

Realistic Forecasting of Groundwater Level, Based on the Eigenstructure of Aquifer Dynamics

Vince Bidwell

Lincoln Environmental

Lincoln Ventures Ltd

New Zealand

What is eigenstructure? - an analogy

Music:

- Sound of a drum
is a mixture (**eigenvector**)
of the modal frequencies (**eigenvalues**)

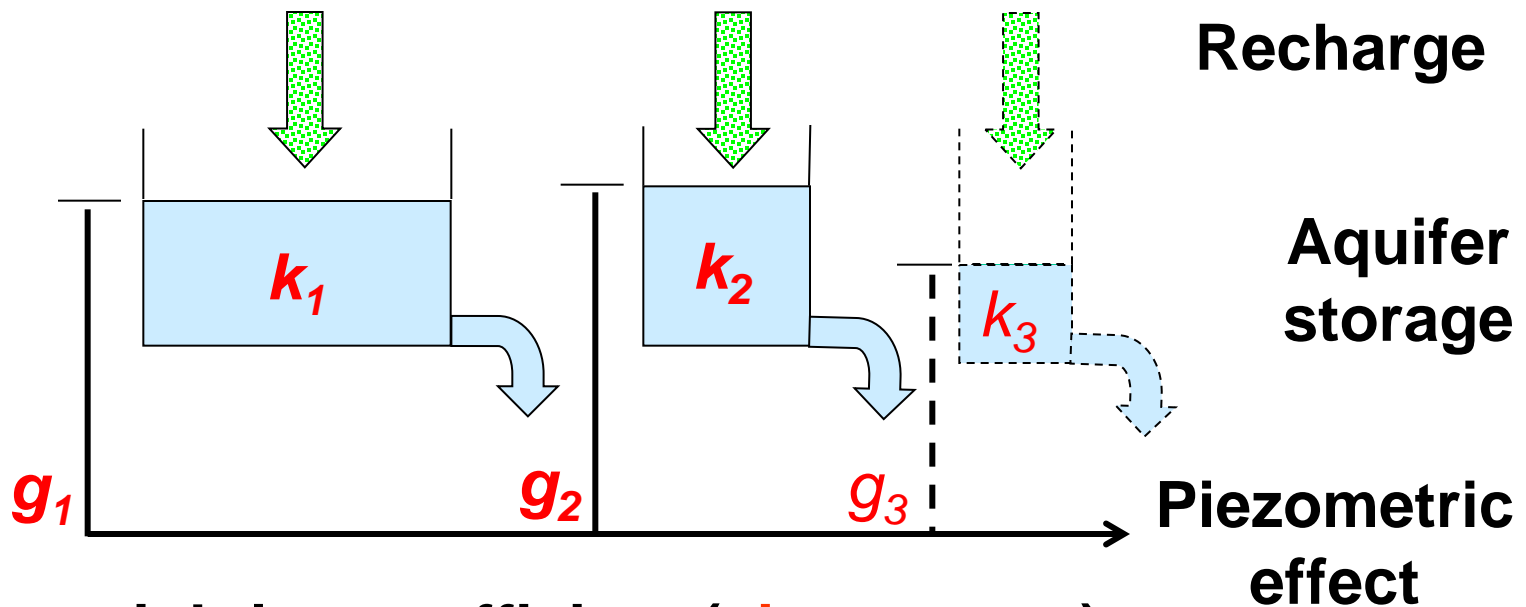
Aquifer:

- Groundwater level response to recharge
is a mixture (**eigenvector**)
of water storage modes (**eigenvalues**)

Water storage modes

k_i - discharge coefficient (**eigenvalue**)

$$\text{discharge} = k_i \times \text{storage}$$

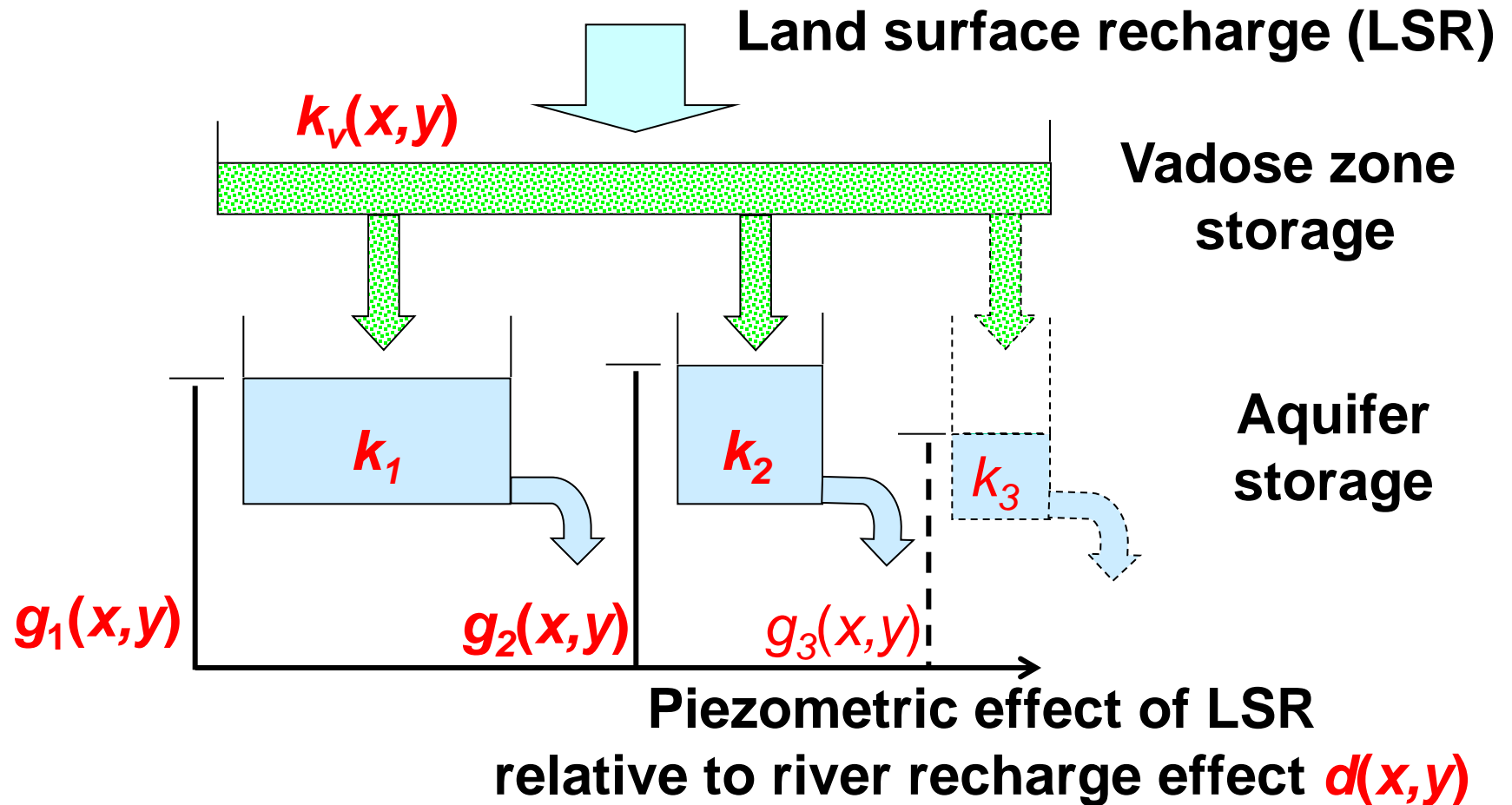


g_i - weighting coefficient (**eigenvector**)

Aquifer eigenstructure properties

- Water storages are conceptual, not physical
- Discharge coefficients (**eigenvalues**) are the same everywhere in the aquifer, and are related to aquifer properties
- Weighting coefficients (**eigenvectors**) depend on observation location and spatial pattern of recharge
- Piezometric response to land surface recharge is the most time-variable, and is usually dominated by the smallest eigenvalues

Conceptual model of piezometric response to land surface recharge



Model assumptions

- **Piezometric effects of river recharge are steady, but spatially variable**
- **Spatial pattern of land surface recharge is fixed, but the magnitude is unsteady**
- **Groundwater abstractions are unknown, and considered to be part of the time-varying model error**

From conceptual model to forecast equation

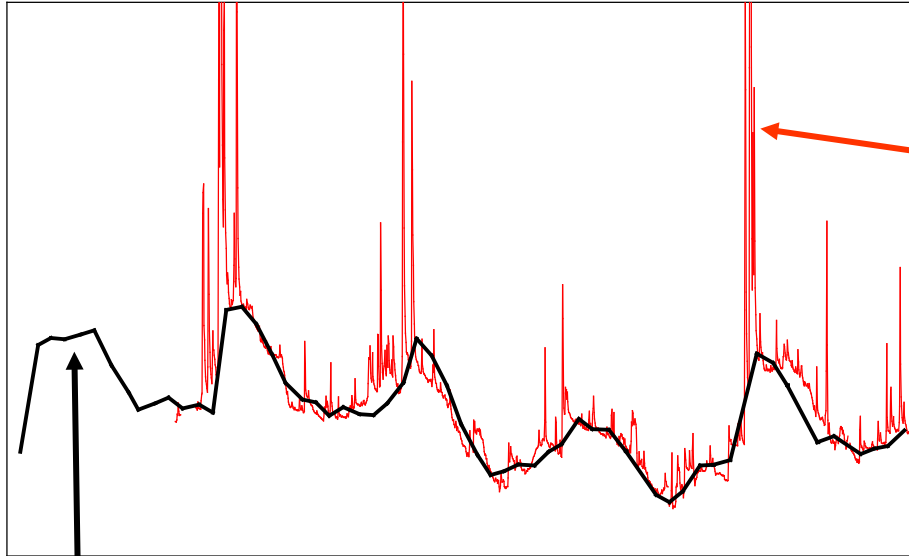
- **Conceptual model of piezometric response to land surface recharge is a linear dynamic system**
- **System structure is described by z-transforms**
- **Z-transform model, with a noise term, is expressed as an ARMAX (Box-Jenkins) stochastic difference equation for forecasting groundwater level**
- **Forecast equations have previous forecast errors as an additional input**

Model calibration

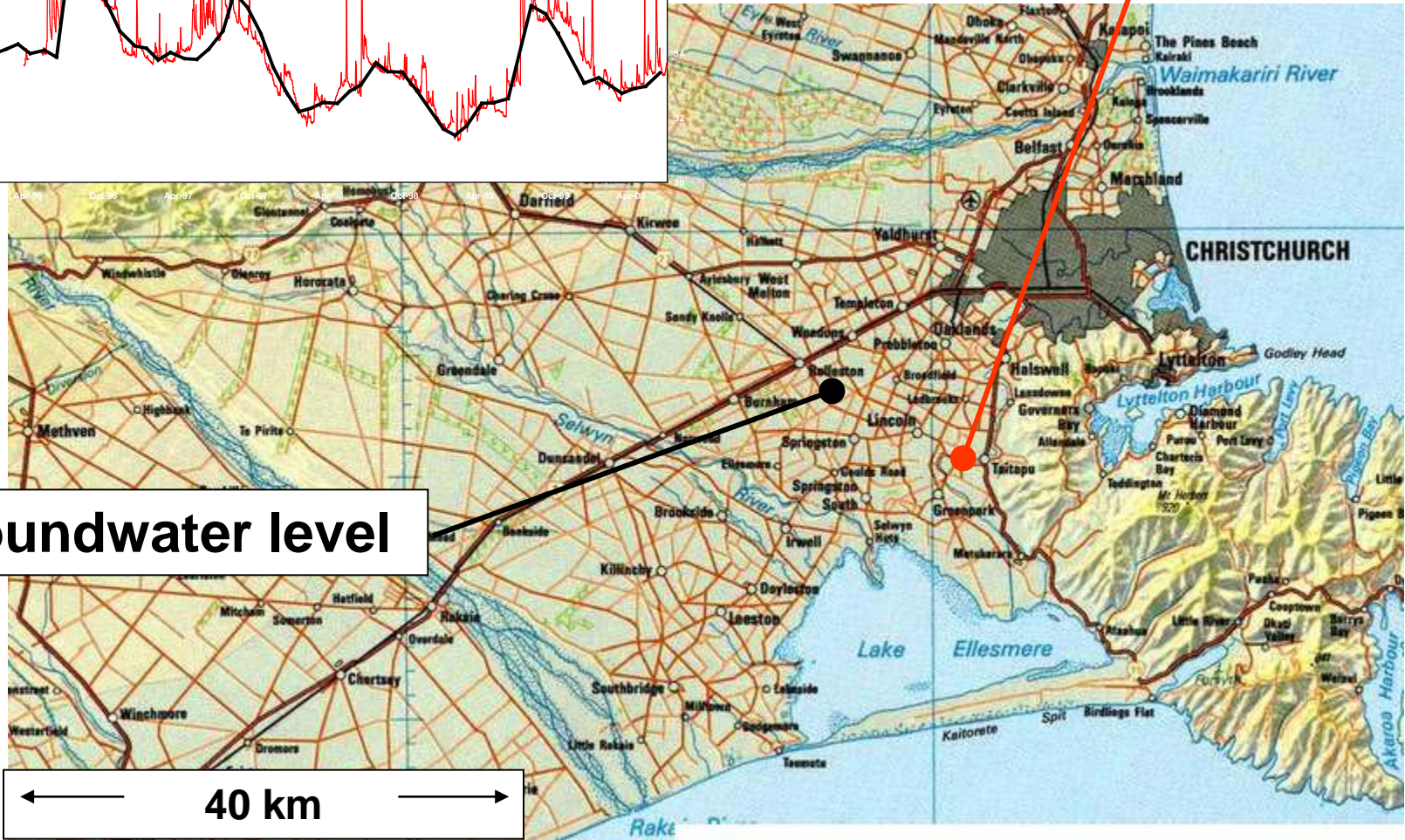
- Parameters of the conceptual model are calibrated, because of structural independence and physical realism
- Noise term has one autocorrelation parameter
- Objective function is minimisation of mean-square forecasting error
- Calibration with the “solver” tool in excel
- Avoids some identification and calibration issues in conventional use of ARMAX equations

Application example

- **Observation well in a 2000 km² aquifer, Central Canterbury Plains, New Zealand**
- **Land surface recharge (one, monthly series) calculated from daily water balance model**
- **Forecasts of groundwater level convert to forecasts of low flow in river supplied by aquifer, under drought conditions**
- **Effect of unknown groundwater abstraction is incorporated into the forecast equation as inputs of previous forecast errors**

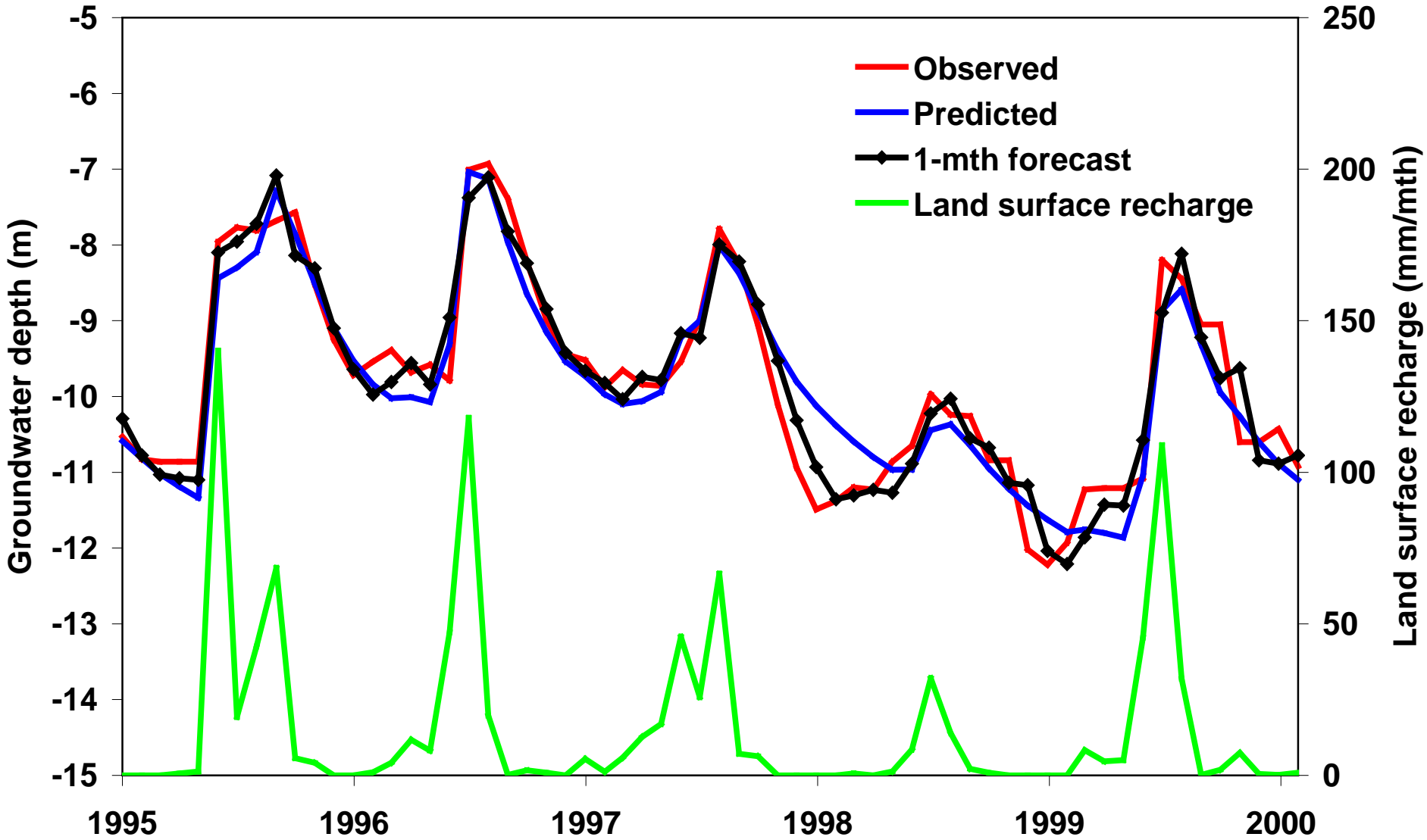


River flow



Groundwater level

← 40 km →



Summary

- **Eigenstructure approach expresses the dynamic behaviour of an aquifer as a simple linear system**
- **System parameters are physically realistic, related to aquifer properties, and structurally independent**
- **System can be expressed as a stochastic difference equation for real-time forecasting**
- **Calibration based on system structure avoids identification issues with ARMAX approach**