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SCIENTIA ET INDUSTRIA CUM PROBITATE

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PROCESSING PLANT LOCATION STUDIES 1: THEORY AND A SIMPLE APPLICATION TO NEW ZEALAND WOOL SELLING CENTRES

by

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I. INTRODUCTION

This paper is the first of a series concerned with that aspect of spatial economics known as facility location. Here, the term facility is assumed to have a wide meaning. It includes raw material processing activities such as livestock slaughter works, services such as schools, electricity substations or ambulance centres and storage such as warehouses or bulk depots. Further, the problem may be within either the public or the private sector. The broad classes of processing, service and storage are distinguished because each has characteristic attributes and requires somewhat different procedures.

The problem discussed in this paper concerns processing. The paper has three major sections. Firstly the relationship of facility location problems to general economic theory is indicated and some empirical plant location studies noted. Secondly, a workable methodology for solving plant location problems is defined and solved, using the methodology discussed.

II. FACILITY LOCATION AND ECONOMIC THEORY

1. Space in Economics

As Kuenne (33) points out[1] there has been relative neglect of space in economic theory when compared to the attention devoted to temporal problems. This is attributed to a number of causes. First, many problems in economics, for example those relating to capital, interest and money, can only be approached within a temporal framework. Second, a number of spatial factors such as transport can be transformed into cost terms and thus lose their spatial characteristics. Third, what are initially conceived of as spatial problems, for example factory location, may more properly be considered time problems. For example, the desire to locate near customers may be based on the need to service them quickly. Fourth, spatial analysis often deals with discrete phenomena and is thus not subject to the niceties of marginal analysis which can be readily applied to continuous functions assumed elsewhere in economics.

What attempts there have been made to introduce space as one determinant of economic activity are usually traced back to von Thunen (59) writing in 1826. However it is only in the last 20 years or so that mathematical economists have formulated operational spatial models capable of providing answers to real world problems.

Traditional location theory is considered to have originated with von Thunen who was concerned with locating different types of agricultural production activities around a central point. The next substantial contribution was that of Weber (64) although meantime other Germans, including Launhardt (35) had published in this field.

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[1] Also Richardson (53) amplifies.

Weber's main contribution was in the location of manufacturing industry. He considered transport costs, regional wage rates and raw material costs important.[2] Hoover (21) extended Weber's work by introducing the theory of the firm and also pioneered empirical content with his shoe and leather industry studies. He is said to have been influenced by Palander(48) who was also concerned with introducing further economic backbone into location analysis.

These early contributors were attempting to explain the influence of space on social and economic activity - an aspect which they considered neo-classical economics had ignored. Moreover they worked within a partial equilibrium framework.

The first effort to work within a general equilibrium framework and thus utilise major neo-classical insights (specifically the Walrasian system) was by Losch (4)[3]. His system had five simple equations, emphasised product demand as a major variable, but tended to ignore or assume away the effect of cost on location.

Isard (25) and Leferber (37) further develop and generalise spatial general equilibrium models. Leferber worked within a linear programming framework.

Nevertheless all these efforts were conceptual rather than operational in nature.

Meantime another group of economists were exploring the problem of introducing space into economic analysis, through the use of linear production models.

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[2] Translations of von Thunen and Weber were not available until 1966 (16) and 1928 (65) respectively, thus delaying their influence in location economics.

[3] Again not available in English until 1954 when translated (41).

The theory, application and problems of such models[4] is conveniently summarised in a set of papers edited by Koopmans (32) in 1951.

Here, Koopmans & Reiter dealt specifically with the transportation (spatial) model while acknowledging earlier work which included the pioneering study of Hitchcock. Later, Samuelson (54), building on Enke (9), generalised the Koopman-Hitchcock problem into a spatial price equilibrium model. This was a significant development in spatial economics, explained by Fox (14) as follows:

> "The "transportation model" has won a prominent place in the linear programming literature and has been applied successfully to the shipping problems of some industrial concerns. In this model specified quantities of a commodity are to be shipped from each of a number of sources and other specified quantities are to be received at each of a number of destinations, total receipts being equal to total shipments. The receipts at each market are determined in advance and do not depend upon price. The objective is to satisfy the set of destination requirements at the least possible total transportation cost.
>
> "The "spatial equilibrium model" differs from these in that prices at each shipping point and destination are continuous functions of the quantities shipped or received plus the quantities produced and retained locally. The spatial equilibrium model, with its price-dependent demand and supply functions, may be quite useful in analysing problems of public policy, most of which arise at the industry level. These include tariff and other policies affecting international trade, excise taxes, freight rate regulation, farm price support policies, and perhaps others. The competitive organisation and continuously varying prices in many agricultural commodity markets make these particularly adaptable to analysis by means of the spatial equilibrium model. The existence of a considerable body of empirical demand analyses for farm products is also a favourable factor. "

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[4] Also called input-output, linear programming or activity analysis.

In fact Fox's model of the U.S. feed-livestock economy was the first empirical application of Samuelson's approach. However other studies which followed, for example Henry & Bishop (18) and Snodgrass & French (55) mostly assumed fixed supply and demand prices, thus simplifying to the original Koopmans-Hitchcock transport model.

A further important development was that of Takayama & Judge (58). Rather than representing regional supplies as linear price dependent functions they introduced an activity analysis formulation for supply.

2. Classification of spatial models

The purpose of this section is to show the relationship of facility location models to spatial models in general. Briefly, most emphasis has been placed on production and resource allocation models so that none of the existing classifications are entirely satisfactory. A simple alternative is suggested.

The most elegant classification and interpretation of spatial models is that of Kuenne (33). In dealing with space within a general equilibrium framework he divides models into:

Interregional trade models.

Locational models.

Interregional trade models assume that the structure of the economy is given in the sense that the spatial co-ordinates of resources, entrepreneurial activity, consumers and transportation channels are fixed. The problem is to determine equilibrium patterns of prices and flows of goods over space.

Locational models differ only in assuming the location of production may vary and seek to determine its pattern. The conditions of consumption and other assumptions are the same.

Thus interregional trade models are mostly concerned with the extent of product flows between points while location models mostly focus on the origins of the flows. In the former case space can be conceived as a friction to the flows of goods and in the latter as a matrix for the placement of economic activities.

Kuenne further distinguishes between non-operational and operational models. The former are constructs in the neo-classical tradition and thus without empirical validation.

Non-operational interregional examples include Walrasian derivations from which may be derived the classical transportation model of Hitchcock (20)/Koopmans (31) and the Ohlin (45) model.

Operational models include the Leontief (38) and Isard-Kuenne (24) input-output models, the interregional models of Isard (23), Chenery-Moses (5), Isard-Stevens (26), and Leferber (37) and the international model of Graham (15).

Location models discussed by Kuenne include those of Weber (65), Losch (41), Launhardt (35), Fetter (13), Palander (48) and Hoover (24).

For the facility location economist Kuenne is best viewed as providing a general theoretical introduction to mainly spatial general equilibrium models as well as a listing and contrasting of the major contributions in this area.

In marked contrast is the approach of Bawden (1). Here, the orientation is pragmatic and although not required by the classification, largely exampled by partial equilibrium studies from agriculture.

Bawden groups spatial models into:

Standard equilibrium models.

Activity analysis models.

Both types are built up from the standard transportation model of linear programming, both are point trading[5] in nature (production and consumption in each region is assumed to take place at a single point) and both normally assume perfect competition.

The difference between the two groups lies in their treatment of production. Standard equilibrium formulations rely on given regional supply (and demand) functions. These are usually based on historical data suitably adjusted. Activity analysis models generate their own supplies via linear production models.[6]

Facility location models fall within the activity analysis group. This may initially appear puzzling, say in the processing plant location problem, because usually raw product supply is predetermined. However the processing plants in effect generate their own final product supplies.

Weinschenck et al. (67) use the same classification as Bawden but also provide a comprehensive review of relevant theory plus empirical examples. However they add a third group which they call dynamic approaches. This allows the introduction of recursive programming models (e.g. Day (8)).

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[5] Assume spatial location problems as either relating to:
Location on a plane.
Location on a network.
In the first case space is regarded as continuous and allows for very small changes in location. This is how von Thunen visualised location problems. Among others, Isard & Dunn have also used the continuous approach. However although Beckmann (2) has shown in theory that there is a solution when the generalised transportation problem is cast in a continuous framework, no satisfactory solution procedure exists. Accordingly the discrete or network approach is used - hence point trading models.

[6] Recall Koopmans (ed.) Activity analysis of Production and Allocation, Wiley 1951.

Weinschenck et al. include some discussion on the location of processing industry but as with Bawden see such problems as falling within the activity analysis approach.

However they do make a useful distinction between models which optimise from the point of view of a central planning board or authority and those which are based on decentralised decision making by individual competitive firms.

Revelle et al. (52), dealing specifically with facility location problems, draw a similar distinction and identify private sector models and public sector models. In the former the objective is to minimise the total cost of transport in a facility operation. In public sector models however the objective is to maximise or minimise some surrogate or substitute measure for utility (e.g. minimise average time or distance by users of the facility).

Probably the most meaningful classifications from the point of view of guiding those interested in carrying out facility location studies are those of Bawden and Weinschenck et al.

However, neither provides sufficient detail. This is partly because facility models after all only deal with a restricted range of spatial economic situations and thus do not normally merit extensive discussion.

A key to classification is the characteristic of facility models which distinguish them from all others. This is the "processing" aspect and its associated nonlinear cost function.

Accordingly consider the following grouping which places facility location models clearly and simply:

Standard equilibrium models.
Standard activity analysis models.
Decreasing cost activity analysis models.

3. Empirical plant location studies

The studies of Stollsteimer (57) and Logan & King (39) are usually regarded as a reference base for agricultural processing.[7] The problem Logan & King investigated was to determine the optimum location and size of cattle slaughter plants in California which would minimise the costs of shipping live animals to slaughtering points, processing them and shipping the dressed beef to consumers. This was the first study to incorporate transshipment and a nonlinear average processing cost function into the transport model. The method of allowing for transhipment was based on earlier work of Orden (47). Because of the assumption of a nonlinear cost curve, the model falls into the decreasing cost activity analysis class.

Prior to Logan & King most plant size, number and location studies first determined the least-cost size of plant and then estimated the number of plants required to satisfy total demand. For example, Olsen (46) considered the problem of optimum location of plants to process milk. Plant size was first determined by estimating the volume of milk to be assembled and processed at lowest average cost. Volume per plant then determined the location of plants within the overall production area.

Cobia & Babb (7), in investigating the processing and distribution of packaged fluid milk in Indiana, derived a number of scale curves for processing and distribution and drew inferences about an optimum plant location network.

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[7] Not discussed here but closely associated are warehouse location studies. Major contributions include Pherson & Firch (49), Keuhn & Hamburger (29) and Feldman, Lehrer & Ray (10).

However Stollsteimer carried the methodology a step further by simultaneously estimating the size, number and location of pear packing facilities in part of California that would minimise the sum of assembling and packing costs.[8]

A number of later studies added further flexibility. Thus Polopolus (50) extended the model to include the multiple product case with a study of vegetable canning plants in Louisiana. Chern & Polopolus (6) attempted to relax the assumption of a linear total processing cost function and worked through an example using the Florida orange industry. Warrack & Fletcher (62, 63) indicated how the computational burden could be reduced and hence enable larger problems (in this case the Iowa feed manufacturing industry), to be solved. Ladd & Halvorson (34) worked out simple procedures to enable parametric solutions to be derived and used the Mid-West turkey processing industry as an example. Kloth and Blakley (30) derived a model for the U.S. dairy processing industry which allowed for scale economies in processing and market sharing by processing firms.

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[8] The solution techniques of Stollsteimer and Logan & King differ considerably. The former estimates costs for all possible location and plant combinations and the minimum cost combination is selected. The Logan & King procedure involves the iterative solution of a heuristically adjusted transhipment model in order to isolate an acceptable solution. The Logan & King method will be described in detail in section III of this paper.

Both approaches have weaknesses. Because all possible plant combinations need to be evaluated when using the Stollsteimer method, the computational burden is prohibitive for all except small problems. Additional shortcomings are the requirement of a linear total processing cost function, and that only processing and either assembly or distribution costs are considered. The Logan & King method is open to criticism because the heuristic procedure by which potential locations are eliminated from the model yields a solution which is not guaranteed to be a global optimum. Rather, it is a local optimum. The Stollsteimer method does produce a global optimum.

Similarly the Logan & King model has benefited from a number of modifications. Hurt & Tramel (22) set out methodology for considering intermediate and final product processing as well as multiple products and more than one plant at a single location. They also indicated how greater economy could be introduced into solution procedures. Leath & Martin (36) also hypothesized multiple plant and product relationships as well as the possibility of storage which in effect introduced the time dimension. Bobst & Wannen (3) derived a restricted model which was relevant for imperfectly competitive situations such as market sharing and applied it to fluid milk processing in the State of Washington. Stammer (56) extended the Logan & King model formulation and suggested improvements to the Logan & King heuristic search procedures.

Empirical studies following the standard Logan & King approach, but reported in more detail, include those of Cassidy et al. (4) on the location of beef slaughter plants in Queensland, Ferguson & McCarthy (11) on Australian wool selling centres, O'Dwyer (44) on dairy manufacturing plants in Ireland and Weichelt (66) on egg packing plants in Germany.

One critical aspect of all these studies is their usefulness for real world decision making, both at the micro and macro level. Thus, some work has been done on the sensitivity of solutions. Papers here include those by Kanbur & Neudecker (27,28), Toft et al. (60) and McCarthy et al. (42).

More importantly, policy makers, when considering plant location decisions, often need to be supplied with alternative plans rather than a single solution derived on the basis of a least-cost objective. This is so because criteria,in addition to those of cost, commonly need to be taken into account when making such decisions. To this end Higham has developed two approaches capable of generating multiple low cost solutions.[9] The first, initially suggested by Cassidy[10], utilises Monte Carlo methods to obtain a number of acceptable solutions often with widely varying spatial characteristics. The second approach is an extension to the Logan & King solution technique, involving interference with the heuristic search so as to produce various solutions which are all local optima. This second method, known as "forcing", is described in detail in the next paper in this series. Both the Monte Carlo and forcing approaches are briefly outlined and applied in Ferguson et al. (12).

While there is considerable scope for improvements on current basic solution techniques, it is suggested that development of facility location methodology appropriate for dynamic and stochastic situations would be major contributions.

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[9] Higham, C.A. Notes on multiple solutions to plant location problems, Unpublished Paper, AERU, Lincoln College, Dec. 1971.

[10] Cassidy, P.A. Personal communication.

III. METHODOLOGY

1. General

The Logan & King technique involves the repeated solution of a successively adjusted transhipment model. To give an appreciation of the approach the methodology is developed below in three stages. To begin with the general transportation model is outlined and illustrated by a simple example. Transhipment is then incorporated using the transportation example and a further example, including intermediate points, is presented. Finally the inclusion of processing costs is discussed and the intermediate point example expanded accordingly.

2. The Transportation Problem

Consider the situation of an homogeneous commodity which is supplied at M different origins and demanded at N different destinations. Let the origins be designated as S1, S2, S3, ..., SM, supplying quantities s_1, s_2, s_3, ..., s_m respectively, and the destinations as D1, D2, D3, ..., DN, demanding quantities d_1, d_2, d_3, ..., d_n respectively.

Supply and demand are assumed fixed for each origin and destination. The cost of transporting one unit of the product from the i^{th} origin to the j^{th} destination, c_{ij}, is known for all i and j (thus c_{11} is the cost of transporting one unit of the product from S1 to D1 and so on).

If x_{ij} represents the amount to be transported from the i^{th} origin to the j^{th} destination, then the problem is to determine all x_{ij} so that the total cost of transport is minimised.[11]

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[11] For a feasible solution to exist the total amount available at all origins should be not less than the total amount required at all destinations. If this is not the case then an artificial origin supplying the deficit can be introduced into the model. Supply from an artificial origin to any destination represents unsatisfied demand at that destination.

While the standard linear programming algorithm can be used to solve this problem, there is a more efficient solution procedure, known as the transportation algorithm, which can be used, provided the total amount demanded by all destinations exactly equals the total amount available at all origins. This requirement, if not initially met, can be satisfied by introducing an artificial destination into the model which demands all excess supply. Now by introducing an artificial origin or artificial destination if necessary[12], the transportation problem can be stated as:

Objective function

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

that is, minimise total transport costs

subject to:

Supply constraints

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad \text{for } i = 1, 2, \ldots m$$

Thus each origin supplies exactly the amount available.

Demand constraints

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad \text{for } j = 2, 3, \ldots n$$

Thus each destination receives exactly the amount demanded.

Non-negativity constraints

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j.$$

That is, negative quantities cannot be transported.

Feasibility condition

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

That is, the total supply should equal total demand.

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12 Transport costs associated with artificial origins and destinations are usually zero.

A table specifying the unit costs of transport from each origin to each destination, the amount supplied by each origin and the amount demanded by each destination, can be constructed and is called the cost/requirements table (see Table 1).

TABLE 1

General Cost/Requirements Table

		Destinations				
	D1	D2	D3	......	DN	Supply
S1	c_{11}	c_{12}	c_{13}	......	c_{1n}	s_1
S2	c_{21}	c_{22}	c_{23}	......	c_{2n}	s_2
S3	c_{31}	c_{32}	c_{33}	......	c_{3n}	s_3
Origins .	.	.	.		.	.
.	.	.	.		.	.
.	.	.	.		.	.
.	.	.	.		.	.
.	.	.	.		.	.
SM	c_{m1}	c_{m2}	c_{m3}	......	c_{mn}	s_m
Demand	d_1	d_2	d_3	......	d_n	

This table shows the input required by the transportation algorithm in order to obtain a solution.

3. A Transportation Example and its Solution

Consider a simple 2 origin, 3 destination problem. The cost/requirements data are outlined in Table 2.

TABLE 2

Cost/Requirements Table,
2 origin, 3 destination problem

Destinations

	D1	D2	D3	Supply
Origins S1	\$2.0	\$1.0	\$1.0	30 units
S2	\$1.5	\$1.5	\$2.0	50 units
Demand	20 units	15 units	45 units	80 units (Total)

This problem in equation form is:

$$\text{Minimise Total Cost} = 2.0x_{11} + 1.0x_{12} + 1.0x_{13} + 1.5x_{21} + 1.5x_{22} + 2.0x_{13}$$

Subject to:

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} &= 30 \\ x_{21} + x_{22} + x_{23} &= 50 \end{aligned}\right\} \quad \text{Supply constraints}$$

$$\left.\begin{aligned} x_{11} + x_{21} &= 20 \\ x_{12} + x_{22} &= 15 \\ x_{13} + x_{23} &= 45 \end{aligned}\right\} \quad \text{Demand constraints}$$

$$\left.\begin{aligned} x_{ij} \geqslant 0 \text{ for } i &= 1,2 \\ j &= 1,2,3 \end{aligned}\right\} \quad \text{Non-negativity constraints}$$

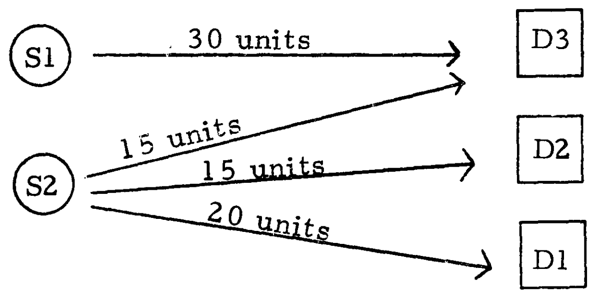
In the above problem total supply (30 + 50 = 80) equals total demand (20 + 15 + 45 = 80), and so the transportation algorithm can be utilised. When this is done the minimum cost solution shown in Table 3 is obtained.

TABLE 3

Solution, 2 origin, 3 destination problem

		Destinations			
		D1	D2	D3	Supply
Origins	S_1	0	0	30	30
	S_2	20	15	15	50
	Demand	20	15	45	

The solution can be represented diagrammatically as:



The total cost of this solution is minimum and is given by:
(\$1.0 x 30) + (\$1.5 x 20) + (\$1.5 x 15) + (\$2 x 15) = \$112.5

The transportation algorithm is not described here. The interested reader will find the procedure discussed in most introductory operations research books, for example Hillier & Lieberman (19).

While the above example is trivial, real world problems involving a great number of origins and destinations are not. For instance the problem discussed in section IV of this paper involves solving a transportation problem whose cost/requirements table has 38 rows and 18 columns.

4. The Transhipment Problem

Consider now the situation where it is possible to transport the product not only direct from an origin to a destination, but also via another origin or destination. It is conceivable that such a route might be cheaper. The possibility of such transhipment can be achieved by allowing each origin to act as a destination without any real demand and each destination to act as an origin without any real supply. Therefore, rather than a problem with m origins and n destinations, the problem now has m + n origins and m + n destinations. As before c_{ij} is the unit cost of transport from the i^{th} origin to the j^{th} destination. Obviously c_{ij} for all i = j is zero.

The amount to be transhipped through any point is not known but its feasible upper limit is, namely the total amount supplied. Thus if $A = \sum_{i=1}^{m} s_i$ and if A is added to the supply available at each of the m + n origins and to the demand at each of the m + n destinations, the model is then capable of allowing transhipment through one or more points between an initial origin and final destination. Referring to the added quantities of A, as stockpiles, Orden (47) states, "the solution to the transhipment problem lies in the fact that withdrawals from and compensating additions to the stockpiles are equivalent to transhipment. The stockpiles do not matter provided they are large enough to permit all desirable shipments which can reduce cost. In the computation, excessively large stockpiles are arbitrarily introduced. The excesses of stockpiles over amounts actually shipped drop out of the final solution (they appear as shipments from a point to itself at zero cost)."

The transhipment cost matrix consists of four submatrices as shown in Table 4. The north-west submatrix contains the per unit transport costs from actual origins to each other. The north-east submatrix contains the per unit transport costs from actual origins to actual destinations. The south-west submatrix is made up of the per unit transport costs from actual destinations to actual origins while the south-east submatrix consists of per unit transport costs from actual destinations to each other.

TABLE 4

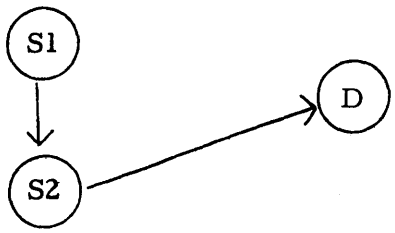
General Cost/Requirements Table of a Transhipment Problem

Destinations

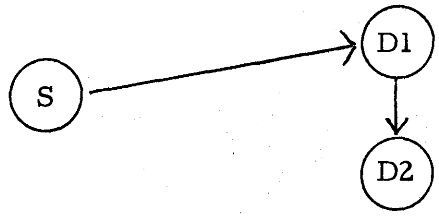
		S1 S2 ...... SM	D1 D2 ...... DM	Supply
Origins	S1			$S_1 + A$
	S2	Origin to origin	Origin to destination	$S_2 + A$
	.			.
	.	Transport costs	Transport costs	.
	SM			$S_m + A$
	D1			A
	D2	Destination to origin	Destination to destination	A
	.			.
	.	Transport costs	Transport costs	.
	DN			A
	Demand	A A ...... A	$d_1 + A$ $d_2 + A$.... $d_n + A$	

A cost/requirements table of this construction enables the following types of transport routes in addition to direct source to destination shipment:

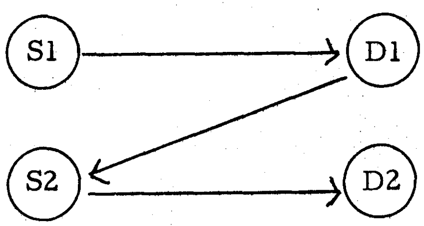
(a) Origin - Origin - Destination



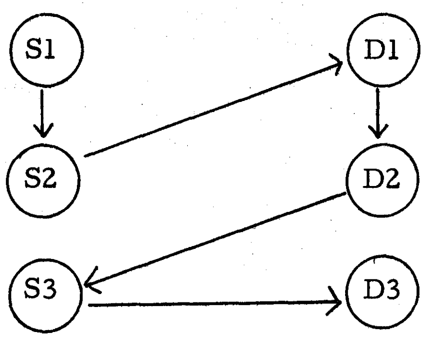
(b) Origin - Destination - Destination



(c) Origin - Destination - Origin - Destination



(d) Some combination of the above, for instance



Transhipment models are solved by the standard transportation algorithm. The following example demonstrates how the model construction given in Table 4 facilitates transhipment.

5. An example of a transhipment problem and its solution

Consider the previous example but now allow for the possibility of transhipment. The expanded cost/requirements table is given in Table 5. The cost matrix is partitioned into the various submatrices described above. Note that cost elements on the main diagonal are all zeros, and that the north-east submatrix corresponds to the cost matrix of the previous transportation example.

TABLE 5

Cost requirements table
2 origin, 3 destination transhipment problem

		Destinations					
		S1	S2	D1	D2	D3	Supply
	S1	\$0.0	\$0.2	\$2.0	\$1.0	\$1.0	30 + 80
	S2	\$0.2	\$0.0	\$1.5	\$1.5	\$2.0	50 + 80
Origins	D1	\$1.5	\$10.0	\$0.0	\$5.0	\$3.0	80
	D2	\$6.0	\$10.0	\$0.2	\$0.0	\$2.0	80
	D3	\$6.0	\$16.0	\$5.0	\$4.0	\$0.0	80
	Demand	80	80	20 + 80	15 + 80	45 + 80	

Applying the transportation algorithm to this problem, the solution given in Table 6 is obtained.

TABLE 6

Solution of 2 origin, 3 distribution
transhipment problem

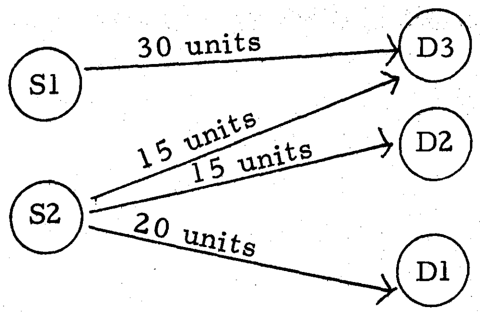
		Destinations					
		S1	S2	D1	D2	D3	Supply
	S1	30	0	0	35	45	110
	S2	50	80	0	0	0	130
Origins	D1	0	0	80	0	0	80
	D2	0	0	20	60	1 0	80
	D3	0	0	0	0	80	80
	Demand	80	80	100	95	125	

Figures in the main diagonal of the solution matrix (i.e. 30, 80, 80, 60, 80) are ignored as they represent artificial shipments from a point to itself. The solution indicates that the 50 units available at 52 are transported to S1, resulting in a total of 80 units being available at 51. Of this 80, 45 units are shipped to D3, and the remaining 35 are transported to D2. Finally 20 units are shipped from D2 to D1. The total cost of this solution is given by:

$$(\$0.2 \times 50) + (\$1.0 \times 45) + (\$1.0 \times 35) + (\$0.2 \times 20) = \$94.$$

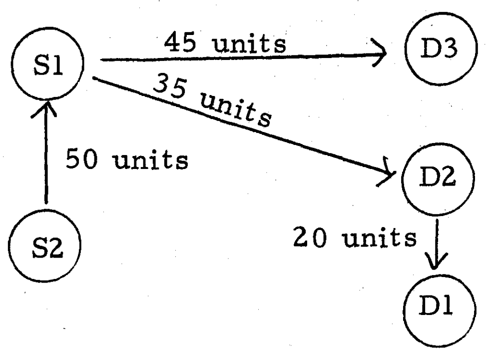
The following diagrams contrast the nature of the transport pattern with and without transhipment.

(a) Without transhipment



Cost = $[redacted] 112·5

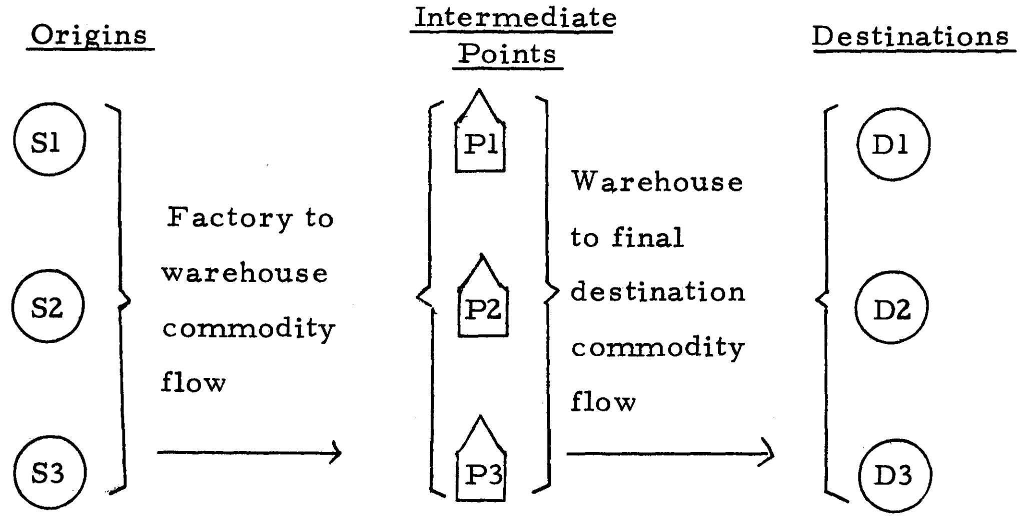
(b) With transhipment



Cost = $94.00

6. <u>Transhipment through intermediate points - an example</u>

The previous example discussed how to allow for transhipment through origins and destinations. But often feasible transhipment points are intermediate points between origins and destinations. For instance a commodity which is manufactured in a number of spatially separate factories, may have to be stored in a number of warehouses, before being transported to various final demand destinations. The warehouses in this case are intermediate transhipment points between origins and destination as shown in the following diagram.



The following numerical example is given to demonstrate how such a situation is modelled so that total transport costs can be minimised.

Supplies of 20, 30 and 30 units are available at origins S1, S2 and S3 respectively. Demands of 20, 15 and 45 units are required at D1, D2 and D3 respectively. Transportation directly from origins to destinations is not possible; rather, the commodity must be transported via one or more of four intermediate transshipment points P1, P2, P3 and P4. These intermediate points have no supply or demand of their own.

The relevant transport costs are given in Tables 7(a) and 7(b).

TABLE 7(a)

Unit Transport Costs - Origins to Intermediate Points

	P1	P2	P3	P4
S1	$9	$7	$5	$3
S2	$5	$3	$1	$9
S3	$1	$9	$7	$5

TABLE 7(b)

Unit Transport Costs - Intermediate Points to Destinations

	D1	D2	D3
P1	$1	$4	$7
P2	$3	$1	$3
P3	$5	$2	$4
P4	$7	$4	$5

The cost/requirement table for the problem is presented in Table 8.

TABLE 8

Cost/requirements table with intermediate transhipment points

Destinations

		P1	P2	P3	P4	D1	D2	D3	Supply
Origins	S1	$9	$7	$5	$3	∞	∞	∞	20
	S2	$5	$3	$1	$9	∞	∞	∞	30
	S3	$1	$9	$7	$5	∞	∞	∞	30
	P1	0	∞	∞	∞	$1	$4	$7	80
	P2	∞	0	∞	∞	$3	$1	$3	80
	P3	∞	∞	0	∞	$5	$2	$4	80
	P4	∞	∞	∞	0	$7	$4	$5	80
Demand		80	80	80	80	20	15	45	

The four submatrices of the partitioned cost matrix are interpreted as follows:

(a) The north-west submatrix includes the per unit transport cost from each origin to each intermediate point. Allocations in the corresponding submatrix of the solution are the least cost shipments from origins to intermediate points.

(b) The north-east submatrix is made up of prohibitively high cost elements, hence preventing direct origin to destination shipment.

(c) The south-west submatrix contains zero cost elements along the main diagonal and prohibitively high costs elsewhere. Allocations in the corresponding submatrix of the solution represent artificial shipments from each intermediate point to itself, and as Orden states, "drop out of the solution".

(d) The south-east submatrix includes the per unit transport cost from each intermediate point to each destination. Allocations in the corresponding submatrix of the solution represent shipments from intermediate points to final destinations.

Note that while intermediate points have no supply or demand of their own, Table 9 shows that each has been given an artificial supply and demand of 80 units. This artificial quantity, which can be viewed as a stockpile, sets the upper limit on the amount that can be transhipped through each intermediate point.

Applying the transportation algorithm to the model presented in Table 8, the solution given in Table 9 is obtained.

TABLE 9

Solution, 3 origin, 3 destination, 4 intermediate points problems

Destinations

		P1	P2	P3	P4	D1	D2	D3	Supply
	S1	0	0	0	20	0	0	0	20
	S2	0	0	30	0	0	0	0	30
Origins	S3	30	0	0	0	0	0	0	30
	P1	50	0	0	0	20	10	0	80
	P2	0	80	0	0	0	0	0	80
	P3	0	0	50	0	0	5	25	80
	P4	0	0	0	60	0	0	20	80
	Demand	80	80	80	80	20	15	45	

The solution is interpreted as follows:

(a) From the north-west submatrix:-

20	units	are	transported	from	S1 to P4
30	"	"	"	"	S2 to P3
30	"	"	"	"	S3 to P1

(b) There are no allocations in the north-east submatrix as expected.

(c) The numbers in the main diagonal of the south-west submatrix are artificial shipments (which can be viewed as unused stockpiles) and are not regarded as part of the "real" solution.

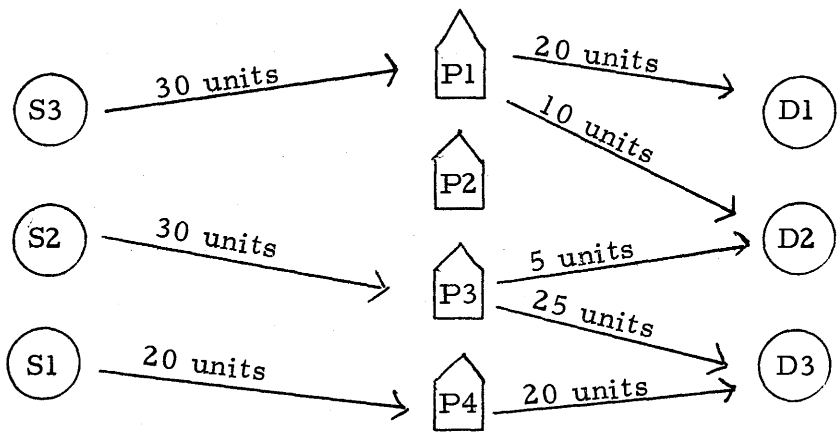
(d) From the south-east submatrix:-

20	units	are	transported	from	P1 to D1
10	"	"	"	"	P1 to D2
5	"	"	"	"	P3 to D2
25	"	"	"	"	P3 to D3
20	"	"	"	"	P4 to D3

The total cost of the solution is given by:-

(\$3 x 20) + (\$1 x 30) + (\$1 x 30) + (\$1 x 20) + (\$4 x 10) +
(\$2 x 5) + (\$4 x 25) + (\$5 x 20) = \$390

The solution is presented in the following diagram.



This diagram clearly shows two features of the solution. Firstly note that intermediate point P2 is not utilised. Secondly, a requirement of feasibility is that the quantity shipped to any intermediate point, should exactly equal the quantity shipped from it. This requirement is satisfied by the solution.

If, for some reason, one or more of the intermediate points are incapable of coping with a throughput exceeding some specific quantity, then this can be incorporated in the model by adjustment of the amount specified as the supply and demand of the appropriate intermediate point. For instance, in the above example, if Pl is incapable of handling a throughput of greater than 15 units, then 15 units rather than 80 is specified as the supply and demand for Pl. This adjustment would ensure that the capacity limit of 15 would not be exceeded. This method of "capacitating" intermediate points is particularly useful in facility location problems.

7. Inclusion of processing costs

The facility location problem usually requires the determination of the number, size and location of some type of facility; for example a wool store (in which wool can be thought of as undergoing various processes). Processing plants are in almost all instances characterised by economies of scale, and sometimes by diseconomies of scale. Economies of scale exist when expansion of the scale of productive capacity of a firm causes total production costs to increase less than proportionately with output. As a result long run[13] average costs of production fall.

[13] Economic theory recognises two "time" periods - the long run and the short run.
The long run is defined as a time period long enough for a firm to be able to vary the quantities of all its factors of production. (This implies that the period is long enough to permit the firm to choose the most efficient combination of inputs to produce any given output.)
The short run is a time period within which a firm is not able to vary all its factors of production (usually plant and machinery).

as throughput increases. For example an automobile plant designed to produce 20,000 cars per year will, under normal circumstances, have a lower per unit production cost than a 10,000 car capacity plant producing the same car, provided both plants are operating at their normal capacities. Conversely, diseconomies of scale result in increasing average costs.

Economies of scale occur for one or more of the following reasons:

(a) Larger scale production allows the use of the most efficient technology and techniques.

(b) Specialisation of management and labour.

(c) Lower per unit administrative costs. (That is, overhead costs tend to rise more slowly than output.)

(d) Ability to make use of by-products.

(e) Ability to utilise more efficient marketing methods.

Diseconomies arise from organisational and administrative problems resulting from too large an enterprise. For very large enterprises, marketing problems may arise. For further discussion of scale relations see for example McConnell (43) and Pratten (51).

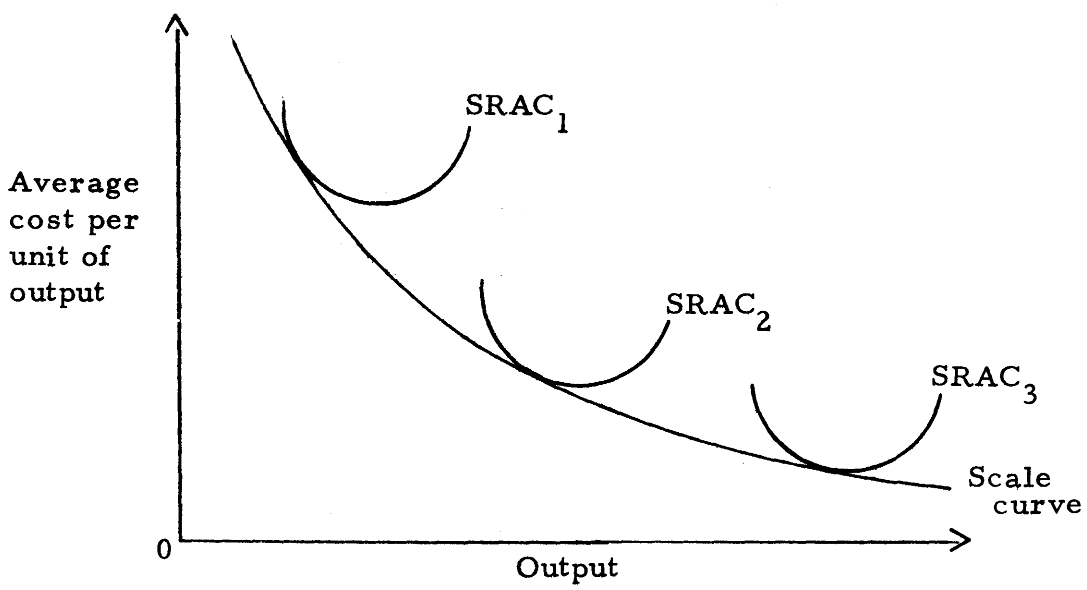
Associated with the short run and long run concepts of economic theory are short run and long run cost curves. The short run cost curve relates average costs of production and the extent to which plant is utilised. Following Viner (61) the long run curve is the envelope of the short run curves. It shows the lowest possible cost per unit of producing various outputs when the firm has time to build any desired scale of plant.

As Pratten points out the long run average cost curve does not show what happens to costs as the scale of production is increased over time. It does show what the effect of scale would be on average costs of production of a series of plants at a particular point in time, each operating at minimum cost.

Thus Pratten concludes it is misleading to talk of a long run average cost curve although this is the accepted term in economics and economics texts. He suggests "scale curve" is more appropriate and that term is used here. Figure 1 illustrates the relationship between short run average cost curves and the scale curve.

FIGURE 1

Short run average cost curves and the scale curve

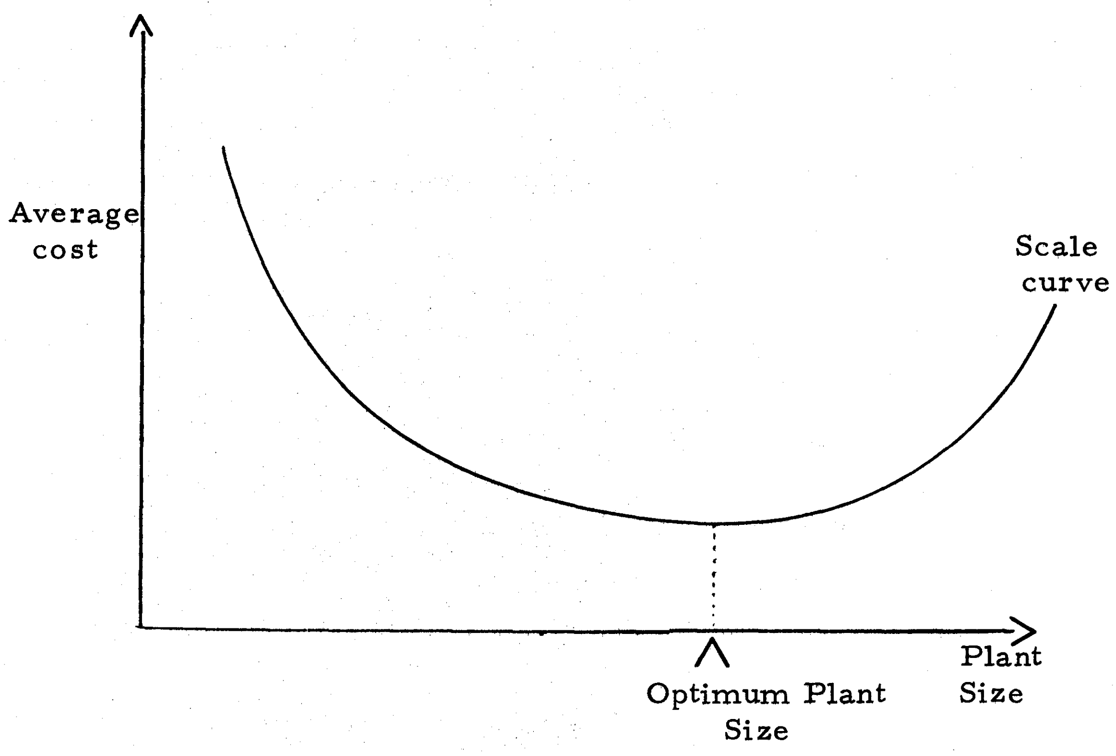


As far as the shape of the scale curve is concerned, economic theory argues in favour of a U-shape, the downward sloping part of the curve being the result of economies of scale in different sizes of plant, each operating at minimum cost, while the upward sloping segment is the result of diseconomies of scale.

The optimal plant size is that indicated by the minimum of this U-shaped curve, as indicated by Figure 2.

FIGURE 2

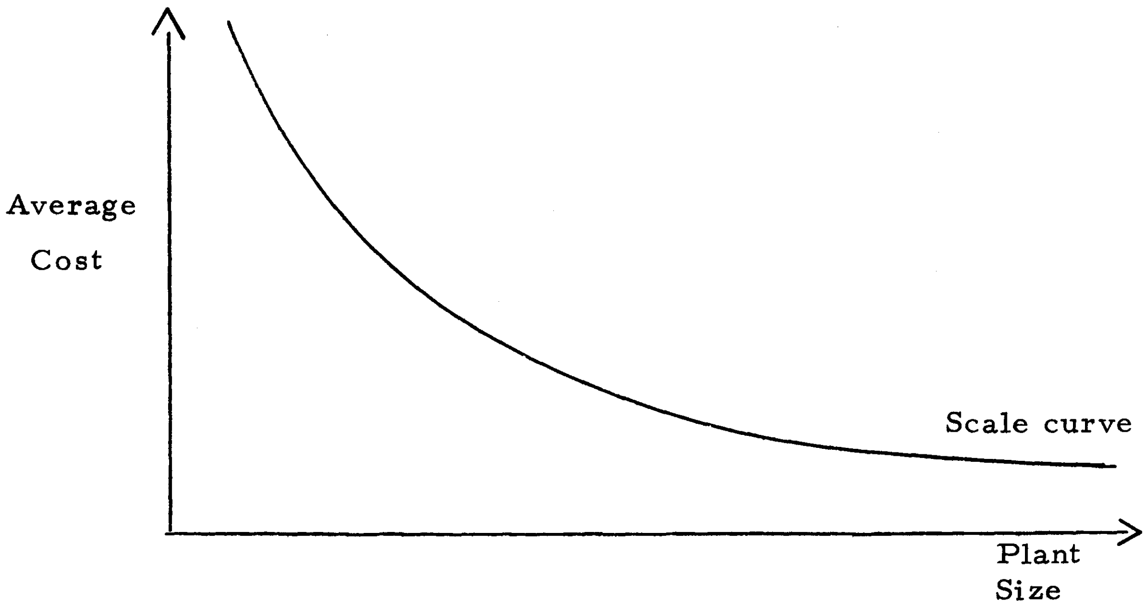
Relation of scale curve and optimum plant size



However, empirical studies, for example as discussed in Pratten (51), have revealed that for many processing plant situations the scale curve is L-shaped; with economies of scale causing the initial decline in average processing costs which then level out with no evidence of diseconomies of scale as indicated in Figure 3.

FIGURE 3

Shape of scale curve as indicated by empirical studies



The essence of the processing plant location problem is minimising total processing plus transport costs. The existence of economies of scale in the processing activity tends to support the use of one or a few large processing plants, while minimising transport costs alone usually tends to favour the siting of a larger number of smaller processing facilities. The problem is to find the solution where the combined cost is minimised.

8. Plant location with processing costs included

The third and final stage of developing the Logan & King methodology requires the inclusion of nonlinear processing costs into the transhipment model. For this purpose the previous example will be expanded by assuming that the commodity involved has to be processed in some way.

Recall that the example involves a commodity which was supplied at three origins S1, S2 and S3 in quantities 20, 30 and 30 respectively, and was demanded at three destinations, D1, D2 and D3 in quantities 20, 15 and 45 respectively. Shipment from an origin to a destination had to be via one of four intermediate points, P1, P2, P3 and P4. The appropriate per unit transport costs are given in Tables 7(a) and 7(b). The problem is now extended by adding the requirement that the commodity has to be processed and that potential processing plant locations are at P1, P2, P3 and P4.

Per unit processing costs are normally subject to economies of scale. In the present example assume that the scale curves for all potential plant locations are the same[14] and given by the equation:

$$\text{Average Cost} = \$(5 + \frac{200}{Q}) \quad ; \text{ where } Q = \text{plant capacity.}$$

For instance, the per unit processing cost of a 50 unit plant operating at normal capacity is: $\$(5 + \frac{200}{50}) = \$9$.

The problem is to determine the size and location of processing plants so that total costs (transport costs + processing costs) are minimised. Knowing the solution to the transhipment problem one might be tempted to choose processing plants at P1, P3 and P4 of sizes 30, 30 and 20 respectively, with a corresponding transport allocation. The cost of such a solution is $1390 ($1000 processing costs plus $390 transport costs). However such an approach takes no account of the per unit processing costs which decline as plant throughput increases, thus producing a trend towards larger plant sizes.

The Logan & King solution procedure takes into consideration the combined effect of transport costs and processing costs by the following heuristic (non-analytical) search method:

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[14] The method is capable of handling curves of different functional forms.

(a) Beginning with a transhipment cost/requirements table (such as that shown in Table 8), the minimum per unit processing cost for each potential plant is added to each cost element in the corresponding column in the north-west submatrix.

(b) The resulting problem is optimised using the transportation algorithm.

(c) The added per unit processing costs are adjusted in accordance with the throughput given in the solution while any potential plant with zero throughput is dropped from the model.

(d) Steps (b) and (c) are repeated until no further adjustments are required (i.e., when a stable solution is achieved).

Applying this procedure to the example the stable solution obtained involves a 30 unit capacity plant at Pl and a 50 unit capacity plant at P3. The total cost of this solution is $1210, made up of $800 processing costs and $410 transport cost. The transport pattern is detailed in Tables 10(a) and 10(b).

TABLE 10(a)

Transport pattern from origins to plants

From Origin	Amount Available	To Plant Pl	P2	P3	P4
Sl	20	-	-	20	-
S2	30	-	-	30	-
S3	30	30	-	-	-
Amount Processed		30	-	50	-

TABLE 10(b)

Transport pattern from plants to destinations

From Plant	Amount Processed	To destination Dl	D2	D3
Pl	30	20	10	-
P2	50	-	5	45
Amount Required		20	15	45

The Logan & King solution procedure yields a local optimum[15]. In the next paper in this series the modified solution procedure suggested by Stammer is outlined and a method for obtaining alternative low cost solutions which may be of interest to decision makers is presented.

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[15] A local optimum is one where a small change in the solution would involve a higher total cost. A solution is a global optimum if no solution with a lower total cost exists.

IV. THE WOOL SELLING CENTRE PROBLEM

1. Nature of the problem

As a textile fibre with a small share of the world fabric market, wool faces strong competition from synthetic fibres. The major advantage of wool is the nature and properties of the fibre but it has disadvantages including a widely fluctuating price, inelastic supply and high cost handling and selling charges from producer to user. This latter area is the concern of this paper. However only wool sold at auction will be considered. In New Zealand this accounts for about 1.3 million out of a total of 2 million bales. Wool to be sold at auction is transported by road or rail to wool brokers' stores at eight selling centres (Auckland, Napier, Wanganui, Wellington, Christchurch, Timaru, Dunedin, Invercargill). There it may be classed, reclassed or binned (combined with other wool) and then displayed for buyers and appraisers prior to auction. After auction it may be scoured and dumped (a number of bales pressed together to reduce size), and is stored to await local or overseas shipment. As far as cost economies prior to export are concerned, the location of wool stores relative to wool production (supply) and demand will affect costs of transport. Further, there are economies of handling wool at stores as throughput increases. The problem investigated here is to determine the optimum number, size and location of wool stores so that total transport and wool store costs are minimised.

2. Method of approach

(a) Solution method and model construction:

The solution method chosen for the current problem is the Logan & King procedure detailed above.

The cost/requirements table involves thirty origins (i.e. supply regions), eight potential selling centre locations (in this study only locations which are current selling centres are considered), and ten demand destinations (nine local destinations and one destination to represent all overseas demand[16]). The supply and demand estimates are set out in Table 11. Capacity limits on each potential selling centre were imposed in obtaining an initial solution and were subsequently lifted. These capacity limits are also included in Table 11. The cost/requirements table structure is given in Figure 4.

FIGURE 4

Cost/requirements table : wool selling centre problem

	Potential Plant Locations 1 2 8	Final Destinations 1 2 10	Supply
Wool supply regions 1 2 30	Origin to selling centre transport costs + processing costs	Inactive (all cost elements set at a prohibitively high level)	S_1 S_2 . . . . S_{30}
Potential Plant Location 1 2 . . . 8	Main diagonal cost elements zero - elsewhere set at a prohibitively high level	Selling centre to destination transport costs	K_1 K_2 . . . K_8
Demand	A_1 A_2 .... A_8	D_1 D_2 .... D_{10}	

S_i represents the quantity supplied by the i^{th} region; D_j represents the quantity demand by the j^{th} destination; and K_t is the upper capacity limit on the t^{th} potential selling centre.

[16] A considerable simplification but as this is mainly an expository study, it is considered acceptable. Work currently under way, disaggregates export demand according to overseas demand patterns.

This model differs from the previous models formulated for the solution of similar problems, for example, Cassidy (4) and Ferguson (11) in that transhipment prior to processing is not allowed for. This does not restrict the model since the cost elements in submatrix I of the current model are calculated on the basis of the lowest possible unit transport cost from the i^{th} source to the j^{th} selling centre. Thus a submatrix allowing transhipment prior to arrival at the selling point would be redundant and its non-inclusion leads to greater computational efficiency.

(b) <u>Estimation of regional supply and demand:</u>

The model requires that supplies and demands be specified by region. As no suitable published data were available estimates were made of supplies (initially on a county basis) and a survey undertaken to estimate local demand.

(i) Regional supply:

The Department of Statistics collects and publishes farm production data on a county basis, including sheep numbers which are broken down into breeds every fifth year, but these do not include wool production. The New Zealand Wool Commission record information on sales and export of wool according to wool selling centre districts. Wool could be assumed to come from around each selling centre but this is a risky assumption and in any case does not provide small enough regions.

The procedure used here was to apply the breed breakdown for the latest period available (1966/7) to the latest data on county sheep numbers (1968/9) and multiply sheep in each breed by an average wool weight to give total county wool production.

The Department of Statistics breaks breeds into four main classes (Down type, Merino, $\frac{1}{2}$ bred (including Corriedale), Romney (including long wools and crossbred). Average fleece weights for each breed were based on estimates made by the Economic Service of the Meat & Wool Boards. These estimates give a wool yield range depending on climate, locality and class of country and have been adjusted to give an average for the breed. Figures used were as follows:

Lamb	3 lbs (irrespective of breed or locality)
Down	4 lbs
Merino	7.5 lbs
½ bred	9.5 lbs
Romney	10.1 lbs North Island only
"	10.0 lbs Harder South Island areas
"	10.5 lbs Easier Otago & Southland areas.

These estimates were aggregated on an island basis and when compared with shorn wool sold were found to be highly accurate. County and total production does of course vary from year to year so if a non typical year was used it could provide a location pattern which was not satisfactory over a period. However the year chosen (1968/9) was considered to be representative.

(ii) Regional demand:

There are two sources of demand for New Zealand wool, domestic and export. The latter is assumed to be the difference between total supply and domestic demand. Thus stockpiling of wool by brokers or the Wool Commission is not allowed for. This is regarded as a reasonable assumption because in the former case quantities are small and no trend is discernible and in the latter the wool passes through the auction system in any case.

While data and total internal demand for wool by mills are available from the Department of Statistics and the Wool Commission, these cannot be disaggregated to determine regional demand.

Accordingly a mail survey of all domestic woollen, spinning and carpet mills was undertaken. The response obtained was satisfactory with only one major user of wool not replying. These data, together with the Department of Statistics aggregated data, enabled demand estimates to be made for each region.

About 30 per cent of wool exported is now scoured before shipment. This mostly occurs post-auction. However because of the exploratory nature of this study scouring was not considered. It is conceded that if scouring had been included transport costs post-auction to local mills or to shipside could change.

A further simplification as far as domestic demand is concerned is that wool is considered a homogeneous product. This means that regional demand is met from within the region or adjacent regions. In practice North Island demand for fine wools has to be met by purchases at South Island sales. However as total domestic demand is only about 6-7 per cent of wool sold at auction this simplification was considered warranted.

(c) Selection of basing points:

The point trading model used here assumes that supply and demand accrue at individual points in space. The ideal approach would be to allow every farm (supply) and mill (demand) to enter the model. However this would render the matrix so large as to be unworkable. In fact for workability counties were aggregated into representative regions.

Within each region a point needs to be chosen (a basing point) where all supplies and demands are assumed to occur. Each basing point usually represents a potential selling centre.

Aggregation of counties into regions requires establishment of criteria to define a region. Harris (17) has suggested theoretical criteria as follows:

(i) Homogeneity, implying the underlying conformity of some element in the region.

(ii) Nodality, in which emphasis is placed on a significant location in the region.

(iii) Regions defined by policy. Additional subjective guidelines include

a. All regions should be much the same size so that within region average unit transport costs will be approximately equal.

b. The volume of wool production within a region should be sufficient to warrant the classification of the basing point as a potential wool selling centre. The Wool Buyers' Association and the Wool Commission have indicated that a minimum of 20,000 bales would be required before buyers would service a sale. However local producers would require more than one sale per season so an annual regional production of 100,000 bales is considered a minimum. Nevertheless in some cases regions have been defined on the basis of geographical isolation despite very small supplies, for example the West Coast of the South Island.

c. Basing points are usually taken to be the most prominent town in the central part of the region. The existence of a railway siding or a port may cause the basing point to be located there rather than at the centre of the region, as it is more realistic to do so.

Figure 5 depicts the 30 regions used for this study together with a list of regional basing points.

Table 11 includes data on wool supply and demand for each region.

(d) Transport costs:

Unrestricted carriage of goods in New Zealand is not permitted by the Transport Act of 1962. Licensing of carrying service has as its objectives better co-ordination of road and rail transport, and the prevention of excessive competition and duplication within the transport industry. In general, goods cannot be carried either by commercial carriers or farmers carting their own goods by road between places where a route is available that includes 40 miles or more of railway.

Accordingly the principal method of transport of wool assumed in this study is rail. All but one of the basing points chosen in this study have a railway siding, so that in all but this case wool can be transported from one point to another by rail. (The exception is Nelson where, even though no railway exists, any goods consigned to or from Nelson by rail are charged the railway rates plus a handling charge at Blenheim.)

Although railways connect all basing points, many of these routes are not direct and road transport is legally competitive. (Where the route that includes the railway is longer by more than one-third than the shortest road route available, the rail protection legislation does not apply.) A survey of transport operators in the appropriate areas was conducted and the resulting road transport cost estimates were compared with corresponding rail charges. The lesser of the two was used in the cost matrix.

TABLE 11

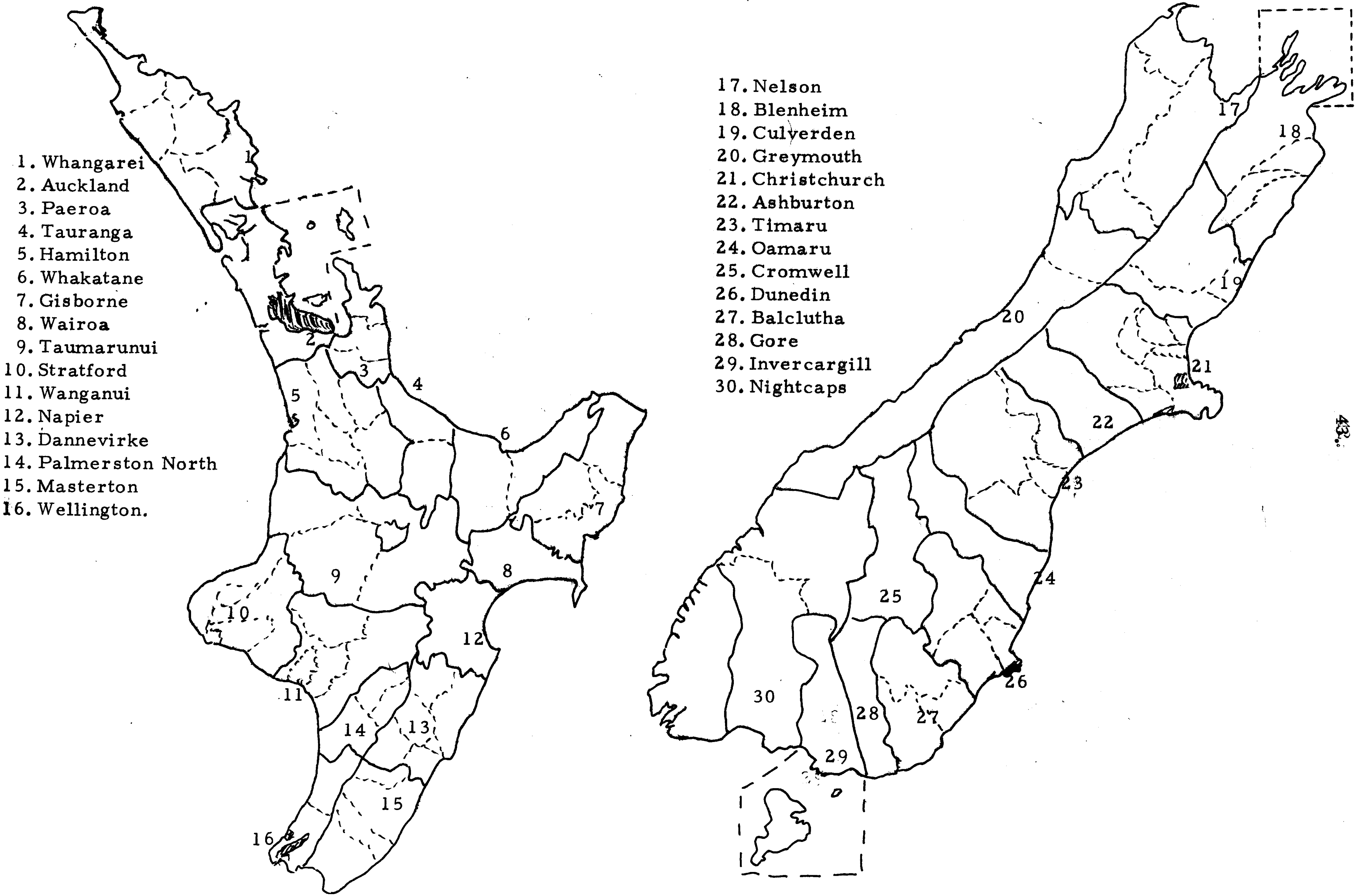
Supplies, demands and basing points for location model

Centre	Supply (100 bale units)	Selling Centre Capacity (100 bale units)	Demand (100 bale units)
Whangarei	386		
Auckland *	425	2,892	117
Paeroa	87		
Hamilton	944		
Tauranga	317		
Whakatane	109		
Taumaranui	874		
Gisborne	628		
Wairoa	233		
Napier *	578	3,701	162
Stratford	426		
Wanganui *	913	1,630	527
Palmerston North	473		
Dannevirke	1,081		
Masterton	642		
Wellington *	120	1,974	21
Nelson	186		
Blenheim	340		
Culverden	392		
Greymouth	50		
Christchurch *	658	2,918	158
Ashburton	586		36
Timaru *	703	1,336	71
Oamaru	230		
Dunedin *	402	2,918	119
Cromwell	162		
Balclutha	834		84
Gore	697		
Nightcaps	694		
Invercargill *	698	2,629	
Export			13,573
Total	14,868		14,868

Note:

1. Selling centre capacities were estimated on the basis of a 50 per cent increase in throughput over the average of the previous three years.
2. Centres marked * are the present wool selling centres.

FIGURE 5 : Regions & Regional Basing Points



(e) The scale curve:

Ideally the scale curves used in plant location models should be based on empirical data. When such information is not available a synthetic approach is a possible alternative. This consists of deriving the required scale curve or curves on the basis of budgeted costs for hypothetical plants of various sizes.

In this study, wool selling centres of up to around 500,000 bales annual throughput had to be allowed for. Since such sizes are far in excess of throughputs of existing New Zealand wool selling centres, the required scale curve had to be synthesized.

In constructing such curves two simplifying assumptions were made.

(i) It was assumed that selling centre costs are independent of the location of the selling centre. Thus, selling centres of the same size in, say, Auckland and Dunedin, have the same unit throughput costs. This may be a significant simplification of the real system, but is regarded as acceptable in this preliminary investigation. This assumption could be dropped if analysis of such costs indicates that there are significant differences.

(ii) The current model is based on the assumption that wool handling and selling methods will not change markedly. This may be an unrealistic assumption in view of current proposed changes within the wool industry.

In constructing budgets, wool selling centre costs were divided into four separate classifications, building costs, equipment costs, labour costs and other costs (materials, administration and so on). These costs were estimated for selling centres of sizes ranging from 50,000 bales annual throughput to 500,000 bales annual throughput. Major economies of scale were found to occur in the area of labour costs, whil no economies of scale were found for building costs.

The following table gives the budgeted per bale cost for selling centres of various sizes.

TABLE 12

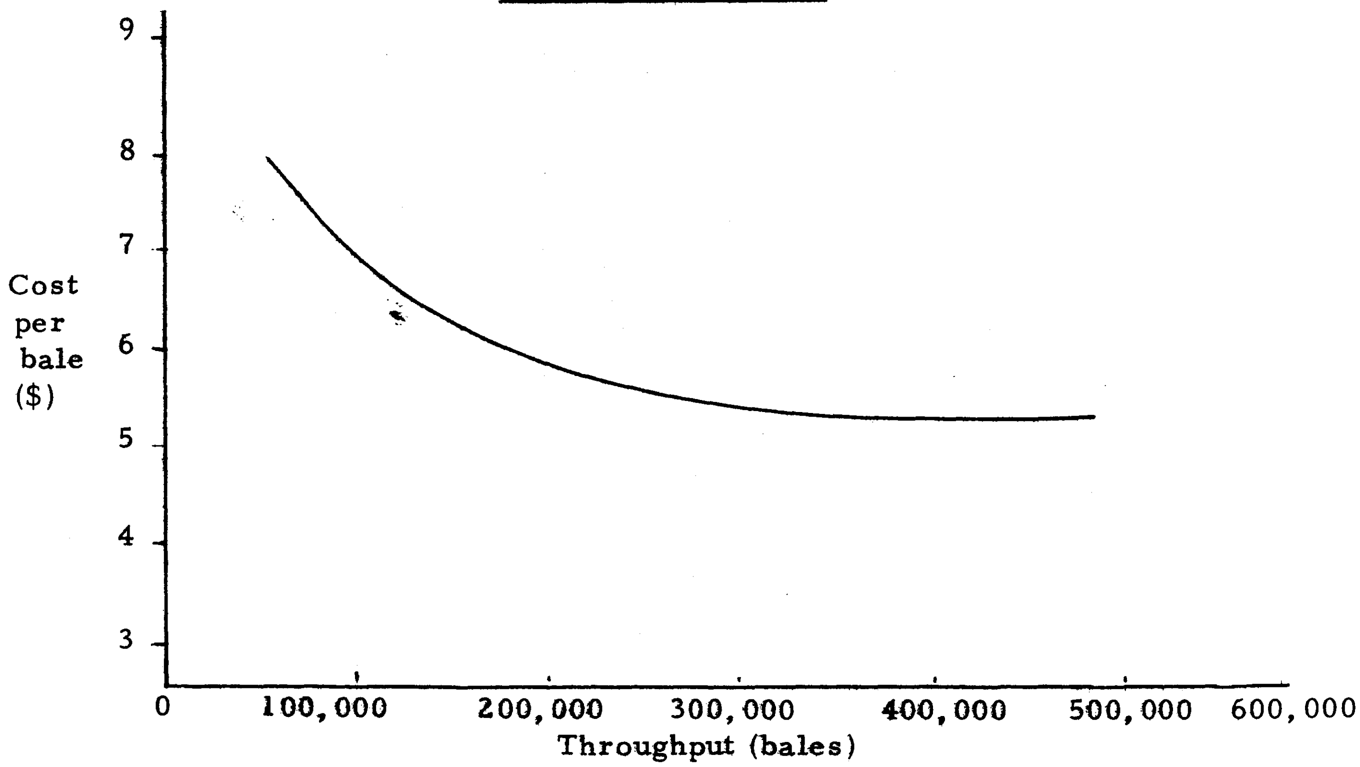
Selling costs per bale for various size centres

Annual Throughput (thous. of bales)	50	100	200	300	400	500
Average Cost/Bale Throughput ($)	7.99	6.76	5.95	5.63	5.54	5.53

As Figure 6 indicates, most economies of scale are exhausted by the 200,000 bale size. There are only small savings above the 400,000 bale size.

FIGURE 6

Average cost curve



When the budgeted per bale costs were graphed against selling centre size, as in Figure 6, the resulting pattern suggested that a curve of the form:

$$Y = \alpha + \frac{\beta}{X}$$

would be the most appropriate. Least squares regression yielded estimates of parameters as follows:

$$\alpha = 5.22$$

$$\beta = 140.62$$

(with throughput expressed in terms of 1,000 bale units).

For example, consider selling centres of various sizes.

Size: 100,000 bales annual throughput

Then Av. cost/bale = $\$(5.22 + \frac{140.62}{100}) = \$6.626$

Size: 200,000 bales annual throughput

Then Av. cost/bale = $\$(5.22 + \frac{140.62}{200}) = \$5.923$

If the budgeted costs were either over- or under-estimated, so long as such costs are not subject to economies of scale, then the optimum spatial pattern yielded by the model is valid even though the estimated total cost of the solution may be in error.

(f) Results & Discussions:

Selling centre size and location obtained from the above data are presented in Table 13.

TABLE 13

Selling centre size and throughput[1]

Selling Centre	Actual Throughput 1968/8	Least Cost Throughput
Auckland	2,506	2,892
Napier	2,426	3,701
Wanganui	1,282	913
Wellington	1,569	730
Christchurch	2,087	1,809
Timaru	1,020	1,336
Dunedin	2,041	1,398
Invercargill	1,937	2,089
Total	14,868	14,868
Cost		12.99 m

[1] Unit = 100 bales.

The results indicate that the selling centres at Auckland, Napier and Timaru enter this solution at the upper limit imposed. This suggests that under the current wool selling system, to minimise transport and handling costs, as much wool as possible should be sold through Auckland, Napier and Timaru. Also, any further investment in additional facilities should occur at these centres rather than at one or more of the remaining locations. To test the validity of this observation, capacity limits on the potential selling centres included in the model were lifted (that is, no upper limit was imposed on the quantity of wool that could be sold through any particular centre). This modified problem yielded the result outlined in Table 14.

TABLE 14

Solution without capacity restraints

Centre	Throughput (100 bale units)
Auckland	2,159
Napier	6,077
Wanganui	Nil
Wellington	Nil
Christchurch	1,626
Timaru	1,519
Dunedin	1,398
Invercargill	2,089
Cost	$12.64 m

These results indicate that, under current wool selling and handling techniques, as much North Island wool as possible should be sold through Auckland and Napier, even to the extent of closing down stores at Wanganui and Wellington. In the South Island as much wool as possible should be shipped through Timaru. Also, any additional investment in wool selling and handling facilities should take place at Napier in the North Island, and Timaru in the South Island.

The result that Napier enters at a level in excess of 600,000 bales may not be a practical solution. A selling centre of this size, particularly at Napier, operating under current selling procedures, may lead to diseconomies of scale. Such a possibility requires further investigation. Certainly with current building and handling facilities at Napier, an annual throughput of 600,000 bales would not be possible.

The cost of $12.99 m of the first solution (capacitated case) is based on the assumption that local demand at Wanganui must be satisfied from a South Island selling centre. If this assumption is dropped, thus allowing local demand at Wanganui to be satisfied from the local selling centre, the cost falls to $12.56 m but the size, number and location of selling centres does not change.

The cost of $12.64 m of the second solution (uncapacitated case) is based on the same assumption. However, when it is dropped, the cost falls to $12.34 m and Wanganui enters the solution at a level of 54,800 bales which in fact satisfies local demand of Wanganui (52,700) and Wellington (2,100) only. This is an unrealistic situation and until further information as to the origin of locally consumed wool is obtained, the assumption of South Island supply should remain.

An indication of cost savings possible is given by comparing actual costs and model estimated costs. New Zealand Wool Board data indicate that costs per bale from farm gate to local mill or shipside average $12. Thus total cost is $17.8 m compared with model costs of $12.99 m or $12.64 m. Hence, in theory, large savings are possible although there would also be heavy capital expenditure involved in changing from present stores to least cost stores.

Finally, be clear that the above results are based on a somewhat simplified model of the wool selling system, including some fairly restrictive assumptions. Policy decisions should not be made on such results. Rather these solutions provide examples of the type of information that can be obtained from plant size and location optimising studies.

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