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Existence Advertising, Price Competition and Asymmetric Market Structure*

B. Curtis Eaton, Ian A. MacDonald, and Laura Meriluoto

Abstract

We examine a two-stage duopoly game in which firms advertise their existence to consumers in stage 1 and compete in prices in stage 2. Whenever the advertising technology generates positive overlap in customer bases, the equilibrium for the stage-1 game is asymmetric in that one firm chooses to remain small in comparison to its competitor. For a specific random advertising technology, we show that one firm will always be half as large as the other. No pure-strategy price equilibrium exists in the stage-2 game, and as long as there is some overlap in customer bases, the mixed-strategy price equilibrium does not converge to the Bertrand equilibrium.

KEYWORDS: existence advertising, price dispersion, Bertrand equilibrium, duopoly

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1 Introduction

Before a customer can contemplate the decision to buy a good from a particular firm he or she must be aware that the firm exists. Indeed, one of the more important functions of a firm's advertising effort is to inform potential customers of its existence. Intuitively, it seems reasonable that firms would want to be known by as many customers as is economically possible.

That this intuition is flawed can be seen by considering a situation in which there are two newly established firms, A and B, capable of serving the market for some new product. Clearly the firms would want to inform potential customers of their existence. If, through advertising, they were to create identical customer bases, price competition between the firms would be fierce, dissipating much or all of the market's potential profit. In contrast, if they were able to create customer bases that did not overlap completely, so that some customers knew only of firm A and others knew only of firm B, both firms would enjoy a degree of market power, and would be able to earn at least some profit. Thus it may make sense for the firms to limit and/or target their advertising efforts if by doing so they are able to manage the overlap in their customer bases. We examine this possibility in a duopoly setting when the firms are able to influence the size and the degree of overlap of their customer bases through their advertising efforts.

Our firms produce identical goods, and their marginal costs are constant and identical. Potential customers have identical demand functions. Any customer who knows of both firms patronizes the firm offering the lower price, and any customer who knows of just one firm patronizes that firm. Competition is modeled as a two-stage game: in stage 1 firms advertise their existence, thereby creating their customer bases and the overlap between them, and in stage 2 they choose prices. We find the subgame perfect Nash equilibrium of the two-stage game using the familiar backward-induction algorithm.

In Section 2 we develop the equilibrium of the stage-2 price game, given arbitrary customer bases and overlap. It is well known that when the overlap of the customer bases of two firms is incomplete, no pure-strategy price equilibrium exists but a mixed-strategy equilibrium does exist (see for example Varian, 1980, Burdett and Judd, 1983, Narasimhan, 1988 and Baye and Morgan, 2001). In the mixed-strategy price equilibrium of our model, the firms randomize over a set of prices where the upper bound is the monopoly price and the lower bound is a function of the monopoly price and the extent of the overlap between the customer bases. The expected equilibrium price is decreasing in the degree of overlap, and increasing in the degree of asymmetry between the sizes of the customer bases.

In Section 3, we examine the firms' stage-1 advertising decisions given an advertising technology that we refer to as the *random advertising technology* – cus-

customer bases are generated by random sampling with replacement from the entire population of potential customers, and the marginal cost of a draw from the population is constant. With the random advertising technology, there are two asymmetric pure-strategy equilibria of the stage-1 customer-base game and in each the customer base of the smaller firm is one half that of the larger firm. It is always the case that the expected prices of both firms in the corresponding stage-2 price equilibrium are large relative to the marginal cost of production – so, with the random advertising technology, by managing the overlap in their customer bases our firms rig an equilibrium that has little resemblance to the Bertrand equilibrium where competition drives price down to marginal cost.

In Section 4 we generalize the asymmetry result, and in Section 5 we present additional results for a special case in which customers have a common reservation price rather than a common demand curve and firms use the random advertising technology.

Our work builds on a pioneering paper that has not received the attention that it deserves. Ireland (1993) finds that when two firms who sell identical goods can costlessly advertise to a population of previously uninformed customers, one firm will advertise to all potential customers and the other firm will advertise to half of them. This results in half of all customers being captive to the large firm and a mixed-strategy price equilibrium where firms put positive probability on all prices between $[\frac{R}{2}, R]$, where R is the reservation price common to all customers. Ireland extends the duopoly model to an n -firm oligopoly and finds that in equilibrium there will be one large firm that advertises to all customers and $(n - 1)$ equally sized smaller firms that advertise to only part of the population. We extend Ireland's duopoly model by examining costly advertising with both a random advertising technology and a general advertising technology. As regards the random advertising technology, we find that in equilibrium the size of either firm's customer base is inversely related to the ratio of the marginal cost of a draw from the population of customers and the per-customer monopoly revenue, and that the larger firm's customer base is always twice that of the smaller firm in a pure-strategy equilibrium. Moreover, the lower bound of the resulting equilibrium price distribution and the magnitude of the expected prices are positively related to this cost-revenue ratio. Ireland's advertising technology is a special case of our random advertising technology in which the marginal cost of an additional draw from the population of customers is zero. As regards the general advertising technology we get a strong result: the pure-strategy customer-base equilibria are asymmetric if there is positive probability that a single customer will be in both customer bases. In other words, if it is not possible to target customers perfectly, the pure-strategy customer-base equilibria are asymmetric.

In our model firms make irreversible advertising decisions in stage 1 that influence the nature of price competition in stage 2. We find that the smaller firm has a strong incentive to, in effect, hide from some of the potential customers in order to manage overlap in customer bases and hence price competition in the stage-2 price game. Others have demonstrated a similar incentive to take action in stage 1 to lessen price competition in stage 2. For example, in Kreps and Scheinkman (1983) firms choose capacities in stage 1 in order to credibly commit to smaller output and therefore above marginal cost prices in stage 2. Similarly, Fudenberg and Tirole (1984) show that an incumbent has an incentive to engage in pre-entry advertising to create loyal customers in order to soften the price competition upon entry. In Gabszewicz and Thisse (1979, 1986), firms choose qualities before entering into price competition, which gives rise to maximum differentiation in qualities in stage 1 and above marginal cost prices in stage 2.

A seminal paper in advertising by Butters (1977) examines a situation where customers become aware of the existence of firms only through random advertising. If informed, customers purchase at most one unit of the good from the firm advertising the lowest price. Butters looks at the cumulative distribution of advertised prices and sale prices in the market, the latter taking into account that each customer chooses the lowest price advertised to her. His focus is on the market and not on the behavior of individual firms, which is our focus. Our equilibrium is asymmetric whereas his is symmetric. Hence, his equilibrium price support, when applied to two firms, is quite different from ours. However, comparative static results for the models are similar in that we both find that the expected transaction price is decreasing in the cost of advertising and that, in the limit as the marginal cost of informing an additional customer approaches zero, all customers buy the good.

McAfee (1994) generalizes Butters by separating the firms' choices of product availability and pricing into a two-stage process. McAfee's product availability can intuitively be thought of as shelf space in stores that are located in malls. McAfee shows that the equilibrium availability rates are asymmetric across firms such that there is one large firm and $(n - 1)$ equally sized small firms and derives the equilibrium mixed strategies for prices. In this way McAfee's model is closer to Ireland and us than it is to Butters. We differ from McAfee in that we focus on how the choice of advertising technology affects the price equilibrium whereas McAfee focuses on cartel formation and mergers. Grossman and Shapiro (1984) examine existence advertising and the resulting price competition when goods are differentiated. Iyer, Soberman and Villas-Boas (2005) study the choice of uniform versus targeted advertising when customers are differentiated in their buying behaviour.

2 The Stage-2 Pricing Problem

If one or both firms in a duopoly have captive customers, it is well known that the price game has no pure-strategy equilibrium but that it does have a mixed-strategy equilibrium. With the exception of a possible mass point at the highest price, the mixed-strategy equilibrium consists of continuous probability density functions over a continuum of prices. Several authors have derived equilibrium price density functions for different models (see for example Narasimhan, 1988, Ireland, 1993 and McAfee, 1994). There are some commonalities. The upper bound of the equilibrium price support is the monopoly price. The lower bound is the price at which the larger firm is indifferent between selling to its captive customers at the monopoly price and selling to all the customers who know of it at the lower bound. Alternatively, the lower bound is equal to the product of the monopoly price and the proportion of captive or loyal customers amongst those who know of the larger firm. All firms randomize over a common set of prices with the exception of the upper bound of the distribution where the larger firm, if one exists, has a mass point and the smaller firm has zero density. No other mass points exist.

Obviously, details of the mixed-strategy equilibrium depend on the assumptions made and especially the notation used. We write the equilibrium mixed strategies used by the firms in our model as functions of the proportions of the two firms' customers that are captive. These proportions depend on the degree of overlap in the firms' customer bases, which in turn depend on the advertising technology used to build the customer bases. Our notation allows us to derive some interesting results about the effect of the advertising technology on the price equilibrium in a straightforward way.

2.1 Notation

There are two firms competing to sell a homogeneous good and many potential customers. Each of these potential customers demands a quantity $Q(p)$ from the firm offering the lowest price amongst the firms whose existence they know of. We assume that the associated per-customer revenue function,

$$R(p) \equiv pQ(p), \tag{1}$$

is continuous and single peaked.¹ Then, letting \bar{p} denote the price at which $R(p)$ attains its maximum value, we see that $R(p)$ is an increasing function of p on the interval $[0, \bar{p}]$.

¹In fact neither continuity nor single peakedness are necessary for the main results we establish in this section, but for expositional purposes they are convenient.

It is convenient to think in terms of a larger and a smaller firm, so we index firms by L and S . The number of customers who know of firms L and S are N_L and N_S , respectively, and the number of customers who know of both firms is M . We assume that $N_L \geq N_S \geq M \geq 0$. The number of customers who know of firm S but not firm L is $N_S - M$ and the number of customers who know of firm L but not firm S is $N_L - M$. The M customers that know of both firms are, of course, *up for grabs*, but the willingness of firms to cut price to grab them is conditioned by the fact that they have captive customers who are unaware of the other firm.

Customers who know of just one firm patronize that firm. Customers who know of both firms patronize the firm with the lower price if prices differ, and if prices are identical they randomly choose one firm or the other, with equal probability. For convenience we assume that the constant marginal costs of the firms are equal to zero. Firms maximize expected profit, which is equal to expected revenue given that marginal cost is zero.

The proportions of the larger and smaller firms' customers who are captive are

$$\lambda_L \equiv \frac{N_L - M}{N_L} \text{ and } \lambda_S \equiv \frac{N_S - M}{N_S}. \quad (2)$$

Define the maximized per-customer monopoly revenue as

$$\bar{R} \equiv R(\bar{p}). \quad (3)$$

Define price \underline{p} as the price that makes the larger firm indifferent between selling to all the customers who know of it at price \underline{p} and selling only to its captive customers at price \bar{p} :

$$R(\underline{p}) \equiv \lambda_L \bar{R}. \quad (4)$$

To derive certain results we consider a special case that we call the *unit-demand* case:

$$Q(p) = \begin{cases} 1 & \text{if } p \leq \bar{R} \\ 0 & \text{if } p > \bar{R}. \end{cases} \quad (5)$$

In the unit-demand case, the per-customer revenue function is

$$R(p) \equiv \begin{cases} p & \text{if } p \leq \bar{R} \\ 0 & \text{if } p > \bar{R}. \end{cases} \quad (6)$$

Both the monopoly price and the per-customer monopoly revenue equal \bar{R} .

2.2 The Mixed-Strategy Price Equilibrium

If the customer bases are the same size ($N_L = N_S$), in the mixed-strategy equilibrium² both firms put positive probability on all prices in $[\underline{p}, \bar{p}]$. If the customer bases are unequal ($N_L > N_S$), the larger firm randomizes on the closed interval $[\underline{p}, \bar{p}]$ with a mass point at $p = \bar{p}$, and the smaller firm randomizes on the open interval $[\underline{p}, \bar{p})$. The equilibrium cumulative density function (CDF) for the smaller firm is

$$F_S(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{1}{1-\lambda_L} \left(1 - \frac{\lambda_L \bar{R}}{R(p)}\right) & \text{if } \underline{p} \leq p \leq \bar{p} \end{cases} \quad (7)$$

and for the larger firm it is

$$F_L(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{1}{1-\lambda_S} \left(1 - \frac{\lambda_L \bar{R}}{R(p)}\right) & \text{if } \underline{p} \leq p < \bar{p} \\ 1 & \text{if } p = \bar{p}. \end{cases} \quad (8)$$

Notice that when $N_S < N_L$, there is a mass point at \bar{p} in the larger firm's density function – the probability that $p_L = \bar{p}$ is $\frac{\lambda_L - \lambda_S}{1 - \lambda_S}$, and the remainder of the probability mass is distributed over the half open interval, $[\underline{p}, \bar{p})$.

The equilibrium expected profits are

$$\Pi_L^* = \lambda_L N_L \bar{R} \quad (9)$$

for the larger firm and

$$\Pi_S^* = \lambda_L N_S \bar{R} \quad (10)$$

for the smaller firm.

2.3 Comments on the Mixed-Strategy Price Equilibrium

Since firms compete over the M customers who know of both firms, comparative statics with respect to M are of considerable interest. Naturally, as M increases expected prices and profits decrease. In fact, expected profits decrease linearly at the rate \bar{R} for the larger firm and at the rate $\frac{N_L}{N_S} \bar{R}$ for the smaller firm. In the limit, as M approaches 0, we have two monopolists with prices equal to \bar{p} enjoying full monopoly profits, $\Pi_L^* = N_L \bar{R}$ and $\Pi_S^* = N_S \bar{R}$. When M approaches its upper bound, N_S , equilibrium profits approach $\Pi_L^* = \left(1 - \frac{N_S}{N_L}\right) N_L \bar{R}$ and $\Pi_S^* = \left(1 - \frac{N_S}{N_L}\right) N_S \bar{R}$.

²The mixed-strategy equilibrium is derived in Appendix A.

If the firms are of equal size, that is if $N_L = N_S = N$, equilibrium profits and prices go to zero as M approaches its upper bound, N . So in the symmetric case, as M transits the $[0, N]$ interval there is a smooth transition from the monopoly outcome where expected prices are \bar{p} to the Bertrand outcome where expected prices are 0.

If the firms are of unequal size, however, as M approaches its upper limit, N_S , equilibrium profits persist and the model does not converge to the Bertrand outcome. For example, if the smaller firm's customer base is half that of the larger firm, equilibrium profits of the two firms converge to one half their monopoly levels as M approaches its upper bound. In fact, $1 - \frac{N_S}{N_L}$ can be regarded as the upper bound on the degree of competitiveness of this model because the profits of the firms can be no less than $\left(1 - \frac{N_S}{N_L}\right)$ percent of full monopoly profits.

Of course, in the larger picture both the sizes of the customer bases (N_L and N_S) and the degree of overlap (M) are endogenous. We turn to this problem in the next section. It is clear, however, that foresighted firms will understand that both overlap and the degree of asymmetry in customer bases have significant impacts on equilibrium prices and will take this into account when choosing customer bases.

For future purposes, let us record some precise results with respect to expected per-customer revenue as well as expected prices in equilibrium.

Result 1. *In equilibrium, the expected per-customer revenue of the smaller firm is*

$$E(R(p_S)) = -\frac{\lambda_L \ln(\lambda_L) \bar{R}}{1 - \lambda_L}, \quad (11)$$

and the expected per-customer revenue of the larger firm is

$$E(R(p_L)) = \frac{(\lambda_L - \lambda_S - \lambda_L \ln(\lambda_L)) \bar{R}}{1 - \lambda_S}. \quad (12)$$

For the unit-demand case, the expected prices of the smaller firm and the larger firm are simply $E(p_S) = E(R(p_S))$ and $E(p_L) = E(R(p_L))$, respectively, the expected minimum price is

$$E(\min(p_L, p_S)) = \frac{\lambda_L \bar{R}}{1 - \lambda_S} \left(2 + \frac{\lambda_L + \lambda_S}{1 - \lambda_L} \ln(\lambda_L) \right),^3 \quad (13)$$

and the expected transaction price is

$$ETP = \frac{\lambda_L (2 - \lambda_L - \lambda_S) \bar{R}}{1 - \lambda_L \lambda_S}.^4 \quad (14)$$

³ $E(\min(p_L, p_S)) = \int_{p_S=p}^{\bar{p}} \int_{p_L=p}^{\bar{p}} (p_L f_L(p_L) (1 - F_S(p_L)) + p_S f_S(p_S) (1 - F_L(p_S))) dp_L dp_S$, where $f_L(p_L) = \frac{\lambda_L}{1 - \lambda_S} \left(\frac{\bar{R}}{p_L^2} \right)$ and $f_S(p_S) = \frac{\lambda_S}{1 - \lambda_L} \left(\frac{\bar{R}}{p_S^2} \right)$.

⁴ $ETP = \frac{N_L - M}{N_L + N_S - M} E(p_L) + \frac{N_S - M}{N_L + N_S - M} E(p_S) + \frac{M}{N_L + N_S - M} E(\min(p_L, p_S))$.

3 The Stage-1 Customer-Base Game with Random Advertising Technology

In our duopoly model, firms compete in a two-stage game: in stage 1 firms create their customer bases and the overlap between them through their advertising efforts, and in stage 2 they choose prices. The mixed-strategy equilibrium presented in the previous section is, of course, the equilibrium of the stage-2 price game. In this section we focus exclusively on the pure-strategy equilibria of the stage-1 game, ignoring any mixed-strategy equilibria that may exist.

We posit a population of size H , composed of *ex ante* identical potential customers. There is an advertising technology that allows firms to manage their customer bases by varying their expenditure on advertising. We assume that initially all the potential customers know nothing about the firms and become aware of them only through the firms' advertising efforts. In this section we work with an advertising technology that allows us to generate closed form solutions for the stage-1 game. We consider more general technologies in Section 4.

3.1 The Random Advertising Technology

We denote the cost of making one's firm known to $N \leq H$ of the potential customers by the function $A(N)$. It is natural to suppose that A is increasing and convex in N and that marginal cost, $A'(N)$, is a decreasing function of population size, H . Given N_L and N_S , what should we assume about M , the overlap in the two customer bases? If both N_L and N_S are small relative to H it seems likely that there will be very little overlap, whereas if both are large relative to H there will inevitably be substantial overlap. It also seems sensible to suppose that the overlap is an increasing function of both N_L and N_S . The random advertising technology we lay out below exhibits these properties.

We assume that customer bases are generated by random drawings with replacement from the entire population of potential customers, and that each draw costs v , where $v \in (0, \bar{R})$. The model is uninteresting when $v \geq \bar{R}$, since the cost of getting a customer is greater than or equal to the maximum revenue the firm could generate from the customer.

Suppose a firm's customer base is initially N . Then, since new customers are generated by drawing with replacement from the population of size H , the probability that an additional draw from the distribution yields a customer that is not already in the firm's customer base is $\frac{H-N}{H}$. The expected number of additional draws needed to get a new customer is $\frac{H}{H-N}$. Since each draw costs v the marginal

cost of an additional customer is $\frac{vH}{H-N}$. That is,

$$A'(N) = \frac{vH}{H-N}.$$

Then, integrating the marginal cost function yields the cost function

$$A(N) = vH[\ln(H) - \ln(H - N)].$$

Notice that $A(N)$ is increasing and strictly convex in N , and that $A'(N)$ is a decreasing function of the population size, H .

Given that customer bases are generated in this manner, any particular person in the population is just as likely to be included in a firm's customer base as is any other person. So, the probability that any person in firm i 's customer base is also included in firm j 's customer base is $\frac{N_j}{H}$ and the expected overlap in customer bases is

$$M = \frac{N_L N_S}{H}. \quad (15)$$

If both N_L and N_S are small relative to H , there will be very little overlap, if both are large relative to H there will be substantial overlap, and the overlap is an increasing function of both N_L and N_S .

3.2 Stage-1 Objective Functions

In our two-stage game, firms simultaneously choose customer bases in stage 1 and price distributions in stage 2. A firm's stage-1 objective function is therefore the expected profit that it will earn in the stage-2 price game minus the cost of creating the customer base in stage 1.

Clearly a firm's expected profit in the stage-2 mixed-strategy equilibrium is a function of the stage-1 choices of customer bases, N_L and N_S . Given the random advertising technology, $M = \frac{N_L N_S}{H}$, so

$$\lambda_L \equiv \frac{N_L - M}{N_L} = 1 - \frac{N_S}{H}.$$

Of course, λ_L is the proportion of people in the larger firm's customer base who are captive – that is, who know nothing of the smaller firm. Notice that λ_L is completely

determined by N_S and H . From (9) and (10), the expected profits of the two firms in the stage-2 price game are then

$$\Pi_S^* = \lambda_L N_S \bar{R} = \left(1 - \frac{N_S}{H}\right) N_S \bar{R} \quad (16)$$

and

$$\Pi_L^* = \lambda_L N_L \bar{R} = \left(1 - \frac{N_S}{H}\right) N_L \bar{R}. \quad (17)$$

For the larger firm, the marginal value of an additional person in its customer base is independent of its choice variable, N_L , whereas for the smaller firm, the marginal value of an additional person in its customer base is a decreasing function of its choice variable, N_S .

Using these results, and changing notation so as to allow either firm to be the larger or the smaller firm, we see that the stage-1 objective function of firm i , $V_i(N_i, N_j)$, is the following:

$$V_i(N_i, N_j) = \begin{cases} \left(1 - \frac{N_i}{H}\right) N_i \bar{R} - A(N_i) & \text{if } N_i \leq N_j \\ \left(1 - \frac{N_j}{H}\right) N_i \bar{R} - A(N_i) & \text{if } N_i \geq N_j. \end{cases} \quad (18)$$

3.3 Concavity of the Stage-1 Objective Functions

In choosing its customer base, firm i must contemplate two regimes, the LT regime where $N_i \leq N_j$ and the GT regime where $N_i \geq N_j$. In the LT regime,

$$\frac{\partial V_i(N_i, N_j)}{\partial N_i} = \left(1 - \frac{2N_i}{H}\right) \bar{R} - \frac{vH}{H - N_i} \quad (19)$$

$$\frac{\partial^2 V_i(N_i, N_j)}{\partial N_i^2} = -\frac{2\bar{R}}{H} - \frac{vH}{(H - N_i)^2} < 0, \quad (20)$$

and in the GT regime,

$$\frac{\partial V_i(N_i, N_j)}{\partial N_i} = \left(1 - \frac{N_j}{H}\right) \bar{R} - \frac{vH}{H - N_i} \quad (21)$$

$$\frac{\partial^2 V_i(N_i, N_j)}{\partial N_i^2} = -\frac{vH}{(H - N_i)^2} < 0. \quad (22)$$

Equations (19)-(22) show that *within* each of these regimes firm i 's objective function is strictly concave in N_i . However, *across* the two regimes, firm i 's objective

function is not concave. When $N_i = N_j$, the the marginal value of N_i in the LT regime in (19) is smaller than its marginal value in the GT regime in (21):

$$\left(1 - \frac{2N_j}{H}\right)\bar{R} - \frac{vH}{H - N_j} < \left(1 - \frac{N_j}{H}\right)\bar{R} - \frac{vH}{H - N_j}. \quad (23)$$

This, of course, means that over the two regimes, firm i 's objective function is not concave. The objective function is illustrated in Figure 1 for four different values of N_j . Firm i 's objective function is continuous in N_i , but it is kinked at $N_i = N_j$; in some neighborhood centered on the kink, $\frac{\partial V_i(N_i, N_j)}{\partial N_i}$ is smaller in the LT regime than it is in the GT regime.

The non-concavity implies that there are no symmetric pure-strategy equilibria. Suppose to the contrary that there was a symmetric pure-strategy equilibrium, $N_i^* = N_j^* = N^*$. For this to be an equilibrium, it must be the case that N^* is a best response to N^* . Supposing that $N_j^* = N^*$, this requires that $\left(1 - \frac{2N^*}{H}\right)\bar{R} \geq \frac{vH}{H - N^*}$, otherwise firm i 's best response in the LT regime would be some $N_i < N^*$, and that $\left(1 - \frac{N^*}{H}\right)\bar{R} \leq \frac{vH}{H - N^*}$, otherwise firm i 's best response in the GT regime would be some $N_i > N^*$. Obviously, it is impossible to satisfy both inequalities simultaneously so we have a result.

Result 2. *Given the random advertising technology, there is no symmetric pure-strategy equilibrium in the customer-base game.*

3.4 The Pure-Strategy Equilibria of the Customer-Base Game

For clarity, we focus on firm i 's best-response function, $BR_i(N_j)$, because the best-response function of the other firm is symmetric. The basic features of $BR_i(N_j)$ are apparent from Figure 1. The derivation is, however, rather cumbersome so we leave it to Appendix B.

Result 3. *The best-response function of firm i is given by*

$$BR_i(N_j) = \begin{cases} \Phi(N_j) & \text{if } N_j \leq \tilde{N} \\ \Delta_1 & \text{if } N_j \geq \tilde{N}. \end{cases} \quad (24)$$

$\Phi(N_j)$ satisfies $\frac{\partial V_i(N_i = \Phi(N_j), N_j)}{\partial N_i} = 0$ in the GT regime and is given by

$$\Phi(N_j) \equiv H \left(1 - \frac{v}{\bar{R}} \frac{H}{H - N_j}\right). \quad (25)$$

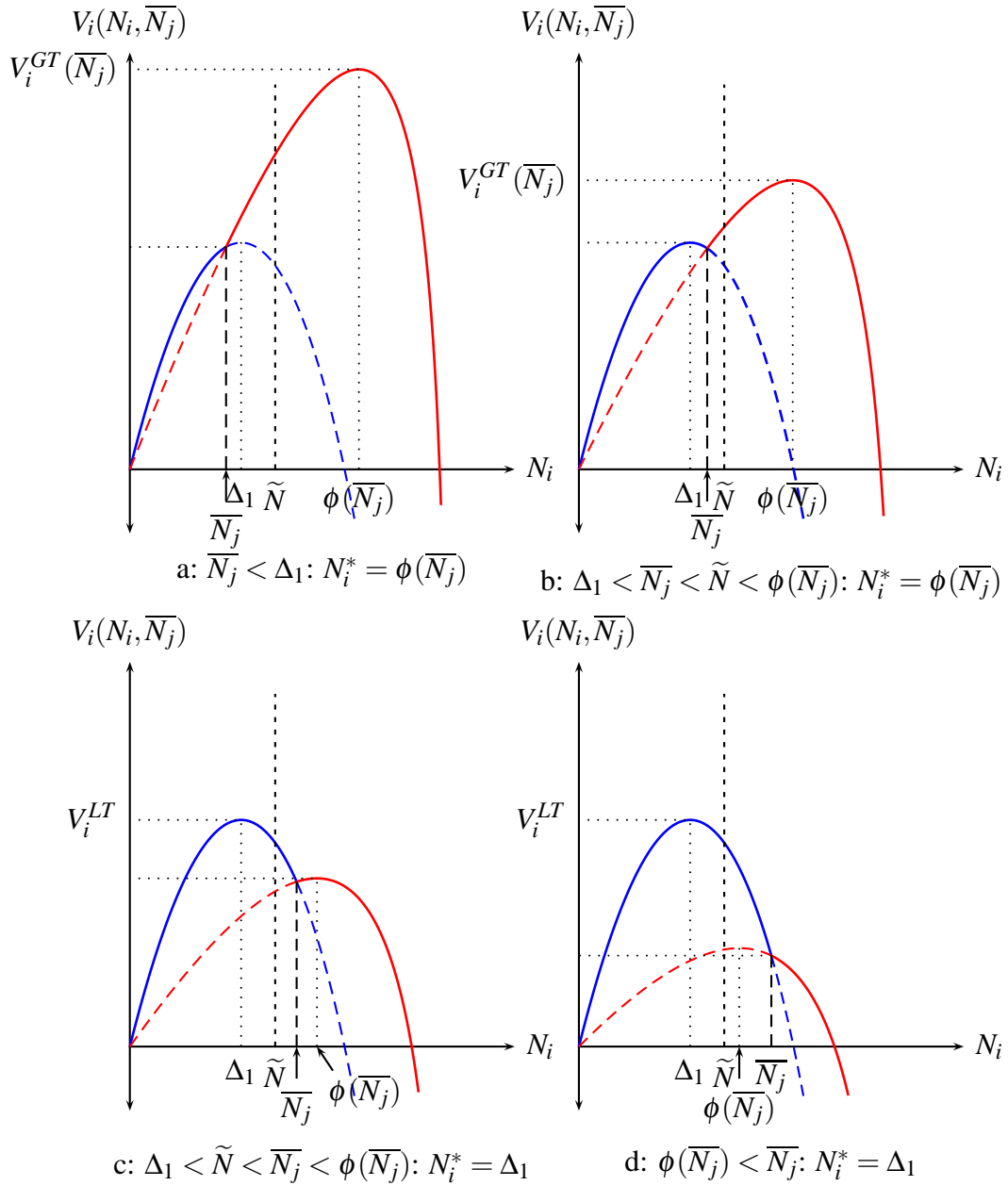


Figure 1: Stage-1 objective function of firm i for four different values of \bar{N}_j . Parts a and b: Firm i chooses $N_i^* = \phi(N_j) > N_j$ when $N_j < \tilde{N}$. Parts c and d: Firm i chooses $N_i^* = \Delta_1 < N_j$ when $N_j > \tilde{N}$.

Δ_1 is the value of N_j that satisfies $\frac{\partial V_i(N_i=N_j, N_j)}{\partial N_i} = 0$ in the LT regime and is given by

$$\Delta_1 \equiv H \left(\frac{3}{4} - \frac{1}{4} \sqrt{1 + 8 \frac{v}{R}} \right). \quad (26)$$

\tilde{N} is the value of N_j such that $V_i^{GT}(\tilde{N}) \equiv V_i^{LT}$ and is implicitly defined by

$$\begin{aligned} \Phi(\tilde{N}) \left(1 - \frac{\tilde{N}}{H} \right) \bar{R} - vH[\ln(H) - \ln(H - \Phi(\tilde{N}))] = \\ \Delta_1 \left(1 - \frac{\Delta_1}{H} \right) \bar{R} - vH[\ln(H) - \ln(H - \Delta_1)]. \end{aligned} \quad (27)$$

When N_j is small (less than \tilde{N}), firm i wants to be large and it maximizes profit by choosing $N_i^* = \phi(N_j)$. This is illustrated in cases a and b in Figure 1. Conversely, when N_j is large (greater than \tilde{N}), firm i wants to be small and it maximizes profit by choosing $N_i^* = \Delta_1$. Notice that Δ_1 is independent of N_j . This is illustrated in cases c and d in Figure 1. \tilde{N} is, of course, the value of N_j such that $V_i^{LT} \equiv V_i^{GT}$.

We have plotted the best-response functions in Figure 2. Notice that there are two pure-strategy equilibria. In each the smaller customer base is Δ_1 and the larger customer base is $\Phi(\Delta_1) = 2\Delta_1$. So, in a pure-strategy equilibrium the larger customer base is twice the smaller customer base.⁵ It is worth noting that one of the equilibria is also the Stackelberg equilibrium in a game where the firms choose their customer bases sequentially in stage 1 and choose prices simultaneously in stage 2. The leader chooses to be the larger firm with customer base $2\Delta_1$ and the follower chooses to be the smaller firm with customer base Δ_1 .

Result 4. *In the subgame perfect pure-strategy equilibrium, the customer base of the smaller firm, N_S^* , is*

$$N_S^* = \Delta_1 = H\Omega, \quad (28)$$

and the customer base of the larger firm, N_L^ , is*

$$N_L^* = 2\Delta_1 = 2H\Omega, \quad (29)$$

where

$$\Omega \equiv \frac{3}{4} - \frac{1}{4} \sqrt{1 + 8 \frac{v}{R}}. \quad (30)$$

⁵When advertising is costless, the larger firm targets the entire population and the smaller firm targets half of the population. This result was shown by Ireland (1993).

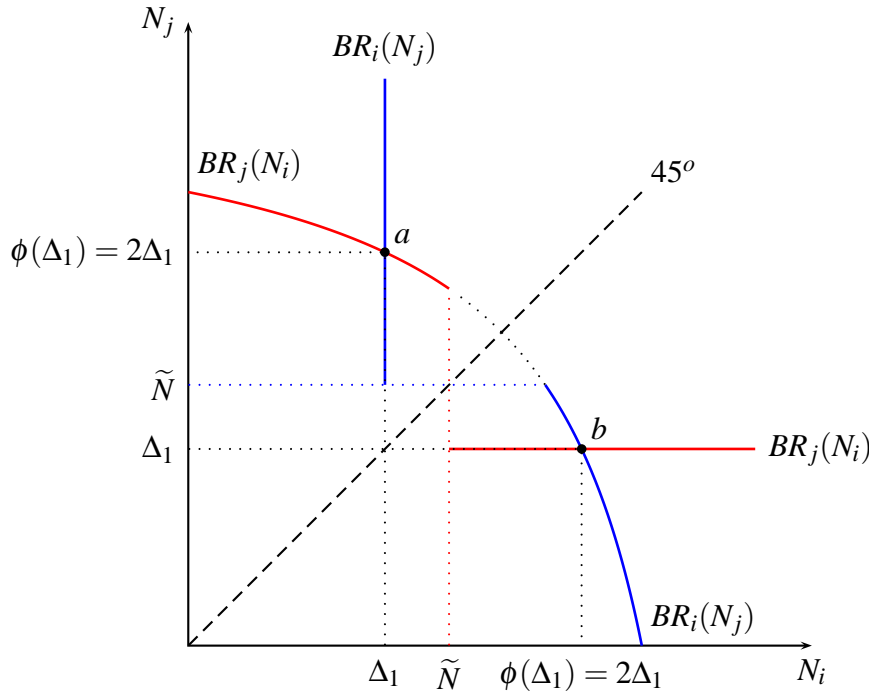


Figure 2: Best-response functions and the two pure-strategy Nash equilibria in the stage-1 customer-base game

The composite parameter Ω is the equilibrium proportion of the total population that is in the smaller firm's customer base. It is inversely related to the ratio $\frac{v}{\bar{R}}$, and given that $0 < \frac{v}{\bar{R}} < 1$, $0 < \Omega < \frac{1}{2}$. Of course, 2Ω is the equilibrium proportion of the total population that is in the larger firm's customer base. As the cost of sending a message to a customer, v , approaches \bar{R} , the ratio $\frac{v}{\bar{R}}$ approaches 1, Ω approaches 0, and both customer bases go to 0. As $\frac{v}{\bar{R}}$ approaches 0, Ω approaches $\frac{1}{2}$ and in this limit the smaller firm targets half the population and the larger firm targets the entire population.⁶

In choosing its customer base, firm i faces conflicting incentives. It can keep the overlap in customer bases small and therefore the expected price high by choosing a small customer base. But it can sell more by choosing a large customer base. When firm j 's customer base is large ($N_j > \tilde{N}$), the first incentive dominates

⁶In Section 5 we show that in the special case of unit demand, the equilibrium is efficient in the sense that the number of customers who are in at least one firm's customer base is equal to the socially optimal number of informed customers.

and firm i chooses to be small to keep the expected price high. And, when firm j 's customer base is small ($N_j < \tilde{N}$), the second incentive dominates and firm i chooses to be large.

3.5 Prices

Now let us explore the stage-2 price equilibrium associated with the equilibrium customer bases. Here, we normalize all measures by dividing them by \bar{R} .

Result 5. *In the subgame perfect equilibrium of this game, the normalized expected per-customer revenues of the smaller firm and the larger firm are*

$$E\left(\frac{R(p_S^*)}{\bar{R}}\right) = -\frac{(1-\Omega)\ln(1-\Omega)}{\Omega} \quad (31)$$

and

$$E\left(\frac{R(p_L^*)}{\bar{R}}\right) = \frac{1}{2} - \frac{(1-\Omega)\ln(1-\Omega)}{\Omega}. \quad (32)$$

With the case of unit demand, the normalized expected prices of the smaller firm and the larger firm are

$$E\left(\frac{p_S^*}{\bar{R}}\right) = -\frac{(1-\Omega)\ln(1-\Omega)}{\Omega} \quad (33)$$

and

$$E\left(\frac{p_L^*}{\bar{R}}\right) = \frac{1}{2} - \frac{(1-\Omega)\ln(1-\Omega)}{\Omega}, \quad (34)$$

the normalized expected minimum price is

$$E\left(\min\left(\frac{p_L^*}{\bar{R}}, \frac{p_S^*}{\bar{R}}\right)\right) = \frac{1-\Omega}{2\Omega} \left(2 + \frac{(2-3\Omega)\ln(1-\Omega)}{\Omega}\right), \quad (35)$$

the normalized expected transaction price is

$$\frac{ETP^*}{\bar{R}} = \frac{3(1-\Omega)}{3-2\Omega}, \quad (36)$$

and the lower bound on normalized prices is

$$\lambda_L^* = 1 - \frac{N_S^*}{H} = 1 - \Omega. \quad (37)$$

The lower bound on normalized prices conveys the flavor of these results well and simply. In the unit-demand case, prices are never less than $1 - \Omega$. Given that $0 < \Omega < \frac{1}{2}$, we see that $\frac{1}{2} < \lambda_L^* < 1$ and in equilibrium normalized prices are never less than $\frac{1}{2}$. As $\frac{v}{R}$ approaches 0, both Ω and the lower bound on the price support approach $\frac{1}{2}$. This seems to be an interesting result because in this limit both advertising and production are costless yet the normalized equilibrium prices are never less than $\frac{1}{2}$. The contrast with the standard Bertrand model, where normalized price would be 0 in equilibrium, is sharp.

In Result 5, normalized per-customer revenues and prices are expressed as functions of the composite parameter Ω , but Ω is itself completely determined by the ratio $\frac{v}{R}$. Naturally, each of the normalized per-customer revenues and prices reported in Result 5 are increasing functions of $\frac{v}{R}$. Table 1 presents detailed results for the unit-demand case.

4 Choosing Customer Bases with a General Advertising Technology

The incentive to avoid overlap in customer bases drives the asymmetry of the equilibrium we found in the previous section. The degree of overlap that results from a particular level of advertising is, in turn, determined by the advertising technology. This raises an obvious question: what can we say about the set of advertising technologies that generate the asymmetry result? We will show that asymmetry in the pure-strategy equilibria of the customer-base game is a very general result.

Let $M(N_1, N_2)$ denote the overlap associated with an arbitrary advertising technology. At the most general level, the only *a priori* restrictions on $M(N_1, N_2)$ would seem to be that overlap is non-negative and that overlap is non-decreasing in the sizes of the customer bases.

Consider the expected profit of firm 1 in the stage-2 price game. In the LT regime (where $N_1 < N_2$), firm 1 is the smaller firm so its stage-2 profit is $\lambda_2 N_1 \bar{R}$, and in the GT regime (where $N_1 > N_2$) it is the larger firm so its stage-2 profit is $\lambda_1 N_1 \bar{R}$. Of course, $\lambda_1 = 1 - \frac{M(N_1, N_2)}{N_1}$ and $\lambda_2 = 1 - \frac{M(N_1, N_2)}{N_2}$, so the stage-2 equilibrium profit is

$$\Pi_1^* = \begin{cases} \left(1 - \frac{M(N_1, N_2)}{N_2}\right) N_1 \bar{R} & \text{if } N_1 \leq N_2 \\ \left(1 - \frac{M(N_1, N_2)}{N_1}\right) N_1 \bar{R} & \text{if } N_1 \geq N_2. \end{cases} \quad (38)$$

Let $A(N_1)$ denote firm 1's advertising cost for a customer base of size N_1 , and assume that $A(N_1)$ is differentiable. Firm 1's stage-1 objective function is simply

$$V_1 = \begin{cases} \left(1 - \frac{M(N_1, N_2)}{N_2}\right) N_1 \bar{R} - A(N_1) & \text{if } N_1 \leq N_2 \\ \left(1 - \frac{M(N_1, N_2)}{N_1}\right) N_1 \bar{R} - A(N_1) & \text{if } N_1 \geq N_2. \end{cases} \quad (39)$$

Differentiating V_1 with respect to N_1 gives us two necessary conditions for the existence of a symmetric pure-strategy equilibrium. The partial derivative of V_1 with respect to N_1 , evaluated at $N_1 = N_2 = N$, must be non-negative in the LT regime and non-positive in the GT regime:

$$\bar{R} - \frac{\bar{R}M(N, N)}{N} - \frac{\partial M(N, N)}{\partial N_1} \bar{R} - A'(N) \geq 0 \quad (40)$$

$$\bar{R} - \frac{\partial M(N, N)}{\partial N_1} \bar{R} - A'(N) \leq 0. \quad (41)$$

If $M(N, N) > 0$, it is impossible to satisfy both of these conditions simultaneously.

Result 6. *If the advertising technologies used by the firms generate positive overlap, there is no symmetric pure-strategy equilibrium in the customer-base game.*

The left-hand-sides of (40) and (41) show clearly that the partial derivative of V_i with respect to N_i at $N_i = N_j = N$ is always smaller on the LT side than on the GT side as long as $M(N, N) > 0$ and therefore that V_i is not concave at that point. The implication of this is that firm i will either choose $N_i < N_j$ or $N_i > N_j$ as was the case with our random advertising technology.

While equilibrium customer bases remain asymmetric for any advertising technology that generates overlap, it seems intuitive that the extent of the asymmetry of the equilibrium decreases as the advertising technology allows for a better selection to avoid overlap.

5 Further Insights

In this section we discuss two further issues – the efficiency of the equilibrium in the stage-1 customer-base game and the incentive for customers to engage in search. We use a special case of our model where the firms' advertising technology is random and consumers have unit demands.

5.1 Efficiency of Equilibrium in the Customer-Base Game

Since demand is perfectly inelastic with respect to price up to the reservation price \bar{R} , the only efficiency issue concerns the number of unique customers in the aggregate customer base. Call this number T (for total). The marginal cost of increasing the aggregate customer base is just $\frac{vH}{H-T}$ since the probability that an additional draw yields a customer not already in the customer base is $\frac{H-T}{H}$ and, therefore, the expected number of draws needed to uncover someone new is $\frac{H}{H-T}$. To maximize total surplus, T must be chosen so that this marginal cost is equal to the reservation price, \bar{R} , since \bar{R} is the surplus that is generated when a new person enters the aggregate customer base. Solving this condition we get the optimal size of the aggregate customer base, T^* :

$$T^* = H \left(1 - \frac{v}{\bar{R}} \right). \quad (42)$$

Obviously, a monopolist would choose price \bar{R} for everyone in its customer base. So to maximize its profit a monopolist would choose its customer base N_m to equate the marginal cost $\frac{vH}{H-N_m}$ to \bar{R} . Therefore, $N_m^* = T^*$ and we see that the monopoly equilibrium is efficient. Since the monopolist captures all of the consumer surplus and incurs all of the costs of making customers aware of its product, this result is not surprising.

What about the duopoly equilibrium? When overlap in the customer bases is taken into account we see that the number of people in at least one customer base is

$$N_S^* + N_L^* - \frac{N_S^* N_L^*}{H} = H\Omega + 2\Omega H - \frac{2\Omega^2 H^2}{H} = H(3\Omega - 2\Omega^2). \quad (43)$$

Then a bit of algebra establishes, perhaps surprisingly, that the duopoly equilibrium is also efficient:

$$H(3\Omega - 2\Omega^2) = T^*. \quad (44)$$

Result 7. *Both the monopoly and duopoly equilibria are efficient, in the sense that social welfare is maximized.*

We can get some insight into the efficiency result for the duopoly equilibrium by working with the first order conditions for the larger firm's profit-maximizing problem and the planner's total surplus-maximizing problem. We show that, given any permissible N_S , the larger firm's profit-maximizing choice results in an aggregate customer base that is efficient.

Given N_S , to maximize its profit, the larger firm chooses N_L to equate the marginal cost and the marginal revenue of increasing its customer base:

$$\frac{vH}{H - N_L} = \left(1 - \frac{N_S}{H}\right) \bar{R}. \quad (45)$$

On the other hand, to maximize social surplus, the planner chooses T to equate the marginal social cost and the marginal social benefit of increasing the total customer base:

$$\frac{vH}{H - T} = \bar{R}. \quad (46)$$

Comparing equations (45) and (46), we see that the larger firm's marginal revenue is a fraction $\left(1 - \frac{N_S}{H}\right)$ of the social marginal benefit (\bar{R}). In light of this, the efficiency result is somewhat surprising. Notice, however, that the marginal cost of adding a new person to the larger firm's customer base is less than the marginal cost of adding a new person to the total customer base – the number of people who are not in the larger firm's customer base ($H - N_L$) is larger than the number who are not in the total customer base ($H - T$), so the larger firm requires fewer draws than the social planner to add a new person to the relevant customer base. With the random advertising technology, $T = N_L + N_S - \frac{N_L N_S}{H}$. Using this fact, one can show that the larger firm's marginal cost is the *same fraction* $\left(1 - \frac{N_S}{H}\right)$ of the social marginal cost ($\frac{vH}{H-T}$); that is

$$\left(1 - \frac{N_S}{H}\right) \frac{vH}{H - T} = \frac{vH}{H - N_L}. \quad (47)$$

Hence, for any N_S , the larger firm's profit-maximizing choice yields the socially optimal total customer base.

5.2 Incentive to Search

Our model is motivated by the observation that in some circumstances not all customers know of the existence of all firms. Surely this is not an uncommon occurrence, so we have modeled the existence-advertising and pricing problems that it raises. But in the equilibrium of our model some customers have something to gain by learning about the existence of firms and in this sense these customers have an incentive to search. Clearly, if customers recognize and act on this incentive the equilibrium will be upset. This is the issue we discuss here.

The usual set-up found in the search literature involves a large number of price setting firms and a large number of customers. Customers engage in either non-sequential search (Stigler, 1961, Burdett and Judd, 1983) or sequential search

(McCall, 1965, Nelson, 1970, Burdett and Judd, 1983) to locate a low price. The search costs of customers determine the intensity of the search as well as the price equilibrium. Common to all these models are the assumptions that customers are aware of the existence of all firms and the distribution of the prices charged by the firms but, that in the absence of search, they are unable to match any price in the distribution with a particular firm. This contrasts sharply with our model where, through the existence-advertising efforts of firms, some customers become aware of the existence and price of one of the firms, others become aware of the existence and price of both firms, and yet others remain unaware of the existence of either firm. It is not at all clear how to model search in our framework. How does a person go about finding a firm the very existence of which the person is not aware? How does one calculate the possible gains from finding such a firm? Further, from our analysis it is quite clear that beyond some point the smaller firm has an incentive to frustrate the attempts of customers to search it out because successful search generates overlap, and overlap leads to the dissipation of profit through price competition. A more complete model would perhaps include both customer search and its obstruction by the smaller firm.

Although we are not in a position to formally model customer search, we can say a little bit about the incentives. In the equilibrium there are four categories of customers: those who know of the smaller firm but not the larger firm; those who know of the larger firm but not the smaller firm; those who know of neither firm; and those who know of both firms. Customers in all but the last category have something to gain from search. Using Result 5, we can quantify potential gains. Those who know of neither firm have the most to gain – a successful search that allowed them to identify one firm would yield a surplus equal to

$$B_0 = \bar{R} \left(1 - \frac{ETP^*}{\bar{R}} \right). \quad (48)$$

From column B_0 in Table 1, we see that this can be as large as $.25\bar{R}$, but is a modest $.07\bar{R}$ when $\frac{v}{\bar{R}} = .5$. Those who know of the larger firm but not the smaller one have the second highest incentive to search – a successful search that allowed them to identify the smaller firm would yield a surplus equal to

$$B_L = \bar{R} \left(E \left(\frac{P_L^*}{\bar{R}} \right) - E \left(\min \left(\frac{P_L^*}{\bar{R}}, \frac{P_S^*}{\bar{R}} \right) \right) \right). \quad (49)$$

From column B_L in Table 1 we see that this could be as large as $.2\bar{R}$, but is only $.07\bar{R}$ when $\frac{v}{\bar{R}} = .5$. Finally, we see from column B_S in Table 1 that the incentive to search for those who know only of the smaller firm, equal to

$$B_S = \bar{R} \left(E \left(\frac{P_S^*}{\bar{R}} \right) - E \left(\min \left(\frac{P_L^*}{\bar{R}}, \frac{P_S^*}{\bar{R}} \right) \right) \right), \quad (50)$$

is no more than $.04\bar{R}$. Obviously, if the cost of search is large relative to its expected benefit there will be no search to upset the equilibrium. This will be the case when $\frac{v}{\bar{R}}$ is sufficiently large because the expected benefit of search is inversely related to $\frac{v}{\bar{R}}$ and goes to zero as $\frac{v}{\bar{R}}$ approaches 1.

Table 1: Normalized expected prices and benefit of search

$\frac{v}{\bar{R}}$	$E\left(\frac{p_S^*}{\bar{R}}\right)$	$E\left(\frac{p_L^*}{\bar{R}}\right)$	$\frac{ETP^*}{\bar{R}}$	$E\left(\min\left(\frac{p_L^*}{\bar{R}}, \frac{p_S^*}{\bar{R}}\right)\right)$	B_0	B_L	B_S
.001	.69	.85	.75	.65	.25	.19	.04
.10	.76	.88	.81	.72	.19	.16	.03
.20	.80	.90	.85	.77	.15	.13	.03
.30	.84	.92	.88	.82	.12	.10	.02
.40	.87	.94	.91	.85	.09	.08	.02
.50	.90	.95	.93	.88	.07	.07	.02
.60	.92	.96	.95	.91	.05	.05	.01
.70	.94	.97	.96	.94	.04	.04	.01
.80	.96	.98	.98	.96	.02	.02	.01
.90	.98	.99	.99	.98	.01	.01	.00
.99	1.00	1.00	1.00	1.00	.00	.00	.00

6 Conclusions

We have investigated existence advertising using a two-stage game in which profit-maximizing firms advertise to manage their customer bases in stage 1 and choose prices in stage 2. There are two firms and they produce and sell a homogeneous good.

In the stage-2 price game, as long as one firm has some customers that know only of it and not of its competitor, there is no pure-strategy equilibrium, but there is a mixed-strategy equilibrium. When there is imperfect or incomplete overlap between customer bases, in the mixed-strategy equilibrium both firms randomize over prices that are strictly above marginal cost. Equilibrium expected prices and profits are linearly and inversely related to the size of the overlap in the customer bases.

In the stage-1 customer-base game, as long as the advertising technology generates any overlap in customer bases there is no symmetric pure-strategy equilibrium. There are, however, two asymmetric pure-strategy equilibria. Because the profit of both firms in the stage-2 price equilibrium is negatively affected by overlap, the firms have clear incentives to limit advertising as a way of managing overlap in the customer bases, and this incentive is stronger for the smaller firm. With the

random advertising technology, the asymmetry in equilibrium customer bases is pronounced – the larger firm’s customer base is twice that of the smaller firm – and firms enjoy substantial positive profits. In the special case where customers have unit demands and the advertising technology is random, the equilibrium is efficient.

A Proof of the Mixed-Strategy Equilibrium

Let $\Pi_i(p_i|F_j)$ ($i \in \{S, L\}, j \neq i$) denote firm i ’s expected profit, given price p_i and the other firm’s CDF, F_j , and let Π_i^* denote the expected equilibrium profit of firm i . A mixed-strategy equilibrium is characterized by the following properties:

- P1:** if $f_i(p_i) > 0$, then $\Pi_i(p_i|F_j) = \Pi_i^*$
- P2:** if $f_i(p_i) = 0$, then $\Pi_i(p_i|F_j) \leq \Pi_i^*$
- P3:** if $\Pi_i(p_i|F_j) < \Pi_i^*$, then $f_i(p_i) = 0$.

In words: the prices that get positive probability in firm i ’s equilibrium density function all yield profit Π_i^* ; all other prices yield an expected profit that is no larger than Π_i^* ; and all prices that yield an expected profit that is less than Π_i^* get zero probability in firm i ’s equilibrium density function.

We first establish some useful results based on the assumption that a mixed-strategy equilibrium exists, and then go on to find one. Notice that for any F_S , $\Pi_L(\bar{p}, F_S) \geq \lambda_L N_L \bar{R}$, because when $p_L = \bar{p}$ the number of customers who patronize the larger firm is no smaller than $\lambda_L N_L$ and revenue per customer is \bar{R} . This establishes a lower bound for the larger firm’s profit in any mixed-strategy equilibrium: $\Pi_L^* \geq \lambda_L N_L \bar{R}$.

Next we show that \underline{p} is a lower bound on the support of the larger firm’s DF. If $p_L < \underline{p}$, then for any F_S , $\Pi_L(p_L|F_S) < N_L R(\underline{p}) = \lambda_L N_L \bar{R} \leq \Pi_L^*$. The strict inequality follows because the number of customers who patronize the larger firm is no larger than N_L and revenue per customer is less than $R(\underline{p})$, the equality follows from the definition of \underline{p} , and the weak inequality was established in the previous paragraph. Property P3 then dictates that, in any mixed-strategy equilibrium, $f_L(p_L) = 0$ for all $p_L < \underline{p}$.

To establish similar results for the smaller firm, assume that $f_L(p_L) = 0$ for all $p_L < \underline{p}$, as must be the case in any mixed-strategy equilibrium. Given this assumption, if $p_S < \underline{p}$, then $\Pi_S(p_S|F_L) = R(p_S)N_S$, which is strictly increasing in p_S . Notice that the limit of $R(p_S)N_S$ as p_S approaches \underline{p} from below is $R(\underline{p})N_S = \lambda_L N_S \bar{R}$, so $\Pi_S^* \geq \lambda_L N_S \bar{R}$. Then, from Property P3 we see that in any mixed-strategy equilibrium, $f_S(p_S) = 0$ for all $p_S < \underline{p}$ (since for any such price $\Pi_S(p_S|F_L) < \lambda_L N_S \bar{R}$).

This suggests that there is a mixed-strategy equilibrium in which $\Pi_L^* = \lambda_L N_L \bar{R}$, $\Pi_S^* = \lambda_L N_S \bar{R}$, and that the prices that get positive probability are in $[\underline{p}, \bar{p}]$.

To prove that the cumulative price density functions in (7) and (8) constitute a mixed-strategy equilibrium, we must verify that properties P1, P2 and P3 set out above are satisfied for both firms. We begin with the larger firm. If $p_L > p_S$, the larger firm's profit is $\lambda_L N_L R(p_L)$ since the customers who know of both firms choose to buy from the smaller firm. The probability that $p_L > p_S$ is just $F_S(p_L)$. On the other hand, if $p_L < p_S$, the larger firm's profit is $N_L R(p_L)$ since the customers who know of both firms now choose to buy from the larger firm, and the probability that $p_L < p_S$ is just $1 - F_S(p_L)$. Since there are no mass points in $f_S(p_S)$, we can ignore the case where $p_L = p_S$. Then, the larger firm's expected profit is just

$$\Pi_L(p_L|F_S) = F_S(p_L)\lambda_L N_L R(p_L) + (1 - F_S(p_L))N_L R(p_L) \text{ for all } 0 \leq p_L \leq \bar{p}. \quad (51)$$

It is straightforward to verify the following:

$$\Pi_L(p_L|F_S) = \lambda_L N_L \bar{R} \text{ for all } \underline{p} \leq p_L \leq \bar{p} \quad (52)$$

$$\Pi_L(p_L|F_S) < \lambda_L N_L \bar{R} \text{ for all } 0 \leq p_L < \underline{p}. \quad (53)$$

It is then clear that properties P1, P2 and P3 are satisfied for the larger firm.

Given the mass point in the density function of the larger firm, the smaller firm's expected profit given p_S and F_L is

$$\Pi_S(p_S|F_L) = \begin{cases} F_L(p_S)\lambda_S N_S R(p_S) + (1 - F_L(p_S))N_S R(p_S) & \text{for all } 0 \leq p_S < \bar{p} \\ \frac{1-\lambda_L}{1-\lambda_S}\lambda_S N_S R(p_S) + \frac{\lambda_L-\lambda_S}{1-\lambda_S}\frac{1+\lambda_S}{2}N_S R(p_S) & \text{for } p_S = \bar{p}. \end{cases} \quad (54)$$

It is straightforward to verify the following⁷:

$$\Pi_S(p_S|F_L) = \lambda_L N_S \bar{R} \text{ for all } \underline{p} \leq p_S < \bar{p} \quad (55)$$

$$\Pi_S(p_S|F_L) < \lambda_L N_S \bar{R} \text{ for } p_S = \bar{p} \quad (56)$$

$$\Pi_S(p_S|F_L) < \lambda_L N_S \bar{R} \text{ for all } 0 \leq p_S < \underline{p}. \quad (57)$$

It is then clear that properties P1, P2 and P3 are satisfied for the smaller firm. *QED*.

B Best-Response Functions

In this appendix we derive the best-response functions expressed in Result 3. First we arbitrarily restrict firm i to one of the two regimes, and find for each regime a

⁷The second condition holds with equality if $N_S = N_L$.

restricted best-response function, $BR_i^{LT}(N_j)$ for the LT regime and $BR_i^{GT}(N_j)$ for the GT regime. Then we splice these restricted best-response functions to get the actual or unrestricted best-response function, $BR_i(N_j)$.

In the LT regime,

$$\frac{\partial V_i(N_i, N_j)}{\partial N_i} = \left(1 - \frac{2N_i}{H}\right) \bar{R} - \frac{vH}{H - N_i}. \quad (58)$$

Notice that because $\bar{R} > v$, $\frac{\partial V_i(N_i=0, N_j)}{\partial N_i} > 0$, so firm i always chooses $N_i > 0$. Then, given the concavity of the objective function within the LT regime, if $\frac{\partial V_i(N_i=N_j, N_j)}{\partial N_i} \geq 0$ firm i 's maximizing choice is $N_i = N_j$, and if $\frac{\partial V_i(N_i=N_j, N_j)}{\partial N_i} < 0$ firm i 's maximizing choice is the N_i such that $\frac{\partial V_i(N_i, N_j)}{\partial N_i} = 0$, or $N_i = \Delta_1$ in (26). The best-response function for the LT regime is therefore

$$BR_i^{LT}(N_j) = \begin{cases} N_j & \text{if } N_j \leq \Delta_1 \\ \Delta_1 & \text{if } N_j > \Delta_1. \end{cases} \quad (59)$$

The maximized objective function for the interior solution ($BR_i^{LT}(N_j) = \Delta_1$) is

$$V_i^{LT} = \Delta_1 \left(1 - \frac{\Delta_1}{H}\right) \bar{R} - vH[\ln(H) - \ln(H - \Delta_1)]. \quad (60)$$

In the GT regime,

$$\frac{\partial V_i(N_i, N_j)}{\partial N_i} = \left(1 - \frac{N_j}{H}\right) \bar{R} - \frac{vH}{H - N_i}. \quad (61)$$

Because $v > 0$, $\frac{\partial V_i(N_i, N_j)}{\partial N_i}$ approaches negative infinity as N_i approaches H , so firm i always chooses $N_i < H$. Given the concavity of the GT objective function, if $\frac{\partial V_i(N_i=N_j, N_j)}{\partial N_i} \leq 0$ firm i 's maximizing choice is $N_i = N_j$, and if $\frac{\partial V_i(N_i=N_j, N_j)}{\partial N_i} > 0$ firm i 's maximizing choice is the N_i such that $\frac{\partial V_i(N_i, N_j)}{\partial N_i} = 0$, or $N_i = \Phi(N_j)$ in (25).

To make further progress in identifying the actual best-response function, it is useful to define a composite parameter, Δ_2 :

$$\Delta_2 \equiv H \left(1 - \sqrt{\frac{v}{\bar{R}}}\right). \quad (62)$$

Δ_2 is the value of N_j such that $\frac{\partial V_i(N_i=N_j, N_j)}{\partial N_i} = 0$. The size of Δ_2 relative to $\Phi(N_j)$ in (25) is easy to establish: when $N_j < \Delta_2$, $\Delta_2 < \Phi(N_j)$, when $N_j > \Delta_2$, $\Delta_2 > \Phi(N_j)$

and when $N_j = \Delta_2$, $\Delta_2 = \phi(N_j)$. A bit of algebra establishes the following useful inequalities:

$$0 < \Delta_1 < \Delta_2 < H. \quad (63)$$

The best-response function for the GT regime is now

$$BR_i^{GT}(N_j) = \begin{cases} N_j & \text{if } N_j \geq \Delta_2 \text{ (or if } N_j \geq \phi(N_j)) \\ \Phi(N_j) & \text{if } N_j < \Delta_2 \text{ (or if } N_j \leq \phi(N_j)). \end{cases} \quad (64)$$

The maximized objective function for the interior solution ($BR_i^{GT}(N_j) = \Phi(N_j)$) is

$$V_i^{GT}(N_j) = \Phi(N_j) \left(1 - \frac{N_j}{H} \right) \bar{R} - vH[\ln(H) - \ln(H - \Phi(N_j))]. \quad (65)$$

Now let us splice the restricted best-response functions to get the actual best-response function, $BR_i(N_j)$. When $N_j < \Delta_1$, N_j is so small that if forced to be in the LT regime firm i would choose $N_i = N_j$. But, it will not voluntarily choose the LT regime, because the non-concavity in $V_i(N_i, N_j)$ at $N_i = N_j$ means that it gets an even larger profit by choosing $N_i > N_j$ in the interior of the GT regime. Hence, when $N_j < \Delta_1$, $BR_i(N_j) = \Phi(N_j)$. This case is illustrated in part a of Figure 1 where we have plotted $V_i(N_i, N_j)$ holding N_j fixed at a value less than Δ_1 . Notice that $V_i(N_i, N_j)$ has a single local maximum, in the interior of the GT regime, where $N_i = \Phi(N_j)$.

When $N_j > \Delta_2$, the story is similar. In this case, N_j is so large that if forced to be in the GT regime firm i would choose $N_i = N_j$. But, it will not voluntarily choose the GT regime, because the non-concavity in $V_i(N_i, N_j)$ at $N_i = N_j$ means that it gets an even larger profit by choosing $N_i = \Delta_1 < N_j$ in the interior of the LT regime. Hence, when $N_j > \Delta_2$, $BR_i(N_j) = \Delta_1$. This case is illustrated in part d of Figure 1—notice that in this case there is a single local maximum in the interior of the LT regime.

The situation is a bit trickier when $\Delta_1 \leq N_j \leq \Delta_2$. In this case, illustrated in parts b and c of Figure 1, N_j is large enough so that $V_i(N_i, N_j)$ has a local maximum in the interior of the LT regime (at $N_i = \Delta_1$), and small enough so that it has a local maximum in the interior of the GT regime (at $N_i = \Phi(N_j)$). So $BR_i(N_j)$ is either Δ_1 in the LT regime, or $\Phi(N_j)$ in the GT regime, depending on which option yields the larger payoff. As V_i^{LT} in (60) is independent of N_j , whereas $V_i^{GT}(N_j)$ in (65) is a continuous, decreasing function of N_j , and because $V_i^{GT}(N_j) > V_i^{LT}$ when $N_j = \Delta_1$ and $V_i^{GT}(N_j) < V_i^{LT}$ when $N_j = \Delta_2$, we know that there must exist a value of $N_j = \tilde{N}$ such that $V_i^{GT}(\tilde{N}) \equiv V_i^{LT}$, implicitly defined in (27). There is no closed

form solution for \tilde{N} , but to find the equilibria of the model it is sufficient to know that

$$\Delta_1 < \tilde{N} < \Delta_2. \quad (66)$$

This completes the splice for firm i : if $N_j \leq \tilde{N}$, firm i 's best response is $\Phi(N_j)$ in the GT regime, as in parts a and b of Figure 1, and if $N_j \geq \tilde{N}$, firm i 's best response is Δ_1 in the LT regime, as in parts c and d of Figure 1. *QED*.

References

- Baye, M. R. and J. Morgan, 2001, "Information Gatekeepers on the Internet and the Competitiveness of Homogenous Product Markets." *American Economic Review* 91(3), 454-474.
- Burdett, K. and K. L. Judd, 1983, "Equilibrium Price Dispersion." *Econometrica* 51(4), 955-969.
- Butters, G. R., 1977, "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies* 44(3), 465-491.
- Fudenberg, D. and J. Tirole, 1984, "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look." *American Economic Review* 74(2), 361-366.
- Gabszewicz, J. J. and J.-F. Thisse, 1979, "Price Competition, Quality and Income Disparities." *Journal of Economic Theory* 20(3), 340-359.
- Gabszewicz, J. J. and J.-F. Thisse, 1986, "On the Nature of Competition with Differentiated Products." *Economic Journal* 96, 160-172.
- Grossman, G. M. and C. Shapiro, 1984, "Informative Advertising with Differentiated Products." *Review of Economic Studies* 51(1), 63-81.
- Ireland, N. J., 1993, "The Provision of Information in a Bertrand Oligopoly." *Journal of Industrial Economics* 41(1), 61-76.
- Iyer, G., D. Soberman and J. M. Villas-Boas, 2005, "The Targeting of Advertising." *Marketing Science* 24(3), 461-476.
- Kreps, D. and J. Scheinkman, 1983, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes." *Bell Journal of Economics* 14(2), 326-337.
- McAfee, R. P., 1994, "Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion." *Journal of Economic Theory* 62, 24-47.
- McCall, J. J., 1965, "The Economics of Information and Optimal Stopping Rules." *Journal of Business* 38(3), 300-317.
- Narasimhan, C., 1988, "Competitive Promotional Strategies." *Journal of Business* 61(4), 427-449.
- Nelson, P., 1970, "Information and Consumer Behavior." *Journal of Political Economy* 78(2), 311-329.

- Stigler, G. J., 1961, "The Economics of Information." *Journal of Political Economy* 69(3), 213-225.
- Varian, H., 1980, "A Model of Sales." *American Economic Review* 70(4), 651-659.