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## On modelling drying of porous materials: analytical solutions to coupled partial differential equations governing heat and moisture transfer

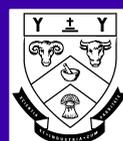
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# RESEARCH REPORT

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**On modelling drying of porous materials: analytical solutions to coupled partial differential equations governing heat and moisture transfer**

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**Summary**

Luikov theory of heat and mass transfer provides a framework to model drying of porous materials. The coupled partial differential equations governing the moisture and heat transfer can be solved using numerical techniques, and in this paper, we solve them analytically in a setting suitable for industrial drying situations. We discuss the nature of the solutions using the property values of pine wood. It is shown that the temperature gradients play a significant role in deciding the moisture profiles within the material when thickness is large, and the models based only on moisture potential gradients may not be sufficient to explain the drying phenomena in moist porous materials.

**Subject classification:** Applied mathematics, Engineering

**Keywords:** partial differential equations, heat and mass transfer, diffusion, modelling

## 1.0 Introduction

Porous materials such as wood, grains, fruits and dairy products have microscopic capillaries and pores which cause a mixture of transfer mechanisms to occur simultaneously when subjected to various processes involving heating and cooling. Transfer of non-condensable gases, vapours and liquids can occur in porous bodies; inert gases and vapour transfer can take place by molecular means in the form of diffusion and by molar means as a filtration motion of the steam-gas mixture under a pressure gradient. Transfer of liquid can occur by means of diffusion, capillary absorption and filtration motion in the porous medium arising from the hydrostatic pressure gradient. The complex interactions of various phenomena occurring within a material undergoing heating and cooling make modelling the transient moisture and temperature within the body a difficult task. The empirical models dealing with the drying of porous materials ignore the temperature variation within the material and formulate the models in terms of a measure of moisture content of the body and the equilibrium moisture content of the material ([1], [2], and [3]). Temperature variations are introduced to the models by relating the coefficients of the models to external temperature and humidity. These empirical relationships give satisfactory results in many industrial situations. In modelling drying, the most widely used mass transport model is the Fick's second law [4] and analytical solutions can be obtained for isotropic and anisotropic conditions ([5] and [4]). Similarly heat conduction equation can be solved analytically [4] and in many cases it is sufficient to solve the governing partial differential equations separately without paying attention to the coupling effects especially when the drying rates are small (see for example, [6]). However, one would expect these coupled heat and mass transfers in porous bodies

could be expressed mathematically and a mechanistic model can be developed. For this end, we make use of the theory developed by Luikov [7] to formulate a model of heat and mass transfer within the material.

Luikov showed the importance of the temperature gradient for moisture migration in capillary porous bodies [7]. He developed a system of coupled partial differential equations using the thermodynamics of irreversible processes. Fulford [8] surveyed work on the drying of solids by Luikov and his colleagues and no attempt will be made here to describe Luikov's rigorous theoretical development. Luikov assumed that both vapour and liquid diffusion are driven by both the total concentration gradient and the temperature gradient. He assumed that molecular and molar transfer of air, vapour and water occurred simultaneously within the porous body. Luikov stated that these coupled equations can not be solved and therefore has to be simplified [7]. When the coupling effects are important, one can solve the coupled equations using the numerical methods such as the finite element method for a given situation; and the use of numerical solutions has been the accepted practice among scientists and engineers. The objective of this paper is to present a model associated with Luikov theories on mass and heat transfer and solve them analytically to explore the behaviour of the model in relation to the coupling parameters and material properties. The analytical solutions for simplified boundary and initial conditions can be used to gain insight into drying behaviour of porous materials and also to develop empirical relationships in industrial situations as most empirical drying models have an underlying theoretical solutions.

## 2.0 Modelling heat and mass transfer

Consider a system consisting of a porous body and a bound substance, which can be in the form of liquid, vapour or inert-gas under positive temperature regimes but can be in the form of a solid (ice), sub-cooled liquid or vapour or a gas. Luikov developed a theory of mass and heat transfer for what he called capillary-porous bodies using the principles of irreversible thermodynamics [7]. In this paper, we consider cellular solids, materials consists of cells, are of this category, though there are significant differences between porous solids such as ceramics and cellular solids such as softwood. We modify the Lukov's equations for positive temperature regimes to a more applicable form to drying situations. Luikov and his co-workers showed that the thermal and moisture potential gradients within a capillary-porous body cause the vapour diffusion and transfer of liquid water. A porous body under above-freezing temperatures can be considered as a moist disperse system consisting of four different components: dry porous skeleton (solid), water vapour, liquid water, and air within capillaries and pores. Using the subscript  $i$  to denote the  $i$ th component ( $i = 0$  for bone-dry solid,  $i = 1$  for water vapour,  $i = 2$  for liquid water, and  $i = 3$  for air), and after neglecting the water vapour and air masses, the mass transfer of bound water can be modelled by the following conservation equation assuming that the moist material can be regarded as a continuum:

$$\frac{\partial(\rho_0 m)}{\partial t} = -\nabla \cdot (J_1 + J_2) \quad (1)$$

where  $m$  is the moisture content of the body (dry basis),  $\rho_0$  is the density of the bone-dry solid,  $J_i$  denotes the mass flux of component  $i$  and  $t$  is time. The heat transfer within the material can be modelled by the energy conservation equation:

$$c\rho_0 \frac{\partial T}{\partial t} = -\nabla \cdot q - (H_1 I_1 + H_2 I_2) \quad (2)$$

where  $c$  is the weighted specific heat of the moist solid referred to unit mass of the dry solid,  $T$  is the temperature of the dispersed system,  $q$  is the total heat flux,  $H_i$  is the enthalpy per unit mass and  $I_i$  denotes the mass formation or disappearance rate during the phase changes.

Solutions to (1) and (2) with appropriate boundary and initial conditions give the moisture and temperature profiles within the material. However, before solving (1) and (2) with appropriate boundary and initial conditions, the mass flux  $J_i$  and the heat flux  $q$  should be expressed in terms of driving forces (moisture potentials and temperature gradients). Using the thermodynamics of irreversible processes and experiments, Luikov [7] proved that existence of two driving forces for mass transfer: the moisture concentration gradient and the temperature gradient for each of the mass fluxes. This means, that the vapour diffusion and transfer of liquid water within the material can occur due to moisture concentration gradient and/or temperature gradient (thermo-diffusion effect). Hence,  $J_1$  and  $J_2$  in (1) can be replaced by

$$J_1 = -a_{m1}\rho_0 \nabla m - a_{m1}^T \rho_0 \nabla T \quad (3a)$$

$$J_2 = -a_{m2}\rho_0 \nabla m - a_{m2}^T \rho_0 \nabla T \quad (3b)$$

where  $a_{m1}$ ,  $a_{m2}$  are the effective diffusion coefficients for water vapour and liquid, respectively, and  $a_{m1}^T$  and  $a_{m2}^T$  are the corresponding thermal moisture diffusion coefficients. Substituting (3a) and (3b) into (1),

$$\frac{\partial m}{\partial t} = \nabla \cdot [a_m \nabla m + a_m \delta \nabla T] \quad (4)$$

where  $a_m$  is the total diffusion coefficient,  $a_m = a_{m1} + a_{m2}$ , and  $\delta$  is the thermo gradient coefficient,

$$\delta = (a_{m1}^T + a_{m2}^T) / (a_{m1} + a_{m2}) .$$

The thermo gradient coefficient is a measure of relative significance of the mass transfer due to the thermal gradient. The total heat flux,  $q$ , in (2) is replaced by

$$q = -k \nabla T \quad (5)$$

where  $k$  is the thermal conductivity of the moist material and the  $(H_1 I_1 + H_2 I_2)$  term is replaced by

$$H_1 I_1 + H_2 I_2 = -R \varepsilon \frac{\partial m}{\partial t} \quad (6)$$

where  $R$  is the specific enthalpy of phase change and  $\varepsilon$  is the phase change coefficient [8].

The phase change coefficient  $\varepsilon$  varies from 0 to 1 as the vapour diffusion increases relative to liquid transfer during drying. After substituting (5) and (6) into (2),

$$c \rho_0 \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + R \varepsilon \frac{\partial m}{\partial t} \quad (7)$$

Luikov compiled experimental values of  $a_m$ ,  $\delta$ ,  $\rho_0$ ,  $k$  and  $c$  for a large number of porous materials. For example, values of  $a_m$  for pine wood varies from  $1.0 \times 10^{-6}$  m<sup>2</sup>/h to  $6.2 \times 10^{-6}$  m<sup>2</sup>/h and  $\delta$  varies from  $0.6 \times 10^{-2}$  to  $2.0 \times 10^{-2}$  for the same temperature.

For all practical purposes, the heat and mass transfer in a porous body can be simplified in to (4) and (7). The system of equations given by (4) and (7) are coupled and non-linear partial differential equations, and behaviour of which can be investigated analytically.

Equations (4) and (7) can further be simplified by assuming constant values for the parameters  $a_m$ ,  $c$ ,  $\rho_0$ ,  $k$  and  $\delta$ , which can be justified as the relationship of these parameters with the temperature and moisture are not known for great many materials. Therefore, we seek to solve the following system of equations:

$$\frac{\partial m}{\partial t} = a_m \nabla^2 m + a_m \delta \nabla^2 T \quad (8)$$

$$c\rho_0 \frac{\partial T}{\partial t} = k\nabla^2 T + R\varepsilon \frac{\partial m}{\partial t} \quad (9)$$

Equations (8) and (9) have another complication: value of  $\varepsilon$  changes from 0 to 1 depending on the significance of liquid transfer relative to the vapour diffusion within the material, and this depends on the nature of the material and we can expect it to increase as the temperature increases. The effect of the phase change coefficient on the moisture and temperature regimes can be explored using the model given by (8) and (9).

### 3.0 Analytical Solutions

Equations (8) and (9) form a pair of coupled non-homogenous second-order partial differential equations (PDEs). The applicability of general methods for solving coupled PDEs is limited by properties of the region over which the problem is defined, as well as by

boundary and initial conditions. For simplicity, let us reduce the problem to a single spatial dimension and substitute

$$\alpha = k/\rho_0 c \quad \text{and}$$

$$\beta = R/\rho_0 c:$$

$$\frac{\partial}{\partial t} m(x,t) = a_m \frac{\partial^2}{\partial x^2} m(x,t) + a_m \delta \frac{\partial^2}{\partial x^2} T(x,t), \quad (10a)$$

$$\frac{\partial}{\partial t} T(x,t) = \alpha \frac{\partial^2}{\partial x^2} T(x,t) + \varepsilon \beta \frac{\partial}{\partial t} m(x,t), \quad 0 < x < l, \quad (10b)$$

with initial and boundary conditions defining the problem over the region  $[0,l]$  along the x axis:

$$m(x,0) = m_i(x) = M_i, \quad T(x,0) = T_i(x) = T_i, \quad (11)$$

$$m(0,t) = m(l,t) = M_\infty, \quad T(0,t) = T(l,t) = T_\infty. \quad (12)$$

One of the widely accepted approaches of solving systems of PDEs is applying integral transforms to reduce the problem to simple differential equations. We found that Laplace and Fourier transforms in time were applicable, but did not significantly simplify the problem. Transformations applied in the spatial domain would have done so, but boundary conditions ruled out both Fourier and Laplace transforms. Fourier transform did not prove viable because of the finite nature of the spatial domain, while Laplace transform could not be utilised due to the lack of information on the derivatives of the solutions at the boundaries. Efforts made using transforms in the spatial domain thus yielded results satisfying the PDEs, but inconsistent with the boundary and/or initial conditions.

The method finally used to obtain satisfactory results is based on solving the non-homogenous heat conduction problem by means of eigenvalues and eigenfunctions [9]. The steps for solving the partial differential equation

$$\frac{\partial F(x,t)}{\partial t} = \gamma \frac{\partial^2 F(x,t)}{\partial x^2} + D(x,t),$$

for  $F(x,t)$  with appropriate initial and boundary conditions is as follows ( $\gamma$  is a constant, and  $D(x,t)$  is the “disturbing” non-homogenous term):

1. solve the appropriate homogenous problem ( assuming  $D(x,t)=0$  ) to find the eigenvalues and eigenfunctions needed to construct the solution as an infinite sum;
2. express all functional terms in the non-homogenous problem in terms of the obtained eigen functions; and
3. solve the resulting equation of infinite sums for the coefficients needed to construct the solution  $F(x,t)$  by exploiting properties of eigenvalues and eigenfunctions.

To apply this method to the original problem of solving a coupled pair of equations, the coupling terms in each equation are treated as the non-homogenous ”disturbing” terms ( $D(x,t)$ s). It is assumed that the solutions for  $m(x,t)$  and  $T(x,t)$  in (10) can be expressed using infinite sums:

$$m(x,t) = C + \sum_{n=1}^{\infty} a_n(t) \Phi_n(x), \quad (13a)$$

$$T(x,t) = D + \sum_{n=1}^{\infty} b_n(t) \Psi_n(x) \quad (13b)$$

where  $\Phi_n(x)$  and  $\Psi_n(x)$  are eigenfunctions independent of time, coefficients  $a_n(t)$  and  $b_n(t)$  are functions of time only, and  $C$  and  $D$  are translating functions derived from the initial and

boundary conditions. In the special case of (11) and (12), where boundary conditions are symmetric (and constant), and the initial functions are constants,  $C$  and  $D$  are constants.

The appropriate homogenous problem to (10) is:

$$\frac{\partial}{\partial t} m_H(x, t) = a_m \frac{\partial^2}{\partial x^2} m_H(x, t), \quad (14a)$$

$$\frac{\partial}{\partial t} T_H(x, t) = \alpha \frac{\partial^2}{\partial x^2} T_H(x, t). \quad (14b)$$

Note that equations in (14) are no longer coupled. Also note that both have the same general form:

$$\frac{\partial}{\partial t} f(x, t) = \sigma \frac{\partial^2}{\partial x^2} f(x, t). \quad (15)$$

The assumption can be made that  $m_H(x, t)$  and  $T_H(x, t)$  have identical eigenvalues and eigenfunctions. Note that homogenous PDEs with the above form can be solved using the method of separation of variables. Doing so suggests that the eigenvalues and eigenfunctions for the corresponding boundary conditions (12) are:

$$\lambda_n = \frac{n^2 \pi^2}{l^2}, \quad \Psi_n(x) = \Phi_n(x) = \sin(\sqrt{\lambda} x) = \sin\left(\frac{n\pi}{l} x\right), \quad n = 1, 2, \dots, \infty. \quad (16)$$

From (13a) and (13b) we thus derive:

$$m_n(x, t) = a_n(t) \Phi_n(x), \quad (17a)$$

$$T_n(x, t) = b_n(t) \Phi_n(x). \quad (17b)$$

Because differential operators are linear,  $m_n(x, t)$  and  $T_n(x, t)$  must satisfy the equations in (10) for all  $n$ . Substituting (17) into (10):

$$\sum_n a_n'(t)\Phi_n(x) = a_m \sum_n a_n(t)\Phi_n''(x) + a_m \delta \sum_n b_n(t)\Phi_n''(x), \quad (18a)$$

$$\sum_n b_n'(t)\Phi_n(x) = \alpha \sum_n b_n(t)\Phi_n''(x) + \beta \varepsilon \sum_n a_n'(t)\Phi_n(x). \quad (18b)$$

Substituting the identity that is easy to verify,

$$\Phi_n''(x) = -\lambda_n \Phi_n(x), \quad (19)$$

and rearranging to one summation the equations become:

$$\sum_n [a_n'(t) + a_m \lambda_n a_n(t) + a_m \delta \lambda_n b_n(t)]\Phi_n(x) = 0, \quad (20a)$$

$$\sum_n [b_n'(t) + \alpha \lambda_n b_n(t) - \beta \varepsilon a_n'(t)]\Phi_n(x) = 0. \quad (20b)$$

The equations in (20) can only hold for all  $0 < x < l$  if the quantities in the square brackets are 0 for all  $n$  because the eigenfunctions form an orthogonal set. Thus to find  $a_n$  and  $b_n$ , we must solve the coupled pair of first order differential equations for each value of  $n$ :

$$a_n'(t) + a_m \lambda_n a_n(t) + a_m \delta \lambda_n b_n(t) = 0, \quad (21a)$$

$$b_n'(t) + \alpha \lambda_n b_n(t) - \beta \varepsilon a_n'(t) = 0. \quad (21b)$$

Initial values for  $a_n(t)$  and  $b_n(t)$  are derived from the original initial and boundary conditions (11) and (12) through a translation [9]:

$$a_n(0) = A_n = \frac{2}{l} \int_{x=0}^l (m_i(x) - M_\infty) \Phi_n(x) dx = \frac{-2(M_i - M_\infty)}{n\pi} [\cos(n\pi) - 1], \quad (22a)$$

$$b_n(0) = B_n = \frac{2}{l} \int_{x=0}^l (T_i(x) - T_\infty) \Phi_n(x) dx = \frac{-2(T_i - T_\infty)}{n\pi} [\cos(n\pi) - 1]. \quad (22b)$$

$A_n$  and  $B_n$  are actually the coefficients used to express the original initial functions as infinite linear combinations of  $\Phi_n(x)$ . Note that the cosine term is 1 if  $n$  is even and -1 if  $n$  is odd. This cancels out all harmonics of  $\Phi(x)$  non-symmetric on the interval  $0 < x < l$ . To solve (21) the Laplace transform in time ( $f(t) \rightarrow F(s)$ ) is applied; simplifying the differential equations to algebraic equations (omitting the indexes  $n$ ):

$$sA - A + a_m \lambda A + a_m \delta \lambda B = 0, \quad (23a)$$

$$sB - B + \alpha \lambda B - \beta \varepsilon (sA - A) = 0. \quad (23b)$$

Solving for  $A$  and  $B$  and applying the inverse Laplace transformation to the results we obtain the solutions for  $a_n(t)$  and  $b_n(t)$ :

$$a_n(t) = \frac{p_1 A_n + C_n}{p_1 - p_2} \exp(p_1 t) + \frac{p_2 A_n + C_n}{p_2 - p_1} \exp(p_2 t), \quad (25a)$$

$$C_n = -B_n a_m \lambda_n \delta + \alpha \lambda_n A_n + \beta \varepsilon A_n a_m \lambda_n \delta$$

$$b_n(t) = \frac{p_1 B_n + D_n}{p_1 - p_2} \exp(p_1 t) + \frac{p_2 B_n - D_n}{p_2 - p_1} \exp(p_2 t), \quad (25b)$$

$$D_n = B_n a_m \lambda_n - \beta A_n a_m \lambda_n \varepsilon$$

where,

$$p_1 = -\frac{1}{2} \left( \beta \varepsilon a_m \delta + \alpha + a_m - \sqrt{(\beta \varepsilon a_m \delta)^2 + 2\beta \varepsilon \delta a_m^2 + 2\beta \varepsilon \delta a_m \alpha + a_m^2 + \alpha^2 - 2a_m \alpha} \right),$$

and

$$p_2 = -\frac{1}{2} \left( \beta \varepsilon a_m \delta + \alpha + a_m + \sqrt{(\beta \varepsilon a_m \delta)^2 + 2\beta \varepsilon \delta a_m^2 + 2\beta \varepsilon \delta a_m \alpha + a_m^2 + \alpha^2 - 2a_m \alpha} \right).$$

(Note, that for positive constants poles  $p_1$  and  $p_2$  are always negative, and the singularity at  $p_1=p_2$  is bounded.) Finally, the solution for  $m(x,t)$  and  $T(x,t)$  consistent with the initial and boundary conditions can be expressed as (since (22) was a solution for the translated initial values, we compensate by defining  $C$  and  $D$  from (13) accordingly.):

$$m(x,t) = M_\infty + \sum_{n=1}^{\infty} a_n(t) \Phi_n(x), \quad (26a)$$

$$T(x,t) = T_\infty + \sum_{n=1}^{\infty} b_n(t) \Phi_n(x). \quad (26b)$$

The results have also been verified by substituting zeros for coupling parameters, and comparing results with those obtained from solving the non-coupled homogenous pair of equations ( $m_H(x,t)$ ,  $T_H(x,t)$ ).

#### 4.0 Exploration of Analytical Solutions

Equations (26) provide analytical solutions to the PDEs that describe simultaneous moisture and heat transfers within a porous body, and the solutions determine the temperature and moisture profiles. It is important to obtain insight into the behaviour of the solutions with respect to the coupling parameters ( $\delta$  and  $\varepsilon$ ) as well as material properties. It is useful to know the conditions under which the moisture and temperature coupling is important, for example. The reduced dimensionality of the problem makes it easy to understand the interactions among various factors without unduly complicating the study.

Let us assume that the following numerical values for the drying of an infinite “panel” of pine wood 0.10 m thick, with drying temperature and humidity held constant along both of its infinite faces:  $R = 2400 \text{ kJ/kg}$ ,  $a_m = 3.0 \times 10^{-6} \text{ m}^2/\text{h}$ ,  $\rho_0 = 500 \text{ kg/m}^3$ ,  $c = 1284 \text{ J/kg } ^\circ\text{C}$ ,  $k = 0.12 \text{ W/(m K)}$ ,  $T_i = 10 \text{ } ^\circ\text{C}$ ,  $T_\infty = 80 \text{ } ^\circ\text{C}$ ,  $M_i = 0.5$  (dry basis), and  $M_\infty = 0.12$  (dry basis). Figures 1 and 2 show the moisture and temperature profiles, respectively, by assuming the values  $\delta = 0.01 \text{ } ^\circ\text{C}^{-1}$ , and  $\varepsilon = 0.1$  for the coupling parameters. The moisture and temperature vs time at  $x = 0.05 \text{ m}$  are given in figures 1b and 2b, respectively. While the rise in temperature is sharp, the decrease in moisture is slow because of relatively low value of total diffusion coefficient,  $a_m$ , and the thermal diffusivity,  $\alpha$ , is relatively high. We can expect the coupling effects to be negligible when the temperature gradients are very small after initial hours of drying. Moisture is increased slightly at the beginning because of the moisture transfer driven by the temperature gradient.

{ figures 1a, 1b, 2a and 2b }

Figure 3 shows the variation of moisture content at  $x = 0.05 \text{ m}$ ,  $t = 200 \text{ hours}$  with  $\delta$  and  $\varepsilon$  for the same set of values given above. While the moisture is insensitive to the changes of  $\varepsilon$ , the thermo gradient coefficient,  $\delta$ , has a significant effect on moisture. For a given total diffusion coefficient,  $a_m$ ,  $\delta$  indicates the significance of moisture transfer due to temperature gradient. Therefore, within realistic limits,  $\delta$  can be expected to influence drying as seen in the transient moisture profiles at the same location as before (Figure 4). For pine wood, nature of thermo diffusion effect has an impact on the moisture content of the material during the first 300 hours.

{ figures 3 & 4 }

To illustrate how the thickness of the panel affect the moisture and temperature profiles, we have produced figures 5, 6 and 7 for a thickness ( $l$ ) of 0.5 m. Temperature at  $x=0.25$  (mid-point) takes almost 1500 hours to reach close to the outside temperature when  $l=0.5$  (figure 6b), whereas the mid-point temperature reaches the outside temperature within 70 hours when  $l=0.1$  (figure 2b). Moisture profile at  $x=0.25$  m is slow to change (figure 2) as compared to figure 1. The thermo gradient coefficient ( $\delta$ ) is significant for a longer period of time (figures 7 and 8), and therefore it can be argued that the drying models purely based on moisture potential gradients without taking dynamic changes in temperature would not give realistic results when the thickness is high.

{ figures 5, 6 & 7 }

It was found that the values of  $\alpha$  and  $a_m$  should be in realistic ranges for moisture profiles to have meaningful values. This is illustrated by figure 9 where moisture content at  $x=0.25$  m,  $t=500$  hours is plotted against  $\alpha$  and  $a_m$  for  $\varepsilon=0.1$  and  $\delta=0.1$ . The regimes of  $\alpha$  and  $\beta$  which would produce realistic moisture contents at  $x=0.25$  m,  $t=500$  hours are given in figure 10, and it is seen that low  $\beta$  (i.e., the density and/or specific heat of the material is high) has a significant effect on moisture profile at low values of  $\alpha$  and vice versa.

{ figures 8 & 9 }

## 5.0 Conclusion

A coupled partial differential equations that govern the drying of porous materials have been solved analytically, and these solutions were explored to show that thermo diffusion effects can not be ignored when drying of moist porous materials is concerned. These solutions enable us to experimentally determine the coupling parameters in a setting where both the moisture and heat transfers are important without resorting to a simplified set of equations that govern heat and mass transfer separately.

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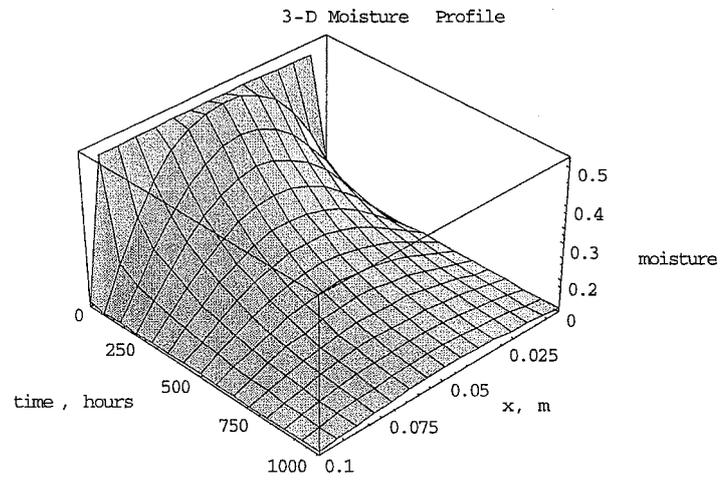


Figure 1a. Moisture content profile for  $l=0.1$  m.

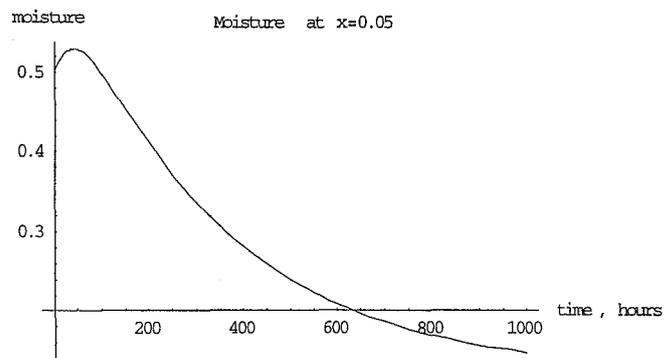


Figure 1b. Moisture content at  $x = 0.05$  m vs time for  $l = 0.1$  m.

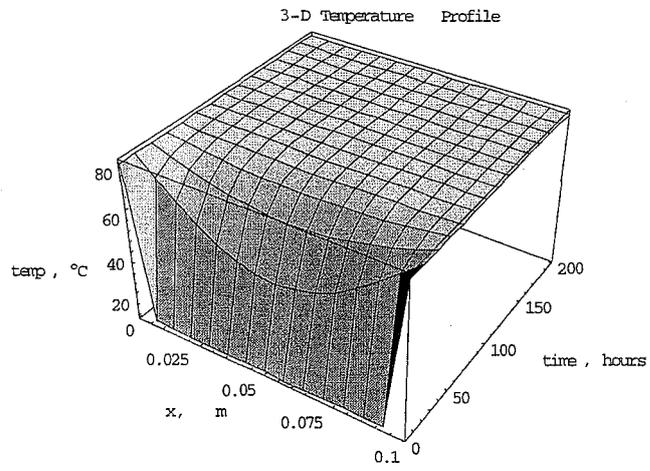


Figure 2a. Temperature profile for  $l = 0.1$  m.

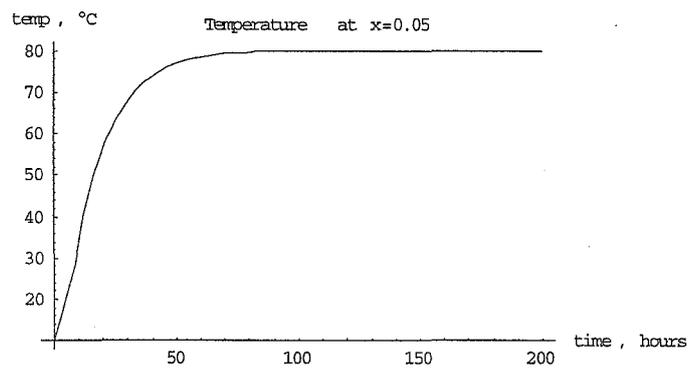


Figure 2b. Temperature at  $x=0.05$  m vs time for  $l=0.1$  m.

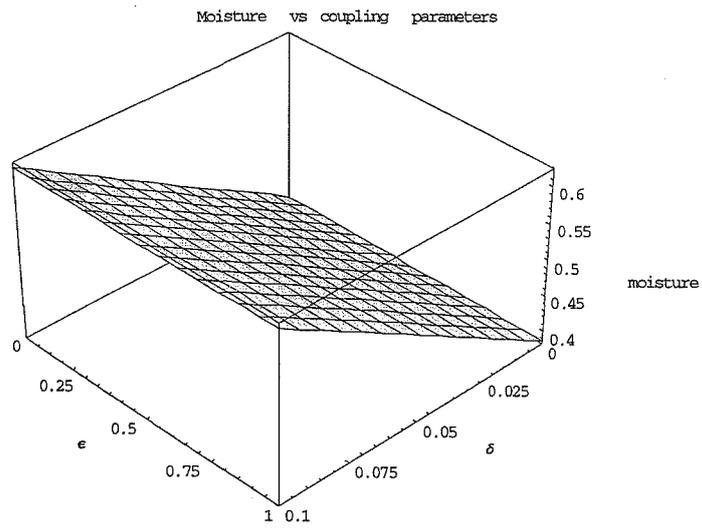


Figure 3. Moisture variation with thermo gradient coefficient ( $\delta$ ) and phase change coefficient ( $\epsilon$ ) for  $x = 0.05$  m,  $t = 200$  hours and  $l = 0.1$  m.

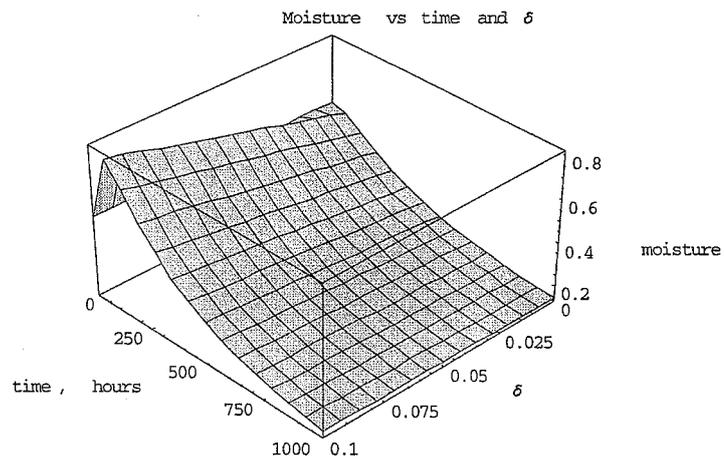


Figure 4. Transient moisture content vs thermo gradient coefficient for  $x=0.05$  m,  $l=0.1$  m and  $\epsilon = 0.1$ .

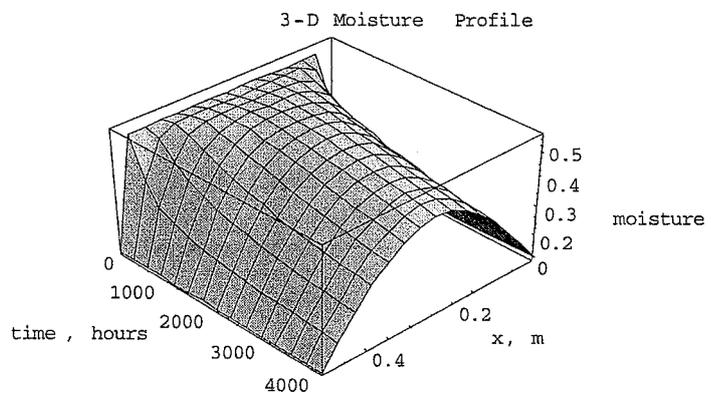


Figure 5a. Moisture content profile for  $l=0.5$  m.

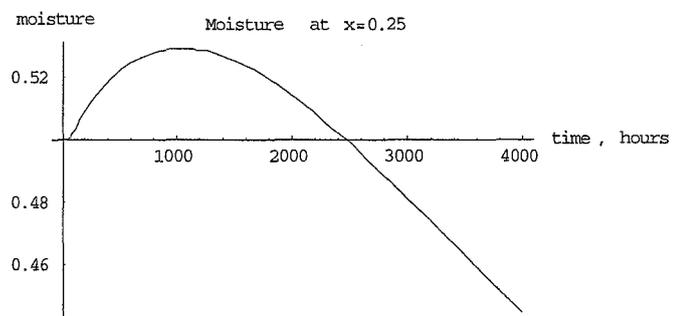


Figure 5b. Moisture content at  $x = 0.25$  m vs time for  $l = 0.5$  m.

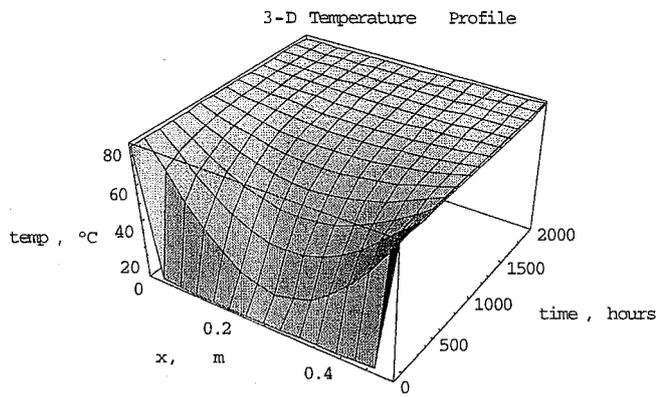


Figure 6a. Temperature profile for  $l=0.5$  m.

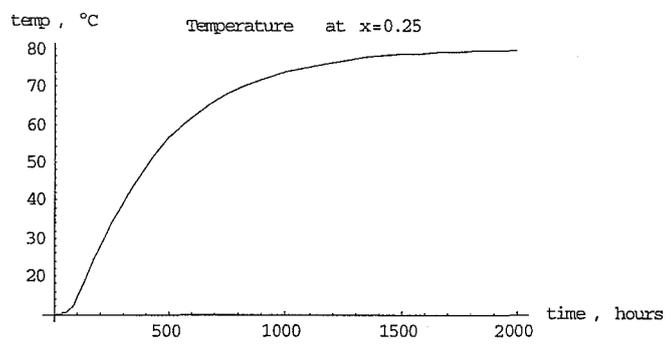


Figure 6b. Temperature at  $x=0.05$  m vs time for  $l=0.5$  m.

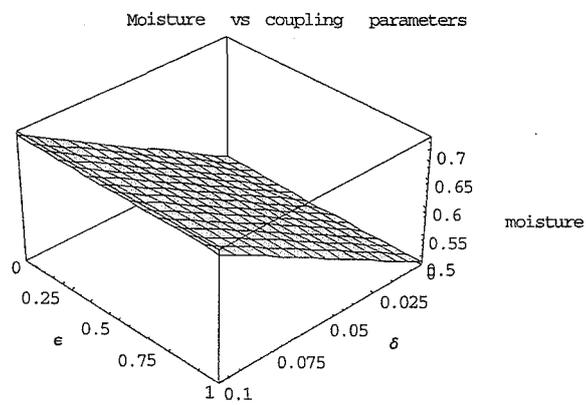


Figure 7. Moisture variation with thermo gradient coefficient ( $\delta$ ) and phase change coefficient ( $\epsilon$ ) for  $x=0.25$  m,  $t=500$  hours and  $l=0.5$  m.

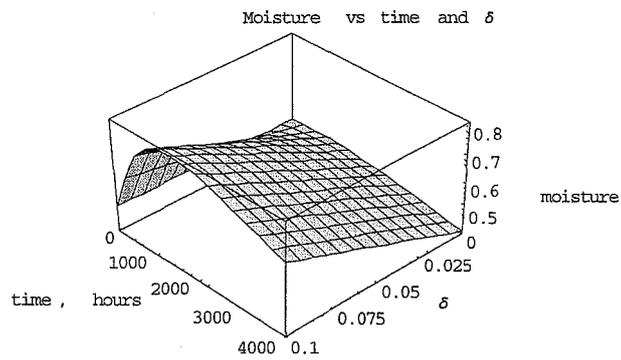


Figure 8. Transient moisture content vs thermo gradient coefficient for  $x=0.25$  m,  $l=0.5$  m and  $\varepsilon = 0.1$ .

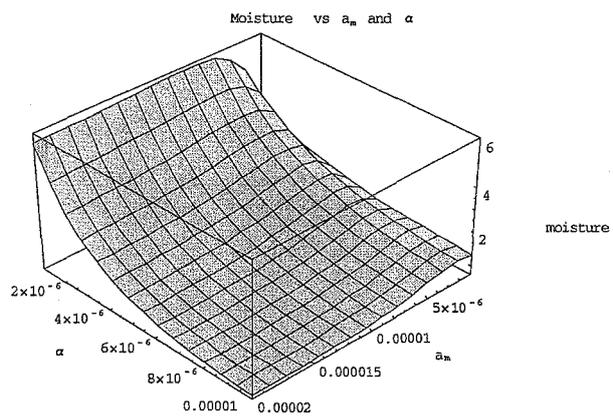


Figure 9. Moisture as a function of  $a_m$  and  $\alpha$  for  $x=0.25$  m,  $t=500$  hours and  $l=0.5$  m.

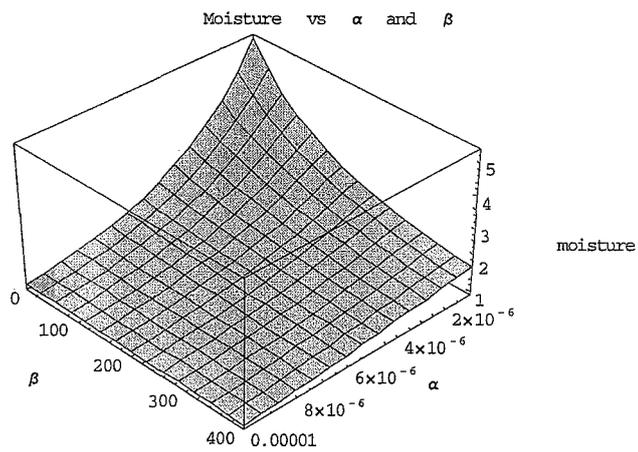


Figure 10. Moisture at  $x=0.25$  m,  $t=500$  hours as a function of  $\alpha$  and  $\beta$  for  $\varepsilon=0.1$ ,  $\delta=0.1$  and  $l=0.5$  m.