

EVOLUTIONARY BARGAINING GAMES

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LIST OF SYMBOLS USED

∞
 \in
 $|$
 \approx
 \leq
 \equiv
 Δ
 α
 \mathbb{R}
 \Rightarrow
'
 \neq
 \cdot
 β
 e
 \square

\subset
 Σ
 \cdot
 \forall
 $*$
 \wedge
 $=$
 $>$
 Γ
 l
 $\{$
 $\}$
 $($
 $)$
 \rightarrow

AUTHOR'S NOTE

Julian Wright has a special interest in bargaining theory. He completed a B Sc with first class Honours in Economics at the University of Canterbury and undertook the research while employed as an assistant lecturer at Lincoln University. Julian Wright will be undertaking a PhD at Stanford University in July 1991.

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SUMMARY

The simple Nash demand game is analysed in an evolutionary context. The evolutionarily stable strategies (ESS) are characterised in the cases where players have symmetric and asymmetric roles. Introducing stochastic effects we show conditions under which the set of Nash equilibria converges to the Nash bargaining solution as the noise becomes negligible. Two evolutionary models are given for which these conditions are satisfied. In each case above the simulated limit outcome of the replicator selection dynamics is given for a range of parameter values. Both approaches give evolutionary support for the Nash bargaining solution.

CHAPTER 1

INTRODUCTION

Rubinstein (1982) analysed a bargaining game where one's bargaining power comes from one's ability to put the onus of waiting entirely on the other party. By applying the concept of sub-game perfection he showed a unique equilibrium existed. However if we relax the assumption of complete information, then we are again faced with a multiplicity of equilibria. Furthermore recent papers including (Muthoo, 1990; Stahl, 1990; Van Damme et al., 1990) have shown that modification or relaxation of some of Rubinstein's technical assumptions will also lead to multiple equilibria.

If this is the case then bargaining becomes a coordination problem. The simplest bargaining game capturing this feature is the Nash demand game.

"Although very simple (the bargaining is reduced to the players making simultaneous demands), it nevertheless clearly captures the strategic essence of a wide variety of bargaining situations." - Binmore (1987, p 63)

In this paper we apply the theory of evolutionary games to describe how players coordinate on a particular equilibrium. The necessity for using such a theory can be seen if we relax the unrealistic common knowledge assumption. That is with a multiplicity of equilibria and a lack of common knowledge it is not clear how a rational player should act.¹

An evolutionary approach says that on average players adapt to strategies which are relatively successful in the previous period. In a symmetric-role game individuals are selected at random from a population and matched, in pairs, to play a two-person Nash demand game. The classic example is bargaining over the division of a cake. The essential difference with an asymmetric-role game is that individuals assume different roles, like "buyer" vs "seller", "employer" vs "employee" or "tenant" vs "landlord". Note that it is not necessary for players to have different pay-offs or strategy choices for a game to be asymmetric in this sense.

We are interested in the limit outcomes of symmetric-role and asymmetric-role Nash demand games. By a limit outcome we mean the limit of an evolutionary path or attractor of the corresponding dynamical system. We examine these outcomes via two approaches. Firstly we characterise the ESS and use the links between ESS and limit outcomes given in the literature. Secondly we provide simulations based on the replicator selection dynamics to find the simulated limit outcomes.

Given that multiple ESS exist we ask the question, which are the most stable? An evolutionary approach leads to a natural justification for discriminating between equilibria by studying their relative stabilities. Thus Nash's approach of studying the limit of equilibrium points of smoothed games can be motivated from an evolutionary viewpoint. In fact we consider two models incorporating stochastic effects, arising naturally in an evolutionary context, for which the set of Nash equilibria converges to the Nash bargaining solution as the noise becomes negligible. Thus this paper can be viewed as a contribution to the Nash program.

The rest of the paper is organised as follows. Sections II and III characterises the ESS in symmetric-role and asymmetric-role Nash demand games. In each case multiple ESS exist. However the simulated limit outcome is shown to be close to the Nash bargaining solution for unbiased initial populations. In section IV we show that when a stochastic process is introduced, the previous result holds even when the initial population is biased against the Nash bargaining solution. Moreover we show how an evolutionary approach gives a natural interpretation of Nash's limit result. Finally section V concludes.

CHAPTER 2
SYMMETRIC-ROLE BARGAINING GAMES

Consider any two players, denoted 1 and 2, selected at random from the population. Then the set of possible agreements is $X = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_i \geq 0 \text{ for } i=1,2 : x_1 + x_2 \leq 1\}$ and $d_i=0; i=1,2$ is the disagreement outcome. Players' preferences over lotteries on X can be represented by the expected value of a utility function u_i with domain $[0,1]; i=1,2$. For simplicity we assume that in symmetric-role bargaining games $u \equiv u_1 \equiv u_2$, although this is not essential for our results.² Furthermore assume $u(a) > u(b)$ if and only if $a > b$ where $a, b \in [0,1]$ and $u(0)=0$, without loss of generality. Note that the Nash equilibria of this game include every Pareto optimal allocation in X , while the Nash bargaining solution corresponds to the equal division allocation (0.50,0.50). For purposes of simulation we require a finite strategy set. Since, in reality, we can only ever measure a finite number of divisions this assumption is not restrictive. Moreover the standard convex negotiation set is used to show the limit result in section IV.

Let the finite set of pure strategies, s_i , available be denoted S , where $S = \{(s_1, s_2, \dots, s_n) \mid s_i \in [0,1]; i=1,2, \dots, n\}$. Assume if $s_i \in S$ then $1-s_i \in S$. A strategy, s , is an element of the set of probability measures over S . That is

$$s \in \Delta S = \{m \in \mathbb{R}^n \mid m: s_i \rightarrow [0,1]; i=1, \dots, n : \sum_{i=1}^n m(s_i) = 1\}.$$

Let A be the $n \times n$ matrix of pay-offs where A_{ij} is the utility to player 1 if he plays his i^{th} pure strategy and player 2 plays her j^{th} pure strategy. Thus

$$A_{ij} = \begin{cases} u(s_i) & \text{if } s_i + s_j \leq 1 \\ u(d_i) & \text{otherwise} \end{cases}$$

Let the expected pay-off to a player who plays $m \in \Delta S$ and whose opponent plays $m' \in \Delta S$ be denoted $E(m, m')$. Thus $E(m, m') = (m)^T A (m')$. A strategy $m^* \in \Delta S$ is an ESS of a symmetric-role Nash demand game if $\forall m' \neq m^* \text{ s.t. } m' \in \Delta S$ we have

either $E(m^*, m^*) > E(m', m^*)$ (1)

or $E(m^*, m^*) = E(m', m^*)$ and $E(m^*, m') > E(m', m')$ (2)

If (1) and (2) hold then m^* is immune to invasion by a mutant strategy. We now characterise the ESS of the symmetric-role Nash demand game.

THEOREM 1. If m^* is an ESS of the symmetric-role Nash demand game then either

- (a) m^* is the equal division allocation, or
- (b) m^* is a mixed strategy for which the support does not contain the equal division allocation.

In particular it has even cardinality of support, r , s.t

$$s_i^* + s_{r+1-i}^* = 1 \text{ for } i=1, \dots, r/2.$$

Proof. Denote the support of m^* as $\{s_1^*, s_2^*, \dots, s_r^*\}$ where $s_1^* < s_2^* < \dots < s_r^*$. Note utility is increasing on the half open interval $(1-s_i^*, 1-s_{i-1}^*]$; $i=2, \dots, r$. Thus if m' is a best response to m^* the pure strategies, t_i , which make up the support of m' must be elements of the set $\{1-s_r^*, 1-s_{r-1}^*, \dots, 1-s_1^*\}$. Since m^* is a best response to itself it must be that $s_i^* \in \{1-s_r^*, 1-s_{r-1}^*, \dots, 1-s_1^*\}$; $i=1, 2, \dots, r$.

$$\text{Then } s_1^* < s_2^* < \dots < s_r^* \rightarrow s_i^* + s_{r+1-i}^* = 1; \text{ for } i=1, \dots, \frac{r}{2} \text{ if } r \text{ even and } i=1, \dots, \frac{r+1}{2} \text{ if } r \text{ odd. (3)}$$

Consider the case where r is odd with $r \geq 3$. We now show the strategy corresponding to the equal division allocation, denoted m' , invades m^* . By Eq.(3) and Bishop and Canning's Theorem $E(m', m^*) = E(m^*, m^*)$. Also by Eq.(3) we can represent the support of m^* as the pure strategies

$$(0.5 - a_k, \dots, 0.5 - a_1, 0.5 + a_0, \dots, 0.5 + a_k) \text{ where } k = \frac{r-1}{2}, a_i > a_{i-1}; i=1, \dots, k \text{ and } a_0 = 0.$$

$$\text{Then } E(m^*, m') = \sum_{i=1}^{k+1} m^*(0.5 - a_{i-1}) u(0.5 - a_{i-1})$$

$$\rightarrow E(m^*, m') < u(0.5) \sum_{i=1}^{k+1} m^*(0.5 - a_{i-1}) < u(0.5) = E(m', m').$$

Thus m' invades m^* . That is if n is odd then $n=1$. The theorem follows from Eq.(3). □

That ESS exist in each case can be seen by noting that the equal division allocation is an ESS and the mixed strategy "play 0.40 with probability $2/3$ and play 0.60 with probability $1/3$ " is an ESS. Since every ESS of a symmetric game is a limit outcome under the replicator selection dynamics it is indeed meaningful to talk of the stability of the above ESS. If we interpret the mixed ESS as vectors

identifying the proportion of the population playing each of the pure strategies in S , then if the current population is to be immune to invasion either everyone demands the equal division allocation or no one does.

Alternatively we can simulate the limit outcome of the replicator selection dynamics, given an initial population. A reasonable initial population is the uniform one since this is not biased in favour of any particular strategy. The strategy set used is

$$S = \left(s_i = \frac{i-1}{100} ; i=1, \dots, 101 \right)$$

and the replicator selection dynamic is

$$s_i(t+1) = s_i(t) \left[\frac{(e_i)^T A(m)}{(m)^T A(m)} \right]$$

where e_i is the i^{th} unit vector. Then a uniform initial population implies $m(s_i) = 1/101; i=1, \dots, 101$.

For this case the simulated limit outcome was precisely the equal division allocation. However for different initial populations different limit outcomes are possible. For example an initial population close to that corresponding to the mixed ESS given above gives a different limit outcome. In section IV we show that even such biased initial populations lead to the equal division allocation, when the evolutionary model contains a stochastic process.

CHAPTER 3
ASYMMETRIC-ROLE BARGAINING GAMES

Many of the interesting bargaining problems are asymmetric in nature, for instance "buyer" vs "seller", "employer" vs "employee" or "tenant" vs "landlord". In these cases the ability to condition strategies on roles can drastically affect the evolutionary outcome - even if pay-offs and strategy choices are identical.

The asymmetric-role Nash demand game is the natural extension of the model used in section II. Subscripts 1 and 2 are used to denote roles 1 and 2 respectively. In particular B is the $n_2 \times n_2$ matrix of pay-offs where B_{ij} is the utility to a player of role 2 when she plays her j^{th} pure strategy, t_j , and the player of role 1 plays his i^{th} pure strategy, s_i . Thus $A = (A_{ij})$, $B = (B_{ij})$ where

$$A_{ij} = \begin{cases} u_1(s_i) & \text{if } s_i + t_j \leq 1 \\ u_1(d_1) & \text{otherwise.} \end{cases}$$

$$B_{ij} = \begin{cases} u_2(t_j) & \text{if } s_i + t_j \leq 1 \\ u_2(d_2) & \text{otherwise.} \end{cases}$$

Note that the negotiation set is

$$X = \{ (x_1, x_2) \in \mathbb{R}^2 \mid u_i(x_i) \geq u_i(d_i) ; i=1,2: x_1 + x_2 \leq 1 \}$$

The characterisation of the ESS for the asymmetric-role Nash demand game follows immediately from knowledge of the strict Nash equilibria. In fact:

THEOREM 2. *If m^* is an ESS of the asymmetric-role Nash demand game then m^* corresponds to a pareto optimal allocation belonging to the interior of the negotiation set.*

Proof. Since the asymmetric-role Nash demand game is a truly asymmetric contest we have that the ESS are equivalent to the strict Nash equilibria.³ But the strict Nash equilibria consist of just the pareto optimal allocations in the interior of the negotiation set. □

That the limit outcome belongs to the set of ESS under certain evolutionary dynamic processes follows from the fact that every ESS is a limit ESS and results on the stability of limit ESS.⁴

Alternatively we can simulate the limit outcome for the replicator selection dynamics given particular

utility functions and initial populations. As in section II the strategy sets used are

$$S = \left(s_i = \frac{i-1}{100} \mid i=1, \dots, 101 \right), \quad T = \left(t_j = \frac{j-1}{100} \mid j=1, \dots, 101 \right)$$

and the replicator selection dynamics are

$$s_i(t+1) = s_i(t) \cdot \frac{(e_i)^T A(m_2)}{(m_1)^T A(m_2)} \quad ; i=1, \dots, 101 \text{ for type 1 players,}$$

$$t_j(t+1) = t_j(t) \cdot \frac{(m_1)^T A(e_j)}{(m_1)^T A(m_2)} \quad ; j=1, \dots, 101 \text{ for type 2 players.}$$

Then a uniform initial population implies $m_1(s_i) = 1/101$, $m_2(t_j) = 1/101$; $i, j=1, \dots, 101$. The results for a number of parameter specifications are contained in table I.

TABLE I

Comparison of Simulated Limit Outcome with Game-Theoretic Solutions - Deterministic Games

Game Number	Parameters	Simulated Limit Outcome	Nash Bargaining Solution	Kalai-Smorodinsky Solution
1	$u_1(x) = x$ $u_2(x) = x$ $d_1 = d_2 = 0$	(0.50, 0.50)	(0.50, 0.50)	(0.50, 0.50)
2	$u_1(x) = \ln(1+x)$ $u_2(x) = x$ $d_1 = d_2 = 0$	(0.47, 0.53)	(0.46, 0.54)	(0.46, 0.54)
3	$u_1(x) = \min(x, 0.60)$ $u_2(x) = x$ $d_1 = d_2 = 0$	(0.50, 0.50)	(0.50, 0.50)	(0.38, 0.63)
4	$u_1(x) = x$ $u_2(x) = x$ $d_1 = 0.50, d_2 = 0$	(0.74, 0.26)	(0.75, 0.25)	(0.75, 0.25)

Notes

- All figures reported to 2.d.p.
- Game 3 gives strong evolutionary support for the independence of irrelevant alternatives axiom over the monotonicity axiom.
- For game 4 we assumed the initial population was uniformly distributed on the interval $[0.5, 1]$ for type 1 players, since these are the only individually rational demands. We also simulated the outcome with the original uniform population and obtained the limit outcome $(0.71, 0.29)$.

While these results clearly support the Nash bargaining solution, by choosing biased initial populations we can obtain vastly different limit outcomes. In the next section we show that this no longer necessarily holds when stochastic effects are introduced.

CHAPTER 4

STOCHASTIC DEMAND GAMES

Nash discriminated between the multitude of equilibria arising in the asymmetric-role Nash demand game by studying their relative stabilities. In fact by taking the limit of the Nash equilibria corresponding to appropriately smoothed games he was able to single out a unique pair of demands - the demands corresponding to the Nash bargaining solution. This work, while setting the way for a non-cooperative approach to bargaining was not generally accepted.

"The weak line in the argument is to single out this particular pair. Nash offers an ingenious and mathematically sound argument for doing so, but we fail to see why it is relevant... isn't it a completely artificial mathematical escape from the troublesome non-uniqueness? Would it have any relevance to the payers?" - (Luce and Ruffalo, 1957, p.141)

In this section we show that in fact Nash's result follows directly from two different evolutionary models. In the first one the cake bargained over varies randomly over time while in the second players make randomly distributed mistakes. In each case if we let the amount of noise become sufficiently small the Nash equilibria can be made arbitrarily close to the Nash bargaining solution. Finally we give an example of an evolutionary process for which the simulated limit outcome is close to the Nash bargaining solution despite our choice of a biased initial population.

In order to give a relevant interpretation of Nash's result we modify his theorem slightly. The essential difference is that the smoothing takes place on both sides of the Pareto boundary of the original game.

$$\text{Let } X_q = \{ (x,y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1 : x+y \leq q \},$$

$$U_q = \{ (u_1(x), u_2(y)) \mid (x,y) \in X_q \}$$

Assumptions

(A1): Let $\{\alpha_n\}$ be strictly increasing sequence of real numbers bounded below by zero and converging to one.

Let $\{\beta_n\}$ be strictly decreasing sequence of real numbers bounded above by two and converging to one.

Then there exists a functional sequence, denoted $\{ p_n(x_n, y_n) \}$ s.t $\forall n$,

$p_n: [0,1] \times [0,1] \rightarrow [0,1]$ is everywhere differentiable s.t

$$p_n(x_n, y_n) = \begin{cases} 1 & \text{for } x_n + y_n \leq \alpha_n \\ 0 & \text{for } x_n + y_n \geq \beta_n \\ \text{decreasing} & \text{for } \alpha_n \leq x_n + y_n \leq \beta_n \end{cases}$$

(A2): $u_1(x_n), u_2(y_n)$ are strictly increasing, differentiable, concave functions.

If we interpret $p_n(x_n, y_n)$ as the probability of x_n and y_n being compatible demands then

$$EU_1(x_n, y_n) = p_n(x_n, y_n) u_1(x_n)$$

$$EU_2(x_n, y_n) = p_n(x_n, y_n) u_2(y_n).$$

Let the set of Nash equilibria corresponding to the probability distribution p_n be denoted B_n . Let an arbitrary sequence $\{(x_n, y_n)\}$ defined by $(x_n, y_n) \in B_n, \forall n$ be denoted $\{(x'_n, y'_n)\}$ and let (x^N, y^N) denote the Nash bargaining solution to the original game.

THEOREM 3. Given (A1) and (A2) every sequence of Nash equilibria of smoothed games tends to the Nash bargaining solution as the smoothing becomes arbitrarily small. That is

$$(x'_n, y'_n) \rightarrow (x^N, y^N) \text{ as } n \rightarrow \infty.$$

Proof. Consider any Nash equilibrium $(x_n^*, y_n^*) \in B_n$. That such an equilibrium always exists follows from the fact, a solution to

$$\max_{(x_n, y_n)} u_1(x_n) u_2(y_n) p_n(x_n, y_n), \text{ exists, and belongs to } B_n.$$

$$\text{Firstly note that } \alpha_n \leq x_n^* + y_n^* < \beta_n \tag{4}$$

Using (A1) and (A2) it is easily seen that a necessary condition for a Nash equilibrium is also the first order condition to the problem,

$$\max_{(x, y) \in X_q} u_1(x) u_2(y) \text{ for some } q \in [\alpha_n, \beta_n].$$

Let the solution to this problem be denoted (x^q, y^q) . It follows from (A2) and the theorem of the maximum (Varian, 1984, p.327) that (x^q, y^q) is a continuous function of q . This together with (4) implies $B_n \subset \{(x^q, y^q) \mid \alpha_n \leq q < \beta_n\}, \forall n$.

Thus $(x'_n, y'_n) \rightarrow (x^N, y^N)$ as $n \rightarrow \infty$.

□

Example 1. Suppose the size of the cake bargained over varies randomly (that is according to the continuous pdf f_n) around unity over time. Thus the bargaining problem is changing over time but in a purely random fashion.⁵

Let F_n be the corresponding distribution function. In this case the probability of a compatible demand when a type 1 player demands x_n , and a type 2 player demands y_n is $p_n(x_n, y_n) = \text{Prob}(x_n + y_n \leq c) = 1 - F_n(x_n + y_n)$. Define the interval

$$I_n = [\inf(c | f_n(c) > 0), \sup(c | f_n(c) > 0)], \forall n \text{ and let } I_1 \subset [0, 2].$$

Then assumption (A1) follows if we take $\{p_n(x_n, y_n)\}$ to be the functional sequence arising as the interval over which the cake size varies becomes smaller. That is $I_{n+1} \subset I_n, \forall n$. Then the conditions of theorem 3 are met and the result follows.

Example 2. Assume the following simple dynamic formulation. Players examine the demands in the previous period and intend to adopt the one with the highest expected pay-off. Suppose however their realised strategies are randomly distributed around these demands. That is, if (x_n, y_n) is the demand with the highest expected pay-off in the previous period then this period's demands are distributed around (x_n, y_n) with the continuous pdf's f_{x_n} and f_{y_n} respectively. Let F_{x_n} and F_{y_n} be the corresponding cumulative distribution functions. Thus, for example, $F_{x_n}(c)$ is the probability of a randomly selected type 1 player's demand being less than or equal to 'c' given that the demand with the highest expected pay-off last period was x_n .

It is natural to assume that both types of players make equivalent mistakes. Thus $F_{x_n}(a) = F_{y_n}(b)$ if $a - x_n = b - y_n$. Let $p_n(x_n, y_n) = F_{x_n}(1 - y_n) = F_{y_n}(1 - x_n)$. Then the demands with the highest expected pay-off this period are the ones which maximise $p_n(x_n, y_n) u_1(x_n)$ over x_n and $p_n(x_n, y_n) u_2(y_n)$ over y_n . Let $I_n = [\sup(a + b | p_n(a, b) = 1), \inf(a + b | p_n(a, b) = 0)], \forall n$ and let $I_1 \subset [0, 2]$. Then assumption (A1)

follows if we take $\{p_n(x_n, y_n)\}$ to be the functional sequence arising as the size of the players' errors becomes smaller. That is $I_{n+1} \subset I_n, \forall n$. Then the conditions of theorem 3 are met and the result follows. In this case it implies the stationary points of the dynamic process approach the Nash bargaining solution as the size of the players' mistakes becomes negligible.

In example 2, above, we considered an evolutionary model for which the dynamic process required players to instantaneously adopt the most successful strategies from the previous period; at least up to a random component. We now relax this requirement and consider the same model with the standard replicator selection dynamics. To do this we simulate the evolutionary path. This also allows us to consider finite strategy sets. To model the players' mistakes we assume, over the population, that the support for a particular strategy is a random convex combination of the supports for the strategies in an interval centred on this strategy. (The exact description of the model is contained in table II.) We are interested in the simulated limit outcome for biased and unbiased initial populations, when the interval is made very small.

In the case of a symmetric-role Nash demand game we chose the initial population to be that corresponding to the mixed ESS considered in section II. That is, " $2/3$ of the population chooses strategy 0.40, while $1/3$ of the population chooses strategy 0.60." Previously we found that this and other similar initial populations lead to simulated limit outcomes vastly different from the equal division outcome. With the introduction of random mistakes we get that the simulated limit outcome is precisely the equal division outcome.

The results for the asymmetric-role Nash demand game are given in Table II. In each game we chose the initial population such that the entire population, of both types of players, was choosing the equal division outcome. This gave two games with biased initial populations and two games without. Table II shows that in each case the simulated limit outcome was close to the Nash bargaining solution.

TABLE 2

Comparison of Simulated Limit Outcome with Game-Theoretic Solutions - Stochastic Games

Game Number	Parameters	Simulated Limit Outcome	Nash Bargaining Solution	Kalai-Smorodinsky Solution
5	$u_1(x) = x$ $u_2(x) = x$ $d_1 = d_2 = 0$	(0.50,0.50)	(0.50,0.50)	(0.50,0.50)
6	$u_1(x) = x^{0.5}$ $u_2(x) = x$ $d_1 = d_2 = 0$	(0.34,0.66)	(0.33,0.67)	(0.38,0.62)
7	$u_1(x) = \min(x, 0.60)$ $u_2(x) = x$ $d_1 = d_2 = 0$	(0.50,0.50)	(0.50,0.50)	(0.38,0.63)
8	$u_1(x) = x$ $u_2(x) = x$ $d_1 = 0.50, d_2 = 0$	(0.74,0.26)	(0.75,0.25)	(0.75,0.25)

Notes

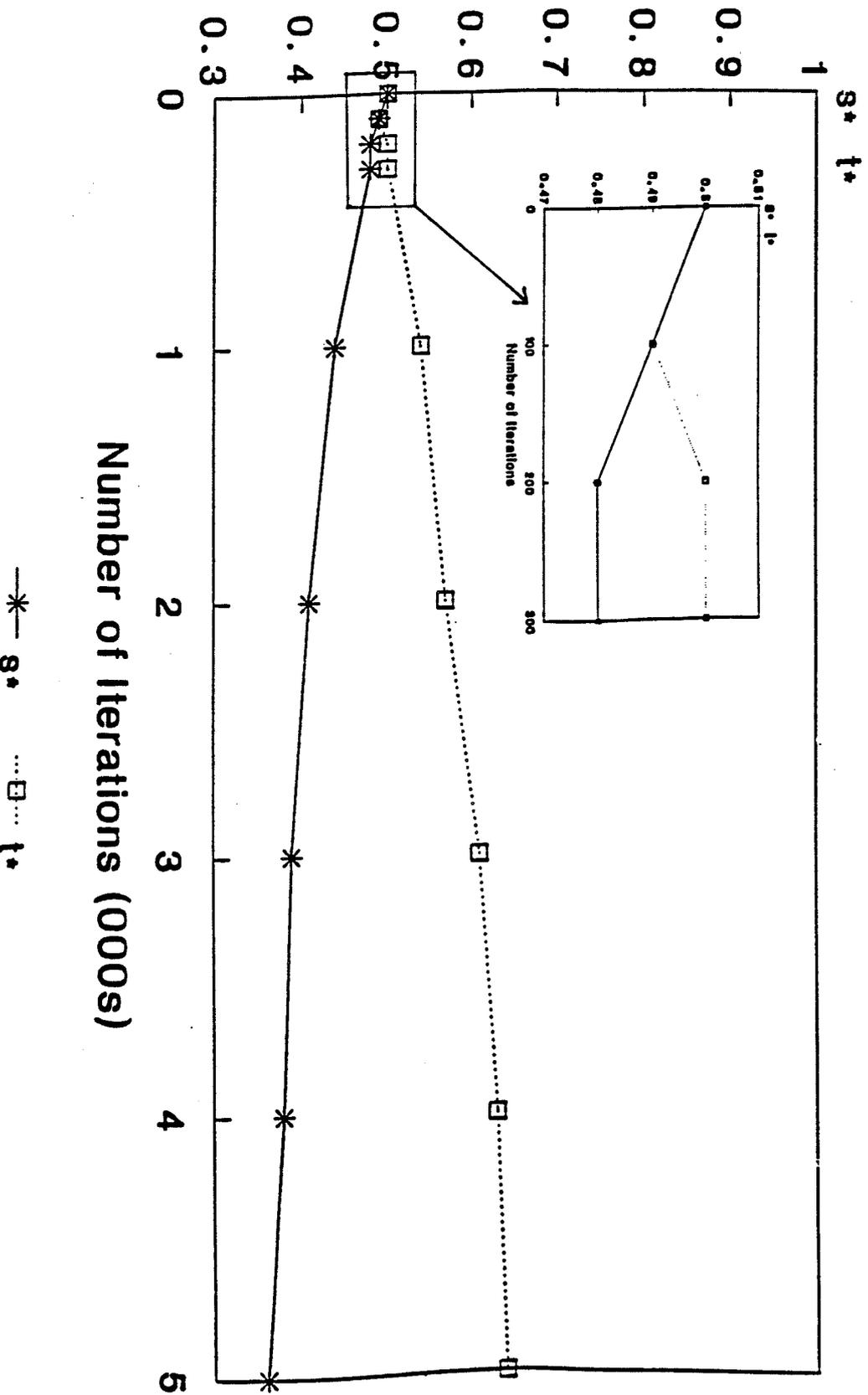
- All figures reported to 2.d.p.
- Description of Evolutionary Model Used
 - The interval for strategy i is $I = [i-2, i+2]$, or truncated, if necessary to make it defined.
 - The weighting, w_i , on the central observation is chosen from the uniform distribution between 0.95 and 1.00.
 - The remaining weightings are chosen so that

$$w_j = w_k \quad \forall j, k \in I, j, k \neq i \quad \text{and} \quad \sum_{j \in I} w_j = 1.$$

- For game 6 we chose $u_1(x)=x^{0.5}$ rather than $u_1(x)=\ln(1+x)$ since this ensured the initial population was biased against the Nash bargaining solution.

Finally consider the evolutionary path taken in the above simulations. If we denote the strategies which have the greatest support in the population at any given time, s^* and t^* for players of type 1 and 2 respectively then figure 1 represents the evolution for game 6.

FIGURE 1
Evolutionary Path for Game 6



The evolutionary path clearly provides an interesting story. Initially both types of players do better adopting lower strategies to ensure coordination. However since type 1 players are more risk averse their gain from ensuring coordination relative to not exceeds that for type 2 players. (A similar story would result if we had chosen type 2 players to have a higher disagreement point.) Thus the proportion of type 1 players adopting lower strategies grows relative to type 2 players. But this, in itself, leads type 2 players to adopt higher strategies. This dynamic process only stops when the gains from adopting 'nearby' strategies relative to not are equalised between the two players; à la the Nash bargaining solution.

CHAPTER 5

CONCLUSION

In this paper we have applied evolutionary techniques to the simple Nash demand game. For both symmetric-role and asymmetric-role bargaining games we found strong support for the Nash bargaining solution as an approximation to the evolutionary limit outcome. Although it is possible for vastly different outcomes to occur in a deterministic framework our results suggest such outcomes are unlikely when stochastic processes are included. Furthermore an evolutionary approach leads to a natural interpretation of the result that the Nash bargaining solution should be regarded as the solution outcome of the simple Nash demand game because it is the 'only necessary limit of the equilibrium points of smoothed games'.

FOOTNOTES

- ¹ A good discussion of these and other related issues is contained in Crawford (1990).
- ² All we require is u_1 and u_2 are selected randomly from the same distribution of utility functions in the population.
- ³ For the technical definition of a truly asymmetric context and proof of the equivalence of ESS, strict Nash equilibria, and locally stable strategies for such contests see Van Damme (1987, pp.234-236).
- ⁴ See for instance theorem 9 in Samuelson and Zhang (1990, p.22).
- ⁵ Alternatively we could have assumed the players beliefs about the actual bargaining problem changed randomly over time.

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