude) joining the nodes numbered 0 to $m$ along the extreme left of the diagram would comprise a bribe chain, if it is a subgame perfect equilibrium. We first show that this profile is a bribe chain under some configuration of parameters.

Consider the subgame at the node $m$. The $m$-level auditor has detected that an auditor at $(m-1)$ level had condoned tax evasion involving, say, an amount $Z$. If there is a bribe negotiation between the two, and $b_m$ is the proportional bribe rate, the gain from successful negotiation of the $m$ level auditor is $(b_m - R)Z$, while that of the $(m-1)$ level auditor is $(f_m - b_m)Z$. The Nash bargaining solution would set $b_m$ so that it maximizes the product of the gains of the two sides, $[(b_m - R)Z] \cdot [(f_m - b_m)Z]$. This gives:
\[ b_m = \frac{f_m + R}{2} \]  \hspace{1cm} (1)

Assume that the parameters are such that \( b_m \) is better than the reservation level for both parties, so that the bargain is struck.

Next consider the subgame at the node \((m-1)\). An \((m-1)\) level auditor, having caught an erring auditor at \((m-2)\) level, is engaged in bribe negotiation. In this subgame, the \((m-1)\) level auditor predicts \( b_m \) as given by (1) and uses it for calculating her payoff. Gains on successful negotiation of the two sides are respectively, \((b_{m-1} - p_m b_m)\) and \((f_{m-1} - b_{m-1})\). The solution value of \(b_{m-1}\) is obtained as earlier to give:

\[ b_{m-1} = \frac{f_{m-1} + p_m b_m}{2} \]  \hspace{1cm} (2)

Using similar backward induction, we have in general:

\[ b_i = \frac{f_i + p_{i+1} b_{i+1}}{2} \]  \hspace{1cm} for \( i = 2 \) to \((m-1)\)  \hspace{1cm} (3)

Now consider the bribe negotiation between an errant taxpayer and a level 1 auditor. It maximizes:

\[ [(f_1 - b_1)Z], [(b_1 - p_2 b_2)] Z \] and yields \( b_1 = \frac{f_1 + p_2 b_2}{2} \).  \hspace{1cm} (4)

Finally consider a taxpayer’s decision about tax evasion. Given her income \( Y \), she chooses an evasion \( Z \) to minimize \( t(Y-Z) + p_1 b_1 Z \). Taxpayers predict \( b_1 \) and assuming:

\[ t > p_1 b_1 \text{ and } f_1 > b_1 \]  \hspace{1cm} (5)

the solution is \( Z = Y \).

Equation (5) is a necessary condition for taxpayers to under-report their income with the premeditated plan that they would bribe the auditor if tax evasion is detected. Similarly, necessary conditions for bribe bargains to be struck at each stage are that the bribe rates defined in (3) and (4) be higher than the reservation pay offs of both sides in the corresponding stages. They require the following inequalities:
\[ f_m > R \]  \hspace{1cm} (6)
\[ f_{m-1} > p_m R \]  \hspace{1cm} (7)
\[ f_{m-2} > p_{m-1} p_m R \]  \hspace{1cm} (8)
\[ \ldots \ldots \ldots \]
\[ f_1 > p_2 p_3 p_4 \ldots \ldots \ldots p_{m-1} p_m R, \]  \hspace{1cm} (9)

Also using these inequalities, condition (5) can be re-written as:
\[ t > p_1 f_1 \]  \hspace{1cm} (10)

If inequalities (6) through (10) are satisfied, the strategy profile (evade, collude, collude,\ldots, collude) joining the nodes numbered 0 to \( m \) along the extreme left of figure 1 is a subgame perfect equilibrium. No player can gain by deviating from this profile in a single stage and conforming to it thereafter. This leads us to posit the following proposition.

**Proposition 1:** A bribe chain exists if inequalities (6) through (10) are satisfied.

Proposition 1 motivates a simple suggestion for stopping a bribe chain by disabling these inequalities by setting:
\[ f_m = R \]  \hspace{1cm} (11)
\[ f_{m-1} = p_m R \]  \hspace{1cm} (12)
\[ f_{m-2} = p_{m-1} p_m R \]  \hspace{1cm} (13)
\[ \ldots \ldots \ldots \]
\[ f_1 = p_2 p_3 p_4 \ldots \ldots \ldots p_{m-1} p_m R, \]  \hspace{1cm} (14)
\[ t = p_1 f_1 \]  \hspace{1cm} (15)

By assumption, a tie between cheating and honest action leads to the latter. So (15) would guarantee that a taxpayer would pay taxes rather than evade and pay fines. She may still find evasion and bribing a possible alternative; however, this action is precluded by the rest of the conditions. Condition (11) ensures that \( m \)-level auditors would not take bribes. This would mean that \((m-1)\)-level auditors would take a bribe from below only if it is larger than expected fines to be paid if caught. Conditions (11) and (12) together prevent this. The argument continues down to auditors at level 1, who would also refuse bribes.
Since the revenue administration can choose \((2m + 1)\) variables, which are constrained by the \((m + 1)\) relations \((11)\) through \((15)\), it has \(m\) degrees of freedom left. Without any loss of generality, the administration can choose \(m\) audit probabilities \(p_i\). Audit probabilities are real numbers in the closed interval between zero and one. Adding the reasonable restriction that all \(f_i\) and \(R\) have to be finite, the authorities can choose \(p_i\), for all \(i\), from the half-open interval \((0, 1]\) and then solve \((11)\) to \((15)\) for \(f_i's\) and \(R\).

**Proposition 2:** Income-revealing equilibria can be generated by any value of \(m \geq 1\).

The proof is obvious in view of the fact that constraints \((11)\) to \((15)\) always leave enough degrees of freedom to choose \(p_i\) \((i = 1, 2, \ldots, m)\), for all values of \(m \geq 1\). As an illustration, consider \(m = 1\). Then \{\(f_1 = t / p_1, R = f_1\) and any \(p\) in \(0 < p \leq 1\}\} ensures income-revealing equilibrium. This proposition implies that hierarchical auditing is superfluous for the purpose of income-revelation alone. In fact, if the marginal cost of an additional level of auditing is positive, hierarchical audits are wasteful and \(m = 1\) is optimal for net revenue.

### 3. Repeated Encounters and Collusion

The structure of fines and rewards discussed above works if there is a single audit encounter. However, if the game is repeated indefinitely and the truth-revealing equilibrium is established each time, no taxpayer ever cheats and no auditor at levels \(l\) to \((m-1)\) misreports. Hence, auditors at the \(m\) level never get a reward either and income of \(m\) level auditors is perpetually lower than in the bribe chain. Intuitively, the \(m\) level auditors could increase their average discounted pay off by encouraging cheating and extracting some bribe. The requirement for a cheating equilibrium to reappear is that the \(m\) level auditors simply ignore the reward, \(R\). By convincing subordinates at level \((m-1)\) that they are willing to accept some rate of bribe \(b_m < R = f_m\), they can make it feasible for the latter to take bribes from the \((m-2)\) level and so on down the hierarchy. Since payoffs for all auditors and taxpayers are higher in a cheating equilibrium, all players are expected to cooperate. The present section develops this intuition by repeating the game of section 2.

Consider first the subgame starting at node \(m\), where the \((m-1)\) level auditor can play either collude, or no action, and the \(m\)-level auditor can play collude, report or no action. If the \((m-1)\) level auditor plays no action, the \(m\)-level auditor plays no action. If the \((m-1)\) level auditor
plays collude, the \( m \)-level auditor has two strategies, either collude or report. We now redefine collude for the \( m \)-level auditor to mean settling for some bribe rate \( b_m < R = f_m. \) The payoffs for the \((m-1)\) and \( m \) level auditors respectively are: \((0,0)\) to the profile report, no action; \((-f_m, R)\) to collude, report; and \((-b_m, b_m)\) to collude, collude.

Suppose that this game is repeated infinitely often and let \( \delta \) denote the common discount factor for both players. Consider the following strategy profile:

\( (m-1) \) level auditor: collude until the \( m \) level auditor plays report; no action for all time afterwards.

\( m \)-level auditor: collude if the \((m-1)\) level auditor plays collude in the current stage game; no action otherwise.

For any \( b_m > R (1-\delta) \), this profile is a subgame perfect equilibrium for the game beginning at node \( m \) and repeated infinitely.

If this is known at the node \((m-1)\), the following profile emerges as the subgame perfect equilibrium for the proper subgame starting at \((m-1)\):

\( (m-2) \) level auditor: collude until the \((m-1)\) level auditor plays report; no action for all time afterwards.

\( (m-1) \) level auditor: collude until the \( m \)-level auditor plays report; no action for all time afterwards.

\( m \)-level auditor: collude if the \((m-1)\) level auditor plays collude in the present stage game; no action otherwise.

This induction can be continued backwards to the taxpayer and, for the game starting at node 0, we have (evade, collude, collude, ..., collude) as the subgame perfect equilibrium in indefinitely repeated encounters. Although we can not characterize the value of \( b_m \) and the related values \( b_i, i = 1, ..., (m-1), \) it appears likely that a bribe chain will be established with \( R > b_m > R (1-\delta) \) from the above arguments.

\[6\] We can not characterize the values of this bribe rate, nor is it necessary for the argument at hand.
4. An Optimal Mechanism Design Perspective

The mechanism suggested in section 2 fails because collusion would undermine it in the long run. We explore a possible alternative that might withstand this tendency for collusion by considering an optimal mechanism design problem. For simplicity, if we assume that there is only one level of audit, we have three parties in the problem. Also suppose there is a single taxpayer and a single auditor.

Parties are characterized by the following attributes. The taxpayer’s income, \( Y \), is her private knowledge. She decides to conceal a portion \( Z \geq 0 \), and sends a message \( (Y-Z) \) to the authorities. Her utility is \( Y - t(Y-Z) - \tau_1 \), where \( \tau_1 \geq 0 \), is a transfer to the authorities. The auditor either learns \( Y \), and therefore also \( Z \), correctly or learns nothing at all. The probability that the auditor learns the true value of \( Y \) is \( p \) and is chosen by the principal. The auditor then sends a report that belongs to the set \{\( Z, 0 \)\} to the authorities, i.e., either stating the correct amount of evasion or certifying that no income has been concealed. The auditor’s utility is \( \tau_2 \geq 0 \), which is a transfer from the authorities. The authority uses the auditor’s report to determine \( \tau_1 \) and \( \tau_2 \) and its utility is given by \( t(Y-Z) + \tau_1 - \tau_2 \).

Consider the following time sequence. First the taxpayer learns \( Y \) privately and sends a message \( (Y-Z) \) and the auditor learns the true values of \( Y \) and \( Z \) with probability \( p \). At the same time the taxpayer is informed of what the auditor has or has not learned. Second, the authority offers the taxpayer and the auditor a contract, which announces \( p \) and specifies \( \tau_1 \) and \( \tau_2 \) as functions of the messages sent by the taxpayer and the auditor. Third, the taxpayer and the auditor can sign a side contract that specifies a side transfer as a function of the messages. Fourth, contracts are implemented.

If auditors and taxpayers can not collude, the third stage of the above game is deleted. In that case, the mechanism discussed at the end of section 2 is optimal for the authority; it offers the contract \{\( \tau_1 = f(Z); \tau_2 = RZ; R = f = t/p \}\}. This contract incorporates sufficient disincentive for the taxpayer to either pay fines or bribe the auditor and for the auditor to take a bribe. However, if it is possible for the taxpayer and the auditor to get into a side contract, providing

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7 Assuming a single taxpayer and auditor makes the interpretation of audit intensity somewhat contrived. We still use it because it makes the presentation of this section simple although we can scale up the model for many taxpayers without changing the conclusions of this section.
sufficient disincentive for the taxpayer and the auditor separately, as in this solution, becomes ineffective (Tirole, 1992). In the last section, we suggested that collusion is likely to emerge in repeated encounters so that the third stage of the above game can not be deleted. Therefore, we need to construct an optimal mechanism for the specified sequence. We use an insight from the optimal mechanism literature that if collusion is possible among agents, incentives need to be related to the aggregate performance of the potentially collusive group (Tirole, 1986; Laffont, 1990). Since agents are linked through potential side contracting, incentives need to be extended to the level of this group. In the present case, we need to consider a mechanism that relates rewards to the aggregate performance of the potentially collusive group of taxpayers and auditors.

5. A Possible Mechanism

One measure of the aggregate performance of taxpayers and auditors together is the total income returned by the system. We propose a mechanism in which an \( m \)-level auditor is rewarded in proportion to the total income returned under her jurisdiction. The rate of the reward will be fixed so as to give an \( m \)-level auditor in a truth-revealing equilibrium the same income that she would have earned in a bribe chain.

To develop this mechanism, we work out the bribe income of \( m \)-level auditors in a bribe chain with no reward for them (\( R = 0 \)). The following two restrictions rule out fine paying as an option for all auditors and taxpayers:

\[
\begin{align*}
pe_i &= b_i, \quad \text{for } i = 2 \text{ to } m. \\
pe_1 &= t
\end{align*}
\]  

(16)

(17)

Conditions (16) can be stated in terms of \( p_i \)'s and \( f_i \)'s once we solve for the \( b_i \)'s; thus, they can be used as elements of the contract provided by the authority.

Equilibrium bribe rates are calculated, starting from level \( m \), as:

\[
b_m = f_m / 2
\]  

(18)

In general:
Using equations (19) and (16), we can solve for $b_i$’s in terms of $f_i$’s. Thus:

$$b_{m-1} = \frac{(f_{m-1} + p_m b_m)}{2} = \frac{f_{m-1}}{2} + \frac{p_m f_m}{4} = \frac{f_{m-1}}{2} + \frac{b_{m-1}}{4}.$$  

Therefore,

$$b_{m-1} = \frac{2}{3} f_{m-1} \quad (20)$$

By similar manipulation, we have:

$$b_{m-2} = \frac{3}{4} f_{m-2} \quad (21)$$

and, in general,

$$b_i = \left[\frac{m-i+1}{m-i+2}\right] f_i \quad \text{for } i = 1 \text{ to } m \quad (22)$$

Bribe rates given by (22) sustain a cheating equilibrium. The amount of bribe extracted by the $j$-th auditor at level $i$ is $p_1 p_2 ... p_i b_i Y^j$, where $Y^j$, denotes the income of the taxpayers under the jurisdiction of this auditor through the chain of sub-auditors. The total bribe accruing to the $k$-th auditor at level $m$ is $p_1 p_2 ... p_m b_m Y^m k$.

As suggested earlier, auditors at level $m$ are to be offered a share of the income returned by taxpayers so that their incomes are invariant between a truth-revealing equilibrium and a bribe chain. Accordingly, each $m$-level auditor is offered a share $s$ of income returned under her jurisdiction, where:

$$s = \frac{p_1 p_2 ... p_m b_m}{(t / f_1)(b_1 / f_2)(b_2 / f_3)(b_3 / f_4)......(b_{i-1} / f_i)......(b_{m-1} / f_m)(f_m / 2)}$$

$$= \frac{t}{f_1} \left[\frac{m}{m+1}\right] \left[\frac{(m-1)}{m}\right] \left[\frac{(m-2)}{(m-1)}\right]...[2/3][1/2]$$

$$= \left[\frac{t}{(m+1)}\right] \quad (23)$$

Given this share, auditors at level $m$ are indifferent between an income-revealing equilibrium and a bribe chain. Therefore, by assumption (6), they act honestly. An auditor at level $(m-1)$
would take a bribe only if it is affordable to pay fines when caught. Since this possibility is precluded by (16), they would not take a bribe from (m-2) level and so on. Because taxpayers can not bribe, they will consider the option of fine payment, but find it precluded by condition (17). Thus, they report their incomes honestly.

For later reference, we write out the restrictions on $p_i$’s and $f_i$’s that define the audit intensity-fine-reward rule. In addition to (23), they comprise $m$ conditions:

$$p_1 f_1 = t$$  
$$p_i f_i = [(m-1)/(m-i+1)] f_{i-1}, \text{ for } i = 2 \text{ to } m.$$  

Condition (24) uses (16) and (22) to eliminate $b_i$’s and make a statement in terms of $p_i$’s and $f_i$’s only. Hence, we have the following proposition.

**Proposition 3:** If rewards are related to tax collection, it is possible to design income-revealing equilibria sustainable over repeated audit encounters.

**Proposition 4:** Such equilibria can be sustained by any value of $m \geq 1$. As for Proposition 4, the proof follows obviously from the construction of the mechanism.

We note that restrictions (17) and (24) determine only the authority’s share of tax revenue, $(t-s)$ uniquely, but not the $2m$ variables $f_i$’s and $p_i$’s. The $m$ degrees of freedom may be utilised to maximize the revenue net of audit cost given by:

$$\Pi = (t-s)y - \{p_1 Nc_1 + p_1 p_2 Nc_2 + \ldots + p_1 p_2 \ldots p_m Nc_m\}$$  

where, $y$ is total income of all tax-payers, $N$ is the number of tax-payers and $c_i$ is the unit audit cost at level $i$. Taking account of (23), we rewrite (25) as:

$$\Pi = [m/(m+1)]ty - \{p_1 Nc_1 + p_1 p_2 Nc_2 + \ldots + p_1 p_2 \ldots p_m Nc_m\}$$  

For any given $m$, maximizing $\Pi$ amounts to minimizing the expression $\{p_1 Nc_1 + p_1 p_2 Nc_2 + \ldots + p_1 p_2 \ldots p_m Nc_m\}$. However, by definition $p_i$’s cannot be zero so that no minimum exists.
The authority works in a political and institutional environment that sets upper limits to acceptable values of $f_i$’s. Let the upper bound of $f_i = F_i$ for all $i$. Note from (17) and (24) that upper bounds on $f_i$’s imply corresponding lower bounds for $p_i$’s. In that case, solving the $m$ equations (17) and (24) for $p_i$’s, after setting $f_i = F_i$ for all $i$, yields audit intensities that minimize audit cost for any given $m$. We should remark that (17) and (24) may not have solutions for all arbitrary $F_i$’s $>0$ in view of the fact that $0 < p_i < 1$ for all $p_i$. We are assuming that the exogenous upper bounds, $F_i$’s, belong to the set of solutions of (17) and (24). A sufficient condition for this is that the $F_i$’s increase with $i$.

**Proposition 5:** For any $m \geq 1$, equations (17), (24) and a further set of $m$ equations $f_i = F_i$ for $i = 1$ to $m$, define $f_i$’s, $p_i$’s and $s$ that sustain a truth-revealing equilibrium with maximum net revenue. Using these latter equations in (26), we can write the maximum net revenue as a function of $m$:

$$\Pi^*(m) = \left[ m/ (m +1) \right] ty - \left\{ (m+1)c_1/F_1 + mc_2/F_2 + (m-1)c_3/F_3 + \ldots + 3c_{m-1}/F_{m-1} + 2c_m/F_m \right\} N$$

(27)

### 6. Net Revenue and a Possible Optimal Hierarchy

We examine the role of $m$ in determining the net revenue of the authority. For a given $m$, the maximum net revenue is given by (27). Suppose that the number of audit levels is increased from $m-1$ to $m$ and that $\Delta \Pi^*_{m-1,m}$ denotes the resulting change in $\Pi^*$. Then:

$$\Delta \Pi^*_{m-1,m} = \left[ ty/m(m+1) \right] - \left\{ c_1/F_1 + c_2/F_2 + \ldots + c_{m-1}/F_{m-1} + 2c_m/F_m \right\} N$$

(28)

The first expression on the right side is the marginal increase in the authority’s share of tax revenues, which is positive but decreasing in $m$. The second expression is the marginal cost of adding another layer of auditing. If it exists, the optimal hierarchy, $m$, is achieved when the following inequality ceases to hold:

$$ty/m(m+1) > \left\{ c_1/F_1 + c_2/F_2 + \ldots + c_{m-1}/F_{m-1} + 2c_m/F_m \right\} N$$

(29)

A sufficient condition for an optimal $m$ to exist is that the marginal audit cost should be non-decreasing with $m$. Comparing the marginal cost for transiting from $(m-1)$ to $m$ with that for $m$ to $(m+1)$, this requires:
\[
\left( \frac{c_1}{F_1} + \frac{c_2}{F_2} + \ldots + \frac{c_{m-1}}{F_{m-1}} + \frac{c_m}{F_m} + 2\frac{c_{m+1}}{F_{m+1}} \right) \geq \left( \frac{c_1}{F_1} + \frac{c_2}{F_2} + \ldots + \frac{c_{m-1}}{F_{m-1}} + 2\frac{c_m}{F_m} \right)
\]

(30)

Simplification yields:

\[
2\frac{c_{m+1}}{F_{m+1}} \geq \frac{c_m}{F_m}
\]

(31)

Note that (31) does not imply that unit audit costs have to increase with \( m \). Even when the sufficient conditions for solving (17) and (24), namely \( F_i > F_{i-1} \) for all \( i \), hold, an optimal \( m \) with unit audit costs falling is feasible. The intuitive explanation is that, as \( m \) increases, the corresponding optimal audit intensities at lower levels generally increase. Thus, an extra level of hierarchy adds to cost not only by creating a new level, but also by increasing the cost of all subordinate levels. The unit audit cost, \( c_m \), represents only the first component of this total cost increase. Therefore, even without increasing unit audit costs, marginal cost can increase fast enough to yield a maximal \( m \). Thus, the condition for the existence of an optimal hierarchy is not very restrictive.

**Proposition 6:** An optimal \( m \) exists for an information-revealing and sustainable equilibrium without bribes under a set of sufficient conditions that are not very restrictive.

7. **Concluding Observations**

Propositions 1 through 6 constitute the results; we conclude with an intuitive summary. If rewards are linked to detection, auditors are rewarded only as long as taxpayers evade taxes. If a mechanism is successful, tax evasion stops and so do the rewards. Auditors then find that they would be better off by allowing evasion to take place and accepting bribes. Thus, auditors have an incentive to undermine the reward mechanism by taking bribes at a rate that makes tax evasion feasible, i.e., at a rate lower than the reward. In effect, the reward is ignored.

The situation changes when rewards are tied to tax revenues. If the reward is no less than the auditors’ share in the cheating equilibrium, auditors are no worse off when they enforce an income-revealing equilibrium. The authority can use this possibility to reduce the total reward money offered to auditors within this sort of mechanism. This reduction is achieved
by increasing the number of levels in the audit hierarchy. In a cheating equilibrium, the bribe of an auditor falls as she is further away from the taxpayer. Therefore, it costs the government less to compensate a higher layer of auditors for not taking bribes than a lower layer. Of course, this consideration has to be weighed against the additional cost of another layer of auditing. Given the income of taxpayers, the government’s share increases with the audit level but at a diminishing rate. Furthermore, audit cost increases as audit levels increase. We can determine an optimal audit level if the marginal increase in audit cost is no less than the marginal increase in government’s share as audit level increases.

The condition for the existence of an optimal $m$ has two components. First, it is necessary that there exist net revenue maximizing audit intensities for any given $m$. Second, condition (31) should hold ensuring that if revenue maximizing audit intensity exists for all values of $m$, then there is an optimal value of $m$. Condition (31) does not appear very restrictive. However, the sufficient condition for the existence of optimal audit intensity for any given $m$, namely that permissible upper bounds of fine rates increase with $m$, may prove restrictive. There are two reasons for concern. First, there may be an upper limit for all acceptable fine rates in any given political and institutional environment. Second, these fines may be contestable in a court of law and contingent on conviction. In that case, effective fine rates are different from $F_i$’s and they may not necessarily be increasing in $m$.

The basic intuition used in this paper is borrowed from the literature on optimal mechanism design. The central result is that, in potentially collusive situations, the principal should devise a contract based on the aggregate performance of the potentially collusive group and not on anyone’s individual performance. In the present context, a measure of aggregate performance of the collusive group of taxpayers and auditors is the total tax returned. In future research, this idea can be pursued in modelling several other forms of collusive evasion or under-performance, e.g., evading commodity taxes, understatement of production and under-invoicing.
References


Figure 1:
Extensive Form of the Game of Section 2
[n.a = no action]