Real Worth Agents,
REH Agents
and the Econometrician

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Abstract

This paper aims to explain from within mainstream theory why incorporating the rational expectations hypothesis in new classical macroeconomics leads to stable market equilibria, while other branches within the mainstream end up demonstrating the impossibility of such equilibria. It argues that Rational Learning describes a learning process where the object of learning, the set of equilibrium prices for example, is unwittingly taken as given data (and so is part of the decision-making kit for the agents from the beginning of the learning process), whereas real life decision making in the markets does not have the benefit of this knowledge.

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Introduction

The literature on Rational Learning (see, for example, the useful surveys provided in Blume et al., 1982, and Savin, 1987) examines a process of learning in the market place: agents observing a time series of the variable whose expectation is to be formed are assumed to have a correctly specified model of the mechanisms generating it. The process of optimal prediction - the necessary tools and incentives are granted to the agents by the hypothesis of rationality - then leads to a rational expectations estimate of the variable.

It is noteworthy that along the learning path, no auctioneer shields the agents from disequilibrium income effects. Neither is there any provision for reconstructing to make the past decisions reversible without cost. Nor in this model are agents supposed to have a knowledge of normal (equilibrium) prices laid down in their subconscious mind as in much of Marshall, but they seem to learn about the equilibrium price merely by trying to economise on the cost of their decision making, and in the final stage by acting with this knowledge so as to navigate the system into equilibrium. This implies that either there is something fundamentally wrong in the present consensus outside new classical macroeconomics that without some extra-market agency competitive markets cannot converge to an equilibrium configuration (even if such a configuration exists), or else the new classical theory overlooks some fundamental aspect of the problem.

One part of the literature critical of Rational Expectation Hypothesis (REH) criticises the set of behavioral and institutional assumptions associated with it. Lucas (1981), commenting on Tobin (1980), expressed dissatisfaction with such critiques, suggesting that a more appropriate criticism would try to point out why the analogy that REH presents to real life decision making should be rejected. Davidson’s (1982-83) reply provided a thorough and fundamental critique of REH based on the non-ergodic nature of the economic process. His critique has implications for much more than just the REH, however, and can be used against much of current orthodoxy in economic theory and econometric practice. The scope of the present paper is less radical. It aims to explain from within mainstream theory why incorporating REH in new classical macroeconomics leads to stable market equilibria, while other branches within the mainstream end up demonstrating the impossibility of such equilibria. It argues that the analogy between real life decision making and the REH is fundamentally mis-specified. Rational Learning describes a learning process where the object of learning, the set of equilibrium prices for example, is unwittingly taken as given data (and so is part of the decision-making kit for the agents from the beginning of the learning process), whereas real life decision making in the markets does not have the benefit of this knowledge. This mistake, it is argued, generates the surprising difference between the conclusions of new classical macroeconomics and much of post-Keynes mainstream theory.
The Argument

The Cobweb model (or one of its variants in the form of firms taking output decisions based on
an expectation of a future price) is generally used in this literature, and will be used for
illustrating the argument of this paper as well. The textbook Cobweb model proceeds as
follows. A supplier bases her supply, $S_t$, on the price expected next period, $p^e_t$:

\begin{equation}
S_t = A + Bp^e_t
\end{equation}

Market demand at time $t$, $D_t$, depends on the actual price, $D_t$:

\begin{equation}
D_t = M - Np_t
\end{equation}

The reduced form equation for this structural system is

\begin{equation}
p_t = (M-A)/N + (B/N)p^e_t
\end{equation}

Suppose that the expectation rule is given by:

\begin{equation}
p^e_t = p_{t-1}
\end{equation}

Then the market clearing price is:

\begin{equation}
p_t = (M-A)/N + (B/N)p_{t-1}
\end{equation}

Equation (5) describes the difference equation path for the price level in a regime of price
expectations given by (4), and generates the famous Cobweb cycles, damped or explosive
depending on the nature of the parameters, $A$, $B$, $M$ and $N$. The steady-state solution, $p^*$,
obtained by equating $p_t$ and $p_{t-1}$, is given by:

\begin{equation}
p^* = (M-A)/(N-B)
\end{equation}

If the supplier develops a rational expectation, then $p^e_t = p^*$, and the market clearing price $p_t$
also is established at $p^*$. That is, the agent is assumed to observe the time series (5),
superimposed by stochastic disturbances arising in the demand and/or the supply equation.
Because she has the correct model of the market, however, she realises that the time series (5)
merely reflects the expectation rule (4), and identifies (3) as the appropriate equation generating the series \( p_t \). She then estimates this reduced form equation. Once this is done, she will seek a \( p^* \), that will be realised if expected; hence solving (3) for a fixed point she gets \( p^* \). When this price is used for decision making, it is realised (within the limits of exogenous disturbances). Thus equilibrium is achieved.

There are a number of assumptions about the information processing ability of agents and the incentive-compatibility of their actions necessary to follow this argument through. The hypothesis of rational expectation encompasses those assumptions, but it is not this paper’s purpose to discuss their realism. Instead, there is a set of theoretical issues related to this formulation which are even more basic.

The structural equations used in the above illustration are not a valid description of the market in question. They feature the notional demand and supply functions of the agents, which are meaningful only if the planned trades materialise at all instants, or if there is a mechanism to prevent out-of-equilibrium trades. Neither is guaranteed in the case under consideration, and hence the appropriate structural equations should be the effective demand and supply functions featuring the income effects on sale and purchase decisions of ongoing (disequilibrium) trade at current prices.

The problem that arises at this stage is how to formulate the structural equations to represent correct behaviour in disequilibrium without any kind of prior knowledge of the equilibrium. The usual approach is to add an income effect to the notional functions in the structural equations. Typically the demand function in the above model may be reformulated as follows:

\[
D_t = M - Np_t + Q(p^* - Lp_t)
\]

where \( L \) is a lag operator, suggesting an income effect that is a function of the divergence between the market price (current or lagged) and the equilibrium price. While this is a reasonable formulation of the income effect, it can not be used in the present model specification, because, until rational learning is complete, \( p^* \) is not known to the agents, and obviously they can not use it as a behavioural parameter. More importantly, the parameters \( M \) and \( N \) in the equation cannot appear either, since they appear in the demand function as a result of the agents’ maximisation under constraints which are consistent with their equilibrium plans, not their extant budgetary constraints. Hence, they should not feature in the behavioral functions in the absence of a Walrasian auctioneer or some prior knowledge about the equilibrium. In fact it is this property of \( M \) and \( N \) (and of \( A \) and \( B \) in the supply equation) that
leads to their information content regarding the equilibrium price. The use of such formulation prior to learning, in a model where learning itself is studied, will constitute an impermissible logical circularity. The equations used actually imply that the agents have a notion of the permanent and the transitory components of their incomes, and that they base their decisions on these two components. The problem is that until the learning process is complete, there is no informational basis to classify income into such components, and this creates the difficulty in writing the structural equations.

If equations of this kind (or their more complicated versions) were permissible, the reduced form equation for the market would contain complete information about the equilibrium, and in principle, the estimation of $p^*$ by agents could be visualised as a possibility. In the absence of such a circular formulation, however, the structural equations of the market will contain current and past prices, but the parameters of these equations are no longer the parameters of the notional equations we had started with. Since these parameters arise from agents’ behaviour without any knowledge of the equilibrium, they cannot have complete information about such equilibrium; and hence the reduced form equation that may emerge from these parameters when estimated correctly, does not confer on the agent any information about the equilibrium price. It is vital for the learning process as studied in the literature that the structural equations contain information about the equilibrium price, and it is this possibility that is shut off when correctly modelling disequilibrium market behaviour if there is no prior knowledge of equilibrium.

The agents thus encounter the same difficulty as economists outside the new classical school consider that the market faces. At the cost of some repetition, the commonsense intuition of the argument can be expressed very simply. Until agents have an idea of the equilibrium price, their market behaviour is not based on that price, and thus whatever demand and supply rule they use does not contain complete information about equilibrium. Thus when the reduced form equation of such a system is estimated, it cannot provide that information either. The usual formulation of demand and supply behaviour based on notional demand and supply functions already incorporates the knowledge of the equilibrium that is being sought. Thus if agents can perform the necessary estimation, a faculty conferred on them by assumption, it is no wonder they generate the equilibrium values.

This argument may be slurred over in the analysis of a single market in partial equilibrium (as in the original Cobweb model, for example) on the excuse that the income effects will be small and the notional curves can be used as approximations to the market curves. But such approximations do not remain valid when the methodology is extended economywide as a tool for analysing macroeconomic theory and policy. Particularly in such a context we are back to the problem of stability faced by general equilibrium theory without tatonnement.
Conclusions

There are several types of problems associated with the Rational Learning process even when it is assumed that the driving process and the solutions are stationary and ergodic. Apart from the obvious questions of realism of allowing agents to possess knowledge of the underlying model and the techniques of estimation, there is the important question of transition from knowledge to action. In an atomistic market, even after an agent happens to estimate the equilibrium price correctly, she cannot practise it. For her individual dissent from the more mundane rules of thumb used by other agents leaves no imprint on the market price. She does not gain and has no incentive to act. Nor can she convince any other agent about her wisdom, since the market experiment will not vindicate her prophesy, unless all agents develop the wisdom simultaneously so as to act in unison. But if they all have knowledge about the relation between the price set by them and the market clearing price, there is no reason why they should anymore pursue the equilibrium price $p^*$. They should try to achieve a price that maximises the collective gain of the supply side. Competitive behaviour transforms into strategic behaviour with the acquisition of knowledge. But it seems that a critique of Rational Learning and Rational Expectation need not be based on the realism of the assumptions regarding agents’ ability to learn and to act. These questions appear to be somewhat secondary, since the very object of learning is not contained in the data used in the learning process. The econometrician will have the same difficulty as her less talented protegé, the agent, in estimating equilibrium values from data which do not contain information about them.

Thus if we have to comment on the nature of the analogy between the REH and learning and decision making process in the real world, we have to admit that the analogy does not do too well. The agents in the real world are handicapped by their lack of information, information that is supplied to agents on the REH path in the form of the parameters of the notional supply and demand functions. When we realise that it is precisely this information that is the object of Rational Learning, the elaborate models of Rational Learning appear vacuous. As for new classical macroeconomic theory, it seems to be as good as the old classical macroeconomic theory, if not on all fours, at least in carrying through the mistakes of the old.
References


