CENTRE FOR
COMPUTING AND BIOMETRICS

On development and comparative study of two Markov models of rainfall in the Dry Zone of Sri Lanka

B.V.R. Punyawardena and Don Kulasiri

Research Report No:96/11
October 1996

RESEARCH REPORT

ISSN 1173-8405

LINCOLN
UNIVERSITY
Te Whare Wānaka O Aoraki

Centre for Computing and Biometrics

The Centre for Computing and Biometrics (CCB) has both an academic (teaching and research) role and a computer services role. The academic section teaches subjects leading to a Bachelor of Applied Computing degree and a computing major in the BCM degree. In addition it contributes computing, statistics and mathematics subjects to a wide range of other Lincoln University degrees. The CCB is also strongly involved in postgraduate teaching leading to honours, masters and PhD degrees. The department has active research interests in modelling and simulation, applied statistics and statistical consulting, end user computing, computer assisted learning, networking, geometric modelling, visualisation, databases and information sharing.

The Computer Services section provides and supports the computer facilities used throughout Lincoln University for teaching, research and administration. It is also responsible for the telecommunications services of the University.

Research Report Editors

Copyright

Correspondence

This paper represents work to date and may not necessarily form the basis for the authors' final conclusions relating to this topic. It is likely, however, that the paper will appear in some form in a journal or in conference proceedings in the near future. The authors would be pleased to receive correspondence in connection with any of the issues raised in this paper. Please contact the authors either by email or by writing to the address below.

Any correspondence concerning the series should be sent to:

The Editor
Centre for Computing and Biometrics
PO Box 84
Lincoln University
Canterbury, NEW ZEALAND

Email: computing@lincoln.ac.nz

On development and comparative study of two Markov models of rainfall in the Dry Zone of Sri Lanka

B.V.R. Punyawardena [a], Don Kulasiri [b *]

[a] *Natural Resources Management Centre, Department of Agriculture, P.O. Box 52, Peradeniya, Sri Lanka*

[b] *Centre for computing and Biometrics, Lincoln University P.O. Box 84, Canterbury, New Zealand,*

Abstract

Being closer to the equator, the most important climatic element for agricultural production in Sri Lanka is rainfall which is erratic and highly unpredictable in nature, especially in the dry zone. This study attempts to model the weekly rainfall climatology of dry zone using Markov processes as the driving mechanism based on the 51 years of past data. The weekly occurrence of rainfall was modelled by two-state first and second order Markov chains while the amount of rainfall on a rainy week was approximated by taking random variates from the best fitted right skewed probability distribution out of Gamma, Weibull, Log-Normal and Exponential distributions. The parameters of the both models namely, elements of transition matrices, and scale and shape parameters of the desired distribution, were determined using weekly data. Both first and second Markov chains performed similarly in terms of modelling weekly rainfall occurrence and amount of rainfall if rain occurred. Use of second order Markov chain did not enhance the representativeness of the simulated data to the observed data in spite of being penalised for its large number of computations. Weekly rainfall data generated with the first-order Markov chain model preserve the statistical and seasonal characteristics that exist in the historical records.

Keywords: Weekly; Rainfall; Markov chains; Dry zone; Sri Lanka

1. Introduction

The soundness of the Sri Lankan economy depends significantly on the agricultural production despite the recent progress of industrialisation. The agriculture of Sri Lanka falls within one of the three agro-climatic zones: dry, intermediate and wet. The wet and the intermediate zones mainly occupy export oriented perennial crops such as tea, rubber and coconut while there is a greater potential for cultivation of arable crops for export and local consumption in the dry zone because of high fertile soils and high insolation. The dry zone having annual rainfall of less than 1800 mm is approximately sixty percent of the total land area of 4.2 millions hectares of the country. The lack of rainfall and relatively high evaporative demand constrain higher crop yields in the dry zone.

* Corresponding author

Rainfall in the dry zone is distinctly bi-modal and it is caused by regional (monsoonal) as well as local (convectional) meteorological phenomenon (Suppiah, 1989). Seventy percent of the total annual rainfall (around 1,200 mm) occurs during a limited rainy season known as *maha* (wet) from October to mid January due to convectional activity (October to mid-November) and northeast monsoonal circulation (mid-November to mid-January) of the atmosphere. Meanwhile, the wet season rainfall is generally augmented by the frequent formation of cyclonic depressions in the Bay of Bengal especially during November and December. The period from late March to mid May, known as the *yala* (dry) season is a minor convective rainy season with a very low rainfall, less than 500 mm, well below the requirement of any crop. Low rainfall during this period is due to the decreasing convectional activity towards north, northeast, east and southeast directions compared to the southwestern part of the country. There are two recognised dry seasons in between the two rainy seasons: from late January to mid March and mid May to mid September. It is common to have these dry seasons, though such climatic conditions are not conducive to promote year around agricultural production. However, extension of the dry season beyond late September or October due to failure of convectional and monsoon rains causes severe consequences in crop production (Somasiri, 1992).

The variability of rainfall in this region should be characterised in order to determine its climatic potential for agriculture, especially to choose a suitable cropping calendar (Mahendrarajah et al, 1996). Although number of studies have already been under taken on the variability of rainfall in Sri Lanka, they are based on the normality assumption in spite of highly skewed frequency distributions of rainfall data. In addition, much of the available information on the variability of rainfall of this area have been based on the monthly time intervals. Indications of variability over relatively long periods such as months have no practical significance because at certain times in the growth season of a crop, presence or absence of water is crucial (Huda, 1994). But, whenever the shorter time intervals of rainfall are considered, frequency distributions of rainfall are not independent of each other and a continuity effect is evident. This tendency in sequences of wet and dry conditions to occur casts doubt on the validity of short term analyses. Mooley (1971) have found that daily, 5-day, and even 10-day periods are not independent of each other under Indian rainfall regimes. It indicates the need to examine the possible dependence of short time intervals of rainfall records. This reality of meteorological persistence or in other words, structure of wet and dry periods can best be described by Markov chains of proper order. Although several authors have discussed the order of Markov chains with daily rainfall models, the issue of choosing the proper order with weekly time interval has not been addressed. Chin (1977) has shown that order of conditional dependence of daily rainfall occurrences depends on the season and the geographical location. He has further concluded that at any station, the rainfall occurrences associated with cyclone passage would most likely to indicate a conditional dependence with Markov order higher than one while rainfall associated with convectional activity may account for the prevalence of first order conditional dependence. Thus, being Sri Lanka' s climate is a combination of several meteorological scenarios, the order of the Markov chain that describes the occurrence of weekly rainfall can not be assumed priori.

This paper compares two-state first and second order Markov chains on describing weekly rainfall occurrence and their amounts with special reference to the dry zone of Sri Lanka based on 51 years of past data. As detailed studies of rainfall modelling in Sri Lanka are scarce, the selected model will form the basis for more comprehensive rainfall generating model in Sri Lanka. In particular, it could be very useful in delineating homogeneous climatic areas where neighbouring locations are either not covered by meteorological recording stations or similar lengths of records do not exist.

2. Review of Markov process

A stochastic process $X = \{X(t), t \in T\}$ is simply a collection of random variables $X_1, X_2,$ .....X_n which can be considered to describe the evolution of a system over discrete instants of time $t_1 < t_2, <......... < t_n$ ..... . It is assumed that there is a common probability space (Ω, A, P) in which the system operates, where Ω is the sample space, A is the σ-field and P is the probability measure (Kloeden, 1994). A realisation, a sample path or a trajectory of the stochastic process is the set of values of X takes for each outcome $\omega \in \Omega$ over the time set T.

If we consider a stochastic process $X = \{X_n = i, n = 0, 1, 2, ...,n \}$ that takes a countable number of possible values for i in the set of non-negative integers $\{0, 1, 2, ...,n \}$, then a fixed probability p_{ij} can be defined to indicate the conditional probability of the process moving from state i to state j when the time changes from the present instance to a future instance. If p_{ij} only depends on the present state and is independent on the past state then the stochastic process is called a Markov chain, and since the transition occurs at discrete time intervals, we can further describe the process as a descrete time Markov chain. It should be noted that, as probabilities are non-negative and the process must make a transition into some state, the transition probability p_{ij} must satisfy the following conditions:

$$p_{ij} \geq 0, \ i, j \geq 0$$

$$\sum_{j=0}^{\infty} p_{ij} = 1, \qquad i = 0,1, \qquad [1]$$

If p_{ij}^k is the probability that a process in state i will be in state j after k additional transitions, then the Chapman-Kolmogorov equations can be used to compute p_{ij}^k using intermediatory transition probabilities (Pfeiffer, 1990):

$$p_{ij}^k = \sum p_{io}^l \, p_{oj}^m \qquad l, m \geq 0, \text{ all } i,j \qquad [2]$$

and

$$l + m = k$$

Equation (2) states that if we denotes P^k as the matrix of k-step transitional probabilities p_{ij}^k, then

$$P^{(k)} = P^{(l)} P^{(m)} \qquad [3]$$

Once the transitional probability matrices at specific time intervals are known, Equation (3) can be used to compute the probability distribution of the states at any given instance (Pfeiffer, 1990).

3. Markov chain in rainfall modelling

As described in the previous section a Markov chain can be defined as a type of time ordered probabilistic process which goes from one state to another according to the probabilistic transition rules that are determined by the current state only. That is, the probability of week being in a certain state (either wet or dry) is conditioned on the state of the previous period where the number of previous periods is termed as the order of the chain (Buishand, 1978). In the first-order, two-state Markov models, the current state is dependent solely on the state of previous period while in the second-order, two-states Markov chains current state is determined by the state of two previous periods.

Markov chains have been widely used with daily rainfall models in hydrological and climatological studies. The first stochastic model of the temporal precipitation with Markov chain (two-state first-order) was introduced by Gabriel and Neuman (1962). Richardson (1981) used a first order Markov chain along with an exponential distribution to describe the daily rainfall distribution in the USA. Brauhn et al (1980) used a similar Markov chain to simulate the daily rainfall occurrence in Geneva and Fort Collins in the USA. A first order Markov chain has also been used by Selavalingam and Miura (1978), Larsen and Pense (1982) and Woolhiser et al (1993) to describe the occurrence of wet and dry day sequences in daily rainfall models. All of these studies have revealed that the generated data using Markov chain along with a suitable probability distribution preserve the seasonal and statistical characteristics of historical rainfall data. Being simple and requiring only two parameters are to be determined, the two-state first-order Markov chain is the most common one referred in literature. Smith and Schriber (1973) have suggested that two-state first-order Markov chains are superior to Bernoulli models which are based on sequential independence for describing wet and dry days. Models of second and higher orders have also been studied by Chin (1977), Singh et al (1981) and Jones and Thornton (1993). When a second-order Markov chain is used, eight separate parameters have to be estimated and this may lead to expensive computational requirements. However, Coe and Stern (1982) prefer choice of either the first or the second order if they fit reasonably well.

4. Development of Models

Although month and longer time bases are very common in agro-climatological studies, the distribution of rainfall within a particular month or longer period may not be favourable for crop growth, allowing crops to be exposed to soil moisture stress. Therefore, use of shorter time intervals (ie, weekly) for agro-climatological studies has been recommended for tropical countries like Sri Lanka (Krishnan, 1980). This is of particular importance in dry zone of Sri Lanka where high intensity, short duration rains are very frequent during the period of October to December due to convectional activity and formation of depressions in the Bay of Bengal (Puwaneswaran, 1983). Considering those aspects, plus the fact that the plant water requirements over a period about one week can usually be met by water stored in the soil, it was decided to use a weekly time interval for this rainfall modelling study.

In order to make an empirical comparison between two different Markov models, the data collected by the Dry Zone Agricultural Research Institute, Dept. of Agriculture, Maha-Illuppallama ($8^0 07'$ N, $80^0 28'$E) were used. This rainfall recording station represents the entire dry zone in terms of soils, cropping pattern and irrigation network. Fifty one consecutive years (1945-1995) of records for weekly rainfall totals were available for the model development. Out of 51 years of data, 30 years of weekly rainfall data were used for the parameter estimation of the models (Wight and Hanson, 1991) while the rest was used for validation purposes (Brauhn et al, 1980).

The determination of whether any particular week is wet or dry necessitates to define a threshold value of rainfall that differentiate the week being wet or dry. The Potential Evapotranspiration (PET) of at least 3 mm/day would make a weekly total of 21 mm and 33% of PET (7 mm) is considered to be the requirement for the crop growth (Hargreaves, 1975). Any rainfall less than 7 mm/week would not make a substantial contribution to the crop growth; therefore, 7 mm of total rainfall during a week was decided to be the threshold value.

If weekly rainfall is modelled by a first-order two-state Markov chain, rain falling on any week depends only on the state (wet or dry) of the previous week. The elements to be estimated are therefore the transition probabilities:

$p_m(W_i \mid W_{i-1})$ = conditional probability of a wet week on week i given a wet week on week (i-1) in a certain period m

$p_m(D_i \mid W_{i-1})$ = conditional probability of a dry week on week i given a wet week on week (i-1) in a certain period m

$p_m(W_i \mid D_{i-1})$ = conditional probability of a wet week on week i given a dry week on week (i-1) in a certain period m

$p_m(D_i \mid D_{i-1})$ = conditional probability of a dry week on week i given a dry week on week (i-1) in a certain period m

Thus, for each week four elements in the transition matrix were to be determined in first order Markov chains. For a second order chain eight elements of the transitional probability matrix were to determined. These were conditional probabilities of a wet week following two wet weeks, $p_m(W_i \mid W_{i-1} W_{i-2})$; a wet week following a wet week and a dry week, $p_m(W_i \mid W_{i-1} D_{i-2})$; a wet week following a dry week and a wet week, $p_m(W_i \mid D_{i-1} W_{i-2})$; a wet week following two dry weeks, $p_m(W_i \mid D_{i-1} D_{i-2})$; a dry week following two wet weeks, $p_m(D_i \mid W_{i-1} W_{i-2})$; a dry week following a wet week and a dry week, $p_m(D_i \mid W_{i-1} D_{i-2})$; a dry week following a dry week and a wet week, $p_m(W_i \mid D_{i-1} W_{i-2})$; a dry week following two dry weeks, $p_m(W_i \mid D_{i-1} D_{i-2})$. As a result of seasonal variations in rainfall, the elements of the transitional matrices vary through the year. The usual method of handling this variation is fitting Fourier series (Richardson, 1981 and Woolhiser et al, 1993) and other periodic functions such as polynomials (Coe and Stern, 1982) at the expense of some accuracy. But, in this study each element of both transition probability matrices of each week was estimated using the respective weekly data as it would reflect the variation more realistically than approximating by a continuous function.

It is customary to assume the amount of rainfall in a given time period follows a particular probability distribution and that it is same for each time intervals. The agro-climatic literature, often uses normal distribution for characterising the rainfall, in which case certain statistics can be calculated from the standard normal distribution. But, over short period of time intervals the distribution of rainfall amounts become skewed with many small and few large amounts (Stern et al, 1982). Hence, normality assumption is rarely appropriate for shorter periods of time intervals and may always lead to over estimation in the model. Thus, in this study, variation of rainfall amounts on wet weeks was determined by using best fitted right skewed distribution out of Gamma, Weibull, Log-Normal and Exponential distributions rather than using a single pre-determined distribution. Weekly rainfall data of 30 years, the same data used for transitional probability matrix estimation in Markov chain, were used to find appropriate distributions. Each distribution was assigned a relative evaluation score from 0 to 100 (best) based on the ranking algorithm of UNIFIT II, a statistical software to determine an appropriate probability distribution for observed data (Law and Vincent, 1993). The higher the score of a distribution, the better it is relative to the other fitted distributions. Out of four probability distributions studied, the one with the highest score was selected to represent the weekly amount of rainfall. Again, the parameters of the desired distribution (location, scale and shape) for each week was determined using Maximum Likelihood Estimation (MLE) method (Appendix 1).

Once the probability of rain occurring on a given week was determined using Equation [3] and an initial probability distribution for the states (Ross, 1993), a random number generated from a uniform probability distribution was used to determine the occurrence of rain during the current week. If the random number exceeds the probability of rainfall, weekly rainfall was zero, literally less than 7 mm of rainfall, otherwise the amount of rainfall was determined by a random variate generated from the desired probability distribution of the current week. Generation process of rainfall occurrence and amount of rain if rain occurred were similar for the both Markov chains.

5. Simulation and comparison

Twenty one arbitrary data generating runs were made with both models in order to compare with the historical data which was not used for parameter estimation of the models (Brauhn et al, 1980). Output consisted occurrence of rainfall, amount of rainfall if rain occurred for each run and mean rainfall occurrence (number of weeks that receive 7mm or more rainfall) and mean amount of rainfall after 21 runs. Additional information such as Maximum rainfall, number of rainfall events greater than 150 mm of rainfall, number of events less than 7 mm of rainfall and total annual rainfall were also collected in order to evaluate the capability of models for reproducing the distribution of annual extreme events. Two types of statistical tests were applied to compare the simulated and the observed data. A non-parametric test (Kolmogorov-Smirnov) was used to test the difference in cumulative distribution functions by calculating the maximum distance between two distribution functions as the test statistics. A two-tailed t-test was applied to test the difference between means of any interested attributes of data records.

6. Results and discussion

The results summarised in Figure 1 show the relationship between weekly mean rainfall generated by the first and second order Markov chains. The Kolmogorov-Smirnov goodness of fit test applied to each of the standard week in the year revealed that only in one week out of 52 weeks (first week of January) the two distribution functions were different from each other at 5% rejection level. Thus, we can conclude that the two distributions came from the same population for all practical purposes. Comparison of simulated mean weekly rainfall of two models again showed that none of the weeks except the standard week number 34 differ significantly with each other (Table 1). The significant difference of amount of rainfall in week number 34 could be due to the use of Log-Normal distribution to represents the frequency distribution of amount of rainfall on this week. Although, it was the best fitted distribution among the four distributions studied, its representation of short time interval rainfall amounts is highly recognised only when the rain is produced with cumulus clouds and weather modification experiments (Mielke and Johnson, 1973). But, in reality, cumulus clouds are hardly evident during this period in the dry zone. Thus, significant difference between two models in terms of rainfall amount at week number 34 could be due to the poor representation of the frequency distribution. The mean number of occurrence of wet weeks at each week was also not significantly different between two models except in standard week 14 (Table 1). In view of these results, it is worthwhile to examine the first order model further whether it is capable of reproducing historical data. In practice low order chains are preferable for two reasons: (a) the number of parameters to be estimated kept minimum so that better estimates for the parameters could be obtained; (b) the subsequent use of the fitted model to calculate quantities such as probability of dry spells is simpler (Coe and Stern, 1982).

Table 2 shows gives a comparison between observed and simulated rainfall data with first order Markov chain and the t-test statistics. The yearly average of 21 years of simulated rainfall was 1504 mm compared to an actual average 1481 mm. Therefore, on a yearly basis, the simulated rainfall values were very close to the observed values. The K-S test comparing cumulative distribution functions of rainfall amounts for the simulated and the observed data were not significantly different during whole year except for the fourth week of January. In all other instances, the maximum deviation statistic was less than the 0.420, the critical value at 5% rejection level. Although there were some measurable differences in weekly mean amount of rainfall between observed and simulated data records (Table 2), these differences were not statistically significant at 5% rejection level due to inherently high variability of the observed data. The highest magnitude of the discrepancy was evident in 18th week mean rainfall, which were 67.2 and 40.8 mm for the historical and observed records respectively. It is noteworthy that the variability of both the simulated and the observed data during rainy periods is higher than that the variability in drier periods (Figure 2).

However, out of 52 weeks, weekly rainfall occurrence was significantly different between simulated and observed data in 12 weeks at 5% rejection level. But, this came down only to a week (last week of January) when the rejection level is one percent. Moreover, it was interesting to note that the weekly rainfall occurrence in major rainy season of dry zone (October to mid January) was not significantly different between observed and simulated data even at 5% rejection level (Table 2). When the occurrence of rainfall was modelled using the second order Markov chain, three weeks out of 52 were significantly different at 1% level from the observed data and one of them was in the major rainy season. Thus, it again confirms the rationale of selecting the first order model for further consideration.

One of the major argument against the stochastic generation of rainfall records is their inability to simulate extreme values (Wight and Hanson, 1991). Therefore, additional comparisons were made with the annual maximum weekly rainfall, mean number of weeks which receive more than 150 mm of rainfall and mean number of weeks which receive less than 7mm of rainfall (Table 3). Although, a discrepancy between simulated and observed values was evident in the attributes tested, none of them was significant at the 5% rejection level, showing the ability of the model to reproduce the extreme values evident in the historical data.

7. Conclusion

Review of the literature revealed that studies had been conducted to describe the daily rainfall process using Markov models at other locations although no similar work was found for weekly time intervals. In relation to modelling weekly rainfall occurrence, both first and second order Markov chains performed similarly. However, use of second order Markov chains, the one with large number of computations, to represent the weekly rainfall occurrence was not justified. This was further confirmed from the results of amount of rainfall as there was no additional improvement to the system in terms of representativeness of simulated data to the observed data by introducing two week dependency.

In general, differences in every aspects between simulated data using first order Markov chain and historical records obtained from the dry zone of Sri Lanka were non-significant despite some measurable discrepancies indicating that parameter estimation of the model was reasonably representative. Thus, results support the recommendations of Richardson et al (1987) made on the adequacy of the data base for parameter estimation of the weather generation models. They recommended that for generating precipitation records, the historical records should be 20 years or more. The results also encourage the determination of weekly parameters of the model rather than fitting to a periodic function.

Based on these comparisons, the overall performance of the two-state first-order Markov model appeared to be adequate for most agricultural applications with good confidence and will be used for studying and projecting the rainfall climatology of dry zone of Sri Lanka.

Acknowledgments

The authors are grateful to Department of Agriculture, Ministry of Agriculture, Land and Forestry, Government of Sri Lanka for the study leave granted and Ministry of Foreign Affairs and Trade, Government of New Zealand for financial assistance for the first author. Thanks are also extended to Centre for Computing and Biometrics at Lincoln University, Canterbury, New Zealand for providing computer facilities during the study.

References

Brauhn, J.A., Fry, W.E. and Fick, G.W., 1980. Simulation of daily weather data using theoretical probability distributions. Journal of Applied Meteorology, 19(9): 1029-1036.

Buishand, T.A., 1978. Some remarks on the use of daily rainfall models. Journal of Hydrology, 36: 295-308.

Chin, E.H., 1977. Modelling daily precipitation occurrence process with Markov chain. Water Resources Research, 13(6): 949-956.

Coe, R. and Stern, R.D., 1982. Fitting models to daily rainfall data. Journal of Applied Meteorology, 21: 1024-1031.

Gabriel, K.R. and Neuman, J., 1962. A Markov chain model for daily rainfall occurrence at Tel Aviv. Quart. Journal of Royal Meteorological Society, 88: 90-95.

Hargreaves, G.H., 1975. Water requirement manual for irrigated and rainfed agriculture. EMBRAPA and Utah State University, Publication No. 75-D158, pp. 40-42.

Huda, A.K.S., 1994. Management strategies to minimise climatic risk to wheat production in low rainfall areas of southern Australia. Agricultural and Forest Meteorology. 69: 125-147.

Jones, P.G. and Thornton, P.K., 1993. A rainfall generator for agricultural applications in the tropics. Agricultural and Forest Meteorology, 63: 1-19.

Larsen, G.A. and Pense, R.B., 1982. Stochastic simulation of daily climatic data for agronomic models. Agronomy Journal, 74: 510-514.

Kloeden, P.E. and Platen, E., 1994. Numerical solutions of stochastic differential equations. Springer-Verlag, Berlin Heidelberg. pp. 26-28.

Krishnan, A., 1980. Agroclimatic classification methods and their application to India. Climatic classification. A consulting meeting, ICRISAT. pp. 59-119.

Law, A.M and Vincent, S.G., 1993. Unifit II user's guide. Averill M. Law Associates, Tuscon.

Mahendrarajah, S., Jakeman, A.J. and Young, P.C., 1996. Water supply in monsoonal Asia: modelling and predicting small tank storage. Ecological Modelling, 84(1996): 127-137.

Mielke, P.W. and Johnson, E.S., 1973. Three parameter kappa distribution, maximum likelihood estimates and likelihood ratio tests. Monthly weather Review, 101:701-770.

Mooley, D.A., 1971. Independence of monthly and bi-monthly rainfall over S.E. Asia during summer monsoon season. Monthly Weather Review, 99(6): 532-536.

Pfeiffer, P.E., 1990. Probability for applications. Springer-Verlag, New York. pp. 616-660.

Puwaneswaran, K.M., 1983. Some contrasting features of the finding in the rainfall fluctuation of Sri Lanka. Climatological Notes 33, Climate, Water and Agriculture in Sri Lanka. Institute of Geoscience, University of Tsukuba. pp. 221-226.

Richardson, C.W., 1981. Stochastic simulation of daily precipitation, temperature and solar radiation. Water Resources Research. 17(1): 182-190.

Richardson, C.W., Hanson, C.L., and Huber, A.L., 1987. Climate generator, Ch. 2., Part I, Documentation, pp. 3-16.

Selvalingam, S. and Miura, M., 1978. Stochastic modelling of monthly and daily rainfall sequences. Water Resources Bulletin, 14(5):1105-1120.

Singh, S.V., Kripalani, R.H., Shaha, P., Ismail, P.M.M. and Dhale, S.D., 1981. Persistence in daily and 5-day summer monsoon rainfall over India. Arch. Meteorol. Geophys. Biokl. Ser. A, 30:261-277.

Smith, R.E. and Schreiber, H.A., 1973. Point processes of seasonal thunderstorm rainfall, 1, Distribution of rainfall events. Water Resources Research, 9(4): 871-884.

Somasiri, S., 1992. Climatic factors in relation to rainfed agriculture in the dry zone of Sri Lanka. Proc. of Sri Lanka Assoc. of Adv. of sci. symposium on Rainfed Agriculture. 19-33.

Stern, R.D., Dennet, M.D. and Dale, I.C., 1982. Analysis of daily rainfall measurements to give agronomically useful results. 1. Direct method. Experimental Agriculture, 18: 223-236.

Suppaiah, R., 1989. Relationships between the southern oscillation and the rainfall of Sri Lanka. International Journal of Climatology. 9, 601-618

Wight, J.R. and Hanson, C.L., 1991. Use of stochastically generated weather records with range-land simulation models. Journal of range Management, 44(3): 282-285.

Woolhiser, D.A., Keefer, T.O. and Redmond, K.T., 1993. Southern oscillation effects on daily precipitin in the Southwestern United States. Water Resources Research, 29(4):1285-1993.

Table 1.
Simulated average rainfall amount and rainfall occurrence (No. of weeks that receive rainfall ≥ 7mm) using two Markov models, Maha-Illuppallama, Sri Lanka.

Standard Week No.	Order 1 rainfall (mm)	Order 2 rainfall (mm)	Order 1 rainfall occurrence	Order 2 rainfall occurrence
1	35.6	18.6	12.0	15.0
2	33.1	47.2	12.3	16.7
3	16.2	17.4	12.3	13.3
4	23.5	25.2	18.0	19.0
5	25.0	15.4	11.7	11.0
6	7.0	5.0	8.7	9.0
7	8.7	14.0	11.7	9.7
8	7.9	17.2	11.7	10.7
9	20.5	13.6	13.3	15.7
10	7.1	8.0	8.3	6.0
11	18.2	29.3	13.0	11.0
12	20.8	33.4	16.3	15.0
13	28.8	38.0	18.3	16.3
14	54.7	44.7	17.0	20.0*
15	40.3	39.0	20.3	19.3
16	39.2	37.4	17.7	19.0
17	53.2	29.7	15.3	15.7
18	67.2	37.4	17.3	16.0
19	24.9	19.3	18.7	16.7
20	33.9	21.9	16.7	17.0
21	7.0	8.7	9.3	9.0
22	11.8	8.3	10.3	10.3
23	6.0	7.2	7.3	5.7
24	6.0	5.7	9.0	7.3
25	4.1	4.7	4.0	2.3
26	1.0	0.8	0.0	0.0
27	3.6	6.0	6.0	6.0
28	9.4	6.4	3.3	5.7
29	7.8	9.6	9.7	8.7
30	4.4	3.4	2.7	4.3
31	3.2	2.4	1.3	1.3
32	6.0	12.0	5.0	6.3
33	6.9	7.2	7.7	8.3
34	1.5	2.2*	0.3	0.0
35	1.2	0.9	0.0	0.0
36	5.8	3.3	4.7	5.3
37	22.0	46.9	12.0	11.3
38	24.1	17.2	10.0	10.0
39	10.5	5.7	8.3	7.3
40	52.4	59.2	15.7	14.7
41	44.9	30.1	15.7	16.3
42	80.6	69.7	19.3	20.3
43	76.8	50.6	19.0	19.3
44	67.7	67.9	21.0	21.0
45	68.4	56.3	20.0	19.7
46	51.6	88.5	17.3	18.0
47	44.7	32.6	19.3	18.7
48	75.7	59.7	16.3	16.7
49	47.5	54.1	16.3	16.0
50	37.0	34.3	17.7	18.7
51	49.9	32.6	18.0	19.0
52	47.6	45.1	17.7	20.0

* The means from two models are significantly different at the 5% level

Table 2

Observed and simulated (first order Markov model) average rainfall amount and rainfall occurrence (No. of weeks that receive rainfall ≥ 7mm), Maha-Illuppallama, Sri Lanka.

Standard Week No.	Simulated rainfall (mm)	Observed rainfall (mm)	Simulated rainfall occurrence - week	Observed rainfall occurrence - week
1	35.6	20.6	12.0	12
2	33.1	25.1	12.3	13
3	16.2	16.0	12.3	11
4	23.5	14.9	18.0	9*
5	25.0	17.6	11.7	8**
6	7.0	4.9	8.7	4
7	8.7	8.6	11.7	4
8	7.9	18.0	11.7	9
9	20.5	13.8	13.3	9
10	7.1	16.4	8.3	8
11	18.2	12.6	13.0	7*
12	20.8	22.2	16.3	11
13	28.8	22.5	18.3	14*
14	54.7	39.8	17.0	17
15	40.3	32.6	20.3	18*
16	39.2	54.2	17.7	19
17	53.2	48.8	15.3	20
18	67.2	40.8	17.3	13
19	24.9	18.7	18.7	11*
20	33.9	37.0	16.7	13
21	7.0	16.5	9.3	5*
22	11.8	10.8	10.3	5*
23	6.0	5.8	7.3	6
24	6.0	3.9	9.0	4
25	4.1	5.2	4.0	5
26	1.0	0.3	0.0	0
27	3.6	2.6	6.0	2
28	9.4	7.0	3.3	2
29	7.8	13.7	9.7	5
30	4.4	6.5	2.7	4
31	3.2	9.0	1.3	3*
32	6.0	4.7	5.0	3
33	6.9	15.3	7.7	4*
34	1.5	6.5	0.3	2*
35	1.2	5.2	0.0	3
36	5.8	7.8	4.7	3
37	22.0	20.0	12.0	7
38	24.1	18.0	10.0	10
39	10.5	12.1	8.3	9
40	52.4	29.4	15.7	7*
41	44.9	53.8	15.7	17
42	80.6	67.2	19.3	20
43	76.8	62.5	19.0	18
44	67.7	82.8	21.0	20
45	68.4	66.7	20.0	19
46	51.6	66.6	17.3	15
47	44.7	72.6	19.3	18
48	75.7	59.4	16.3	17
49	47.5	70.3	16.3	19
50	37.0	59.4	17.7	18
51	49.9	65.0	18.0	19
52	47.6	69.2	17.7	17

* The means of simulated and observed values are significantly different at the 5% level

** The means of simulated and observed values are significantly different at the 1% level

Table 3
Simulated and observed annual rainfall and other extreme attributes, Maha-Illuppallama, Sri Lanka.

	Simulated	Observed	Significance*
Annual rainfall (mm)	1504	1481	n.s
Annual weekly maxima (mm)	210	201	n.s
Mean No. of weeks ≥ 150 (mm)	1.8	1.6	n.s
Mean No. of weeks ≤ 7 (mm)	21.6	26.5	n.s

n.s - Means are not significantly different at 5% level

Figure 1
Simulated weekly mean rainfall using first and second order Markov chain models

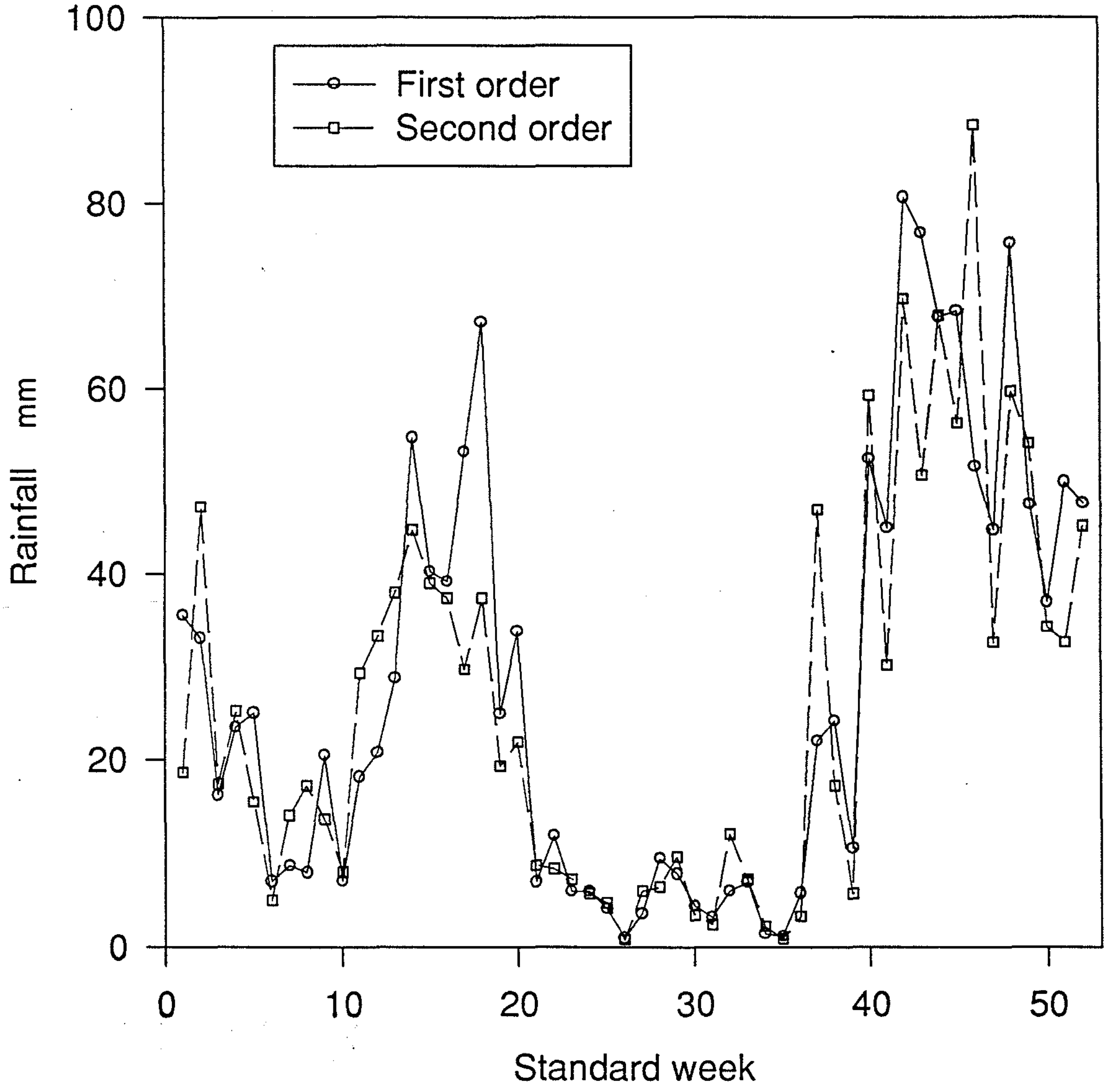
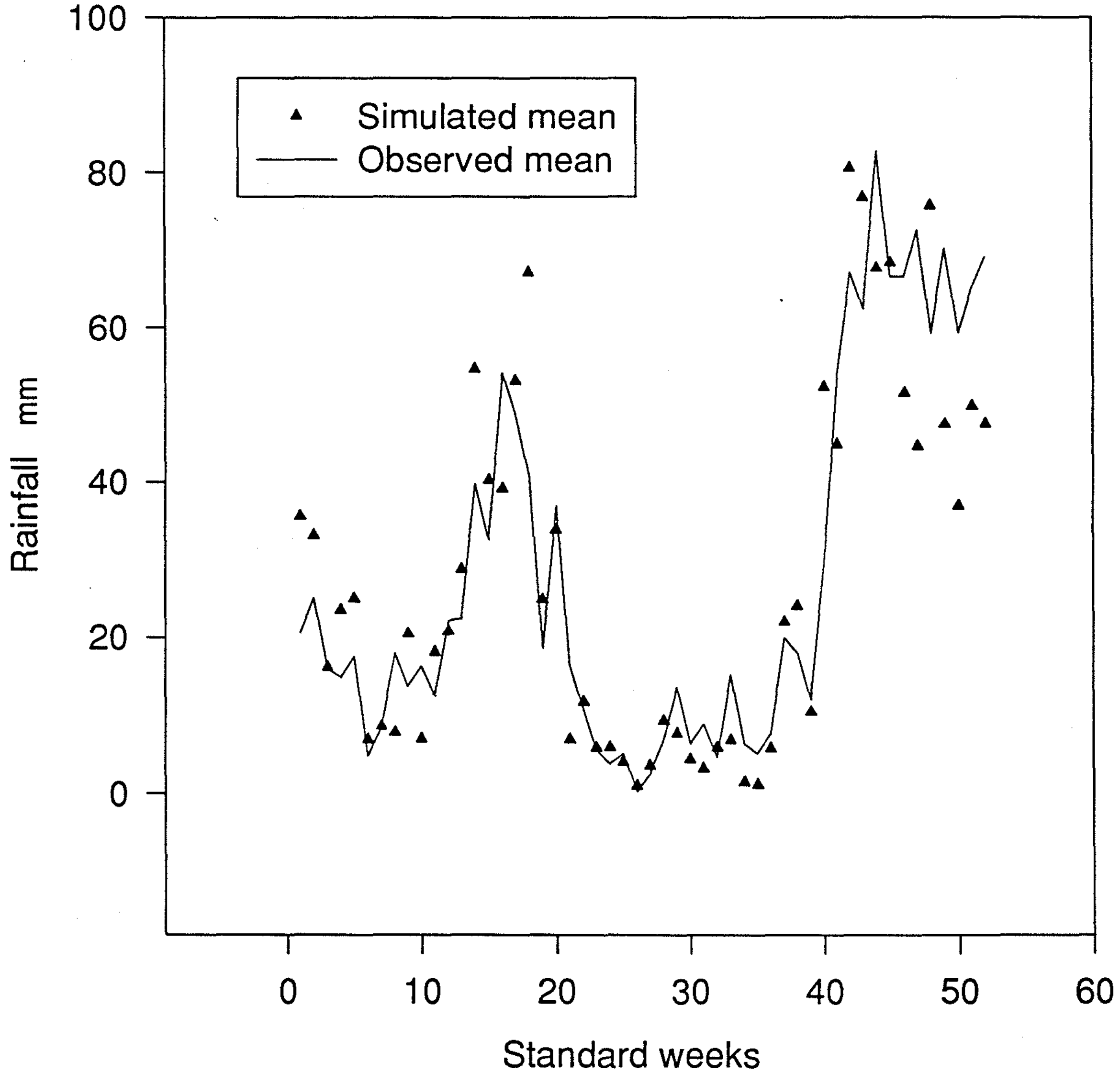


Figure 2
Observed and simulated means of rainfall using first order Markov model



Appendix 1
Best fitted probability distribution and its Maximum Likelihood Estimates (MLE) for each week in the year, Maha-Illuppallama, Sri Lanka.

Week No.	Distribution	Scale (β)	Shape (α)
1	Gamma	76.69	0.4296
2	Gamma	72.56	0.5685
3	Gamma	37.96	0.6505
4	Weibull	22.15	1.4116
5	Weibull	16.04	0.7806
6	Weibull	6.76	1.5615
7	Exponential	9.86	-
8	Weibull	13.13	0.7971
9	Weibull	21.28	0.6679
10	Weibull	6.74	0.7836
11	Weibull	19.68	0.7396
12	Weibull	19.36	0.9921
13	Exponential	33.17	-
14	Weibull	50.63	1.04
15	Weibull	40.25	1.39
16	Gamma	47.61	0.9526
17	Gamma	45.15	0.7620
18	Gamma	71.98	0.6932
19	Weibull	32.16	1.57
20	Gamma	30.41	0.8503
21	Exponential	8.23	-
22	Weibull	9.22	0.7873
23	Weibull	5.86	0.8611
24	Exponential	6.53	-
25	Exponential	4.19	-
26	Exponential	0.78	-
27	Gamma	7.27	0.6706
28	Weibull	3.41	0.5621
29	Log-Normal	12.67	8.97
30	Exponential	3.26	-
31	Weibull	2.19	0.6189
32	Gamma	33.15	0.2646
33	Exponential	7.10	-
34	Log-Normal	1.46	1.90
35	Log-Normal	1.72	1.36
36	Weibull	3.65	0.703
37	Weibull	18.42	0.4974
38	Gamma	31.26	0.4368
39	Weibull	6.80	0.8278
40	Weibull	36.97	0.7478
41	Gamma	51.74	0.6265
42	Exponential	66.33	-
43	Exponential	57.37	-
44	Weibull	85.90	1.55
45	Gamma	51.02	1.38
46	Gamma	86.86	0.7793
47	Weibull	42.53	1.37
48	Weibull	45.98	0.6764
49	Gamma	69.19	0.6288
50	Gamma	56.70	0.8618
51	Weibull	44.22	1.05
52	Exponential	58.82	-