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**AN APPLICATION OF DEMAND THEORY IN
PROJECTING NEW ZEALAND RETAIL
CONSUMPTION**

by

R.H. Court

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IN PROJECTING NEW ZEALAND RETAIL CONSUMPTION

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R. H. COURT
Research Officer
Agricultural Economics Research Unit
Lincoln College
(University of Canterbury)

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P R E F A C E

In this paper Mr Court develops a new method for measuring demand relationships subject to the restrictions which can be derived from the theory of consumer behaviour. These methods are then applied, largely by way of example, in an analysis and projection of New Zealand retail consumption data.

The methods were developed for use in the Research Unit's work on demand analysis and projection of New Zealand exports in overseas markets; and also in connection with the long run interindustry projection model which the Unit is developing and in which of course New Zealand consumer demand projections are of salient importance.

This paper is the first in a new series of Technical Papers to be released by the Research Unit. As compared with the general publications which are prepared for a very wide distribution, the Technical Papers will be confined to a limited professional audience because of their specialised or technical nature, or again because they represent provisional and tentative results of work in progress. Comment and criticism is therefore invited and welcomed.

B. P. Philpott.

Lincoln College,
October 1966.

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1. INTRODUCTION

The increasing emphasis on economic planning in recent years has made it important that planners and policy-makers should know, or at least have some idea of, the likely future courses of leading economic variables.

This paper is a study of retail trading in New Zealand and its object is to explain, and produce projections of, domestic consumption of certain commodity groups at the retail level, and also for all groups as a whole. The individual commodity groups, chosen on grounds of general interest, data availability and computational feasibility, are called (1) meat, (2) other food, (3) apparel, (4) household operation and (5) miscellaneous. Point projections and tolerance limits in both "real" and current value terms are given for 1970 and 1975 for each individual group and for the aggregate of the groups. Some short-term forecasts are also given to show the possible usefulness of the projection method in this direction.

The general procedure used here, as elsewhere, is to explain consumption within the framework of a model whence, if the parameters of this model are stable and known, consumption can be determined at any time in the future under appropriate assumptions about determining variables. The model used here is the theory of consumer demand, based on

utility maximization, and its parameters are estimated by the method of generalized least squares, modified where necessary to produce parameters falling within the framework of the demand theory.

At the outset it is well to distinguish between a projection and a forecast. The view taken here is similar to that expressed in the introduction to (3), where it is emphasized that a projection is a reflection of assumptions made whereas a forecast is an unconditional statement about the future value of a variable. A projection is a conditional prediction, a forecast is an unconditional prediction. It is thus possible to construct exact tolerance limits of projection which are also conditional upon assumptions about values taken on by explanatory variables. With a forecast the explanatory variables must themselves be forecast, generally with an unknown degree of error.

2. DESCRIPTION OF THE PROJECTION METHOD

Consumer demand theory considers an individual maximizing some function $u(q)$ subject to a linear restraint $pq = \mu$, where q is a vector of quantities, p a corresponding price vector and μ is the available income of the consumer. This theory is well known, see (10) for example, and provides a set of equations relating quantities demanded to prices and income:

$$q = d(p, \mu)$$

which are the demand equations of the individual. Some properties of these demand equations which are also derived

as part of the theory are elucidated for the special case considered below.

If the functional form and parameters of d are known then the quantities q can be projected under any given assumptions about prices p and income μ . In practice the functional form must be assumed and the parameters estimated, which introduces some error into the projections.

This theory strictly applies only to an individual consumer, but for empirical purposes it is often extended to an aggregate of consumers and has sometimes (see (4) and (12)) been derived specifically with aggregate applications in view. This approach is also taken in the present study, although here the applicability of the theory to the data used is tested, rather than being assumed without further justification.

It is assumed here that the demand functions are of the linear logarithmic type, such that

$$\log q_i = \sum_{j=1}^n e_{ij} \log p_j + E_i \log \mu \quad i = 1, 2 \dots n$$

where q_i is the demand for the i th commodity, p_i its price, μ is income and e_{ij} and E_j the price and income elasticities. For convenience, units of measurement are assumed chosen so that the constant term vanishes. This particular functional form is chosen because it possesses sufficient parameters to provide a reasonable degree of generality, but few enough and in such a way as to be readily estimated without excessive demands upon degrees of freedom.

But some further relationships between the elasticities are also implied by the demand theory. We have the

well known homogeneity conditions $\sum_{j=1}^n e_{ij} + E_i = 0$ ($i=1, 2 \dots n$)

and the symmetry conditions $s_j e_{ij} + E_i = s_i e_{ji} + E_j$ ($i, j=1, 2 \dots n$) which should be imposed upon elasticity estimates if they are to provide an adequate representation of the basic demand theory. The s_j are defined as $\mu/p_j q_j$, or the reciprocal of the proportion of income spent on commodity j .

If the estimation problem can be formulated as that of estimating linear equations subject to linear restrictions on the coefficients, then convenient statistical methods are available. The above demand functions are linear in elasticities, as are the homogeneity conditions, but the symmetry conditions have as coefficients the s_j which are themselves functions of the elasticities by way of the definition of q_j from the assumed demand functions. However, the s_j generated in this way are in general not compatible with the restraint $p q = \mu$, hence can only be regarded as approximations to the "true" s_j . It is more convenient in this study to approximate the s_j by assuming them constant, whence estimating both the elasticities and the s_j themselves is considerably simplified.

The estimation problem can now be formally stated as that of estimating all coefficients in n linear equations, each with the same set of k independent variables, where there are p ($\leq nk$) linear restrictions upon the coefficients. A time series sample of T ($\geq k$) observations on each variable is assumed.

Denoting the vector of observations on the i th dependent variable by y_i , the $(T \times k)$ matrix of observations on the independent variables by X , the i th coefficient vector

by β_i and the vector of random errors in the i th equation by e_i , then all observations for the i th equation can be written

$$y_i = X\beta_i + e_i \quad i = 1, 2 \dots n$$

and all equations can be written together as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} X & 0 & \dots & 0 \\ 0 & X & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

$$\text{or } y = (I_n \otimes X)\beta + e$$

where y and e are $(nT \times 1)$ vectors, β is a $(nk \times 1)$ vector, I_n is the unit matrix of order n and \otimes denotes a Kronecker matrix product. The p homogeneous linear restrictions upon the coefficients are written $R\beta = 0$, where R is a $(p \times nk)$ matrix of known elements and 0 is the $(p \times 1)$ null vector.

Under appropriate assumptions about the independence of X and e_i , a suitable estimation method is generalized least squares. The optimal properties of this method under linear a priori restrictions are given in (13), pp.536-538. The e_i are assumed to have zero means and can be contemporaneously correlated with covariance matrix Ω , but are serially uncorrelated, whence the covariance matrix of e , given by $E(ee')$, is $(\Omega \otimes I_T)$. E is the expected value operator and I_T is the unit matrix of order T .

The required estimate of β is obtained from minimizing the "generalized sum of squares" $e'(\Omega^{-1} \otimes I_T)e$

subject to $R\beta = 0$, or using the Lagrange multiplier technique, by setting to zero the partial derivatives of

$$S = e'(\Omega^{-1} \otimes I_T)e + 2\lambda'R\beta$$

with respect to β and λ and solving the resulting equations.

λ is a $(p \times 1)$ vector of Lagrange multipliers.

Substituting $y - (I_n \otimes X)\beta$ for e in S , simplifying and obtaining $\frac{\partial S}{\partial \beta}$ and $\frac{\partial S}{\partial \lambda}$ as

$$\frac{\partial S}{\partial \beta} = -2(\Omega^{-1} \otimes X')y + 2(\Omega^{-1} \otimes X'X)\beta + 2R'\lambda$$

and $\frac{\partial S}{\partial \lambda} = 2R\beta$,

a vector b of estimates obeying $Rb = 0$ is obtained by solving the linear equations

$$\begin{bmatrix} \Omega^{-1} \otimes X'X & R' \\ R & 0 \end{bmatrix} \begin{bmatrix} b \\ \lambda \end{bmatrix} = \begin{bmatrix} (\Omega^{-1} \otimes X')y \\ 0 \end{bmatrix}$$

for b . 0 is the $(p \times p)$ null matrix.

If $(\Omega^{-1} \otimes X'X) = A$, say, then the solution for b is

$$b = \left[A^{-1} - A^{-1}R'(RA^{-1}R')^{-1}RA^{-1} \right] (\Omega^{-1} \otimes X')y$$

$$= C(\Omega^{-1} \otimes X')y \quad \text{say,}$$

where C is the matrix $A^{-1} - A^{-1}R'(RA^{-1}R')^{-1}RA^{-1}$. It is clear that $Rb = 0$, as required.

It can be shown, see (13) p.538, that the covariance matrix of b is the matrix C above, thus

$$E(b - \beta)(b' - \beta') = C$$

a result which is used to calculate standard errors for b , and is used later to obtain standard errors of projection.

But the above method requires a prior knowledge of Ω which is unlikely in practice. If we are prepared to assume normality for e , the likelihood of the sample is a monotonic transformation of

$$e'(\Omega^{-1} \otimes I_T)e - \log.\det(\Omega^{-1} \otimes I_T)$$

and maximization of this function subject to the restrictions provides the same equations as previously for b . But maximization with respect to the elements of Ω also provides an estimate $\hat{\Omega}$ such that

$$\hat{\Omega} = \frac{1}{T} \sum_{t=1}^T (\hat{e}_t \hat{e}_t')$$

where \hat{e}_t is a vector of calculated residuals from all equations in the t th observation period. Some iterative calculation is needed to produce b and $\hat{\Omega}$ consistent with one another, such that the b used to calculate $\hat{\Omega}$ is the same as the b resulting from using $\hat{\Omega}$ in the estimation equations for b above, whence b and $\hat{\Omega}$ are maximum likelihood estimates with the condition $Rb = 0$ imposed. The asymptotic covariance matrix of b is given by $\hat{A}^{-1} - \hat{A}^{-1}R'(R\hat{A}^{-1}R')^{-1}R\hat{A}^{-1}$, where $\hat{A} = (\hat{\Omega}^{-1} \otimes X'X)$.

The first step in the demand projections of this paper is to obtain estimates b of β in

$$y = (I_n \otimes X)\beta + e$$

such that $Rb = 0$, and it is shown above how this may be done.

A projection, in the sense used here, consists of predicting y when the values of the variables in X are known exactly. Thus let X_* be a $(1 \times k)$ vector of X values for the projection period whose effect on y is to be evaluated, and let \hat{y}_* be a $(n \times 1)$ vector of projections for y when X_*

prevails. The projections are then obtained from the equation

$$\hat{y}_* = (I_n \otimes X_*)b$$

or, replacing $(I_n \otimes X_*)$ by Z for convenience, by

$$\hat{y}_* = Zb$$

where Z is known and b has been estimated.

If y_* denotes the actual values of y when X_* prevails, this actual value is given by

$$y_* = Z\beta + e_*$$

where e_* is a vector of disturbances in the projection period assumed to have the same properties as the e in the observation periods. The error or projection is then

$$\hat{y}_* - y_* = Z(b - \beta) - e_*$$

and the covariance matrix of projection errors is (b and e_* being independent)

$$\begin{aligned} E(\hat{y}_* - y_*)(\hat{y}_*' - y_*') &= Z \left[E(b - \beta)(b' - \beta') \right] Z' + E(e_*e_*') \\ &= ZCZ' + \Omega \end{aligned}$$

which is estimated by $Z\hat{C}Z' + \hat{\Omega}$

Projection intervals cannot be obtained in the same sense as confidence intervals, since the problem involves estimating the value of a random variable rather than a fixed parameter. However we can define tolerance limits, between which at least δ per cent of nonsample values of the random variable can be expected to fall with probability γ . Values of these tolerance limits are tabulated for normal distributions in (2), chap. 2.

It has been mentioned earlier that restrictions derived from demand theory should be tested to see if utility maximization is a hypothesis capable of explaining the observed demand data. A test for linear restrictions of this nature is indicated in (1), chapter 8. From $y = (I_n \otimes X)\beta + e$ we can obtain both unrestricted and restricted estimates of Ω by minimizing

$$e'(\Omega^{-1} \otimes I_T)e - \log.\det(\Omega^{-1} \otimes I_T)$$

both unconditionally and subject to the restrictions $R\beta = 0$, to obtain $\hat{\Omega}_0$ (unrestricted) and $\hat{\Omega}_R$ (restricted). The determinantal ratio $\frac{\det \hat{\Omega}_R}{\det \hat{\Omega}_0}$ then provides a test criterion

for the null hypothesis $R\beta = 0$. Unfortunately the small sample distribution of this ratio is not known for the case considered, but an asymptotic test is obtained from the result that $T \log_e \frac{\det \hat{\Omega}_R}{\det \hat{\Omega}_0}$ is distributed asymptotically as χ^2

with p degrees of freedom. T is the sample size and p the number of restrictions to be tested.

3. SOURCES OF INFORMATION

In choosing the degree of breakdown of commodity groups for projection purposes, points to consider are the purpose for which projections are required, the data available and the computational cost of producing the projections.

The data for the present study are derived from figures published by the New Zealand Statistics Department in (8).

Quarterly value figures of sales from a sample of retail establishments are given for items classified by both store-type and commodity-type. The commodity classification is conceptually the more satisfactory for projection purposes, but published series begin in 1959, whereas the store-type classification is published from the March quarter, 1954, onwards. The latest figures available at the time of writing refer to the December quarter, 1965, thus the store-type sales figures provide 48 quarterly observations for estimation purposes.

Price data are obtained from published components of the New Zealand consumers' price index (8). There is not much of a tie-up between these prices and the retail sales figures, hence some reconciliation is needed before proceeding. Five groups is the maximum number that can be handled with the available computational facilities, hence five groups are sorted out for which projections are of interest and for which price and sales data show a reasonable correspondence.

The groups chosen are

1. Meat
2. Other food
3. Apparel
4. Household operation
5. Miscellaneous

and together these include the value of all retail sales. These groups are related to the published value of sales and price data as follows:

Value of store-type sales.

- | | |
|-------------------------|--|
| 1. Meat: | sales of "butcher, poulterer etc." type stores. |
| 2. Other food: | "grocer" and "other food and drink" |
| 3. Apparel: | "footwear" and "other apparel" |
| 4. Household operation: | "furniture and soft furnishings", household appliances. |
| 5. Miscellaneous: | "chemist", "general department and variety" and "other". |

Prices

- | | |
|-------------------------|--|
| 1. Meat: | "meat and fish" component of consumers' index. |
| 2. Other food: | "fruits, vegetables and eggs" aggregated with "other foods" using weights 0.33 and 0.67 respectively. |
| 3. Apparel: | "apparel" component of consumers' index. |
| 4. Household operation: | "home furnishing" aggregated with "domestic supplies and services" using weights 0.64 and 0.36 respectively. |
| 5. Miscellaneous: | "other supplies" component. |

The aggregation weights are proportional to the official weighting of these components in the overall index.

Volume of retail sales in each group (in constant prices, base 1955=1000) is obtained by dividing each value series by the appropriate price series.

Other variables used in the demand equations besides the above prices and volumes (or "quantities") are income, population and a price index of all goods in the consumers' budget not included above.

The income figure used is private disposable income, for which annual figures are published in New Zealand Official Yearbooks. These annual figures were interpolated to obtain estimates of quarterly income.

Estimated population figures at the end of each quarter for New Zealand are published in (9) and are averaged to give average quarterly population.

The price index of all other goods, or "other prices" consists of all components of the consumers' index, other than those of the five groups above, aggregated according to the official weights.

To impose linearized symmetry restrictions upon estimated elasticities, it is necessary to know the s_j , or the reciprocal of the proportion of income spent on each commodity group. It is not possible to estimate the budget proportions directly because the retail trading sample is an unknown proportion of total consumption expenditure. The values of s_j used were estimated from the official expenditure weights used to construct the consumers' price index, together with the knowledge that this index covers about 85% of total consumption expenditure in New Zealand. In the case of "miscellaneous" some intuitive judgment also had to be used. The values of the s_j actually used in the demand estimation were

1. Meat	$s_1 = 15$
2. Other food	$s_2 = 5$
3. Apparel	$s_3 = 8$
4. Household operation	$s_4 = 12$
5. Miscellaneous	$s_5 = 5$

implying that about $1/15$ of total consumption expenditure goes on meat, $1/5$ on other foods, etc.

In addition to projecting individual retail groups, total retail trading figures are also projected. The value of all retail sales in the sample is given by the sum of the above five groups, price is obtained by aggregating the five individual prices using official weights, and volume of sales comes from dividing total value by price. The income, population and "other price" variables are the same as before.

The aim of the study is to project retail trading figures to 1970 and 1975 under specified assumptions about levels of prices, income and population prevailing in these years. Government has given no indication of possible targets for these years regarding price, income or migration policy, so the best we can do is assume that things will continue on much as they have in the past and produce a set of illustrative projections that can be modified wherever necessary.

Between now and 1975 therefore, prices are assumed to rise by about 2.5% per annum, and real disposable income by about 5% per annum (rather optimistically, judging by past experience) whence disposable income in current prices will rise by about 7.5% per annum. The Government Statistician's population projections (8), assuming 15,000 per year net immigration and that average 1965 specific age-of-mother and marital status birth-rates will continue, are accepted.

Numerically, estimated disposable income for the December quarter, 1965, is £416 million, whence 7.5% annual growth will result in £597 million and £857 million for the December quarters, 1970 and 1975 respectively.

Individual prices for the December quarters 1970 and 1975, obtained by extrapolating from linear trends fitted for the years 1960 to 1965 are, together with assumed income and population, as follows:-

	December quarter	
	<u>1970</u>	<u>1975</u>
Price meat	1724	2035
Price other food	1250	1347
Price apparel	1242	1329
Price household operation	1284	1381
Price miscellaneous	1490	1677
Price "other items"	1680	1896
Disposable income	£597 mill.	£857 mill.
Population	2.973 mill.	3.320 mill.

All prices are indexes with base 1955=1000.

It is emphasized that the above figures are assumptions, not predictions.

To project aggregated retail demand, the first five prices above are aggregated as before, using official weights, to obtain assumed price indexes for total retail sales in the December quarters, 1970 and 1975, as 1371 and 1515 respectively. The price of "other items", disposable income and population are as above.

4. ESTIMATION AND PROJECTION

The methods of this study require parameter estimation as a logically prior step to obtaining projections. Demand equations in linear logarithmic form, explaining volume of consumption per head in terms of deflated prices, real disposable income per head and dummy seasonal variables,

are estimated below using both least squares and the restricted maximum likelihood procedure. The least squares results are given as they are of intrinsic interest as well as being used to test restrictions and provide a first approximation to the maximum likelihood estimates.

All price and income coefficients are elasticities and obey the homogeneity conditions of demand theory as all prices and income have been deflated by "other prices" before estimation. Only the second set obey the symmetry conditions since the following ten restrictions have been imposed on this set but not on the first,

$$\begin{aligned}
 5e_{12} + E_1 - 15e_{21} - E_2 &= 0 \\
 8e_{13} + E_1 - 15e_{31} - E_3 &= 0 \\
 12e_{14} + E_1 - 15e_{41} - E_4 &= 0 \\
 5e_{15} + E_1 - 15e_{51} - E_5 &= 0 \\
 8e_{23} + E_2 - 5e_{32} - E_3 &= 0 \\
 12e_{24} + E_2 - 5e_{42} - E_4 &= 0 \\
 5e_{25} + E_2 - 5e_{52} - E_5 &= 0 \\
 12e_{34} + E_3 - 8e_{43} - E_4 &= 0 \\
 5e_{35} + E_3 - 8e_{53} - E_5 &= 0 \\
 5e_{45} + E_4 - 12e_{54} - E_5 &= 0
 \end{aligned}$$

where the e_{ij} are price elasticities and the E_i income elasticities. 15, 5, 8, 12 and 5 are the estimated reciprocals of budget proportions.

LEAST SQUARES

		P ₁	P ₂	P ₃	P ₄	P ₅	Income	S ₁	S ₂	S ₃	Const.	R ²	d
Meat	Q ₁	-.719 (.094)	.133 (.151)	.739 (.216)	-.122 (.360)	-.058 (.544)	.086 (.135)	-.083 (.009)	-.029 (.009)	-.010 (.008)	.8697	.916	1.56
Oth. food	Q ₂	-.288 (.091)	-.999 (.146)	.516 (.209)	-.171 (.349)	.272 (.527)	.377 (.131)	-.097 (.009)	-.104 (.009)	-.085 (.008)	.9721	.956	.71
Apparel	Q ₃	.295 (.119)	-.122 (.191)	.102 (.273)	-.317 (.456)	.342 (.688)	.142 (.171)	-.244 (.011)	-.044 (.011)	-.196 (.011)	1.4477	.958	1.07
House Op.	Q ₄	.255 (.171)	-.519 (.274)	1.152 (.392)	-3.209 (.654)	3.219 (.988)	.186 (.245)	-.195 (.016)	-.127 (.016)	-.116 (.015)	1.3785	.919	1.23
Misc.	Q ₅	.058 (.132)	.047 (.212)	.665 (.303)	-.109 (.506)	-1.350 (.764)	.666 (.190)	-.178 (.012)	-.150 (.012)	-.172 (.012)	.1833	.922	1.20

MAXIMUM LIKELIHOOD (RESTRICTED)

		P ₁	P ₂	P ₃	P ₄	P ₅	Income	S ₁	S ₂	S ₃	Const.	R ²	d
Meat	Q ₁	-.721 (.080)	-.115 (.088)	.695 (.093)	.031 (.128)	.232 (.168)	.034 (.064)	-.088 (.009)	-.032 (.009)	-.009 (.009)	1.1041	.906	1.40
Oth. food	Q ₂	-.057 (.031)	-.867 (.070)	.095 (.050)	-.032 (.080)	.359 (.109)	.306 (.057)	-.092 (.009)	-.097 (.009)	-.082 (.009)	1.2721	.941	.49
Apparel	Q ₃	.359 (.051)	.171 (.076)	-.590 (.126)	.264 (.143)	-.015 (.178)	.216 (.077)	-.242 (.011)	-.040 (.011)	-.195 (.011)	1.1033	.949	.97
House Op.	Q ₄	-.010 (.105)	-.126 (.182)	.353 (.222)	-1.898 (.402)	1.929 (.492)	.555 (.149)	-.199 (.016)	-.129 (.016)	-.117 (.016)	-.2720	.903	.85
Misc.	Q ₅	.025 (.054)	.258 (.094)	-.083 (.107)	.783 (.190)	-1.884 (.318)	.810 (.100)	-.179 (.012)	-.149 (.012)	-.171 (.012)	-.4726	.909	1.13

Estimated standard errors are in brackets, d is the calculated Durbin-Watson statistic for each equation and R_i^2 is a statistic calculated according to the formula $R_i^2 = 1 - \frac{\hat{e}_i' \hat{e}_i}{Y_i' Y_i}$, for the

i th equation. This R_i^2 is not the square of a multiple correlation coefficient in the case of the restricted equations and is given here for whatever it is worth. In such cases it is constrained only to the closed interval minus infinity to plus one.

The seasonal variables S_1 , S_2 and S_3 take on values of one in the March, June and September quarters respectively and zero at other times. Seasonal variation in the December quarter is taken up in the constant term in each equation.

From 48 observations on each variable, 38 degrees of freedom were used to calculate the residual covariance matrix (and hence the standard errors) for the unrestricted or least squares equations. Imposing ten restrictions upon the coefficients in five equations is assumed to increase the degrees of freedom by two per equation, whence 40 degrees of freedom were used to calculate the residual covariance matrix and standard errors for the restricted estimates.

To test the linearized symmetry restrictions, the unrestricted estimate of Ω is

$$\hat{\Omega}_0 = \frac{10^{-2}}{38} \begin{bmatrix} 1.51 & .56 & .61 & 1.05 & 1.13 \\ .56 & 1.42 & 1.41 & .96 & 1.29 \\ .61 & 1.41 & 2.42 & 1.15 & 1.86 \\ 1.05 & .96 & 1.15 & 4.99 & 1.89 \\ 1.13 & 1.29 & 1.86 & 1.89 & 2.98 \end{bmatrix}$$

with determinantal value $\det \hat{\Omega}_0 = 1.041 \times 10^{-17}$. The

maximum likelihood estimate of Ω with restrictions imposed is

$$\hat{\Omega}_R = \frac{10^{-2}}{40} \begin{bmatrix} 1.69 & .55 & .56 & 1.02 & 1.17 \\ .55 & 1.92 & 1.80 & .92 & 1.55 \\ .56 & 1.80 & 2.91 & 1.52 & 2.30 \\ 1.02 & .92 & 1.52 & 5.92 & 2.41 \\ 1.17 & 1.55 & 2.30 & 2.41 & 3.47 \end{bmatrix}$$

with determinantal value $\det \hat{\Omega}_R = 1.934 \times 10^{-17}$.

Choosing $T = 38$ as the effective sample size, the asymptotic test criterion $T \log_e \frac{\det \hat{\Omega}_R}{\det \hat{\Omega}_0}$ becomes

$38 \log_e \frac{1.934}{1.041}$ or 23.5. This is very close to the critical value of 23.2 for the χ^2 variate with 10 degrees of freedom at the 1% level, so, although doubt is cast on the restrictions, they should not be definitely rejected, especially as the test is asymptotic.

Demand theory, in addition to requiring the substitution matrix of terms such as $s_j e_{ij} + E_i = K_{ij}$, say, to be symmetric, also requires this matrix to be negative definite¹ (10).

The matrix of K_{ij} calculated from the least squares estimates need not be symmetric, and has elements

$$\begin{bmatrix} -10.7 & .8 & 6.0 & -1.4 & -.2 \\ -3.9 & -4.6 & 4.5 & -1.7 & 1.7 \\ 4.6 & -.5 & 1.0 & -3.7 & 1.9 \\ 4.0 & -2.4 & 9.4 & -38.3 & 16.3 \\ 1.5 & .9 & 6.0 & -.6 & -6.1 \end{bmatrix}$$

¹ Negative semi-definite if all items in the consumers' budget are included, instead of the five considered here.

To determine definiteness or otherwise, it is sufficient to consider the characteristic roots of the symmetric matrix with elements defined by $(K_{ij} + K_{ji})/2$. The roots of this matrix are -13.4, -4.0, 5.5, -40.4 and -6.4. With one positive root it cannot be negative definite.

The corresponding matrix calculated from the restricted elasticities is required to be symmetric and has elements

$$\begin{bmatrix} -10.8 & -.5 & 5.6 & .4 & 1.2 \\ -.5 & -4.0 & 1.1 & -.1 & 2.1 \\ 5.6 & 1.1 & -4.5 & 3.4 & .1 \\ .4 & -.1 & 3.4 & -22.2 & 10.2 \\ 1.2 & 2.1 & .1 & 10.2 & -8.6 \end{bmatrix}$$

The characteristic roots are -14.0, -3.2, .7, -28.1 and -5.5, so, with one positive root, this matrix is not negative definite either. A rigorous test is outside the scope of this paper, but this positive root is small enough to indicate that negative definiteness of the matrix is within the bounds of possibility.

Thus, although it seems unlikely that a utility maximization theory explains the observed demand data, it is still possible that this is so.

It has been shown by Theil (13), pp.331-334, that imposing even incorrect restrictions may provide better estimates than no restrictions. The important point about the restrictions of this paper is not whether they are correct, but whether they reduce standard errors of projections, and the fact that they do so in the empirical analysis of this paper is considered sufficient to justify their use.

The formula for the estimated covariance matrix of projection errors, derived previously, is

$$Z\hat{C}Z' + \hat{\Omega}$$

$$\text{where } Z = (I_n \otimes X_*)$$

For least squares estimates we have

$$\hat{\Omega} = \hat{\Omega}_0$$

$$\text{and } \hat{C} = \hat{C}_0 = \hat{\Omega}_0 \otimes (X'X)^{-1}$$

and for restricted maximum likelihood estimates

$$\hat{\Omega} = \hat{\Omega}_R$$

$$\text{and } \hat{C} = \hat{C}_R = \hat{A}^{-1} - \hat{A}^{-1}R'(R\hat{A}^{-1}R')^{-1}R\hat{A}^{-1}$$

$$\text{where } \hat{A}^{-1} = \hat{\Omega}_R \otimes (X'X)^{-1}$$

Standard errors of projection can thus be calculated for either case using the assumed vector X_* .

The size of the elements in the projection error covariance matrix is obviously influenced by two factors, one arising from the precision of estimating the vector β and the other from the size of the variance-covariance of residuals. For plausible restrictions (even if rejected by tests) the decrease in the first factor may be sufficient to outweigh the increase in the second factor, resulting in lower standard errors and (if this is the criterion) improved projections.

It is possible to calculate projection regions, see (5), but this study is only interested in standard errors whence only the diagonal elements of the error matrices are relevant. These diagonal elements are given below for all five items using both the no restriction and restriction estimation methods

and the X_* vectors assumed for the December quarters, 1970 and 1975.

(1) 1970, no restrictions:

$$\begin{array}{rcc}
 \hat{z}\hat{C}_0z' & + & \hat{\Omega}_0 \\
 \frac{1}{38} \begin{bmatrix} .0240 \\ .0226 \\ .0385 \\ .0794 \\ .0475 \end{bmatrix} & + & \frac{1}{38} \begin{bmatrix} .0151 \\ .0142 \\ .0242 \\ .0499 \\ .0298 \end{bmatrix} & = & 10^{-4} \begin{bmatrix} 10.29 \\ 9.69 \\ 16.50 \\ 34.01 \\ 20.35 \end{bmatrix} & \begin{bmatrix} .0321 \\ .0311 \\ .0406 \\ .0583 \\ .0451 \end{bmatrix} \\
 \text{Variance} & & \text{S.E.} & & &
 \end{array}$$

(2) 1970, restricted:

$$\begin{array}{rcc}
 \hat{z}\hat{C}_Rz' & + & \hat{\Omega}_R \\
 \frac{1}{40} \begin{bmatrix} .0159 \\ .0169 \\ .0268 \\ .0612 \\ .0340 \end{bmatrix} & + & \frac{1}{40} \begin{bmatrix} .0169 \\ .0192 \\ .0291 \\ .0592 \\ .0347 \end{bmatrix} & = & 10^{-4} \begin{bmatrix} 8.20 \\ 9.02 \\ 13.98 \\ 30.08 \\ 17.17 \end{bmatrix} & \begin{bmatrix} .0286 \\ .0300 \\ .0374 \\ .0548 \\ .0414 \end{bmatrix} \\
 \text{Variance} & & \text{S.E.} & & &
 \end{array}$$

(3) 1975, no restrictions:

$$\begin{array}{rcc}
 \hat{z}\hat{C}_0z' & + & \hat{\Omega}_0 \\
 \frac{1}{38} \begin{bmatrix} .0378 \\ .0355 \\ .0606 \\ .1248 \\ .0747 \end{bmatrix} & + & \frac{1}{38} \begin{bmatrix} .0151 \\ .0142 \\ .0242 \\ .0499 \\ .0298 \end{bmatrix} & = & 10^{-4} \begin{bmatrix} 13.91 \\ 13.09 \\ 22.30 \\ 45.97 \\ 27.52 \end{bmatrix} & \begin{bmatrix} .0373 \\ .0362 \\ .0472 \\ .0678 \\ .0525 \end{bmatrix} \\
 \text{Variance} & & \text{S.E.} & & &
 \end{array}$$

(4) 1975, restricted:

$$\frac{1}{40} \begin{bmatrix} .0284 \\ .0298 \\ .0471 \\ .1046 \\ .0585 \end{bmatrix} + \frac{1}{40} \begin{bmatrix} .0169 \\ .0192 \\ .0291 \\ .0592 \\ .0347 \end{bmatrix} = 10^{-4} \begin{bmatrix} 11.31 \\ 12.25 \\ 19.06 \\ 40.94 \\ 23.30 \end{bmatrix} \quad \begin{matrix} \text{Variance} \\ \text{S.E.} \end{matrix} \begin{bmatrix} .0336 \\ .0350 \\ .0437 \\ .0640 \\ .0483 \end{bmatrix}$$

In all cases it is evident that the decrease in \widehat{ZCZ}' as a result of applying restrictions is more than sufficient to offset the increase in $\widehat{\Omega}$, thus giving smaller standard errors (even without the degrees of freedom correction).

The point projections (for December quarters) calculated from the estimated equations using appropriate X_* vectors are also listed. The corresponding standard errors are in brackets.

	Meat	Other Food	Apparel	House op.	Miscell.
1970 Unrestr.	1.0397 (.0321)	2.9210 (.0311)	2.1841 (.0406)	2.5564 (.0583)	3.3472 (.0451)
1970 Restr.	1.0360 (.0286)	2.9276 (.0300)	2.2044 (.0374)	2.5920 (.0548)	3.3673 (.0414)
1975 Unrestr.	.9791 (.0373)	2.9831 (.0362)	2.2301 (.0472)	2.7000 (.0678)	3.4081 (.0525)
1975 Restr.	.9741 (.0336)	3.0002 (.0350)	2.2594 (.0437)	2.7366 (.0640)	3.4340 (.0483)

The restricted results are considered to be the better since standard errors are smaller, so further details are given for the projections under restriction only.

To obtain projection intervals, or tolerance limits within which at least 90% of nonsample observations should fall with probability 0.9, we will assume normality and find from the tables in (2), chap. 2, that (for 40 degrees of freedom) the limits are 1.959 standard errors on either side of the point projections. Thus for the projections under restriction

	Meat	Other Food	Apparel	House op.	Miscell.
1970					
Point	1.0360	2.9276	2.2044	2.5920	3.3673
Lower limit	.9800	2.8688	2.1311	2.4846	3.2862
Upper limit	1.0920	2.9864	2.2777	2.6994	3.4484
1975					
Point	.9741	3.0002	2.2594	2.7366	3.4340
Lower limit	.9083	2.9316	2.1738	2.6112	3.3394
Upper limit	1.0399	3.0688	2.3450	2.8620	3.5286

These figures are in natural logarithms of volume of consumption per head and for practical purposes must be transformed to more useful units. Using assumed population and prices in 1970 and 1975 we can readily transform the above figures to volume of consumption (in 1955=1000 constant prices) and total expenditure on consumption (1970 and 1975 prices) of each item to obtain, in units £ million, *the following table.*

The expenditure figures are calculated from assumed price multiplied by projected volume. Observed 1965 figures are given for comparison.

<u>Volume</u>	<u>Meat</u>	<u>Other food</u>	<u>Apparel</u>	<u>House op.</u>	<u>Miscell.</u>
1970 Point	8.4	55.5	27.0	39.7	86.2
Lower	7.9	52.4	25.0	35.7	79.5
Upper	8.9	58.9	29.0	44.2	93.5
1975 Point	8.8	66.7	31.8	51.2	102.9
Lower	8.2	62.3	29.2	45.2	93.6
Upper	9.4	71.4	34.6	58.1	113.2
1965 Observed	7.9	44.3	22.1	27.6	66.1
<u>Expenditure</u>					
1970 Point	14.5	69.4	33.5	51.0	128.4
Upper	13.6	65.5	31.1	45.8	118.5
Lower	15.3	73.6	36.0	56.8	139.3
1975 Point	17.9	89.8	42.3	70.7	172.6
Upper	16.7	83.9	38.8	62.4	157.0
Lower	19.1	96.2	46.0	80.2	189.8
1965 Observed	11.8	51.4	25.7	33.5	87.6

It is perhaps of more interest to calculate projections for the entire years 1970 and 1975 rather than just for the December quarters, as above. Assuming that average prices, income and population are the same in each quarter as in the December quarter, the only corrections required for other quarters are seasonal. Projected volumes of consumption of each item in all quarters of 1970 and 1975 in 1955=1000 constant prices and without tolerance limits are

	Qtr.ended	Meat	Other food	Apparel	House op.	Miscell.
1970						
(Proj.)	March	7.7	50.6	21.2	32.5	72.1
	June	8.1	50.4	25.9	34.9	74.3
	Sept.	8.3	51.1	22.2	35.3	72.6
	Dec.	8.4	55.5	27.0	39.7	86.2
	Total	32.5	207.6	96.3	142.4	305.2
1975	Qtr.ended					
(Proj.)	March	8.1	60.8	25.0	42.0	86.0
	June	8.5	60.5	30.6	45.0	88.7
	Sept.	8.7	61.4	26.2	45.5	86.7
	Dec.	8.8	66.7	31.8	51.2	102.9
	Total	34.1	249.4	113.6	183.7	364.3
1965	Qtr.ended					
(Observed)	March	7.1	38.2	16.9	22.1	53.7
	June	7.5	39.2	20.7	24.0	52.7
	Sept.	7.6	40.3	18.1	24.2	52.5
	Dec.	7.9	44.3	22.1	27.6	66.1
	Total	30.1	162.0	77.8	97.9	225.0

Projected retail consumption expenditure figures in 1970 and 1975 prices are

	Meat	Other food	Apparel	House op.	Miscell.
1970 (Proj.)	56.0	259.5	119.6	182.8	454.7
1975 (Proj.)	69.4	335.9	151.0	253.7	610.9
1965 (Obs.)	43.7	186.9	89.9	117.9	296.1

In addition to the commodity groupings considered so far, retail demand as an aggregate is also projected, using similar methods. There is only one equation, hence there are

no restrictions to be applied other than deflation.

The estimated equation for total retail demand is

	log p	log μ	S ₁	S ₂	S ₃	Const.	R ²	d
log q	-.384	.475	-.161	-.110	-.133	1.9294	.926	.77
	(.044)	(.053)	(.009)	(.009)	(.009)			

Using assumed prices and income provides point projections and standard errors for the December quarters 1970 and 1975 as

	Proj.
1970	4.2805 (.0310)
1975	4.3504 (.0360)

From the tables for 42 degrees of freedom, at least 90% of nonsample observations have 0.9 probability of falling within 1.949 standard errors of the point projections. The projections, with tolerance limits, are

	Proj.	Lower	Upper
1970	4.2805	4.2201	4.3409
1975	4.3504	4.2802	4.4206

Converted to volume in constant prices (1955=1000) and expenditures in the given years, these become (in £ million)

	Point	Lower	Upper
Volume			
1970	214.9	202.3	228.2
1975	257.3	239.9	276.1
Expenditure			
1970	294.6	277.4	312.9
1975	389.8	363.4	418.3

The expenditure projections here can be compared with the sum of the five individual expenditure projections (for December quarters) as in the following table

	<u>1970</u>	<u>1975</u>
Projection of total	294.6	389.8
Sum of restricted projections	296.8	393.3
Sum of unrestricted projections	291.4	383.7

Projections from the unrestricted equations have not previously been given. Both the restricted and the unrestricted sums are well within the tolerance limits for the total, but in both years the restricted sum is closer to the point projection for the total.

Finally, projecting the volume of total retail demand for each quarter of 1970 and 1975 (assuming that the December prices etc., prevail throughout the year) gives (constant prices, 1955=1000)

<u>Qtr ended</u>	<u>1970</u>	<u>1975</u>
March	182.7	218.8
June	192.5	230.5
Sept.	188.1	225.2
Decr.	214.9	257.3
Total	778.2	931.8

In 1970 and 1975 prices the totals become £1067 million and £1412 million respectively, which compare with the summed individual expenditures of £1073 million and £1421 million.

5. DISCUSSION OF METHODS AND RESULTS

The projection method employed here can be usefully compared with alternative methods in a recent volume of contributions on similar topics (11).

Methods of obtaining demand elasticities for projection work suggested by Professor Frisch (4) have been found particularly useful and for systems with many commodities may provide the only practical procedure. The use of these methods implies both a utility function that is want-independent (Frisch's term) or directly additive (a term used by Houthakker (6)), and also the validity of concepts that are not invariant under monotonic increasing transformations of this function. Such assumptions may be valid, but users do not appear to test whether their assumptions are consistent with their observed data, and one might feel much happier with their results if this could be demonstrated.

The contribution by Stone et al., in the above volume (11) perhaps shows the greatest similarity to the present study. The authors use an additive utility function, $u = \sum b_i \log(q_i - c_i)$ with b_i and c_i constants, and only consider concepts that are invariant under monotonic transformations of this function. The theoretical properties of Stone's system are much tidier than those of the present study (although his independent parameters may be too few to adequately represent reality), but the positions are reversed when considering statistical procedures. Theoretical tidiness and empirical usefulness need not go hand-in-hand and the statistical superiority of the present study could well outweigh the theoretical superiority of that of Stone et al.

It is very desirable to estimate the degree of reliability of parameter estimates and especially of projections, as knowledge of possible errors in the latter may be just as important as their point values. It is equally important to test whether assumptions used are consistent with observed data as this will tend to prevent the use of either inappropriate assumptions or of very inaccurate data.

The present study uses restrictions that are derived approximately from a utility maximization hypothesis. No additivity assumptions are made about the utility function and only invariant results are used, hence it should be possible to test for both utility maximization and additivity.

It has been previously shown that utility maximization is doubtful, but possible. Accepting utility maximization, we can use a result of Houthakker's (6), to consider additivity. Slightly modified, Houthakker's equation 10 states that under direct additivity, the cross-price elasticities are proportional to the income elasticities or

$$e_{ik}/e_{jk} = E_i/E_j \quad (i \neq k, j \neq k)$$

Again, a rigorous test is outside the scope of this paper, but the estimated elasticities of the previous section seem to indicate that this result, and hence additivity, is unlikely, although doubtless it is possible.

Commenting on the numerical results of this paper, the low income elasticity for meat is perhaps unusual, but not surprising since New Zealand's present rate of per capita meat consumption is among the highest in the world, so the typical New Zealander may be closer to physical saturation

than elsewhere. The high cross-elasticities between meat and apparel indicate a high degree of substitution between these groups that is probably spurious. If wished, these elasticities could be reduced to any plausible a priori level (say about 0.1) by way of the restriction matrix R during estimation. Estimation and projection results for the "miscellaneous" grouping and especially the "household operation" grouping appear worse than those for the first three groupings. This is probably due to the vaguer definition and data reconciliation of these two groups, and also to the fact that "household operation" includes consumer durables, the demand for which can not be explained very well by a static utility maximization hypothesis.

Judging by the Durbin-Watson statistics, autocorrelation seems likely in the residuals of most equations, causing standard errors of estimation and projection to be suspect. This can in principle be overcome by suitable transformation of variables during the generalized least squares or maximum likelihood estimation, that is by minimizing the "generalized sum of squares" $e'Me$ subject to $R\beta = 0$, where M is a suitable ($nT \times nT$) matrix, or alternatively by finding the extreme points of $e'Me - \log.\det M$, subject to $R\beta = 0$, with respect to β and certain elements of M that are not assumed or known a priori.

An alternative attempt to reduce autocorrelation in the residuals of the equation explaining total retail demand, by introducing quadratic as well as linear explanatory variables (apart from seasonal), was unsuccessful due to very high intercorrelations between the linear and quadratic terms.

A criticism of the projection method used in this paper is that a one-way dependence of demand upon prices and income is assumed when this may not be so in practice. It is quite possible that retailers set prices at least partially in accordance with demand for the goods which they sell. Total retail sales form quite a significant proportion of national expenditure and hence of national income and therefore disposable income.

If two-way or simultaneous relationships of this type are very strong they will have two effects on the results of this paper. The first is the well-known least-squares or simultaneous equation bias whereby the estimation methods used will produce estimates of demand parameters that are both biased and non-consistent. This effect may not be very important here to the extent that the use of restrictions (if valid) forces the parameters into their theoretical framework, but it could well account for the poor performance of the utility maximization test. The second effect is perhaps more serious with its implication that we have not really explained the level of demand, but only obtained a relationship between demand, prices and income. The dangers of this for projection work can be seen by considering the simplest demand/supply model simultaneously determining price and quantity purchased. If $d(q,p) = 0$ is the demand function and $s(q,p) = 0$ the supply function then these two equations together determine the level of both variables. The above projection method has effectively estimated the demand function, substituted an assumed level for p and thus obtained q . But there is no reason to assume that the p and q arrived

at in this way are consistent with the relationship $s(q,p) = 0$ which must exist if the model is correct. Under such circumstances, both variables should ideally be projected together.

But if disposable income has only a weak dependence on the value of retail sales and if New Zealand retail prices are cost determined rather than demand determined then the above criticism loses most of its force on both grounds. In the absence of evidence to the contrary it is very convenient to assume away possible difficulties of this sort. However, for projections to be free from any doubts on this question, it would be desirable to include and test both price and income determination equations in the model. This has so far been outside the scope of the research undertaken.

6. SOME SHORT-TERM FORECASTS

If the model can give reasonable projections for up to ten years ahead, there is no reason why it should not do so for one year ahead. Testing the model against reality is one way to have it quickly rejected or provisionally accepted, so in this section some forecasts of retail consumption are given whose accuracy can be checked as data becomes available.

The "forecasts" of this section are strictly projections in the same sense as those already given, but over such a short period as one year estimates of prices, income and population should be fairly close to actual levels, so the projections should be quite good as forecasts.

On the basis of recent past experience, plausible assumptions for 1966 (the first year outside the sample)

are that all prices and population will rise by 0.5% per quarter and disposable income will rise by 1.8% per quarter, whence "real" disposable income per head rises by .8% per quarter.

From the December quarter, 1965, levels of

aggregate retail price	=	1248	(1955=1000)
other consumer prices	=	1490	(1955=1000)
population	=	2.667 million	
disposable income	=	£367 million	

aggregate retail demand for each quarter in 1966 is forecast from the aggregate regression equation as

Qtr.ended	March	June	Sept.	Dec.
Forecast	3.9913 (.0248)	4.0451 (.0248)	4.0259 (.0250)	4.1631 (.0249)

These forecasts are in natural logarithms of volume of consumption per head, with standard errors in brackets.

Obtaining (90%, .9) tolerance limits and converting all figures to total volume of retail trading provides (in £ million, 1955=1000 constant prices)

1966	Forecast	Lower	Upper
March qtr.	£145.1 mill.	138.2	152.2
June "	£153.9 mill.	146.6	161.5
Sept. "	£151.7 mill.	144.5	159.3
Dec. "	£174.9 mill.	166.6	183.6

and converting to actual value of forecast sales in each quarter we have

1966	Forecast	Lower	Upper
March qtr.	£182.0 mill.	173.3	190.9
June "	£194.1 mill.	184.9	203.7
Sept. "	£192.2 mill.	183.1	201.8
Dec. "	£222.6 mill.	212.1	233.7
Total	£790.9 mill.		

The forecast 1966 total of £790.9 million compares with the observed 1963, 1964 and 1965 totals of £650.2 million, £694.9 million and £734.5 million respectively.

During the preparation of this paper, official retail trading figures for the quarter ended March 1966, came to hand. The observed value of total retail sales is £181.041 million, which is well within the tolerance limits and very close to the forecast value of £182.0 million

Proceeding similarly for the five commodity groupings provides forecasts from the restricted equations (given only for the March quarter, 1966, without tolerance limits), in £ million worth of sales in the top line of the following table:

March Qtr. 1966

Commodity	Meat	Other food	Apparel	House op.	Miscell.
Forecast	10.8	47.7	20.7	28.2	74.2
Observed	10.9	46.6	20.0	27.9	75.7
% error	-.9%	+2.4%	+3.5%	+1.1%	-2.0%
March 1965	10.0	44.2	19.4	26.4	69.8

The observed figures for the quarter ended March 1965 are given for comparison.

The errors are considered small enough for the model to be adequate for the projections (provisionally at least) and useful as a source of short-term forecasts. As relative prices can be expected to change very little over short periods it is evidently real disposable income per head and especially the seasonal effects that cause variation in retail sales in the short-term.

CONCLUSION

The chief aim of this paper is to provide some numerical projections that should be of use to those involved with economic policy in New Zealand. Towards this purpose it has been found necessary to develop some rather interesting multi-variate statistical analysis and to test, even if only approximately, one of the basic postulates of economic theory, that of utility maximization.

The projections given could undoubtedly be improved by the inclusion of price and income determination equations, thus explaining relevant *variables* within the framework of a simultaneous model (which should ideally be a complete macro-decision model). Imposing the restrictions of this paper upon parameters in a simultaneous model presents no great problem, see (14), p.78 for instance, and provides restrictions upon the reduced form coefficients additional to those implied by simultaneity in the usual sense, thus improving the model from the viewpoint of prediction or projection, see (7), pp.249-264.

As they are, the projections given provide a valuable addition to the present state of knowledge on the topic, both with regard to methods used and results obtained.

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