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Modelling TDR Response in Heterogeneous Composite Materials

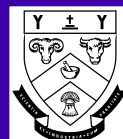
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MODELLING TDR RESPONSE IN HETEROGENEOUS COMPOSITE MATERIALS

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ABSTRACT. The use of time domain reflectometry (TDR) techniques for measuring the moisture content of composite materials is a mature art but usually makes assumptions about the homogeneity of the material, since the response of a parallel transmission line to transverse dielectric variation is non-linear. Further, because the electric field distribution depends on the moisture distribution within the material, an analytic solution to the sensitivity distribution in the transverse plane is impracticable, so numerical solutions are required. We describe an integral equation approach to model the response of the TDR system to a heterogeneous dielectric body. Then, in conjunction with a suitable dielectric model of the composite material, the TDR response to moisture content distribution may be quantified.

Keywords: TDR, heterogeneous, dielectric, spatial.

1. INTRODUCTION

TDR (time domain reflectometry) is used extensively for measurement of θ , the volumetric moisture content in soil, and applicable but less widely used in other materials such as grains, powders, and minerals. For measurement of θ , a short open-ended transmission line that is typically 300 mm long, is buried in the material under test. The travel time of a pulse with very short risetime (typically < 300 ps) is measured and provides the mean propagation velocity v , on the line of known length. Since most biological and composite materials make no contribution to the permeability of the region, v indicates the mean relative permittivity ϵ_r of the material surrounding the transmission line. When the loss tangent is small, and the relative permeability is one, ϵ_r is:

$$\epsilon_r \approx \frac{c^2}{v^2} \quad (1)$$

where c is the velocity of light. Since ϵ_r for most biological and composite materials is typically in the range of three to five whereas that of water is typically 80, ϵ_r of a material forms a useful surrogate for its moisture content. Frequently, empirical calibration techniques are used since practical dielectric models are usually unable to account for the subtle interactions between water molecules and the material that affect the water's polarisability. For example Topp et al [1] developed a polynomial relating the measured ϵ_r to the moisture content of soil. This calibration is applicable to quite a wide range of soil types (and hence orders of magnitude variation in particle size with their attendant variable interactions with water molecules) and typically has an accuracy of better than 2% in θ over the range 5 to 50%.

It has been shown [2] that longitudinal variation in moisture content is fairly accurately integrated by TDR systems. However the lateral sensitivity is not a linear function of distance from the transmission line, so lateral variations are not integrated linearly. Knight [3] derived analytically, an approximate lateral sensitivity weighting function that is useful for a nearly uniform permittivity distribution. For an exact solution with all but the simplest permittivity distributions, a numerical approach is required. Knight et al [4] used a numerical method that agreed well with previous analytical calculations of Annan [5] and Knight [2]. The method used a finite element algorithm adapted to the electrostatic case since this approach adequately represents a TEM transmission on parallel lines provided the line is essentially lossless.

Here an IE (integral equation) method is chosen as the forward model for use with a tomographic inversion algorithm which we are currently developing. This application exploits a key characteristic of IE methods, namely that where the region of anomalous permittivity is surrounded by free space, only the anomalous region need be calculated. The IE approach also has advantages in tomography, since matrix recalculation is not necessary for changes to the impressed electric field. However a disadvantage is that when applied to arbitrary permittivity distributions, IE methods require volume integration even where one dimension is invariant. Under these circumstances and in the present case, quasi 3-D or 2.5-D variants reduce the computational burden of the IE method to compare more favourably with the DE approach. In our application, we use the output from the model to directly calculate the velocity of propagation on the transmission line

2 INTEGRAL EQUATION AND DISCRETISATION

The polarisation of a discretised zone or cell within a dielectric material may be represented by a dipole at its geometric centre. In most dielectric materials, there is no net polarisation until generated by an external or impressed field. When applied to this quasi-static electric field problem, the method of moments may be considered as the summation in each cell, of the electric field contributions due to the polarisation in all other cells. The potential ϕ_p at point $p(x,y,z)$ generated by a dipole with dipole moment or polarisation P , is:

$$\phi_p = \frac{\tilde{P} \cdot \hat{r}}{4\pi\epsilon_0 r^2} \quad (2)$$

where \hat{r} is a unit vector pointing from the centre of the dipole to p [6]. In Cartesian 3-space:

$$\phi_p = \frac{\tilde{P} \cdot (\tilde{x} + \tilde{y} + \tilde{z})}{4\pi\epsilon_0 r^3} \quad (3)$$

where \tilde{x} , \tilde{y} and \tilde{z} are the rectangular components of \tilde{r} . The potential arising from many such dipoles in a region is:

$$\phi_p = \iiint \frac{\tilde{\mathbf{P}} \cdot \hat{\mathbf{r}}}{4\pi\epsilon_0 r^2} dv \quad (4)$$

where dv is the differential volume over which each $\mathbf{P} \cdot \hat{\mathbf{r}}$ applies. Reverting to the single dipole case, its electric field is the space rate of change of potential ($-\text{grad}\phi_p$) so that:

$$\mathbf{E}_{px} = \frac{-\tilde{\mathbf{P}}}{4\pi\epsilon_0 r^5} \cdot \left[\hat{x}(r^2 - 3x^2) - \hat{y}(3xy) - \hat{z}(3xz) \right] \quad (5)$$

and with corresponding equations for \mathbf{E}_{py} and \mathbf{E}_{pz} , may be expressed as a dyadic equation

$$\mathbf{E}_p = \frac{\tilde{\mathbf{P}}}{4\pi\epsilon_0 r^5} \cdot \begin{bmatrix} \hat{x}\hat{x}(3x^2 - r^2) + \hat{x}\hat{y}(3xy) + \hat{x}\hat{z}(3xz) \\ \hat{y}\hat{y}(3y^2 - r^2) + \hat{y}\hat{x}(3xy) + \hat{y}\hat{z}(3xz) \\ \hat{z}\hat{z}(3z^2 - r^2) + \hat{z}\hat{x}(3xz) + \hat{z}\hat{y}(3zy) \end{bmatrix} \quad (6)$$

We may combine the above in an integral equation describing the electric field \mathbf{E}_p at a point p :

$$\mathbf{E}_p(x, y, z) = -\nabla \left(\iiint \frac{\mathbf{P} \cdot \hat{\mathbf{r}}}{4\pi\epsilon_0 r^2} dv \right) \quad (7)$$

The polarisation region may now be discretised, and following the method of moments [7], we calculate the matrix of polarisation vectors $\mathbf{P}(x, y, z)$ using:

$$\begin{aligned} L(\mathbf{P}) &= -\mathbf{E}_i(x, y, z) \\ &= \mathbf{E}_p(x, y, z) - \frac{\mathbf{P}(x, y, z)}{\epsilon_0 \chi(x, y, z)} \end{aligned} \quad (8)$$

where L is a linear operator, \mathbf{E}_i the external impressed field and $\chi(x, y, z)$ the electric susceptibility ($\epsilon_r(x, y, z) - 1$). Equation 8 is converted to matrix form and solved for the vector of polarisations \mathbf{P} , and the electric field strength in each cell is recovered from the polarisation:

$$\mathbf{E}(x, y, z) = \frac{\mathbf{P}(x, y, z)}{\epsilon_0 \chi(x, y, z)} \quad (9)$$

The inputs required for the method are: a vector comprising sets of three elements describing the impressed field, a matrix describing the permittivity within each cell, and the dimensionality of the problem. While the above method applies to any impressed field distribution, in this case \mathbf{E}_i is the vector of impressed field components due to a parallel transmission line. To obtain the potential difference between the two lines and hence determine line capacitance, the matrix \mathbf{E} is

integrated along a path connecting the two lines (along the x-axis for example). Then to obtain the velocity of an electrical edge on the transmission line (assumed lossless), the standard transmission line formula is used:

$$v = \frac{1}{\sqrt{LC}} = \sqrt{\frac{\pi \int E(x, y, z) \cdot dl}{q\mu \cosh^{-1}\left(\frac{b}{a}\right)}} \quad (10)$$

Here dl is the length element of the numerical integration (the cell length in this discretised case), q the same initial line charge density that defined the impressed field, μ the total permeability, b the transmission line rod spacing, and a the rod diameter.

3 VERIFICATION

We chose to verify the above calculation by comparison with propagation times from a Tektronix 1502C connected with 0.8 m of URM 43 coaxial cable to a 1:4 balun. Balun construction followed [8], but omitted the initial 1:1 transformer, and used a single, grade S3 ferrite toroid. A relay (similar to Teledyne 172) switched the balanced line to either a reference transmission line or the measuring line. The 6 mm diameter stainless steel rods were spaced 60 mm apart, with the measuring rods 300 mm longer than the reference rods. At the end of the transmission lines 6 x 1 mm steel shorting straps provided sharper, better defined reflections than unterminated lines. Waveform data retrieved from the 1502 were smoothed and differentiated using 25 point routines [9]. The intersection between the tangents to the maximum negative slope and the immediately preceding stationary point defined the edge of the pulse. Finally, the reading from the reference line was subtracted from that of the measuring line to obtain the actual travel time of the edge.

A rectangular thin walled plastic container 150 by 500 by 80 mm and filled with water, formed the phantom dielectric body. The transmission line was located near the phantom and used computer readable position sensing with 1 mm precision to record relative positions.

4 RESULTS AND DISCUSSION

Table 1. Comparison of measured and calculated data

Position (mm)	Measured (ns)	Model (ns)	Discrepancy (ps)	Normalised (ps)
-30, 5	1.071	1.076	5	-16
-30, 10	1.016	1.039	23	2
-30, 20	1.001	1.017	18	-3
-30, 30	0.992	1.010	18	-3
45, 5	1.172	1.204	32	11
45, 10	1.047	1.103	56	35
45, 20	1.009	1.035	26	5
45, 30	0.998	1.019	21	0
distant	0.987	1.008	21	0

The position is defined as the (x, y) distances (mm) between top edge of the container and the geometric centre of transmission line. A 'distant' separation provided a reference reading to normalise the data. Model predictions were calculated using 5 mm cubic cells and a quasi 3-D approach that includes the influence of the neighbouring cells in the z direction within the 2-D (xy) matrix. The constant portion of the discrepancy between the model prediction and the measurements is attributed to the imperfect measuring system. The value used for ϵ_r of water in the model, took account of the water temperature in the container.

5 CONCLUSIONS

We describe an integral equation method for determining the electric field distribution in a low loss, inhomogenous dielectric material given a pre-determined impressed field, E_i . In the case of a parallel transmission line generating E_i , the procedure has been extended to calculate line parameters and hence the propagation velocity of a pulse on the line. Thus the procedure enables prediction of the impact of arbitrary dielectric (or moisture content) distributions on a TDR measurement system, and quantification of its sensitivity distribution. Experimental verification using water as a dielectric body, provided good agreement with the model predictions.

REFERENCES

- [1] G. C. Topp, J. L. Davis, and A. P. Annan, Electromagnetic determination of soil water content: measurements in coaxial transmission lines, *Water Resources Research*, vol. 16, pp. 574-582, 1980.
- [2] P. A. Ferrer, D. L. Rudolph, and R. G. Kachanoski, Spatial averaging of water content by time domain reflectometry: Implications for twin rod probes with and without dielectric coatings, *Water Resources Research*, vol. 32, pp. 271-279, 1996.
- [3] J. H. Knight, Sensitivity of time domain reflectometry measurements to lateral variations in soil water content, *Water Resources Research*, vol. 28, pp. 2345-2352, 1992.
- [4] J. H. Knight, P. A. Ferrer, D. L. Rudolph, and R. G. Kachanoski, A numerical analysis of the effects of coatings and gaps upon relative dielectric permittivity measurement with time domain reflectometry, *Water Resources Research*, vol. 33, pp. 1455-1460, 1997.
- [5] A. P. Annan, Time-domain reflectometry - air gap problem for parallel wire transmission lines, *Geol. Surv. Can, Report of activities 77-1B*, 1977.
- [6] A. F. Kip, *Fundamentals of Electricity and Magnetism*, 2 ed: McGraw-Hill, 1962.
- [7] R. F. Harrington, *Field Computation by Moment Methods*: R E Kreiger, 1968.
- [8] E. J. A. Spaans and J. M. Baker, Simple Baluns in Parallel Probes for Time Domain Reflectometry, *Soil Sci Soc Am J*, vol. 57, pp. 668-673, 1993.
- [9] A. Savitsky and M. J. E. Golay, Smoothing and Differentiation of Data by Simplified Least Squares Procedures, *Analytical Chemistry*, vol. 36, pp. 1627-1639, 1964.

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